

The background image is a composite of financial and mathematical symbols. It features a silver calculator with black buttons, a silver pen, and several financial charts. One chart on the right shows a line graph with a fluctuating trend. Another chart on the left shows a table of data with columns for dates and values. The overall theme is finance and mathematics.

2DF30 Week 1: The Cramér-Lundberg model

Fiona Sloothak (many thanks to Marko Boon)

Today's schedule

- Insurance risk: the Cramér-Lundberg model
- How to simulate this in Python?

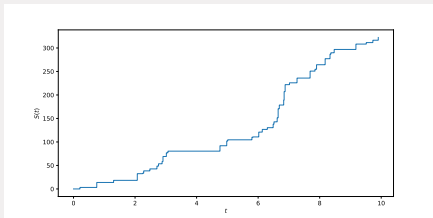
The Cramér-Lundberg model

Also known as the **Compound Poisson risk model**.

A compound Poisson process $\{Y(t), t \geq 0\}$ is similar to an ordinary Poisson process, but at each event, the counter is increased by a *random* variable that may have any continuous distribution function:

$$Y(t) = \sum_{i=1}^{N(t)} X_i,$$

where $\{N(t), t \geq 0\}$ is a Poisson process and $\{X_i, i = 1, 2, \dots\}$ are iid random variables, independent of $N(t)$.



The Cramér-Lundberg model

$$U(t) = u + ct - \sum_{i=1}^{N(t)} X_i.$$

$U(t)$ is the capital of the insurance company at time t

c is the premium income rate

$N(t)$ is total number of claims up to time t , which is a Poisson process with rate λ

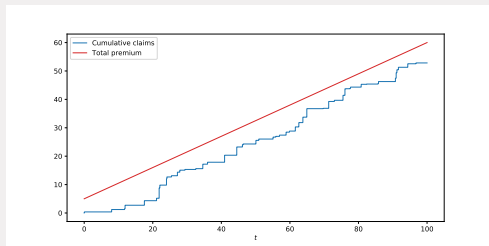
The X_i are i.i.d. claims, with common distribution function F and mean

$\mu_1 := \mathbb{E}[X_1]$.

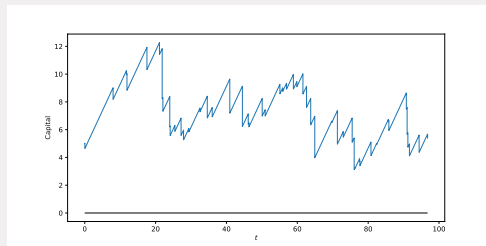
The premium income rate c is often chosen such that

$$c = (1 + \theta)\lambda\mu_1, \tag{1}$$

The Cramér-Lundberg model



$u + ct$ and $\sum_{i=1}^{N(t)} X_i$ plotted separately



$U(t)$

The Cramér-Lundberg model

Main performance characteristic in insurance mathematics is the probability of ruin

$$\psi(u) := \mathbb{P}(T(u) < \infty),$$

where

$$T(u) := \inf\{t \geq 0 : U(t) < 0\}$$

