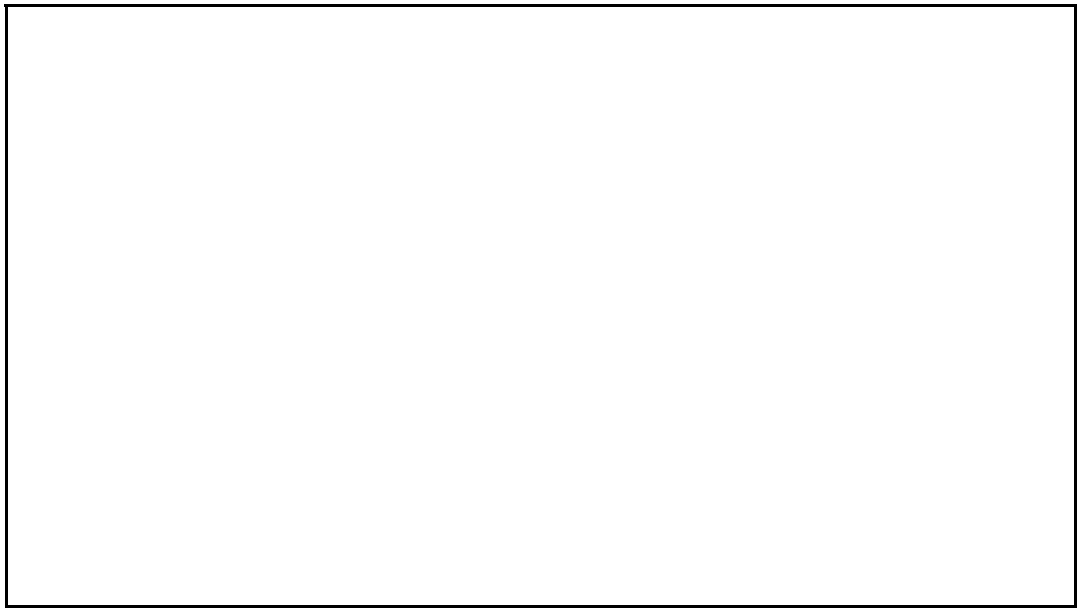
SINGLE SAMPLE T-TEST



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The easiest transition into the use of the t-distribution

is to look at it with an example.

Now we already know the idea of the z-test,

and the use of the normal distribution.

Remember, use of that test requires knowledge

of the Standard Deviation of the population or sigma.

Let's take a look at how to solve a similar

"one sample"

question, but this time without the knowledge of sigma

- and using the correct distribution of t.

In keeping with our knowledge of Gosset and the t-distribution,

let's look at an example that may have actually

been relevant at the time period.

According current standards of bitterness in beer, the International

Bitterness Unit (or IBU), Guinness has an IBU of 40.

The type of barley - the roasting of said barley

- can have an effect on this number.

So let's see what happens when we use a new type of barley for some small batch

brewing.

Now, to put this into perspective, beer can take anywhere from 2 to 6 weeks

to ferment - then there's an aging process

that has to happen again once it's bottled.

Then testing can occur.

Hopefully you see why a labor intensive undertaking, such as brewing beer,

would lead to questions on the best use of small sample sizes.

Here's the data of IBU scores for 12 small batches of beer,

and we want to compare this sample mean to the comparison of an IBU of 40.

Now we know nothing about the population - we don't know sigma.

Help

So, we'll use what's called a single sample

t-test because we don't know sigma of the population,

and we're only looking here at ONE sample

and comparing it to some reference value.

Now before we jump right in to the test, we've

got to talk about some assumptions first.

We need to have drawn a random sample from the population

- we can assume that.

Second: The items in the sample are independent from one another.

In our example, that means each brew of beer was done independently.

We can confirm this so our assumption holds.

And finally, third, the population needs to be approximately normal.

Now here's the nice thing about the t-test.

It's robust to deviations of the normality assumption.

After all, looking at a histogram of 12 items (although we can do it)

really can't inform us about the population normality of a measure.

But for most continuous measures, the population distribution

should be fairly normal.

Now that we have our sample data, and we've confirmed our test assumptions,

we can get down to the mechanics of the test.

This is a hypothesis test, so the first thing we'll do

is define the Null and Alternative Hypotheses.

Since our question of "Will a new crop of barley effect

the bitterness of Guinness beer?" isn't directional

we'll establish a simple difference or "not equal to" Alternative hypothesis.

Second, we'll establish our alpha level.

Now this is simple.

We just use the standard of 0.05.

And now, on to the analysis.

We'll use the same basic set up as the z-test, with some alterations.

Notice that the numerator of the equation

of the statistic is the same

- our sample mean minus some value of comparison or our Null Hypothesis

value.

But, our denominator is different.

In a z-test, we would be using the Standard

Error of the mean, denoted by "sigma x bar."

Which is actually calculated from the population Standard Deviation.

But with our current sample, we don't have the population Standard Deviation,

so we need to estimate it using our sample Standard Deviation.

Once we have that, we can also ESTIMATE the Standard Error of the mean.

This time denoted by SE x bar and called the "estimated Standard

Error of the mean."

Notice the distinction - one from sigma, one

estimated from the sample Standard Deviation.

Now, on to using the formula.

We have a sample mean of 37.67.

And, we know that the comparison or Null value is 40.

Our sample has a Standard Deviation of 2.02,

giving us an estimated Standard Error of the mean of 0.58.

Solving for t then, we get -4.011.

Now for the fourth step of the hypothesis test:

our conclusion based on our analysis.

Help

Is this t-statistic of -4.011 a large enough t value

to claim a difference between the sample value and our Null value?

Now, to answer that, we turn to our critical values.

But remember, we need degrees of freedom to use the t-distribution.

In the case of the single sample t-test, the degrees of freedom is equal to n-1,

so we have 11 degrees of freedom in our example.

For a t-distribution with 11 degrees of freedom,

the critical values to cut off the lower and upper 2.5% are -2.20 and 2.20.

Since our calculated t-statistic falls outside these two values

and into the critical region of the t-distribution for 11

degrees of freedom, we can Reject the Null Hypothesis.

The IBU value for this particular sample of

beer batches brewed

with this new barley is significantly different than the tested IBU of 40.

Now, if an IBU of 40 is what solely distinguished the taste of Guinness,

then it would be safe to say that this new barley would not be good to use.

Now, to expand a little.

What is the probability of getting a t-statisic of -4.011 or smaller

and, of couse, 4.011 or larger, since we have a non-directional Hypothesis Test?

Well, using the pt() in R, we find that the probabilities are only 0.001.

We add those two probabilities together, and we

get our non-directional p-value of 0.002.

Since this value is in fact less than our alpha level,

we can again Reject the Null Hypothesis.

Now this p-value is the same p-value that you would find if you ran this

test in R using the t.test().

In fact, this method (using the t.test()) is the easiest method to run

the single sample t-test.

It even gives us the 95% confidence intervals, which we can interpret:

We are 95% confident that the true population

value of IBU for this type brewing process with this type of barley

is between 36.39 and 38.95.

And this range does not capture our Null Hypothesis value -

exactly what we would expect since we Rejected our Null Hypothesis.

I think that calls for a pint of beer.

Help

Comprehension Check

1. A necessary condition for a one-sample t-test is that the sample must consist of "independent" observations. What does this mean?

(1 point possible)

- The data must have been collected by an objective, non-biased investigator.
- There must be no predictable relationship between one subject's score and any other subject's score. ✔
- The subjects in the study must be individuals that don't know each other.
- The sample cannot have come from the population that is under investigation.

Help

Hide Answer

2. Researchers are interested in whether or not the average person consumes 2,000 calories per day. Their random sample of 25 people consumed an average of 1,891 calories, with a standard deviation of 251 calories.

(1 point possible)

2a. What is the t-statistic? *(Report to 2 decimal places.)*

Answer: -2.17

Hide Answer

(1 point possible)

2b. What is the **absolute** critical t value, assuming $\alpha = 0.05$?

- 2.064 ✔
- 2.060
- 1.708
- 1.960

Hide Answer

(1 point possible)

2c. What should the researchers conclude?

- People consume 2,000 calories per day on average.
- People consume more than 2,000 calories per day on average.
- People do not consume an average of 2,000 calories per day. ✔

Hide Answer

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3. Scientists fear that polar bears are slowly starving due to their shrinking habitat. A healthy male polar bear weighs about 900 pounds. A new expedition was able to estimate the weight of 7 male polar bears. They found an average weight of 861 lbs with a standard deviation of 59 pounds.

Help

(1 point possible)

3a. What is the alternative hypothesis for this test?

- $Ha : \mu < 900$ ✔
- $Ha : \mu > 900$
- $Ha : \mu = 900$
- $Ha : \mu \neq 900$

Hide Answer

(1 point possible)

3b. How many degrees of freedom are there?

Answer: 6

Hide Answer

(1 point possible)

Help

3c. What is the value of the standard error? *(Report to 1 decimal place.)*

Answer: 22.3

Hide Answer

(1 point possible)

3d. What is the t-statistic? *(Report to 3 decimal places.)*

Answer: -1.749

Hide Answer

(1 point possible)

3e. What is the t-critical value, assuming $\alpha = 0.05$?

- -1.943 ✔
- -1.895
- -1.96
- 1.943

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(1 point possible)

3f. Do you reject the null hypothesis?

[ ] No

Hide Answer

(1 point possible)

3g. What should the researchers conclude?

- The average weight of polar bears has dropped significantly.
- There is no evidence that polar bears weigh less than 900 lbs on average. ✔
- We must conclude the polar bears are slowly starving because the sample mean was less than the hypothesized mean.

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