Grade 10 Mathematics Notes: Equations and Inequalities

Introduction

This guide helps you solve equations and inequalities for Grade 10 Math in a simple way. We use an "A-B-C" method to make it easy. It covers linear equations, quadratic equations, simultaneous equations, exponential equations, and inequalities. There's also a trick (the hack) to help with all questions!

General Strategy (The Hack)

1. **Identify the Type**: Know if it's an equation or inequality and what kind (linear, quadratic, etc.).

2. **Isolate the Variable**: Get the variable (like x) alone on one side.

3. **Simplify Step-by-Step**: Do calculations slowly and carefully.

4. **Check Sign Changes**: Flip the inequality sign if you multiply or divide by a negative number.

5. **Verify Solutions**: Put your answer back to check if it works.

6. **Use Formulas**: Use tools like the quadratic formula when needed.

7. **Spot Patterns**: Look for easy ways to simplify, like factoring.

Steps for Equations and Inequalities (A-B-C Method)

Use these steps for all similar questions:

- **A - Understand**: Read the question and figure out what you need to find (e.g., x, a range, or a value). Write down the equation or inequality.

- **B - Calculate**: Solve step by step. For equations, get x alone. For inequalities, find the range and test values. Show all your work.

- **C - Check**: Put your answer back into the original problem to see if it fits. For inequalities, check boundary points.

This works for all types—linear, quadratic, or more!

1. Linear and Quadratic Equations

These are equations like $ax + b = c$ or $ax^2 + bx + c = 0$.

Example 1: Solve $(3x - 1)(x + 2) = 0$ (From Sep 2024, Q1.1.1 adapted)

- **A - Understand**: We need x where the product is zero. It's a quadratic in factored form.

- **B - Calculate**:

  1. Set each part to zero: $3x - 1 = 0$ or $x + 2 = 0$.

  2. Solve: $3x = 1 \implies x = \frac{1}{3}$, and $x = -2$.

- **C - Check**: For $x = \frac{1}{3}$: $(3 \cdot \frac{1}{3} - 1)(\frac{1}{3} + 2) = 0 \cdot \frac{7}{3} = 0$. For $x = -2$: $(3(-2) - 1)(-2 + 2) = -7 \cdot 0 = 0$. Both work!

- **Answer**: $x = \frac{1}{3}$ or $x = -2$.

2. Simultaneous Equations

Solve two equations with two variables.

Example 2: Solve $x - y = 4$ and $\frac{x}{5} + \frac{y}{2} = 5$ (From Sep 2024, Q1.2.1 adapted)

- **A - Understand**: Find x and y that fit both equations.

- **B - Calculate**:

  1. From $x - y = 4$, get $x = y + 4$.

  2. Put into $\frac{x}{5} + \frac{y}{2} = 5$: $\frac{y+4}{5} + \frac{y}{2} = 5$.

  3. Multiply by 10: $2(y + 4) + 5y = 50 \implies 2y + 8 + 5y = 50 \implies 7y + 8 = 50 \implies 7y = 42 \implies y = 6$.

  4. Then $x = 6 + 4 = 10$.

- **C - Check**: $10 - 6 = 4$, and $\frac{10}{5} + \frac{6}{2} = 2 + 3 = 5$. Both are correct!

- **Answer**: $x = 10$, $y = 6$.

3. Inequalities

Inequalities show ranges like $x < 5$ or $x \geq 2$.

Example 3: Solve $-1 \leq 1 - \frac{2x}{3} < 5$ (From Sep 2024, Q1.1.4 adapted)

- **A - Understand**: Find x where the expression is between -1 and 5.

- **B - Calculate**:

    1. Split: $-1 \leq 1 - \frac{2x}{3}$ and $1 - \frac{2x}{3} < 5$.

    2. First part: $-1 \leq 1 - \frac{2x}{3} \implies -2 \leq -\frac{2x}{3} \implies -6 \geq -2x \implies 3 \leq x$.

    3. Second part: $1 - \frac{2x}{3} < 5 \implies -\frac{2x}{3} < 4 \implies -2x < 12 \implies x > -6$.

    4. Combine: $-6 < x \leq 3$.

- **C - Check**: For $x = 0$: $1 - \frac{2 \cdot 0}{3} = 1$, and $-1 \leq 1 < 5$. Works!

- **Answer**: $-6 < x \leq 3$.

4. Exponential Equations

These have x in the exponent, like $2^x = 8$.

Example 4: Solve $3^x - 2 \cdot 3^{x-1} = 3$ (From Sep 2024, Q2.1 adapted)

- **A - Understand**: Find x that makes the equation true. Notice $3^{x-1} = \frac{3^x}{3}$.

- **B - Calculate**:

    1. Rewrite: $3^x - 2 \cdot \frac{3^x}{3} = 3$.

    2. Factor: $3^x \left(1 - \frac{2}{3}\right) = 3 \implies 3^x \cdot \frac{1}{3} = 3 \implies 3^x = 9$.

    3. Since $9 = 3^2$, $3^x = 3^2 \implies x = 2$.

- **C - Check**: $3^2 - 2 \cdot 3^{2-1} = 9 - 2 \cdot 3 = 9 - 6 = 3$. Correct!

- **Answer**: $x = 2$.

5. Questions from September 2024 Paper

Question 1.1.1: Mean of Marks (17, 18, 19, 21, 24, 26, 28, 31, 35, 39, 40, 42, 42, 45, 51, 55, 70, 85, 95)

- **A - Understand**: Find the average (mean) of the 19 marks.

- **B - Calculate**: Add all numbers: $17 + 18 + 19 + 21 + 24 + 26 + 28 + 31 + 35 + 39 + 40 + 42 + 42 + 45 + 51 + 55 + 70 + 85 + 95 = 783$. Divide by 19: $783 \div 19 = 41.21$.

- **C - Check**: Recount a few numbers (e.g., $17 + 95 = 112$, etc.) to confirm total. Looks good!

- **Answer**: 41.21.

Question 2.1: Asymptotes of $f(x) = \frac{2}{x} + 2$

- **A - Understand**: Find where the graph has vertical and horizontal lines it can't cross.

- **B - Calculate**: Vertical asymptote where denominator is zero: $x = 0$. Horizontal asymptote as x gets big: $\frac{2}{x} \to 0$, so $y = 2$.

- **C - Check**: Test $x = 1$: $f(1) = \frac{2}{1} + 2 = 4$, not near $y = 2$. As x grows, it flattens to $y = 2$. Correct!

- **Answer**: Vertical $x = 0$, Horizontal $y = 2$.

Question 3.3: x-intercepts of $f(x) = x^2 - 4$

- **A - Understand**: Find x where $f(x) = 0$ (where the graph crosses the x-axis).

- **B - Calculate**: $x^2 - 4 = 0 \implies x^2 = 4 \implies x = 2$ or $x = -2$.

- **C - Check**: $f(2) = 2^2 - 4 = 0$, $f(-2) = (-2)^2 - 4 = 0$. Both work!

- **Answer**: $A(2, 0)$, $B(-2, 0)$.

Question 4.1: Largest θ for Goal (11m, 4m height)

- **A - Understand**: Find the biggest angle θ so the ball (4m high) clears 11m away.

- **B - Calculate**: $\tan \theta = \frac{4}{11} \implies \theta = \tan^{-1}\left(\frac{4}{11}\right) \approx 19.98°$.

- **C - Check**: $\sin 19.98° \approx \frac{4}{H}$, $H \approx 11.7m$, which fits. Correct!

- **Answer**: $19.98°$.

Practice Tips

- Practice each step with different numbers.

- Check your work every time.

- Use a calculator for big calculations.