2DF30 Week 1: The Cramér-Lundberg model

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Today’s schedule

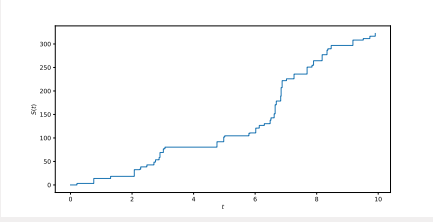
- Insurance risk: the Cramér-Lundberg model
- How to simulate this in Python?

The Cramér-Lundberg model

Also known as the Compound Poisson risk model.
A compound Poisson process $\{Y(t), t \geq 0\}$ is similar to an ordinary Poisson process, but at each event, the counter is increased by a *random* variable that may have any continuous distribution function:

$$Y(t) = \sum_{i=1}^{N(t)} X_i,$$

where $\{N(t), t \geq 0\}$ is a Poisson process and $\{X_i, i = 1, 2, \dots\}$ are iid random variables, independent of $N(t)$.



The Cramér-Lundberg model

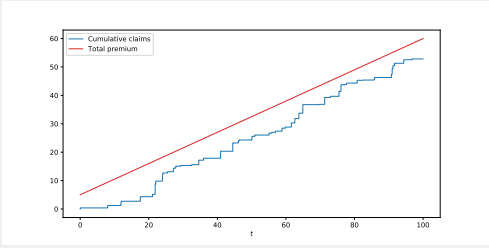
$$U(t) = u + ct - \sum_{i=1}^{N(t)} X_i.$$

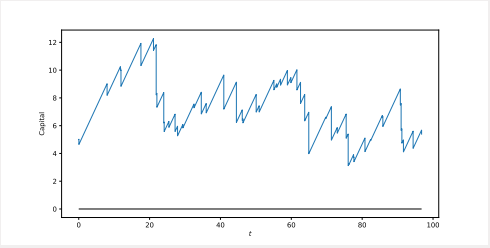
$U(t)$ is the capital of the insurance company at time t
c is the premium income rate
$N(t)$ is total number of claims up to time t, which is a Poisson process with rate λ
The X_i are i.i.d. claims, with common distribution function F and mean $\mu_1 := \mathbb{E}[X_1]$.

The premium income rate c is often chosen such that

$$c = (1 + \theta)\lambda\mu_1, \tag{1}$$

The Cramér-Lundberg model





$u + ct$ and $\sum_{i=1}^{N(t)} X_i$ plotted separately

$U(t)$

The Cramér-Lundberg model

Main performance characteristic in insurance mathematics is the probability of ruin

$$\psi(u) := \mathbb{P}\left(T(u) < \infty\right),$$

where

$$T(u) := \inf\{t \geq 0 : U(t) < 0\}$$