THE LOGISTIC GROWTH MODEL

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Linear and Exponential functions are great.

Tthey help us to describe our data by giving us

a good picture of how the data behaves.

But they each have a fatal flaw - the linear function assumes

constant linear growth or constant linear decay,

and the exponential function assumes

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constant percent growth

or constant percent decay.

But is it reasonable to make this assumption of consistency?

I love the exponential model.

In my fall semester classes, I've introduced

the idea of the exponential growth with some references to zombie movies

- because the topic usually falls around Halloween.

Let's take the following example: The RainCoat Coalition

has been developing a new virus and are examining it under locked conditions.

After 3 days, there are 3,200 cells in a petrie dish.

Two days later and there are 4,350 cells.

Now we can come up with an exponential model of growth using these two points,

and the model will be: f(t) = 2019.02 * 1.166^t.

A this point I usually talk about the virus getting out

and Milla Jovovich or Brad Pitt coming to save the day.

But let's just think about this for a moment -

what if the virus didn't get out and was in

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fact

restricted to growing just in the petrie dish.

Would it KEEP growing as the exponential model dictates it should?

Now there are limited resources and nutrients in the dish.

Say there is only room for about 10,000 cells in the dish.

Given our model, and our knowledge of how

to solve for "t" in the exponential function,

we can see that in just over 10 days, the cells will reach that upper limit.

As exciting for Hollywood as it would be for the zombie virus

to break free or outgrow the petrie dish, it probably won't happen.

There is a natural upper limit to almost all biological growth

and biological systems.

And that's where the logistic growth function comes in.

Instead of seeing this - an exponential model of zombie cell growth -

we're more likely to see something like this - logistic growth.

Let's look at some real world data: It's been all over the news:

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the ebola outbreak of 2014 in West Africa.

I've gotten some data from the Columbia

Prediction of Infectious Diseases - specifically for Liberia.

Here are the cumulative ebola cases from the beginning of May

to the second week of November.

Stop!

Stay right there - at this point on the graph,

we're up to the last week of August.

At this point, the semester was starting at UT,

and folks were attuned to what was happening.

If we take a look at this data, we would be VERY tempted

(I would be very tempted) to think "I see an exponential model here."

And get very nervous.

In fact if we fit an exponential model to just this range of this data,

we get a really good fitting model.

But remember what we said earlier: There is a natural upper limit.

So let's look at the data - up till the second week of November.

We see that yes - there is in fact a slowing down -

a limiting to the number of cases.

Now, if we try to fit an exponential model,

it just doesn't really look right - and to be honest,

if I saw this raw data, with this shape, I

wouldn't even think of an exponential function.

So let's examine the logistic function that we get.

Here's the general form of the logistic function.

We see things that we recognize: an "a" parameter, and a "b" parameter

- we'll get back those in a little bit.

But first, let's take a look at this "C" parameter.

This parameter is possibly the most important of the logistic function.

It distinguishes it from the others.

It is the Carrying Capacity - the upper limit that we've been talking about.

That's right, the function will do it's best

to estimate where that upper limit is.

In our data's case, that value is 4,681.98 cases.

Now to get to that limit, the curve of our model needs to "turn over"

- and we call the point at which it turns over

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the "inflection point" -

defined as the Carrying Capacity divided by 2.

Now, we can use our log rules to find out where on our input variable

that inflection point happens.

Or we can simply use the formula of log(a) / log(b).

Speaking of "a" and "b" - although they look familiar,

they are in fact very different from what we're used to with the exponential

function.The "a" parameter is a "helper."

It's used to help define where the f(t) is when our input variable is at zero.

Solving for an input value of zero, we see that the logistic function reduces

to C / (1+a).

The "b" parameter is the same.

It looks like "b" from exponential, but it's not.

In the case of logistic growth, b must be greater than 1.

So, why is the logistic growth model good to know?

Because unlike linear or exponential functions,

the logistic growth function takes into

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account

the natural "upper limit" we see in most instances of biological processes.

But it comes at a price: Better fitting to more complex data

means a more complex function, with parameters

that are unfortunately less intuitive to interpret.

Comprehension Check

1. The spread of this season's flu virus can be modeled logistically. A group of 500 people were initially infected in a town of 75,000 people. One month later, 750 people were infected.

Use this data to construct a logistic growth model (shown below) for the spread of the flu in this town and answer the following questions.

$$f(t) = \frac{C}{1 + ab^{-t}}$$

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(1/1 point)

1a. What is the value of **C**?

- 500
- 750
- 15,800
- 75,000 ✔

Check | Show Answer

(1/1 point)

1b. What is the value of **a**, if you know that **f(0) = 500**? *(Round to zero decimal places.)*

149

149

Check Show Answer

Help

(1/1 point)

1c. What is the value of **b**, if you know that **f(1) = 750**. *(Round to 3 decimal places.)*

1.505

1.505

Answer: 1.505

Check Hide Answer

(1/1 point)

1d. The point that the model begins to show signs of slowing down (or “flipping over”) is called the

- ( ) "a" parameter
- ( ) log of "b"
- ( ) Carrying Capacity
- (•) Inflection Point ✔

Check Show Answer

(1/1 point)

1e. After how many months will the spread of the virus begin to slow down? *(Round to 2 decimal places.)*

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12.24

12.24

Answer: 12.24

Check Hide Answer

(1/1 point)

1f. How many people will have been infected with the flu when growth begins to slow down?

- ( ) 18,900
- ( ) 22,800
- (•) 37,500 ✔
- ( ) 41,650

Check Show Answer

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