# INTERPRETING THE LINEAR MODEL

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Now that we know what a model is, and what a linear model is,

and how a linear model comes about, we need to take a breather

- we need to take a step back.

Let's look at the linear model more closely

and see what each of these parameters are actually tell us.

Because, if we're not careful, it's good

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fitting model

may give us some very bad results.

We recently found a model that fits the relationship

between socioeconomic status and bicycle helmet wearing

behavior in elementary age children.

We were able to discover that the "best line"

to predict the percent of children wearing bicycle

helmets from the percent of children receiving

a free lunch was this linear function.

So let's talk about interpreting this function.

First, we'll start with the slope, this value right here.

We see that it's actually negative: a -0.51.

In fact, we can write the linear function as either this or this,

with the negative represented as part of the equation.

But what does this value actually mean?

Well, let's put our function to use and take

a look at some results of solving for a specific value of x.

What would the predicted amount of percent of helmet usage

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be when the percent of free lunches is 20 percent?

Solving for our function using the 20 value,

we find a predicted value of 36.68.

Ok, now, let's go ahead and solve for a percent of free lunches of 21%,

just one percent point more - we get a predicted value of 36.17.

And the difference from the first predicted value to the second

is a reduction of 0.51 - the same as our slope.

A handy little formula to keep in the back of our minds

is the rate of change formula: the difference in the function of b

and the function of a, over the difference of b and a.

As long as the denominator is 1 - as long

as we move ONE WHOLE UNIT on our input variable

- then the numerator will be our slope of our function - our rate of change.

And this is true regardless of where we are on the x-axis -

regardless of what two values of x we use.

We can find the difference between 30 and 31,

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and it will be the same as the difference between 31 and 32

or 32 and 33.

In fact, this is a stable feature of the linear model:

a constant rate of change over equally spaced distances of the input variable.

Now, let's tackle the intercept value and how to interpret it.

Go ahead an plug zero in for X in the linear model.

We're left with an outcome or predicted value of 46.91 - our intercept value.

So, at a percent of free lunch value of ZERO,

we'll have a predicted percent of bicycle helmet usage of 46.91%.

Sounds pretty good right?

But there might be a problem.

Where's the school or schools that have percents of free lunches close to zero?

Right here.

Everything looks OK, but if we examine a little closer,

we see that there are in fact no schools with a percent of free lunch

that is actually ZERO - we get close: the minimum value for the range of x

is 2 percent.

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So unfortunately, nothing really seems off here.

Let's examine this idea of an intercept a little more.

Here's some data based on

Yellowstone's re-introduction of the grey wolf, starting in 1995.

We can see that as time goes by, the number of wolves increases.

Let's fit our linear model to this data.

We see our results of running the linear model: An R-squared of 0.840,

an Intercept value of -26,982, and a slope of 13.54.

Now on to the interpretation: Using a linear model,

we can account for 84% of the growth in grey wolves by knowing years -

that's pretty good.

We can see that for every year that the wolves have been back in Yellowstone,

the predicted numbers of grey wolves increases by 13.54 individuals.

And we see that at an x-value - or a year value of ZERO -

we would have -26,982 wolves in Yellowstone.

Wait, what?

You heard that right.

At an x-value or year value of ZERO - there will be -26,982 wolves

in Yellowstone.

Now that's just silly.

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That doesn't even make sense.

A negative wolf population?

So what's going on here?

Well, were is a Year value of ZERO on the graph?

Well it's all the way down here to the right.

Remember, our data spans the years of 1995 to 2007.

Those values are up here.

Our value of year zero is WAY outside our range of the x-axis -

our independent variable.

Being outside of the range of our x values is called extrapolation,

and in general we really want to avoid it.

Why?

Because as we can see, predicting a value outside our range

can give us some really weird results.

- Results that are for the most part not very reliable.

And, in some cases like our current one,

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results

that are really not interpretable either.

So how do we fix this?

Well, the easiest way, is to make the x-axis -

the values of our independent variable - more usable to us.

So instead of using just years, let's start our years, for our example,

at the time that the wolves were reintroduced back into Yellowstone -

at 1995.

So we'll make a new x variable, that will effectively make 1995 OUR year

zero - making the x-axis the years since reintroduction, or years since 1995.

Now, if we re-run the linear model with the new independent variable,

we get the following: Notice that our R-squared value and our Slope value

have not changed.

We haven't altered the relationship between years and number of grey wolves

at all.

We've just made a zero value of our x-axis variable - and our intercept -

more meaningful.

Our new intercept is 27.23.

What would the predicted numbers of grey wolves be at 1995 -

or our current linear model, year 0?

If we solve for our linear model, we get a predicted number of grey wolves

of 27.23 - the same as our intercept.

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So, what do the values of our linear model actually mean?

Our slope is the rate of change in our predicted outcome variable

we expect to see with one whole unit increase in our x-axis variable.

Our intercept is the predicted value of our outcome variable

when our x-axis variable is ZERO.

And it's up to us to make sure that that value of zero makes sense.

# Comprehension Check

1. Here’s our millionaire data again:

Do states with higher populations have more millionaires? Here is data from 2008. The variable,

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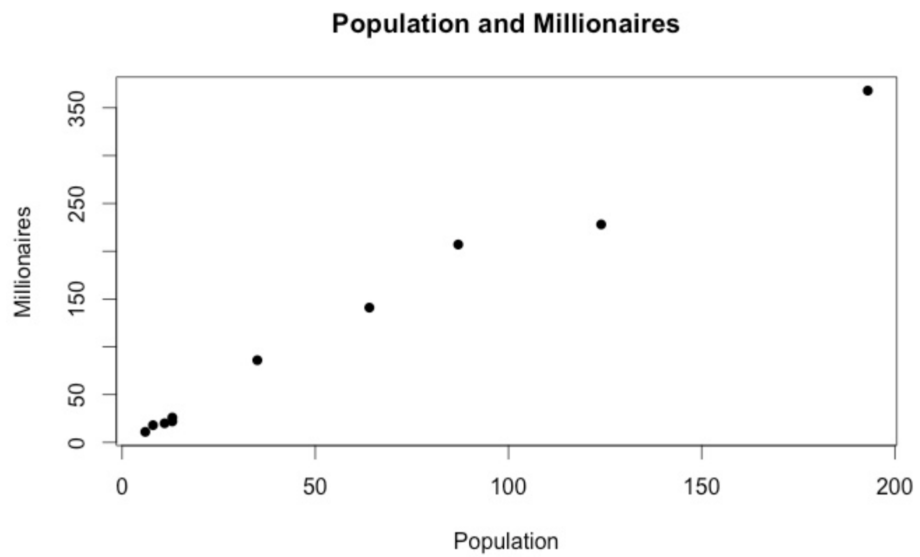
"Population," in the table and scatterplot will be referred to as State.Population in the questions that follow so as to avoid confusion with the meaning of "population" as a statistical concept.

| State | Millionaires *(in thousands)* | Population *(in hundreds of thousands)* |
|---|---|---|
| Connecticut | 86 | 35 |
| Delaware | 18 | 8 |
| Maine | 22 | 13 |
| Massachusetts | 141 | 64 |
| New Hampshire | 26 | 13 |
| New jersey | 207 | 87 |
| New York | 368 | 193 |
| Pennsylvania | 228 | 124 |
| Rhode Island | 20 | 11 |
| Vermont | 11 | 6 |

Using linFit(), the following linear model is found:

Millionaires = 6.296 + (1.921 * State.Population)

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(2/2 points)

1a. What is the interpretation of ŷ for this model, if y represents the variable, Millionaires?

- ( ) It is the amount of variability in the number of millionaires that can be explained by state population.
- ( ) It is the average number of millionaires for the entire sample.
- (•) It is predicted number of millionaires, based on a population in a state. ✔
- ( ) It is the slope of the line.

1b. This linear model crosses the y-axis at **6.296**. What is the interpretation of this point?

- (selected) A state with a population of 0 is expected to have 6,296 millionaires. ✔
- The average income of a millionaire in the US is $6.2 million.
- A state with 6.2 thousand people will have one millionaire.
- States earn $6.2 more for every millionaire that resides in them. A state with 6.2 thousand people will have one millionaire.

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1c. You create a new variable, subtracting the lowest Population value in the sample from each Population value: **new_pop <- State.Population – min(State.Population).**

This gives a new result from linFit():

**Millionaires=17.82 + (1.921 * State.Population)**

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(2/2 points)

1d. What is the interpretation of **17.82** in this model?

- (•) On average, a state with a population equal to the lowest population has 17,820 millionaires. ✓
- ( ) The average income of a millionaire in the US is $17.82 million.
- ( ) The intercept is not meaningful.
- ( ) A state with 17.82 thousand people will have 1.9 millionaires.

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1e. Interpret **1.921** in the above model (with an intercept of 17.82).

- ( ) As the population of a state increases by one whole person, it will gain 1.921 millionaires.
- (•) As the population of a state increases by 100,000, they will gain 1,921 millionaires. ✓
- ( ) States can earn $1.921 for every millionaire that resides within them.
- ( ) A state with zero millionaires will have a population of 1,921,000.

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