

Design Methodology

DATE:

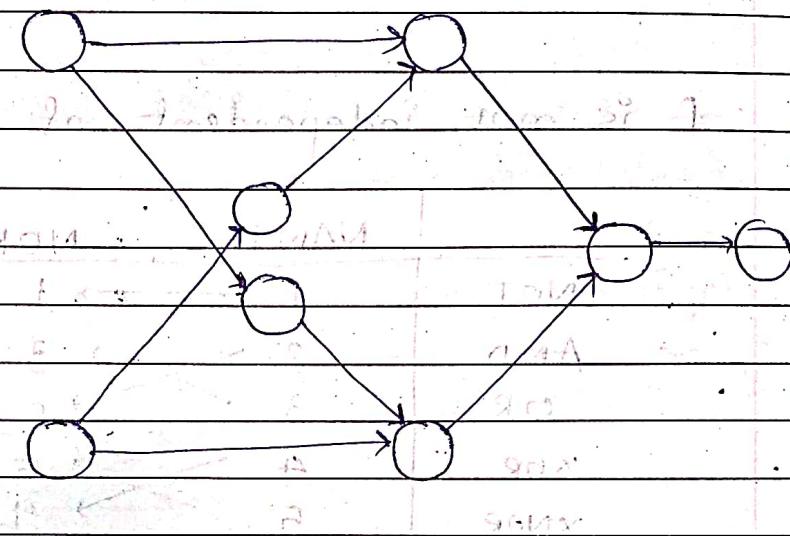
PAGE:

→ Abstraction (Hiding) the data

- Create level (least form of abstraction)
- Register level
- Processor level

→ System representation:

Representation:

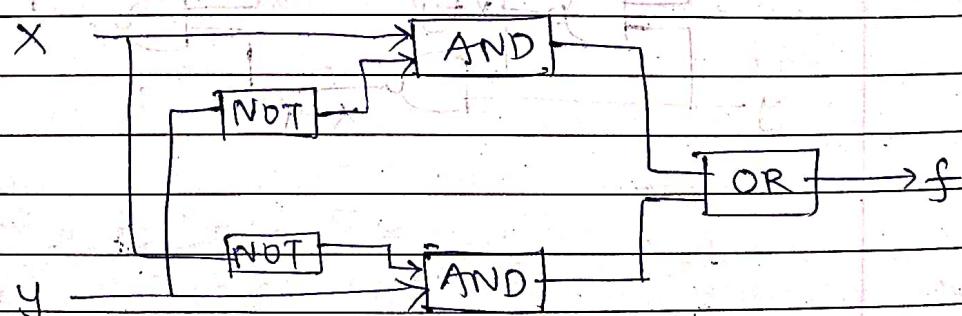


$$V = \{v_1, v_2, \dots\}$$

$$E = \{e_1, e_2, \dots\}$$

$$e_1 = \{v_1, v_2\}$$

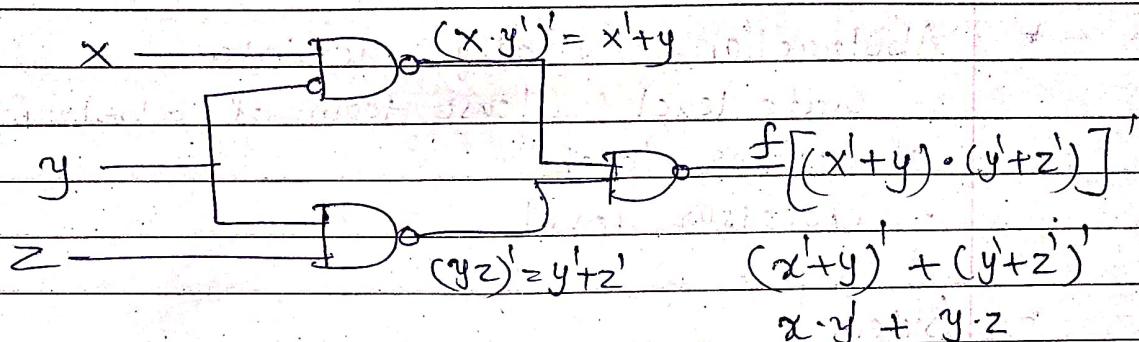
→ Schematic diagram:



Structure \neq Behaviour

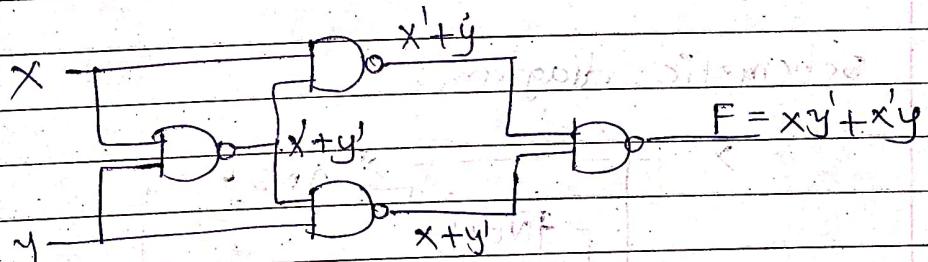
mathematical truth
eg: truth table,

two ways of representation



f is not independent of any variable.

	NAND	NOR
NOT	1 \rightarrow 1	
AND	2 $\times \times \rightarrow$ 3	
OR	3 \rightarrow 2	
XOR	4 $\times \times \rightarrow$ 5	
XNOR	5 \rightarrow 4	
NAND	1 $\times \times \rightarrow$ 4	
NOR	4 $\times \times \rightarrow$ 1	



→ Hardware Description Language (HDL)

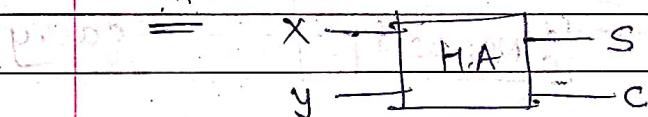
VHDL, VERILOG

- Advantage:- They are very precise and technology ^{indepⁿ}
 - Used to draw prototype (simulation)

- Disad. - They are very complex
 - They are not insightful (not able to know internal thing).

① Entity

ex.



② Architecture

Structure Behaviour

→ Behavioural VHDL description

Entity is half-adder
 port (x, y : in bit; sum, carry : out bit);
 end half-adder;

architecture behaviour of half-adder is
 begin

$$\text{sum} \leftarrow x \text{ XOR } y$$

$$\text{carry} \leftarrow x \text{ AND } y \text{ after}$$

end behaviour

Assumption :

XOR and AND
are already in

VHDL

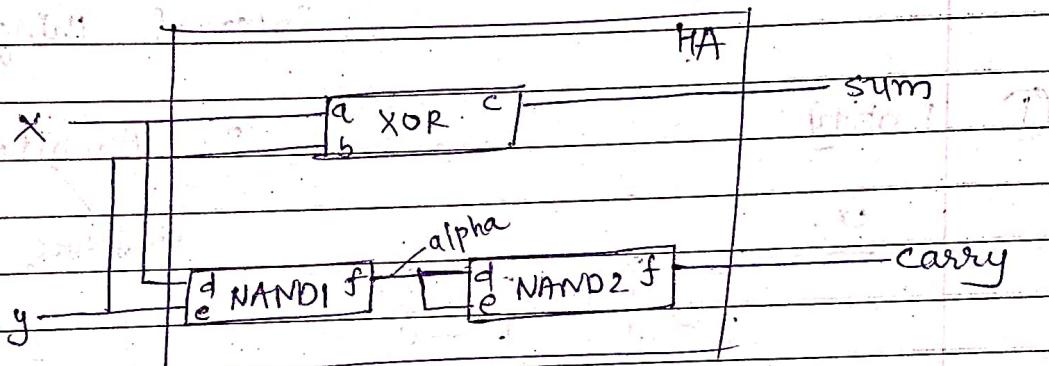
(IMP) (If que. is of 5 or 6 marks draw circuit,
find eq² using K-map and write above)

→ Structural VHDL description

↔ Signal assignment

In half adder we can write,
half adder is

if $x \oplus y = 1$ then $\text{carry} \leftarrow 1$ else $\text{carry} \leftarrow 0$



Define Architecture structure of

entity half adder is
component XOR-circuit port (a,b : in bit, c : out bit)
end component;

component NAND-gate port (d,e : in bit, f : out bit)
end component;

signal alpha : bit

end half adder

Network

begin

1ST (Netlist) XOR : XOR-circuit portmap (a \Rightarrow x, b \Rightarrow y, c \Rightarrow sum)

NAND1 : NAND-gate portmap (d \Rightarrow x, e \Rightarrow y, f \Rightarrow alpha)

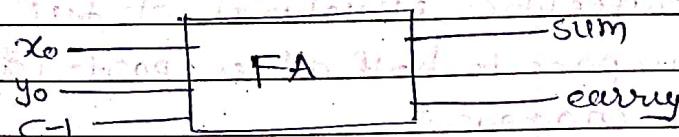
NAND2 : NAND-gate portmap (d \Rightarrow alpha, e \Rightarrow alpha, f \Rightarrow carry)

end

end structure

VHDL Description of Full adder

1. Behavioural VHDL Desc.



$$\text{sum} = x_0 \oplus y_0 \oplus c_{-1}$$

$$\text{carry} = x_0y_0 + x_0c_{-1} + y_0c_{-1}$$

entity is full-adder

port (x_0, y_0, c_{-1} : in bit ; sum, carry : out bit)
end full adder

Architecture behaviour of full adder is

begin

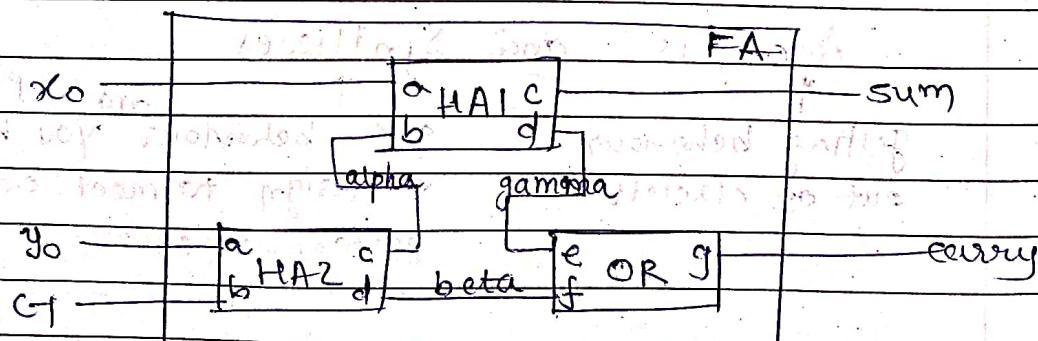
$$\text{sum} \leftarrow x_0 \text{ XOR } y_0 \text{ XOR } c_{-1}$$

$$\text{carry} \leftarrow (x_0 \text{ AND } y_0) \text{ OR } (x_0 \text{ AND } c_{-1}) \text{ OR }$$

$$(y_0 \text{ AND } c_{-1})$$

end behaviour

2. Structural VHDL Desc.



Entity is full adder

port (x_0, y_0, c_1 : in bit; sum, carry out bit);
end full adder

Architecture structural of full adder is

component half adder port (a, b : in bit, c, d : out bit);
end component

component OR gate port (e, f : in bit, g : out bit);
end component

signal α, β, γ : bit)

end full adder

begin

HA1 : half adder portmap ($a \Rightarrow x_0, b \Rightarrow \alpha, c \Rightarrow \text{sum}, d \Rightarrow \gamma$);

HA2 : half adder portmap ($a \Rightarrow y_0, b \Rightarrow c_1, c \Rightarrow \alpha, d \Rightarrow \beta$);

OR : OR gate portmap ($e \Rightarrow \gamma, f \Rightarrow g, g \Rightarrow \text{carry}$)

(1) and all

end structure

Design Process

Analysis and Synthesis

getting behaviour
out of circuit

given behaviour you have
to design to meet cost &
performance

Goals: Design should be correct

Performance maximize

cost minimize

power consumption

reliability

compatibility

→ CAD → synthesizes a design

prototype of the system editor

given out actual hardware implementation

→ Design levels:

level	Component	IC	Info. units	Time unit
(1) gate (logic)	logic gates ff	SSI	bits	10^{-12} to 10^{-9} ns
(2) Register (Reg. trans- fer-RTL)	Reg, Counter comb. circuit seq. circuit	MSI	word	10^{-9} to 10^6 s
(3) Processor (architectural) behaviour system	CPU, Memories, I/O devices	VLSI	block of words	10^3 to 10^3 s

(1)

gate level / hardware designFull adder

$$S_0 = x_0 y_0 c_{-1} + x_0 y_0 c_{-1}' + x_0' y_0' c_{-1}$$

$$+ x_0 y_0' c_{-1}'$$

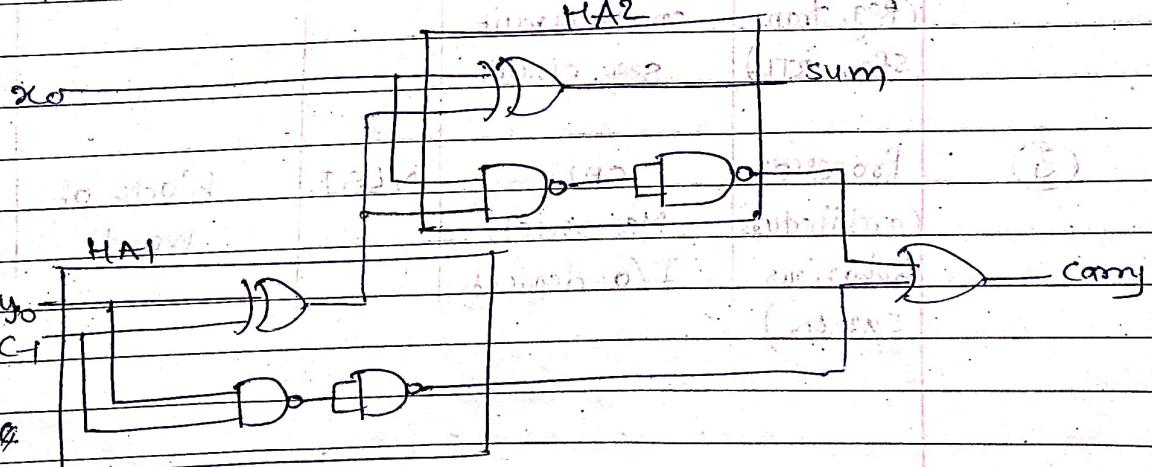
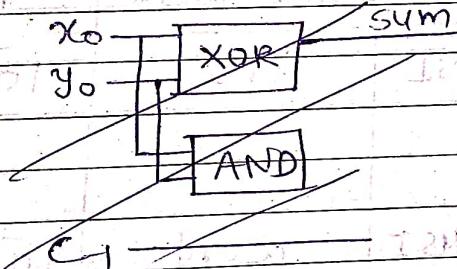
$$C_0 = x_0 y_0 + x_0 c_{-1} + y_0 c_{-1}$$

$$= (x_0 + y_0) \cdot (x_0 + c_{-1}) \cdot (y_0 + c_{-1})$$

Half adder

$$S_0 = x_0 y_0' + x_0' y_0$$

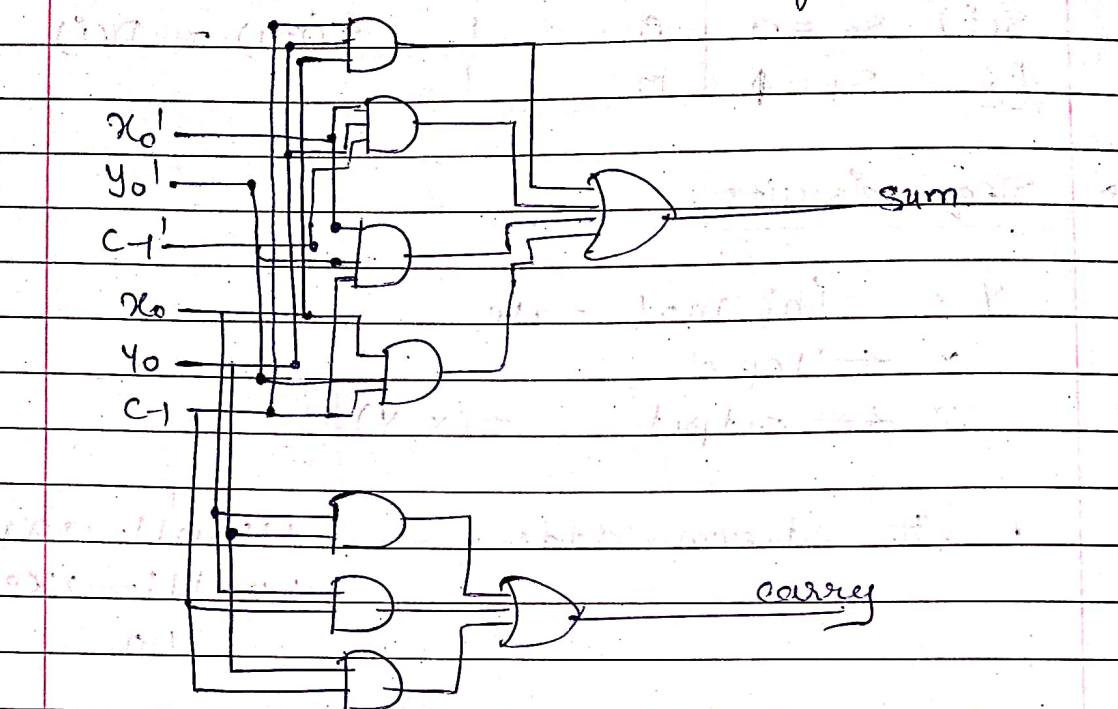
$$C_0 = x_0 y_0$$



4 gate level.

Using AND -OR

& gate level



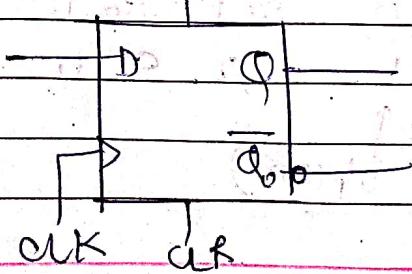
Fan-in \Rightarrow How many input lines is given

Fan-out \Rightarrow How many output lines does there

→ convert into NAND-NOR

1. take bar of each term
2. convert AND \rightarrow OR \Leftrightarrow OR \rightarrow AND
3. take whole bar

→ flipflops



level trigger \Rightarrow problem

transient noise

it will change the O/P

! !
CLR, PRE \rightarrow asynchronous signal

	D_i	s_0, s_1	s_{i+1}
	0	1	input
$s(i)$	$s_0 = 0$	0	$s(i+1) = D(i)$
$P.S.$	$s_1 = \phi$	0	1

→ Seq. circuits:

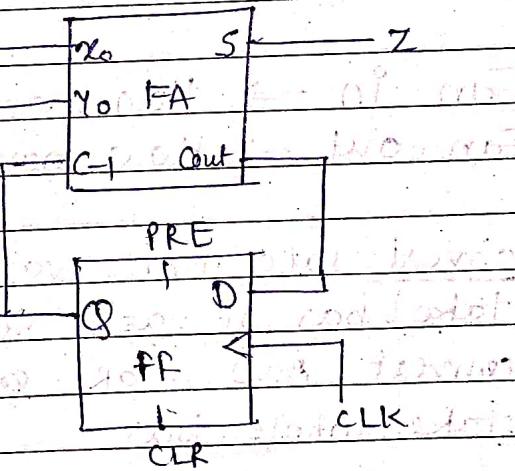
$y \leftarrow$ Internal state

$x \leftarrow$ input

$z \leftarrow$ output, $z(x, y)$

(1) 2 bit stream adder

$$\begin{array}{r} 1101011 \rightarrow x_1 \\ 100111 \rightarrow x_2 \\ \hline 10 \end{array}$$

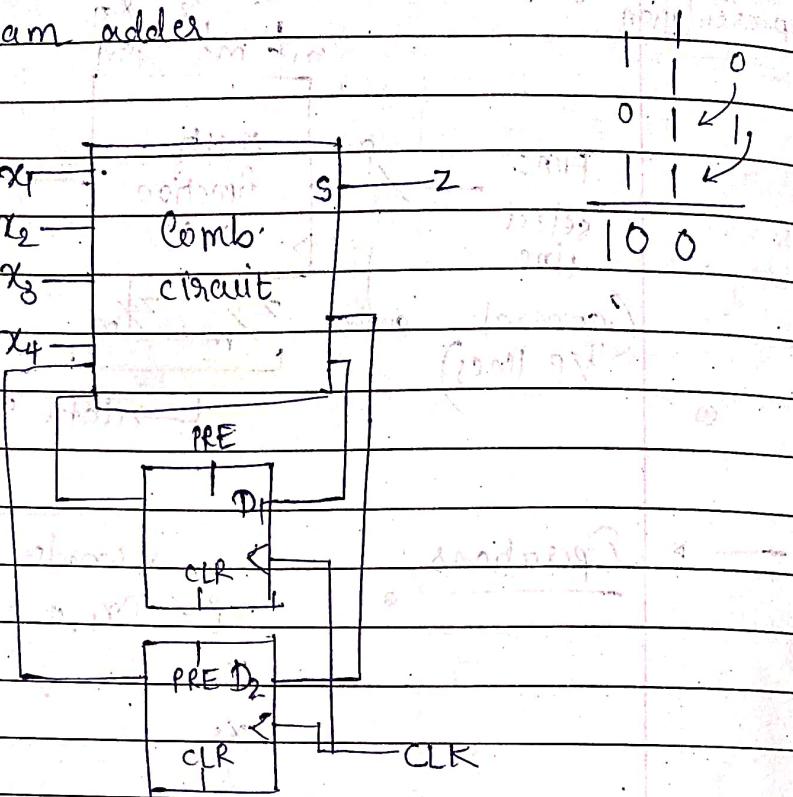


truth table:

Present State	$s_0 = 0$	$s_0, 0$	$s_0, 1$	$s_0, 1$	$s_1, 0$
	$s_1 = 1$ (carry)	$s_0, 1$	$s_1, 0$	$s_1, 0$	$s_1, 1$

01 (sum)

(2) 4 bit stream adder



	0	1	2	3	4	x_1	x_2	x_3	x_4	5	12	15
$S_0 = 00$												
$S_1 = 01$												
$S_2 = 10$												
$S_3 = 11$												

(2)

Register level \rightarrow Inf. unit = word

components : word gates

Mux / Demux

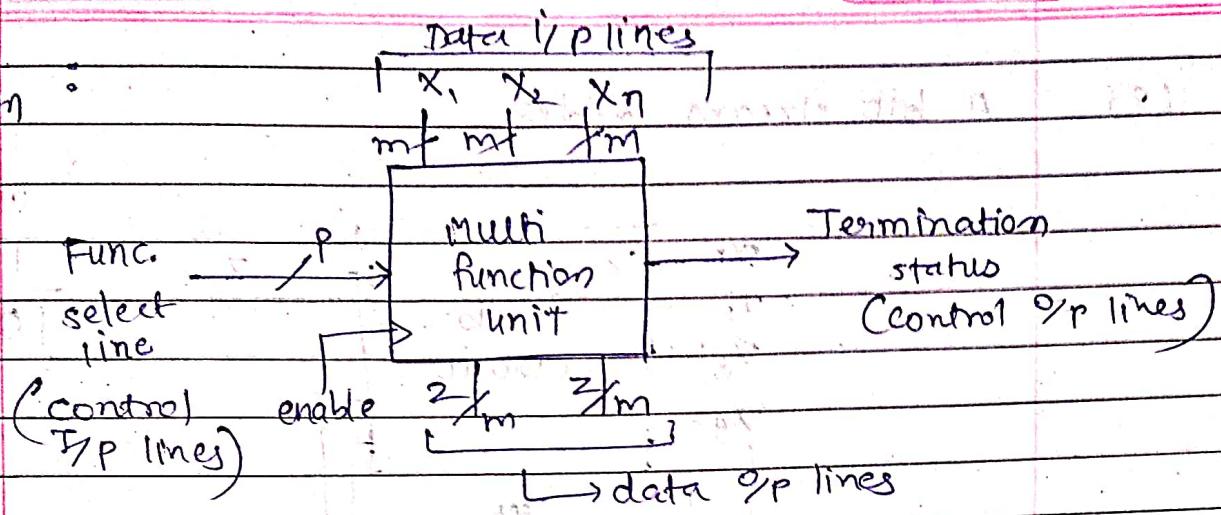
Decoder / Encoder

address

ALU, PLD

shift reg, counter

Block representation



Operations

n words m bits

$$x^i = (x_{1,0}, x_{1,1}, x_{1,2}, \dots, x_{1,m-1})$$

$$x_1 = ($$

$$x_2 = ($$

$$\vdots$$

$$x_n = ($$

$$x_{n,m-1})$$

$$z = ($$

$$z.(x_1, x_2, \dots, x_n) =$$

$$[z.(x_{1,0}, x_{2,0}, \dots, x_{n,0}), \dots]$$

bit by bit operation

$$z(x_{1,1}, \dots, x_{n,1}), \dots$$

$$z(x_{1,m-1}, \dots, x_{n,m-1})]$$

$n = 2$ variables $2^n = 16$

$$m = 2^k \text{ bits} \quad 2^k = 2^2 = 4$$

$$\text{control gate: } x = x_1 x_2 \dots x_m$$

$$y = y_1 y_2 \dots y_m$$

$$f = \overline{xy}$$

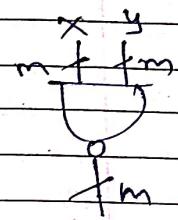
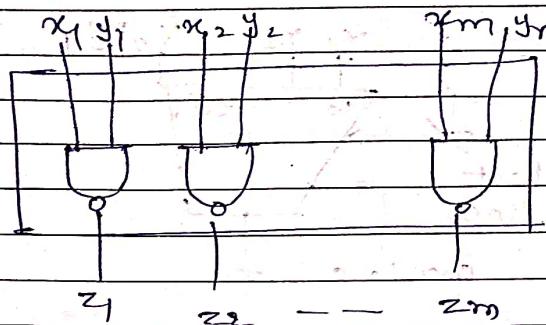
$$z(\overline{xy}) = z_1(\overline{x}y_1), z_2(\overline{x}_2 y_2), z_3(\overline{x}_3 y_3), \dots, z_m(\overline{x}_m y_m)$$

$$z = (z_1, z_2, \dots, z_m) = (\overline{x}_1 y_1, \overline{x}_2 y_2, \dots, \overline{x}_m y_m)$$



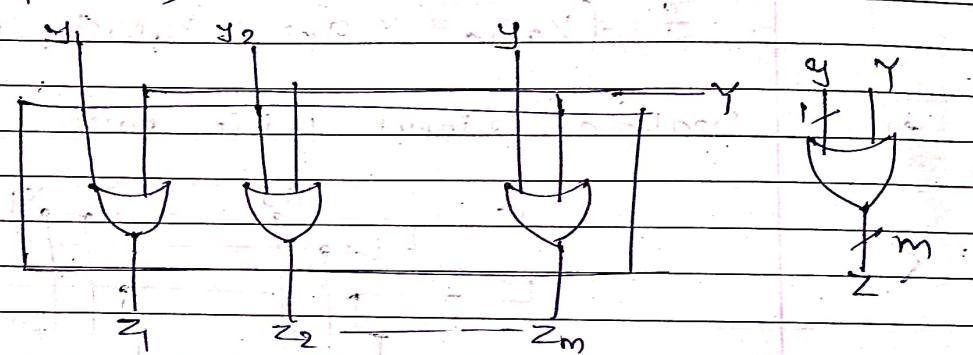
Block diagram

logic
diagram



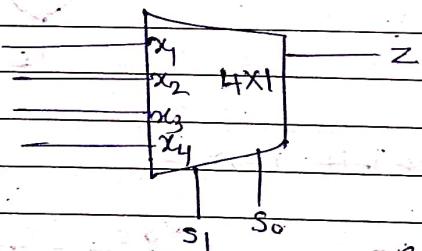
(symbol)

$$f = y + \gamma \text{ (word)}$$



→ Multiplexers

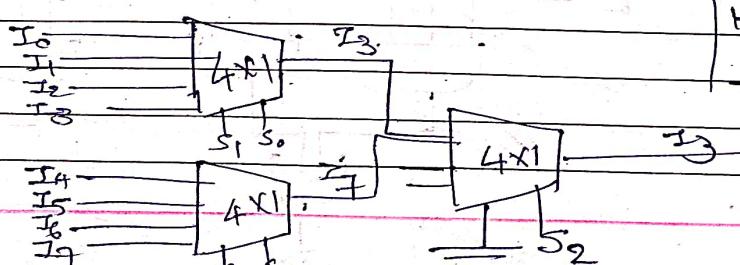
- In OS
- data selecting
- switching

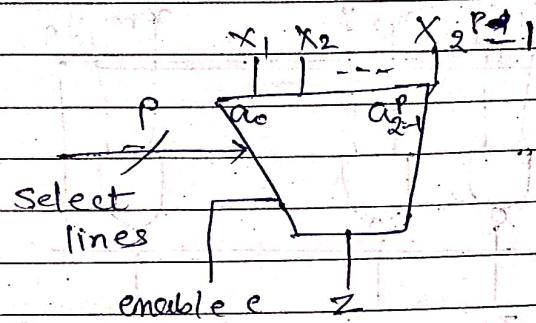


max. no. of data source = k and each IO data line carries m bits

k input (k way)
m bit MUX

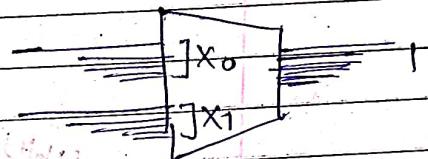
8x1 using only 4x1 MUX



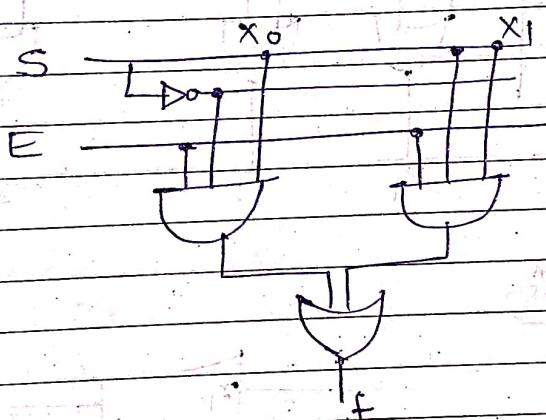


Recall m bit
registers

2 in/p. 4 bit MUX

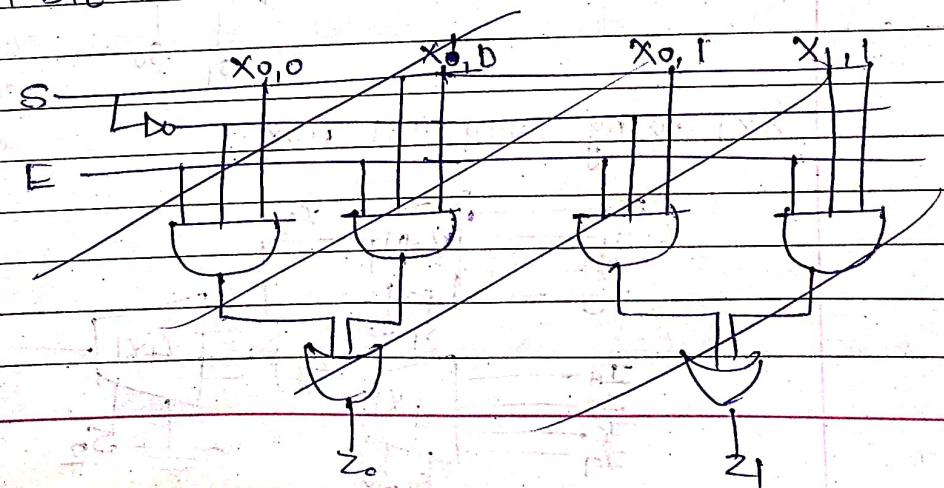


Realize 2 input 1 bit MUX



$$f = (sx_0 + ex_1)E$$

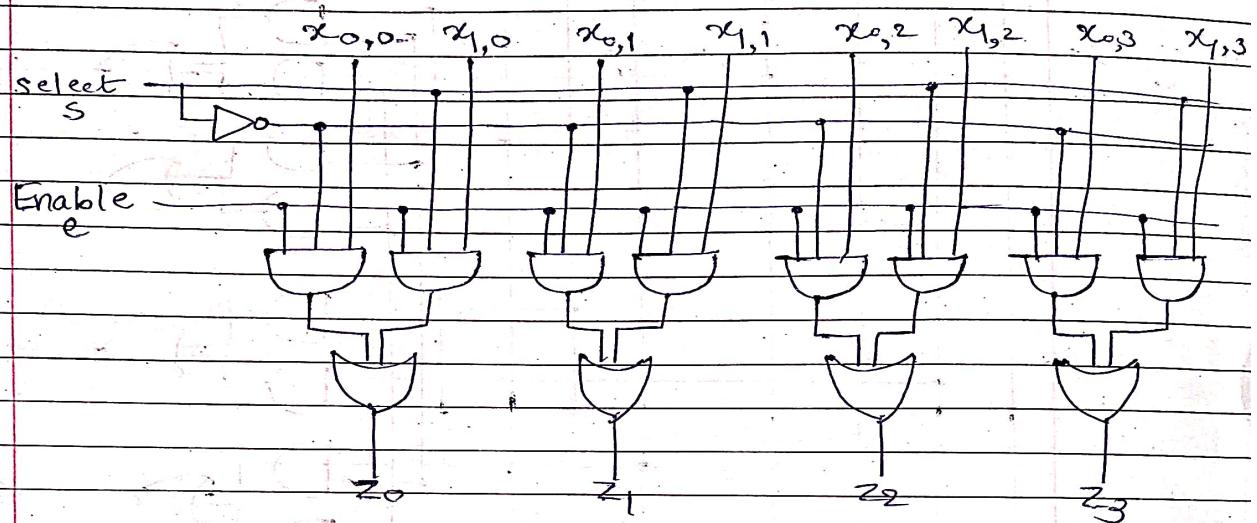
2 in/p. 4 bit MUX



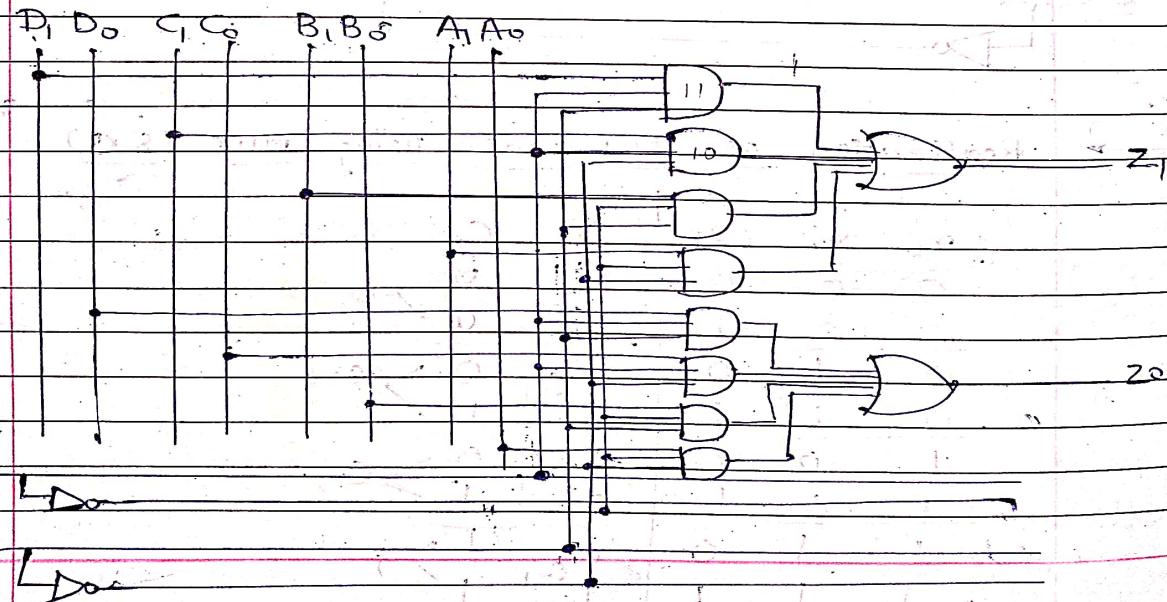
Yp

~~4 bit & 2 bit MUX~~

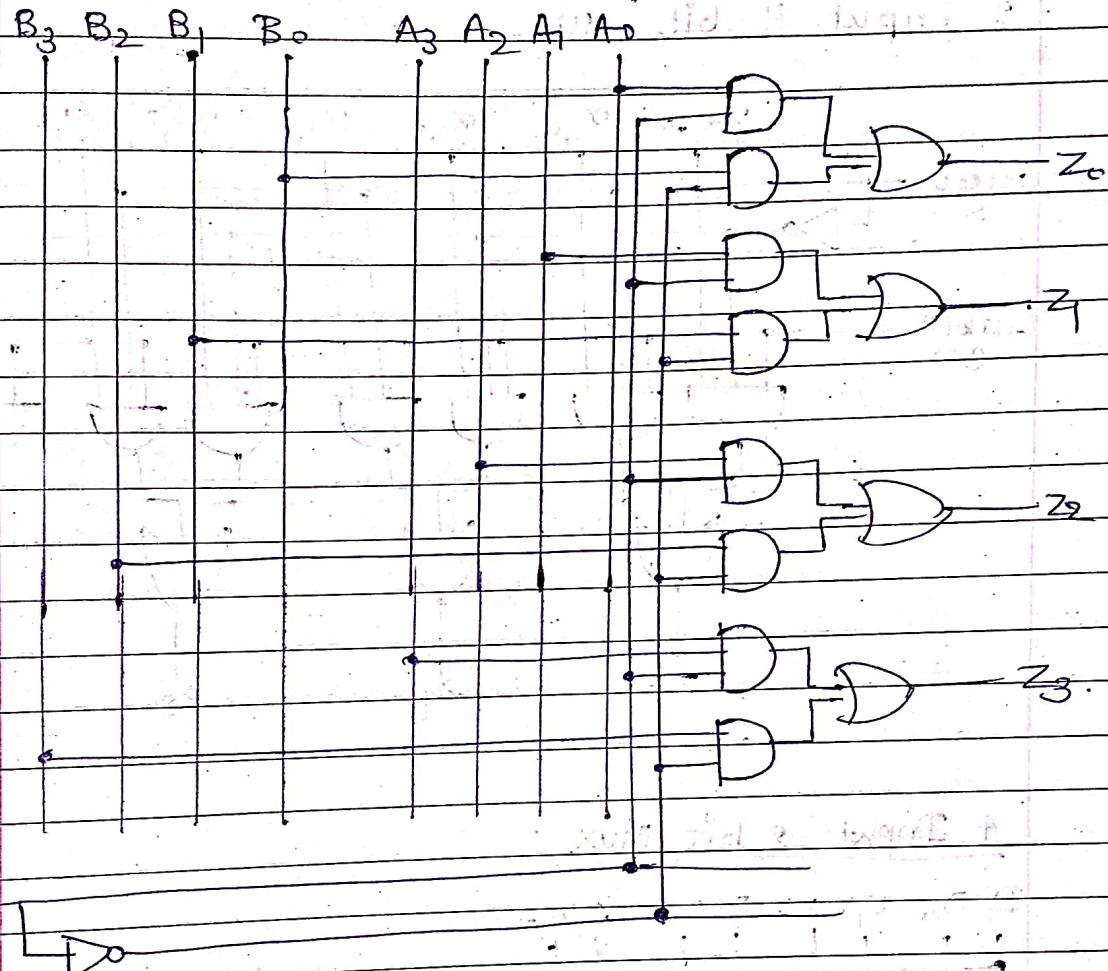
* 2 Input 4 bit MUX.



* 4 Input 2 bit MUX



Another Representation of 2 Input 4 bit MUX

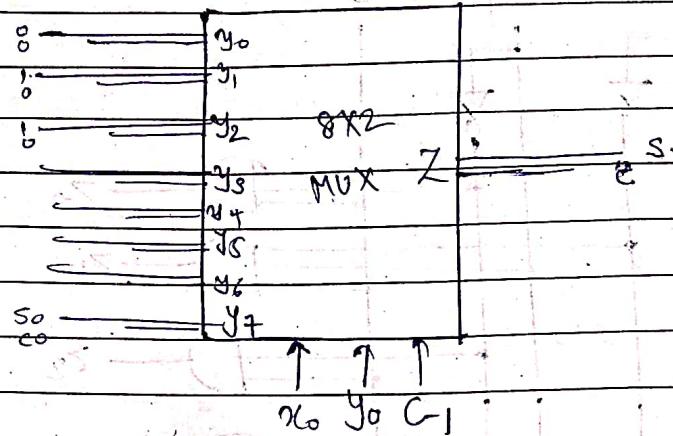
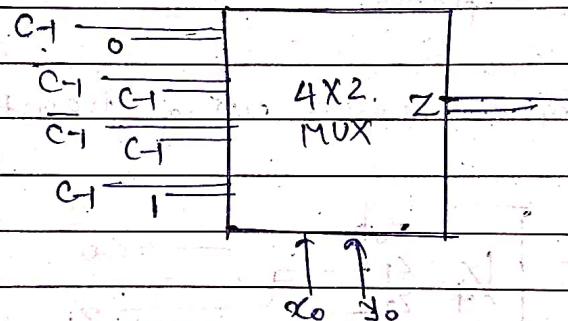
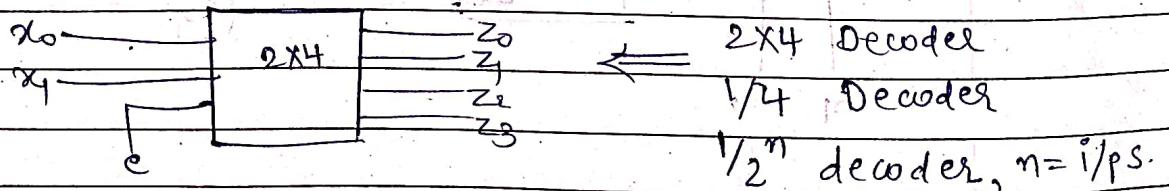


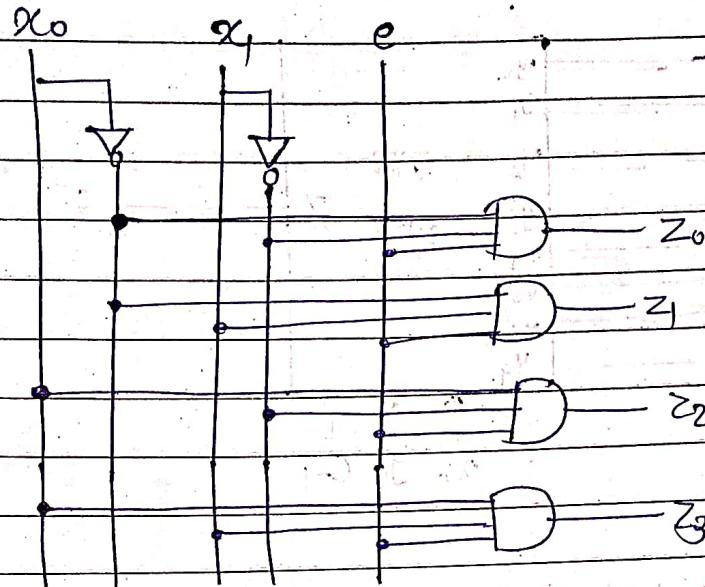
→ Realization of full adder using MUX (8x1) 8 i/p 2 bit

x_0	y_0	C_{-1}	S_0	C_0
0	0	0	$\begin{cases} 0 & C_1 = 0 \\ 1 & C_1 = 1 \end{cases}$	0
0	0	1	$\begin{cases} 1 & C_1 = 0 \\ 0 & C_1 = 1 \end{cases}$	C_{-1}
0	1	0	$\begin{cases} 1 & C_1 = 0 \\ 0 & C_1 = 1 \end{cases}$	C_{-1}
0	1	1	$\begin{cases} 0 & C_1 = 0 \\ 1 & C_1 = 1 \end{cases}$	C_{-1}
1	0	0	$\begin{cases} 1 & C_1 = 0 \\ 0 & C_1 = 1 \end{cases}$	C_{-1}
-1	0	1	$\begin{cases} 0 & C_1 = 0 \\ 1 & C_1 = 1 \end{cases}$	C_{-1}
1	1	0	$\begin{cases} 0 & C_1 = 0 \\ 1 & C_1 = 1 \end{cases}$	1
1	1	1	$\begin{cases} 1 & C_1 = 0 \\ 0 & C_1 = 1 \end{cases}$	1

must be
2 bit

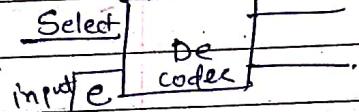
8 way

4^{1/p} 2 bit MUXDecoder / Encoder :

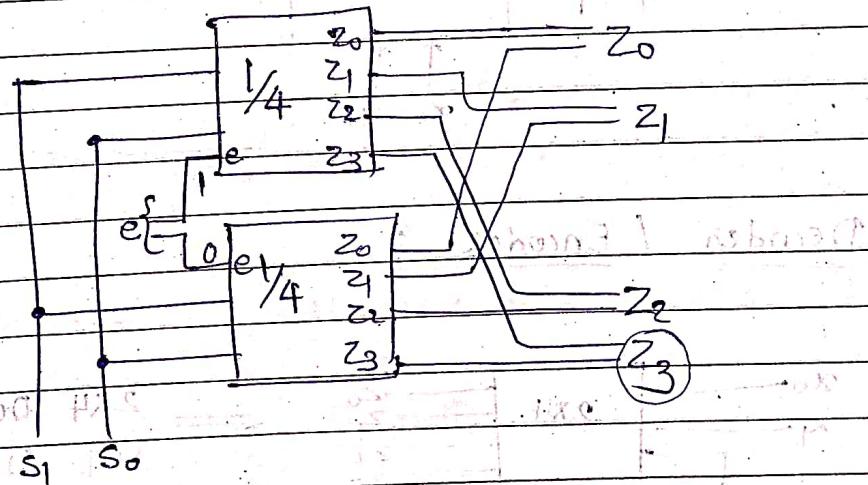


use : p-ROM (select the address)

In Demultiplexer



ex. 4 O/p 2 bit DeMUX using Decoder

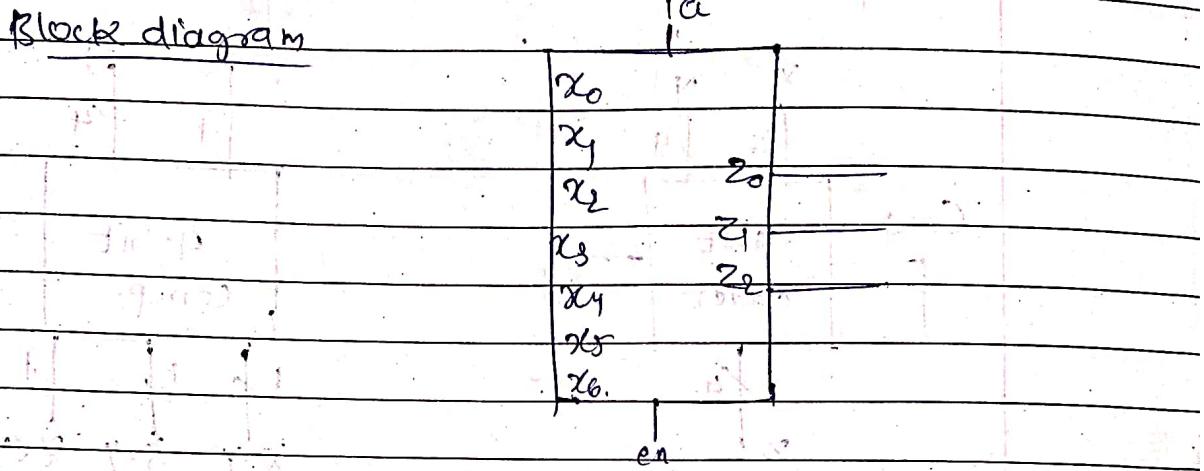


ex. 8 O/p 3 bit DeMUX.

→ Priority Encoder

ex. 8x3 Priority Encoder

Block diagram



e	x ₇	x ₆	x ₅	x ₄	x ₃	x ₂	x ₁	x ₀	z ₂	z ₁	z ₀	ia
0	x	x	x	x	x	x	x	x	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	1	0	0	0	1
1	0	0	0	0	0	0	1	x	0	0	1	1
1	0	0	0	0	0	0	0	x	0	1	0	1
1	0	0	0	0	0	1	x	x	0	1	1	1
1	0	0	0	0	1	x	x	x	1	0	0	1
1	0	0	0	1	x	x	x	x	1	0	1	1
1	0	0	1	x	x	x	x	x	1	1	0	1
1	1	1	x	x	x	x	x	x	1	1	1	1

$$\begin{aligned} z_2 &= \overline{x_7} \overline{x_6} \overline{x_5} x_4 + \overline{x_7} \overline{x_6} x_5 + \overline{x_7} x_6 + x_7 \\ &= x_7 + x_6 + x_5 + x_4 \end{aligned}$$

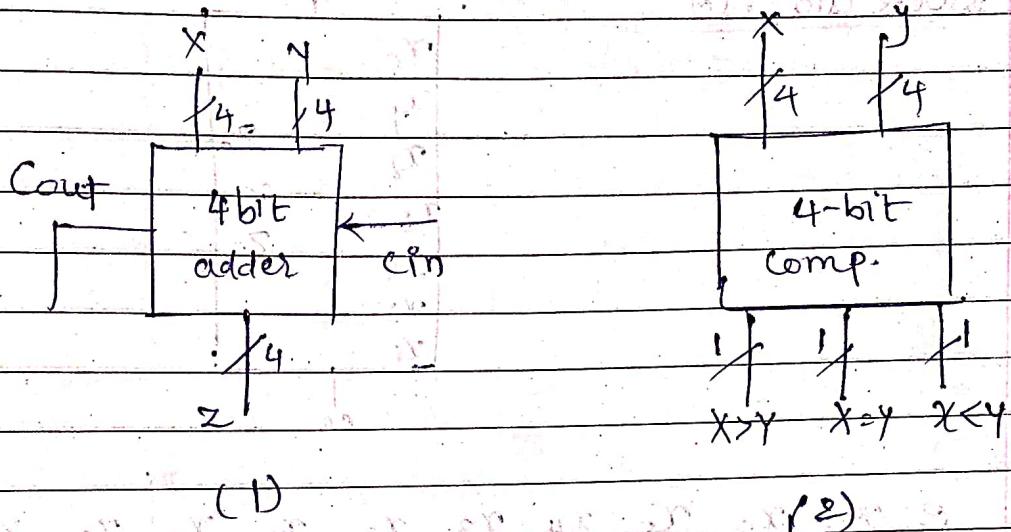
$$z_1 = x_7 + x_6 + x_3 + x_2$$

$$z_0 = x_7 + x_5 + x_2 + x_0$$

Arithmetic elements

1. Adder

2. Comparator



$$\text{case 1 } x > y \Rightarrow x - y > 0 \quad (1)$$

$$y = (2^n - 1) - \bar{y} \quad (2)$$

From (1) & (2),

$$x - [(2^n - 1) - \bar{y}] > 0$$

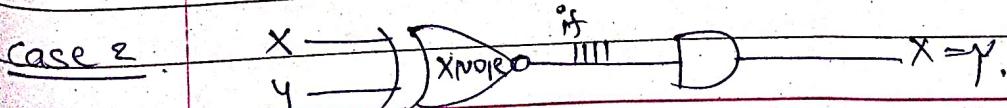
$$x + \bar{y} > 2^n - 1$$

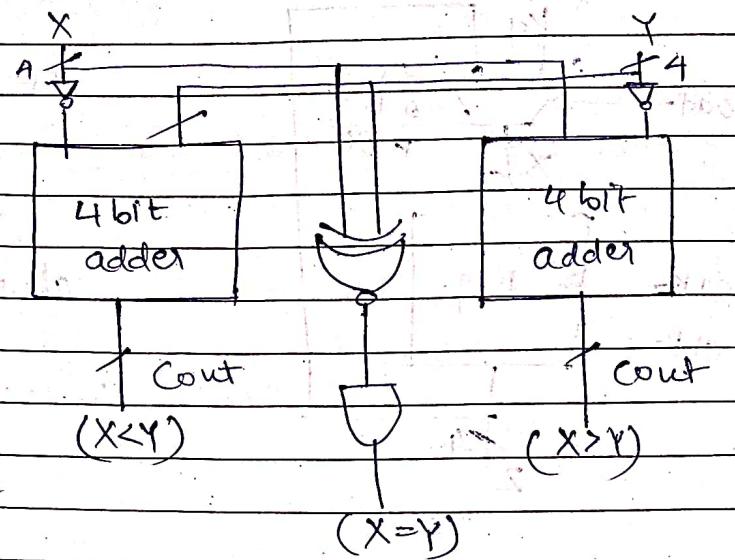
\hookrightarrow if cout is 1, then $x > y$

$$\text{case 3 } x < y \Rightarrow y > x$$

$$\therefore y + \bar{x} > 2^n - 1$$

\hookrightarrow if carry generated,





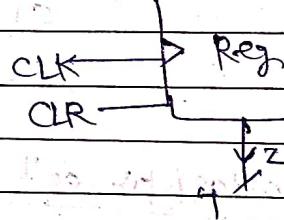
Sequential Elements

Block diagram

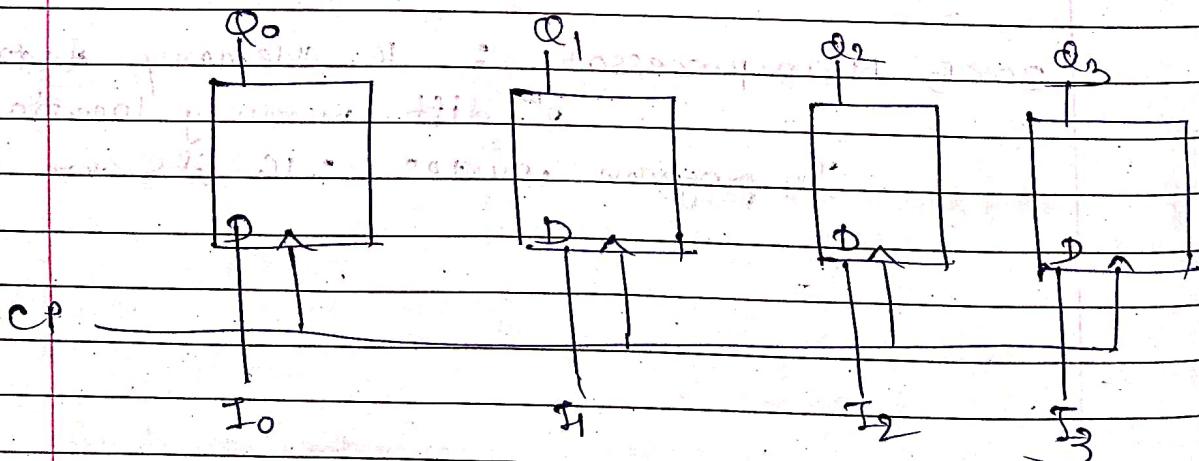
X
x4

Registers

- parallel
- shift left
- right

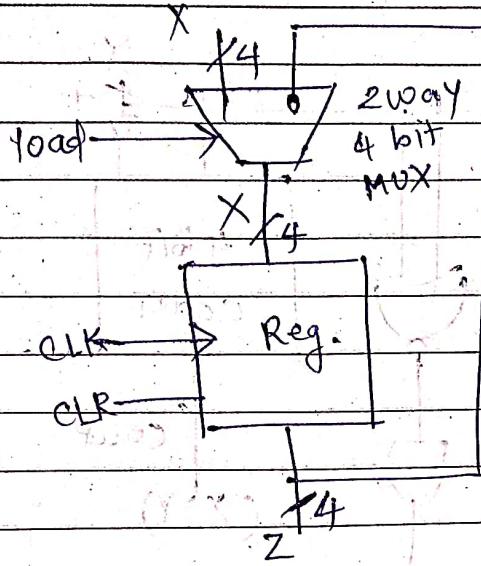


Ex. 4-bit parallel register



left = $x, z_{m-1}, z_{m-2}, \dots, z_0$

right = $z_{m-1}, z_{m-2}, \dots, z_0, x$

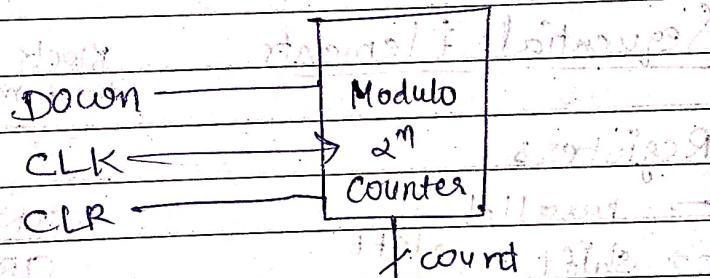


JF i/p is 0 \Rightarrow no change

i/p is 1 \Rightarrow i/p (new)

Characteristic eqn. = ?

Counter



Characteristic eqn?

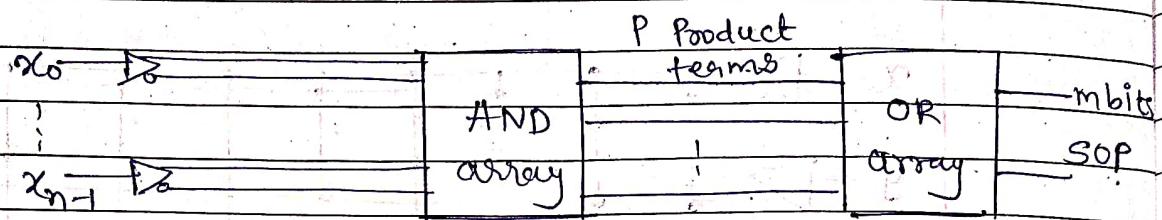
up counter $S_{IT1} = S_1$ plus 1

down counter $S_{IT1} = S_1$ minus 1

8085 Microprocessor : 16 Memory Address

2^{16} diff. memory locations

In program counter : 16 bits are required

PLD

n inputs $\times p$ product terms $\times m$ outputs

n inputs $\times p$ terms $\times m$ outputs

- divided into three types.

	AND	OR
PROM	fixed	programmable
PLA	prog.	prog
PAL	prog.	fixed

Inflexible

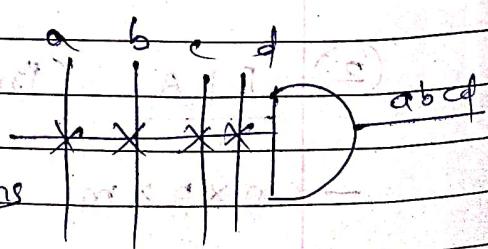
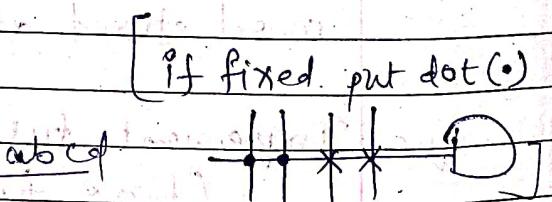
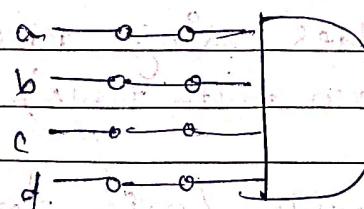
①

PROM (Programmable Read only memory)

- n to 2^m decoder.
- n I/P max. product terms \times^n AND gates (fixed)

8×2 bit

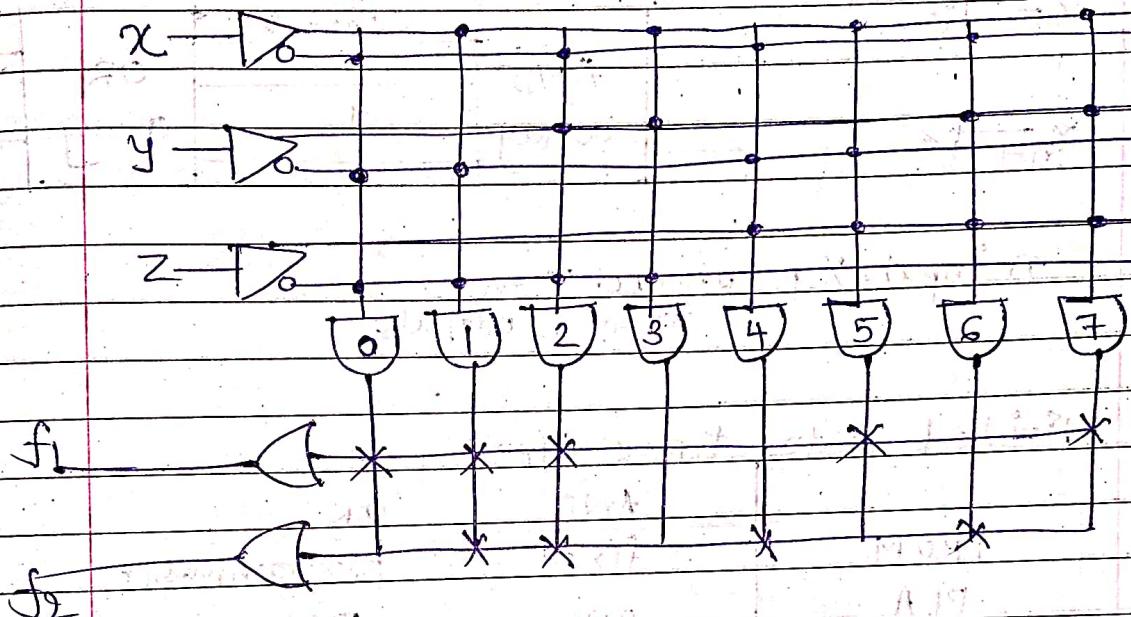
[if fixed, put dot (•)]



field programmable - user programs
mask programmable - using sheet

PLA
(Reprogrammable)

ex 8x2 (8 i/p s and 2 bit) (PROM)



que. Implement $f_1(x, y, z) = \sum(0, 1, 2, 5, 7)$
 $f_2(x, y, z) = \sum(1, 2, 4, 6)$

$\Rightarrow f_1 \& f_2 = 2$ OR gates

$x, y, z = 3$ i/p s and 8 AND gates.

- word stored at loc. 6 is 01

word stored at loc. 2 is 11

que. Implement full adder using PROM.
 (have to draw truth table)

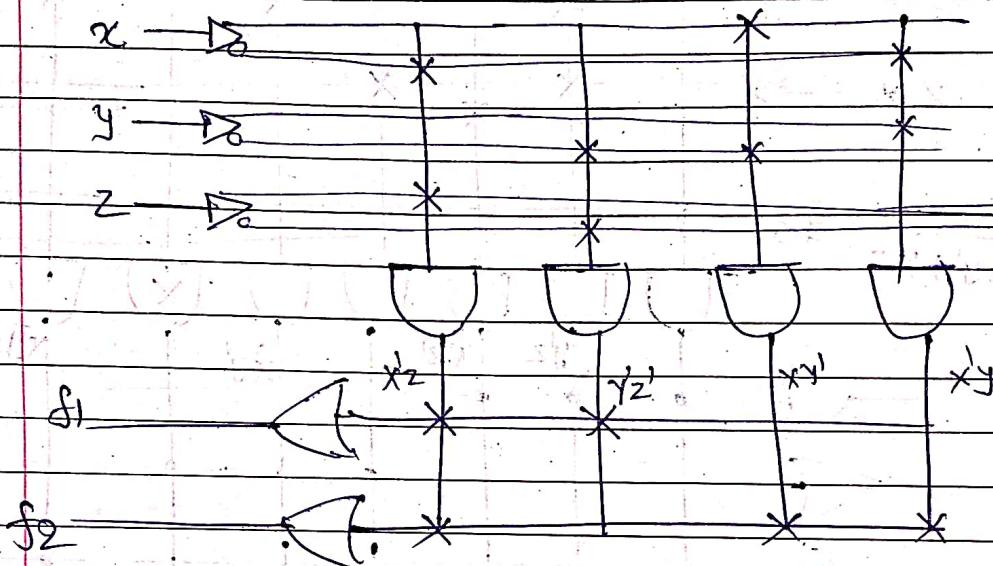
(2) PLA (Programmable Logic Array)

$\rightarrow n \times p \times m$ (AND & OR are pre-g)

que $f_1 = \sum(0, 1, 3, 4)$

$$f_2 = \sum(1, 2, 3, 4, 5)$$

Implement above using $3 \times 4 \times 2$ PLA.



	xz	yz	xy'	$x'y$
0	0	0	1	1
1	1	1	0	0

	xz	yz	xy'	$x'y$
0	0	0	1	1
1	1	1	0	0

$$f_1 = y'z' + \underline{x'z}$$

common

$$f_2 = xy' + \underline{xz} + \underline{x'y}$$

common

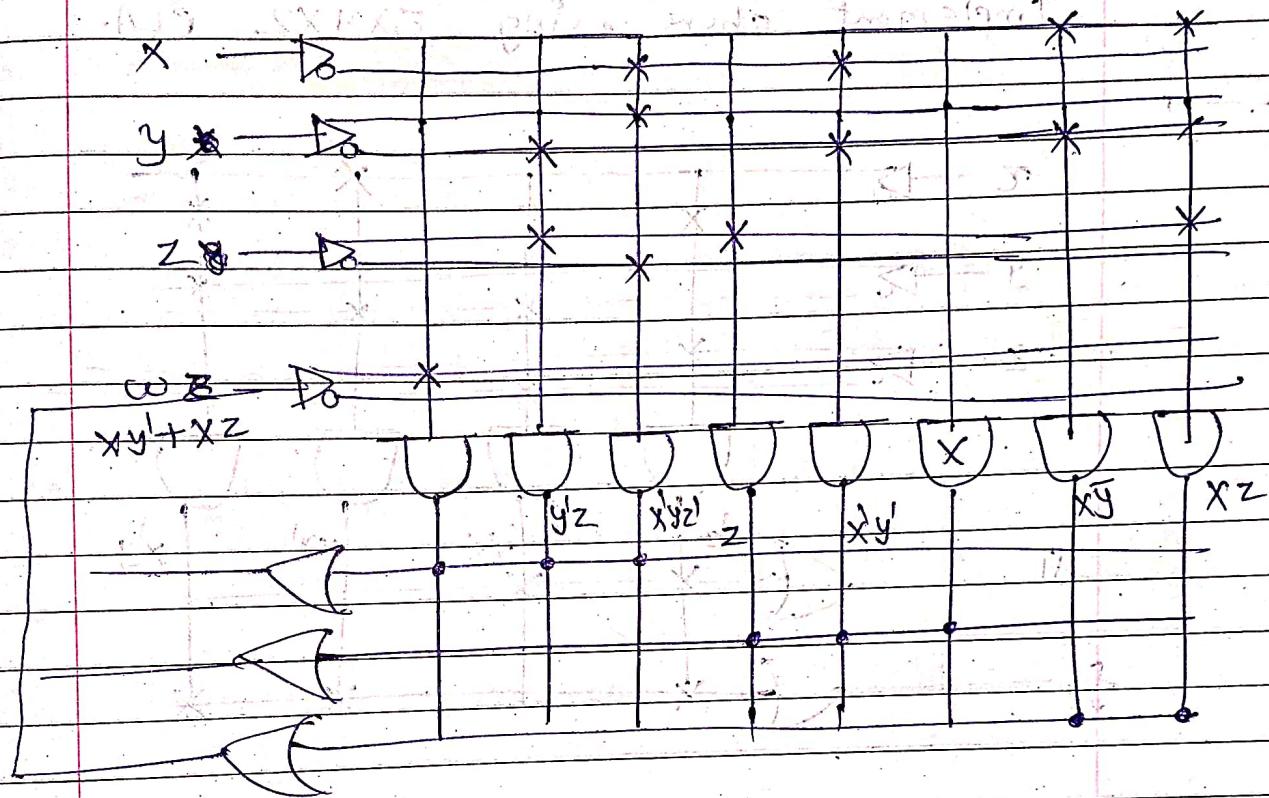
$$\text{Minterms} = \underset{3}{x'y'} + \underset{2}{y'z'} + \underset{1}{x'z} + \underset{4}{x'y}$$

(3) PAL (Programmable Array Logic)

$m \times p \times m$ (OR is fixed)

que. $f_1(x,y,z) = \sum(1,2,4,5,7)$ $\Rightarrow 4 \times 8 \times 3$

$$f_2(x,y,z) = \sum(0,1,3,5,7)$$



		$x^y z$	00	01	11	10
0	0	1	1	1	1	1
1	1	1	1	1	1	1

$$f_1 = y'z + \underline{xz} \\ + \underline{xy'} + \underline{x'y'z}$$

		$x^y z$	00	01	11	10
0	0	1	1	1	1	1
1	1	1	1	1	1	1

$$f_2 = y'z + xz \\ + z + x'y'$$

queImplement the f^M using PLA

$$f_1 = \sum m(0, 1, 3, 5)$$

$$f_2 = \sum m(3, 5, 7)$$

3X4X2

$$f_1 : x'y'z$$

0	1	1	1
1	1	1	1

$$f_2 : x'y'z$$

0			
1	1	1	1
	1	1	1

$$f_1 = x'y' + x'z + xy'z$$

$$f_2 = xy'z + yz.$$

que

$$f_1 = \sum m(3, 5, 6, 7)$$

$$f_2 = \sum m(0, 2, 4, 7)$$

8X4X2

$$f_1 : x'y'z$$

0	1	1	1
1	1	1	1

$$f_2 : x'y'z$$

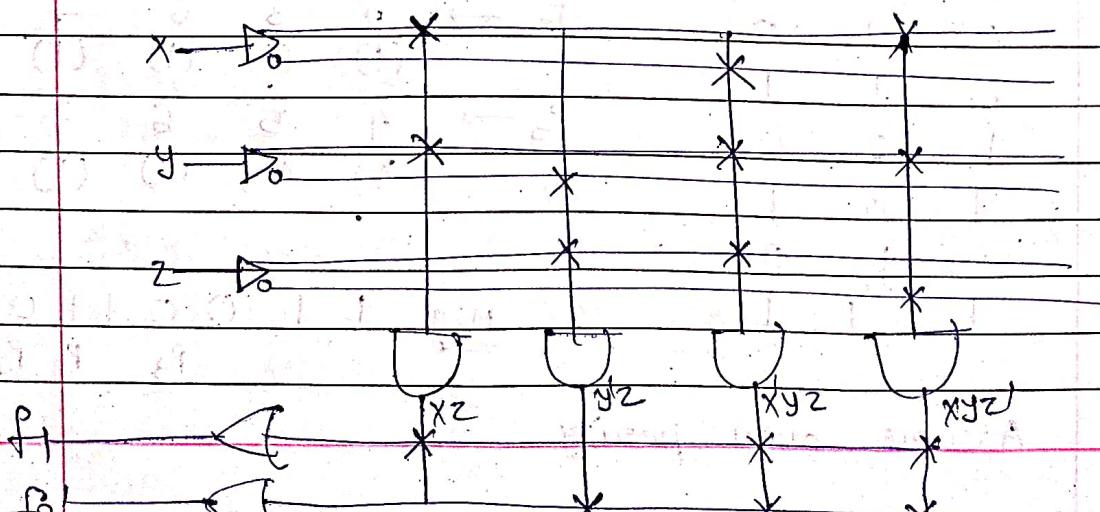
0	1	1	1
1	1	1	1
	1	1	1

$$f_1 = xz + x'y'z + xy'z'$$

$$f_2 : x'y'z$$

0	1	1	1
1	1	1	1
	1	1	1

$$f_2' = y'z + x'y'z + xy'z'$$



*

Hamming Code

two variations : (i) single error correction (SEC)

(ii) single error correction double
error detection (DED)

TxDRxD

1. Calculation of
Redundant data

$$2^r \geq m+r+1$$

where m = data bits

$$r = \text{redundant bits}$$

$$\begin{aligned} \text{eg. } m &= 4, 2^r \geq 5+r \\ m+r &= 4+3 \\ &= 7 \text{ total no. of bits} \end{aligned}$$

(7,3) Hamming code

2. Bit position
table

2^2	2^1	2^0
4	6	5
3	4	2
2	1	1
D ₄	D ₃	D ₂
P ₃	P ₂	P ₁

(redundant)

- parity bits are
always placed at
position of 2^n .

P ₃	P ₂	P ₁
0	0	0
0	0	1

P ₁	1	3	5	7
(0)	(1)	(0)	(1)	P ₁ =0

0	1	0
0	1	0

P ₂	2	3	6	7
(1)	(1)	(1)	(1)	P ₂ =1

0	1	1
1	0	1

P ₃	4	5	6	7
(0)	(0)	(1)	(1)	P ₃ =0

1	1	0
1	1	0

msg. 1 1 0 0 1 1 0

P₃ P₂ P₁

Assume even parity

RX1. Recalculate Redundant bits

$$2^r \geq m+r+1$$

$$\text{but } m+r=7 \Rightarrow r=3$$

2. Bit position table

7	6	5	4	3	2	1	
D ₄	D ₃	D ₂	P ₃	D ₁	P ₂	P ₁	

1	1	1	0	1	1	0	
---	---	---	---	---	---	---	--

$$P_1'' \rightarrow \begin{matrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 1 & 1 \end{matrix} \quad [P_1'' = 1]$$

$$P_2'' \rightarrow \begin{matrix} 2 & 3 & 6 & 7 \\ 1 & 0 & 1 & 1 \end{matrix} \quad [P_2'' = 0]$$

$$P_3'' \rightarrow \begin{matrix} 4 & 5 & 6 & 7 \\ 0 & 1 & 1 & 1 \end{matrix} \quad [P_3'' = 1]$$

$$P_3'' P_2'' P_1'' \neq 0$$

1	0	1
---	---	---

gives position of error bit which was going to flip.

$$\text{Ex. } r = 8 \Rightarrow 2^8 \geq m+9$$

$$m \leq 247$$

$$\text{total bits} \Rightarrow 2^8 - 1 = 255 \quad [(m+r)]$$

$$\text{no. of data bits} \Rightarrow [m \leq 247]$$

Ex. msg = 101101

encode using hamming code

(I)

$$2^r \geq m+r+1$$

$$\text{here, } m=6$$

r

$$2^r \geq 6+r+1$$

$$[r=4]$$

$$m+r=10$$

(II)

	3	2	1	0	7	6	5	4	3	2	1	0
D ₆	1	0	0	1	1	1	0	1	1	1	1	1
D ₅	1	0	0	1	1	1	0	1	1	1	1	1
D ₄	1	0	0	1	1	1	0	1	1	1	1	1
D ₃	1	0	0	1	1	1	0	1	1	1	1	1
D ₂	1	0	0	1	1	1	0	1	1	1	1	1
D ₁	1	0	0	1	1	1	0	1	1	1	1	1
P ₁	1	0	0	1	1	1	0	1	1	1	1	1
P ₂	1	0	0	1	1	1	0	1	1	1	1	1
P ₃	1	0	0	1	1	1	0	1	1	1	1	1
P ₄	1	0	0	1	1	1	0	1	1	1	1	1

(III)

$$P_1 \rightarrow 1 \quad 3 \quad 5 \quad 7 \quad 9 \quad [P_1=0]$$

$$P_2 \rightarrow 2 \quad 3 \quad 6 \quad 7 \quad 10 \quad [P_2=0]$$

$$P_3 \rightarrow 4 \quad 5 \quad 6 \quad 7 \quad [P_3=0]$$

$$P_4 \rightarrow 8 \quad 9 \quad 10 \quad [P_4=1]$$

transmitted msg: 1011100100

Flipping bit at position 8 (P₄)

Received msg: 1011000100
(RX)

(I)

$$2^r \geq m+r+1$$

$$\text{but } m+r = 10$$

$$\therefore 2^r \geq 11$$

$$\Rightarrow r=4$$

(II)

10 9 8 7 6 5 4 3 2 1

1 0 1 1 0 0 0 1 0 0

D₆ D₅ D₄ D₃ D₂ D₁ P₃ P₁ P₂ P₁

$$P_1'' = 1 \quad 3 \quad 5 \quad 7 \quad 9 \quad \boxed{P_1'' = 0}$$

$$P_2'' = 2 \quad 3 \quad 6 \quad 7 \quad 10 \quad \boxed{P_2'' = 1}$$

$$P_3'' = 4 \quad 5 \quad 6 \quad 7 \quad \boxed{P_3'' = 1}$$

$$P_4'' = 8 \quad 9 \quad 10 \quad \boxed{P_4'' = 0}$$

$P_4'' P_3'' P_2'' P_1'' \neq 0$

$\underbrace{0 \ 1 \ 1 \ 0}_{\rightarrow 6^{\text{th}} \text{ bit is flipped.}}$

Ex.

msg = 11011100111100101

(I)

$$2^r \geq m+r+1$$

(14)

$$2^r \geq 18$$

$$\Rightarrow r=5$$

(II)

$$17 \quad 16 \quad 15 \quad 14 \quad 13 \quad 12 \quad 11 \quad 10 \quad 9 \quad 8 \quad 7$$

$$D_{12} \quad P_5 \quad D_{11} \quad D_{10} \quad D_9 \quad P_8 \quad D_7 \quad D_6 \quad D_5 \quad P_4$$

(1) (1) (0) (1) (1) (1) (0) (0) (1) (1) (1)

$$6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1$$

$$D_3 \quad D_2 \quad P_3 \quad D_1 \quad P_2 \quad P_1$$

(1) (0) (0) (1) (0) (1)

$$P_1'' = 3 \quad 5 \quad 7 \quad 9 \quad 10 \quad 13 \quad 15 \quad 17$$

(1) (1) (0) (1) (1) (0) (0) (1)

$$P_2'' = 2 \quad 3 \quad 6 \quad 7 \quad 10 \quad 11 \quad 14 \quad 15$$

(0) (1) (1) (1) (0) (0) (1) (0)

$$P_3'' = 4 \quad 5 \quad 6 \quad 7 \quad 12 \quad 13 \quad 14 \quad 15$$

(0) (0) (1) (0) (1) (1) (1) (0)

$$P_4'' = 8 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15$$

(1) (1) (0) (0) (1) (1) (1) (0)

$$P_5'' = 16 \quad 17$$

(1) (1)

$$P_5'' P_4'' P_3'' P_2'' P_1'' \neq 0$$

$$0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1$$

→ 12th bit

Ex. Encode the msg: 101010011101

(I)

$$2^r \geq m+r+1 \Rightarrow r \geq 4$$

$$2^r \geq 13 \Rightarrow r = 4$$

(II)

$$\begin{matrix} 12 & 11 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ D_8 & D_7 & D_6 & D_5 & P_4 & D_4 & D_3 & D_2 & P_3 & D_1 & P_2 & P_1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{matrix}$$

$$\begin{matrix} P_1'' = 1 & 3 & 5 & 7 & 9 & 11 \\ (1) & (1) & (1) & (0) & (0) & (0) \end{matrix}$$

$$\begin{matrix} P_2'' = 0 & 2 & 3 & 6 & 7 & 10 & 11 \\ (0) & (1) & (0) & (0) & (1) & (0) \end{matrix}$$

$$\begin{matrix} P_3'' = 1 & 4 & 5 & 6 & 7 & 12 \\ (1) & (1) & (0) & (0) & (1) \end{matrix}$$

$$\begin{matrix} P_4'' = 1 & 8 & 9 & 10 & 11 & 12 \\ (1) & (0) & (1) & (0) & (1) \end{matrix}$$

$$P_4'' P_3'' P_2'' P_1'' \neq 0$$

$$1 \ 1 \ 0 \ 1$$

(I)

$$2^r \geq m+r+1$$

$$2^r \geq 12+r+1 \Rightarrow r=5 \quad m+r=17$$

$$17 \ 16 \ 15 \ 14 \ 13 \ 12 \ 11 \ 10 \ 9 \ 8 \ 7 \ 6 \ 5 \ 4$$

$$\begin{matrix} D_{12} & P_5 & D_{11} & D_{10} & D_9 & D_8 & D_7 & D_6 & D_5 & P_4 & D_4 & D_3 & D_2 & P_1 \\ (1) & (0) & (0) & (0) & (0) & (0) & (0) & (0) & (0) & (1) & (1) & (1) & (1) & (0) \end{matrix}$$

$$8 \ 12 \ 1$$

$$D_1 \ P_2 \ P_1$$

$$(1)$$

$$P_1 = 1 \ 1 \ 3 \ 5 \ 7 \ 9 \ 11 \ 13 \ 15 \ 17 \quad [P_1=0]$$

(0) (1) (0) (1) (1) (0) (0) (0) (1)

$$P_2 = 2 \ 3 \ 6 \ 7 \ 10 \ 11 \ 14 \ 15 \quad [P_2=0]$$

(0) (1) (1) (1) (0) (0) (1) (0)

$$P_3 = 4 \ 5 \ 6 \ 7 \ 12 \ 13 \ 14 \ 15 \quad [P_3=0]$$

(0) (0) (1) (1) (1) (0) (1) (0)

$$P_4 = 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \quad [P_4=1]$$

(0) (1) (0) (0) (1) (0) (1) (0)

$$P_5 = 16 \ 17 \quad [P_5=1]$$

(1) (0)

transmitted msg: 1101010011100100.

Extended Hamming Code

msg: 11011101
 mtr → redundant bits
 (12, 4)

I $[s=4]$

$$\begin{array}{ccccccccc} 12 & 11 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ D_8 & D_7 & D_6 & D_5 & D_4 & D_3 & D_2 & D_1 & P_1 & P_2 & P_3 & P_4 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{array}$$

$$P_1 = 1 \ 3 \ 5 \ 7 \ 9 \ 11 \quad [P_1=0]$$

(0) (1) (0) (1) (1) (1)

$$P_2 = 2 \ 3 \ 6 \ 7 \ 10 \ 11 \quad [P_2=0]$$

(0) (1) (1) (1) (0) (1)

$$P_3 = 4 \ 5 \ 6 \ 7 \ 12 \quad [P_3=1]$$

(1) (0) (1) (1) (1) (0) (1)

$$P_4 = 8 \ 9 \ 10 \ 11 \ 12 \quad [P_4=1]$$

(0) (1) (0) (1) (1)

Encoded

msg: 110111101100

Received msg: 110111111100

(I)

$$P_1'' = 1 \ 3 \ 5 \ 7 \ 9 \ 11$$

(1) (1) (1) (1) (1) (1)

$$P_1'' = 1$$

method
Same as
trans-
mitter
side)

$$P_2'' = 2 \ 3 \ 6 \ 7 \ 10 \ 11$$

(0) (1) (1) (1) (0) (1)

$$P_2'' = 0$$

$$P_3'' = 4 \ 5 \ 6 \ 7 \ 12$$

(0) (0) (1) (1) (1)

$$P_3'' = 0$$

$$P_4'' = 8 \ 9 \ 10 \ 11 \ 12$$

(1) (1) (0) (1) (1)

$$P_4'' = 1$$

$$P_4'' P_3'' P_2'' P_1''$$

$$\begin{array}{r} 1 \ 0 \ 0 \ 1 \\ \oplus \\ \text{XOR} \end{array}$$

$$P_4'' P_3'' P_2'' P_1''$$

$$1 \ 1 \ 0 \ 0 = 1010$$

5th bit

Syndrome calculator

SECDED

1101111011000 ← ok Encoded msg

(RP even no.)

case-I :

$$P_4'' P_3'' P_2'' P_1''$$

$$\text{code / syndrome} = 0 \quad \& \quad \text{RP} = 0$$

∴ No error

case-II :

$$\text{code / syndrome} = 0 \quad \& \quad \text{RP} = 1 \text{ only}$$

↳ flipped RP

case-III

syn. $\neq 0$, $csp = 1111011$ Single error detection

case-IV

syn $\neq 0$, $csp = 0111011$ Double error detection

	P_4	P_3	P_2	P_1	csp	
	1	1	0	1	1110110	0
	1	1	0	1	111100	0
	1	1	0	1	111100	0

$$P_1'' \leftarrow 1 \ 3 \ 5 \ 7 \ 9 \ 11 \\ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \quad P_1'' = 0$$

$$P_2'' \leftarrow 2 \ 3 \ 6 \ 7 \ 10 \ 11 \\ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \quad P_2'' = 1$$

$$P_3'' \leftarrow 4 \ 5 \ 6 \ 7 \ 12 \\ 1 \ 1 \ 0 \ 1 \ 1 \quad P_3'' = 1$$

$$P_4'' \leftarrow 8 \ 9 \ 10 \ 11 \\ 1 \ 1 \ 0 \ 1 \quad P_4'' = 1$$

$$P_4'' P_3'' P_2'' P_1'' \\ + \\ \begin{array}{r} 1 \\ 1 \\ 1 \\ 0 \end{array} \\ \hline 0 \ 0 \ 1 \ 0$$

Double error detection

12 11 10 9 8 7 6 5 4 3 2 1

D₈ D₇ D₆ D₅ P₄ P₃ P₂ P₁ P₂ P₁

1 1 0 1 1 1 0 1 1 0 0 CCP = ? CCP = ?

0 0

P₁^{II} = 1 3 5 7 9 11 [P₁^{IK} = 0]

(0) (1) (0) (1) (0) (1)

P₂^{II} = 2 3 6 7 10 11 [P₂^{II} = 0]

(0) (1) (0) (1) (0) (1)

P₃^{II} = 4 5 6 7 12 [P₃^{II} = 0]

(0) (0) (0) (1) (1)

P₄^{II} = 8 9 10 11 12 [P₄^{II} = 0]

(0) (0) (0) (1) (1)

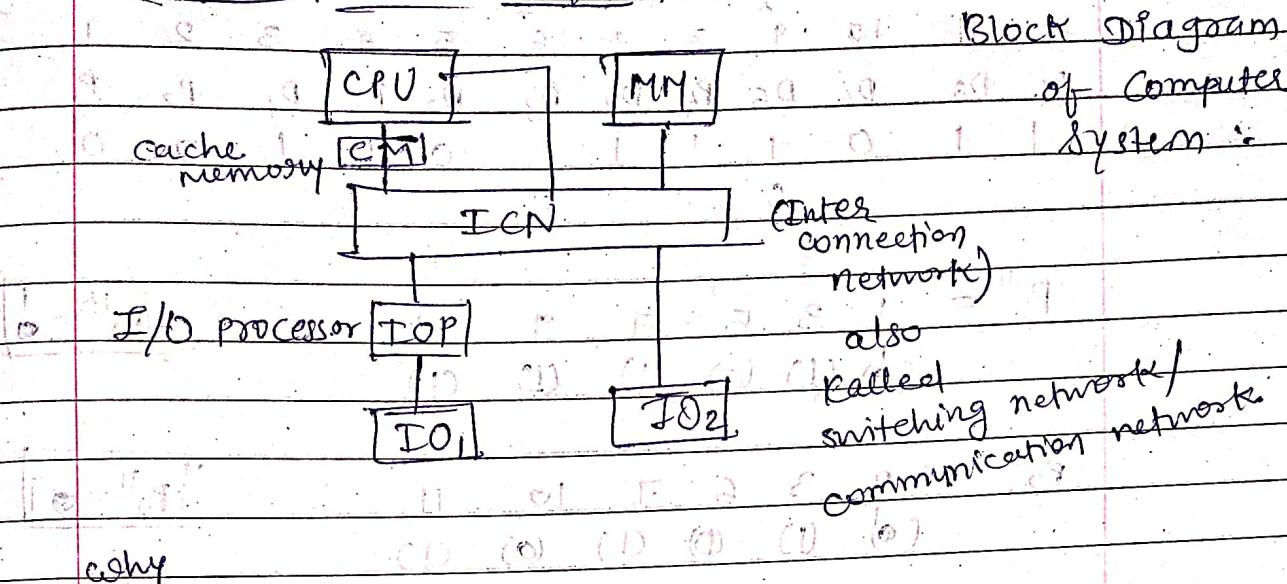
P₄^{II} P₃^{II} P₂^{II} P₁^{II} CCP
 0 0 1 1 0 Received
 + 1 1 0 0 0 Transmitted
 ——————
 1 1 1 1 0

syn ≠ 0, CCP = 0 (Double error detection)

Processor Level:

1. CPU
2. Memories
3. I/O
4. Interconnection network.

Single processor System



copy

Synchronization Problem:

- CPU task and IOP task is different
- CPU is faster (\gg Main memory \gg I/O device)
therefore there is no synchronization betⁿ them.

Multi processor System

