Appendix A
The minimum error of the rank prediction without prior knowledge

Given a set of ranks that uniformly spread across a limited range, what is the minimum prediction error we can obtain without any prior knowledge except for the range? Assume that the range is $[0, K]$ and the set's size is infinite. The mean absolute error (MAE) and mean squared error (MSE) can be derived as follows.

Without knowledge of the ground truth value, one may *randomly* take values from the range $[0, K]$ as the predicted ranks. In this case, the errors are:

$$E_r^{MAE} = \frac{1}{K}\int_0^K \frac{1}{K}\left[\int_0^{\hat{p}} (\hat{p}-p)\mathrm{d}p + \int_{\hat{p}}^K (p-\hat{p})\mathrm{d}p\right]\mathrm{d}\hat{p} = \frac{K}{3}, \tag{1}$$

$$E_r^{MSE} = \frac{1}{K}\int_0^K \frac{1}{K}\int_0^K (\hat{p}-p)^2 \mathrm{d}p\mathrm{d}\hat{p} = \frac{K^2}{6}, \tag{2}$$

where p and $\hat{p}$ are the ground truth and predicted rank, respectively.

In addition to random prediction, one may use a fixed value as the predicted rank. In this case, the errors are:

$$E_f^{MAE} = \frac{1}{K}\left[\int_0^{\hat{p}} (\hat{p}-p)\mathrm{d}p + \int_{\hat{p}}^K (p-\hat{p})\mathrm{d}p\right] = \frac{\hat{p}^2}{K} - \hat{p} + \frac{K}{2}, \tag{3}$$

$$E_f^{MSE} = \frac{1}{K}\int_0^K (\hat{p}-p)^2 \mathrm{d}p = K\hat{p}^2 - K^2\hat{p} + \frac{K^3}{3}. \tag{4}$$

It is easy to derive their minimum with respect to $\hat{p}$:

$$\min E_f^{MAE} = \frac{K}{4}, \text{ when } \hat{p} = \frac{K}{2}, \tag{5}$$

$$\min E_f^{MSE} = \frac{K^3}{12}, \text{ when } \hat{p} = \frac{K}{2}. \tag{6}$$

We set $K = 100$ in our experiments; therefore, the minimum MAE should be the $\min E_f^{MAE} = 100/4 = 25$, and the minimum MSE should be the $E_r^{MSE} = 100^2/6 = 1666.66$. The former can be used as a baseline to compare the MAE obtained by the APN, i.e., MAE in Tables 1, 2, and 3. The latter is where the 1666.66 in Eq. 19 comes from.