EXPONENTIAL GROWTH AND DECAY

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Modeling is great.

It first allows us to first makes sense of a relationship between two

numerical variables in a more hands on way,

and then, second, allows us to share what we've found in a succinct form.

We simply share our model parameters.

But the linear model is not in fact the "end

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all" of models.

Yes - it is in fact the most common, and, if you take a regression course,

it will be the primary focus.

But there are other models out there.

And sometimes those models fit our data better.

One such model is the exponential model.

I'll get this definition out of the way, so we

can move on to more interesting things.

Here's the big difference between the linear and the exponential model:

one is straight and the other is not.

Now that "not straight" has a very specific property

that makes it exponential.

In the linear model, for every one unit change in x, the change in y

(or our outcome variable) is constant.

It's the same here as it is here.

But for an exponential model, that idea of constant change is not true.

Or is it?

Here's some data based on solar power installation

from the past several years - 2007 to 2014.

And here's the linear model for the data.

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Remember, we want to make our intercept a bit more interpretable,

so we'll make a new x-axis variable called "years since 2007."

We find that we have a linear model with the following parameters.

Let's start predicting based on our model.

At year one we have an outcome of 5.48.

At year two, we have 11.99.

Now, if we finish out until year 7, we have the following outcome values.

We can confirm that for every one unit change in x,

the model change in y is constant - in this case it 6.50.

Now let's take a look at the same data, but now fitting an exponential model.

The first thing we notice is that the model is indeed not straight.

We also notice that the R-sqaured value for the exponential model

is slightly better than that of the linear,

informing us that the exponential model may

be a better choice for this data than the linear model.

Let's quickly interpret the two parameters of the exponential model.

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Our "a" parameter is the value predicted for the outcome at year zero

- much like our intercept to our linear model.

If we predict the outcome at year zero, we'll

get 4.168, the value of parameter "a."

Now the exponential model suffers from the same problem as the linear:

the value of zero on the x-axis needs to be meaningful - no extrapolation here.

Now what about the "b" parameter?

Well let's see what happens when we use it for prediction.

At year one, we get a predicted outcome of 6.077.

At year two, we get 8.859.

And here are again the rest of the predicted values,

up through 7 years since 2007.

Now we can see right away that as we change one value on the x-variable,

the change in the outcome is NOT a constant change.

But let's come back to that nagging question - "Or is it?"

Let's take a look at the change that does happen.

From year one to year two, we see a change

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from 6.077 to 8.859.

Now that's an actual change of 2.782.

But it's a precent change of 45.8% And what

about the change from 8.859 to 12.917?

That's an actual change of 4.058.

But a percent change of - you guessed it - 45.8%.

So does an exponential model have a constant rate of change.

No, not like linear.

An exponential model has a constant PERCENT rate of change.

Let's get back to the interpretation of our "b" parameter.

we see that it's 1.458.

Do you see it?

The "b" parameter holds within it the percent rate of change.

Formally, "b" is expressed as 1+r, with r being the proportional change

of the outcome variable for every whole unit increase on the input variable.

In our model's case that proportional change in the outcome variable

is 0.458, making "b" 1.458.

Now what about data that has a decrease across time?

Here's some data from a man-made

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radioactive compound, it's Geiger

count, and time in seconds.

Now like all radioactive material, this compound

shows a decay over the time period.

Let's run this though R and see what our parameters are.

First, since Time is seconds and a zero value here makes sense,

we'll leave the input values just as they are.

When we run the exponential model, we see parameter estimate of 94.95 for "a"

and 0.937 for "b."

Let's go ahead and interpret: At second zero,

we have an estimated Geiger count of 94.95.

As we move one whole second through time,

the Count outcome will decrease by 6.3%.

How did we get 6.3%?

Remember the definition of the growth factor "b":

1+r (the growth rate as indicated by a proportion of change).

If we solve for "r" the growth rate, we find

that we end up with a proportion of change of -0.063.

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Let's see what the predicted values are to get

a better idea of what's going on.

At 1 second, the predicted Geiger count is 88.964.

At 2 seconds, the predicted Geiger count is 83.353.

The proportional change in the Geiger count outcome is -0.063.

We decrease our Geiger count by 6.3%.

And that's how the exponential models differ from the linear models.

An "a" parameter instead of an intercept,

but the interpretation is pretty much the same.

And a "b" parameter of a growth factor which

has the growth rate or "r" as part of its make up

- and we can sort of think of this as a "slope" to the model.

When "b" is above 1, we have growth, and when "b" is less than 1, we have decay.

And the biggest, clearest, difference between exponential and linear models:

a constant rate of change for the linear,

but a constant "percent" rate of change for the exponential model.

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Comprehension Check

1. What is the growth factor in each of the following scenarios? *(Assume time is measured in the units given.)*

(1/1 point)

1a. Water usage is increasing by 3% per year.

1.03

1.03

Answer: 1.03

Check Hide Answer

(1/1 point)

1b. A city grows by 28% per decade.

1.28

1.28

Answer: 1.28

Check Hide Answer

(1/1 point)

1c. A diamond mine is depleted by 1% per day.

0.99

0.99

Answer: .99

Check Hide Answer

(1/1 point)

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1d. A forest shrinks 80% per century.

0.2

0.2

Answer: .2

Check Hide Answer

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2. The amount (in milligrams) of a drug in the body **t** hours after taking a pill is given by:

$$A(t) = 25(0.85)^t$$

(1/1 point)

2a. What is the initial dose given (in milligrams)?

25

25

Answer: 25

Check Hide Answer

(1/1 point)

2b. What percent of the drug leaves the body each hour? *(Report without the **%** sign.)*

15

15

Answer: 15

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(1/1 point)

2c. What is the amount of the drug left after 10 hours? *(Round to 2 decimal places.)*

4.92

4.92

Answer: 4.92

Check Hide Answer

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3. If the population grows by 10 people per year, what is the formula for the population, P, at time t?

(1/1 point)

- (•) $P(t) = 100 + 10t$ ✔
- ( ) $P(t) = 100t^{10}$
- ( ) $P(t) = 100(1.10)^t$
- ( ) $P(t) = 100 + 1.10t$

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Check Hide Answer

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4. If the population grows by 10% each year, what is the formula for the population, P, at time t?

(1/1 point)

- ( ) $P(t) = 100 + 10t$
- ( ) $P(t) = 100t^{10}$
- (•) $P(t) = 100(1.10)^t$ ✔
- ( ) $P(t) = 100 + 1.10t$

Check Hide Answer

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5. Which scenario will result in a larger population in 10 years:

   a. 10% growth per year or

b. an increase of 10 people per year?

(1/1 point)

- 10% growth ✔
- 10 people per year
- 10% growth will outpace the 10 people per year, but then 10 people per year will grow faster.
- They will be the same.

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