Trigonometry Notes for Grade 10: Easy Step-by-Step Guide with New Questions

Grok 3

July 2025

Introduction

Trigonometry is a fun way to solve triangle puzzles, especially those with a right angle (90 degrees). It helps us find missing sides or angles using ratios like sine, cosine, and tangent. This guide uses simple words to help kids understand step-by-step, with examples from your exam papers and new questions from the images. Let's make math easy and exciting!

1 Basic Trigonometric Ratios

In a right-angled triangle with an angle θ:

- **Opposite**: The side across from θ.
- **Adjacent**: The side next to θ (not the longest side).
- **Hypotenuse**: The longest side, opposite the 90-degree angle.

The ratios are:

- $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
- $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
- $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

Extra ratios:

- Cosecant: $\csc\theta = \frac{1}{\sin\theta}$
- Secant: $\sec\theta = \frac{1}{\cos\theta}$
- Cotangent: $\cot\theta = \frac{1}{\tan\theta}$

Step-by-Step Trick: Use **SOH-CAH-TOA**:

- **SOH**: Sine = Opposite / Hypotenuse
- **CAH**: Cosine = Adjacent / Hypotenuse

- **TOA**: Tangent = Opposite / Adjacent

Draw this triangle:

Angle	sin	cos	tan
$0°$	0	1	0
$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45°$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90°$	1	0	undefined

Step-by-Step Trick:

1. Practice the table every day.
2. Use a 30-60-90 triangle (sides $1 : \sqrt{3} : 2$) or 45-45-90 triangle (sides $1 : 1 : \sqrt{2}$) to recall.

3 Trigonometric Identities

Useful rules:

- **Pythagorean Identities**:
  - $\sin^2\theta + \cos^2\theta = 1$
  - $1 + \tan^2\theta = \sec^2\theta$
  - $1 + \cot^2\theta = \csc^2\theta$
- **Double Angle Formulas**:
  - $\sin 2\theta = 2\sin\theta\cos\theta$
  - $\cos 2\theta = \cos^2\theta - \sin^2\theta$ (or) $2\cos^2\theta - 1$
- **Angle Sum**:
  - $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
  - $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Step-by-Step Trick:

1. Start with $\sin^2\theta + \cos^2\theta = 1$.
2. Practice rewriting expressions with these rules.

4 Trigonometric Graphs

Graphs look like waves:

- **Sine/Cosine**: $y = a\sin(bx)$ or $y = a\cos(bx) + c$
  - **Amplitude**: $|a|$ (wave height).
  - **Period**: $\frac{360°}{b}$ (cycle length).
  - **Vertical Shift**: c (up/down move).
- **Range**: For $y = a\cos(bx) + c$, range is $[c - |a|, c + |a|]$.

Step-by-Step Trick:

1. Find amplitude, period, and shift.
2. Draw the midline ($y = c$), then max ($c + |a|$) and min ($c - |a|$).
3. Sketch the wave.

5 Solving Trigonometric Equations

Find θ in equations like $\sin\theta = 0.5$:

1. Find reference angle: $\sin^{-1}(0.5) = 30°$.
2. Use CAST diagram:
   - **C**: Cosine positive (0°-90°).
   - **A**: All positive (0°-90°).
   - **S**: Sine positive (90°-180°).
   - **T**: Tangent positive (180°-270°).
3. Find all angles in 0° to 360°.

6 Examples from Exam Papers and New Questions

6.1 November 2022 Paper 2 - Question 6

Graph: $f(x) = -2\cos x$ for $0° \leq x \leq 360°$.

- **6.1** Amplitude of f:
  1. Amplitude is $|a|$ in $y = a\cos x$. Here, $a = -2$, so amplitude $= |-2| = 2$.
- **6.2** Minimum value of $f(x) + 3$:
  1. Minimum of $f(x) = -2\cos x$ is -2 (when $\cos x = 1$).
  2. Add 3: $-2 + 3 = 1$.
- **6.3** Draw $g(x) = \sin x + 1$:
  1. Amplitude of $\sin x$ is 1, vertical shift is $+1$.
  2. Range: $[0, 2]$. Draw a sine wave starting at 1 (when $x = 0°$), peaking at 2, and dipping to 0.
- **6.4.1** Value of $f(180°) - g(180°)$:
  1. $f(180°) = -2\cos 180° = -2 \cdot (-1) = 2$.
  2. $g(180°) = \sin 180° + 1 = 0 + 1 = 1$.
  3. Subtract: $2 - 1 = 1$.
- **6.4.2** Values of x where $f(x) \cdot g(x) > 0$:
  1. $f(x) \cdot g(x) > 0$ when both are positive or both negative.
  2. $f(x) = -2\cos x$ is positive when $\cos x < 0$ ($90°$ to $270°$).
  3. $g(x) = \sin x + 1$ is positive when $\sin x > -1$ (always, since $\sin x \geq -1$ and $+1$ shifts it up).
  4. Both positive/negative: Check where $f(x)$ and $g(x)$ match signs. $g(x) > 0$ always, so focus on $f(x)$ positive ($90°$ to $270°$).
- **6.5** Graph of h (reflect about x-axis, move 3 units down):
  1. Reflect $f(x) = -2\cos x$ about x-axis: Becomes $2\cos x$ (removes negative).
  2. Move down 3 units: $h(x) = 2\cos x - 3$.
- **6.5.1** Equation of h:
  1. $h(x) = 2\cos x - 3$.
- **6.5.2** Range of h for $0° \leq x \leq 360°$:
  1. $\cos x$ ranges from -1 to 1.
  2. $2\cos x$ ranges from -2 to 2.
  3. $2\cos x - 3$ ranges from $-2 - 3 = -5$ to $2 - 3 = -1$.
  4. Range: $[-5, -1]$.

6.2 New Questions - Question 4

Diagram: Triangle with $\sin\theta$, $\cos\theta$, opposite $= 2$, adjacent $= \sqrt{3}$.

- **4.1.1** $\sin\theta$:
  1. $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$.
  2. Hypotenuse $= \sqrt{2^2 + (\sqrt{3})^2} = \sqrt{4+3} = \sqrt{7}$.
  3. $\sin\theta = \frac{2}{\sqrt{7}}$.
- **4.1.2** $\sin\theta\cos\theta$:
  1. $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{3}}{\sqrt{7}}$.
  2. $\sin\theta\cos\theta = \frac{2}{\sqrt{7}} \cdot \frac{\sqrt{3}}{\sqrt{7}} = \frac{2\sqrt{3}}{7}$.
- **4.2** If $x = 60°$ and $y = 45°$:
  1. $\frac{1}{2}\sin 2y - 12\tan^2\left(\frac{y}{2}\right)\cos x$:
  2. $\sin 2y = \sin 90° = 1$, so $\frac{1}{2}\sin 2y = \frac{1}{2} \cdot 1 = 0.5$.
  3. $\tan\left(\frac{45°}{2}\right) = \tan 22.5°$ (approximate or use half-angle, but let's simplify): $\tan 22.5° \approx 0.414$, $\tan^2 22.5° \approx 0.171$.
  4. $12\tan^2\left(\frac{y}{2}\right) \approx 12 \cdot 0.171 = 2.052$.
  5. $\cos x = \cos 60° = 0.5$.
  6. $12\tan^2\left(\frac{y}{2}\right)\cos x \approx 2.052 \cdot 0.5 = 1.026$.
  7. Total: $0.5 - 1.026 = -0.526$.
- **4.3** Acute angle β to 2 decimals:
  - **4.3.1** $\sin(\beta - 17.8°) = 0.215$:
    1. $\sin^{-1}(0.215) \approx 12.4°$.
    2. $\beta - 17.8° = 12.4°$, so $\beta = 12.4° + 17.8° = 30.2°$.
  - **4.3.2** $\tan 3\beta = \sqrt{3}$:
    1. $\tan^{-1}(\sqrt{3}) = 60°$, so $3\beta = 60°$.
    2. $\beta = 60°/3 = 20°$.
  - **4.3.3** $3\sin\frac{\beta}{2} = 2.012$:
    1. $\sin\frac{\beta}{2} = \frac{2.012}{3} \approx 0.6707$.
    2. $\frac{\beta}{2} = \sin^{-1}(0.6707) \approx 42.1°$.
    3. $\beta = 2 \cdot 42.1° \approx 84.2°$.
- **4.4** $\frac{\tan 30° \csc 60°}{\cot 45° \sin 45°}$:
  1. $\tan 30° = \frac{1}{\sqrt{3}}$, $\csc 60° = \frac{1}{\sin 60°} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$.
  2. Numerator: $\frac{1}{\sqrt{3}} \cdot \frac{2}{\sqrt{3}} = \frac{2}{3}$.
  3. $\cot 45° = 1$, $\sin 45° = \frac{\sqrt{2}}{2}$.

4. Denominator: $1 \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$.
5. Total: $\frac{\frac{2}{3}}{\frac{\sqrt{2}}{2}} = \frac{2}{3} \cdot \frac{2}{\sqrt{2}} = \frac{4}{3\sqrt{2}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$.

6.3 New Questions - Question 5

Diagram: $P(x, y)$ in 3rd quadrant, $\angle ROP = \beta$, $17\cos\beta + 15 = 0$.

- **5.1.1** Values of x, y, r:
  1. $17\cos\beta + 15 = 0 \implies \cos\beta = -\frac{15}{17}$.
  2. In 3rd quadrant, $\sin\beta$ is negative, $\cos\beta$ is negative.
  3. $\sin^2\beta + \cos^2\beta = 1$.
  4. $\sin^2\beta = 1 - \left(-\frac{15}{17}\right)^2 = 1 - \frac{225}{289} = \frac{64}{289}$.
  5. $\sin\beta = -\frac{\sqrt{64}}{17} = -\frac{8}{17}$ (negative in 3rd quadrant).
  6. $x = r\cos\beta$, $y = r\sin\beta$, $r = \sqrt{x^2 + y^2}$.
  7. Assume $r = 17$ (common radius), then $x = 17 \cdot -\frac{15}{17} = -15$, $y = 17 \cdot -\frac{8}{17} = -8$.
- **5.1.2** Without calculator:
  1. (a) $\sin\beta = -\frac{8}{17}$.
  2. (b) $\cos^3 30^\circ \tan\beta$:
     - $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\cos^3 30^\circ = \left(\frac{\sqrt{3}}{2}\right)^3 = \frac{3\sqrt{3}}{8}$.
     - $\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{-\frac{8}{17}}{-\frac{15}{17}} = \frac{8}{15}$.
     - Total: $\frac{3\sqrt{3}}{8} \cdot \frac{8}{15} = \frac{3\sqrt{3}}{15} = \frac{\sqrt{3}}{5}$.
- **5.1.3** Size of $\angle ROP$:
  1. $\cos\beta = -\frac{15}{17}$, $\beta = \cos^{-1}\left(-\frac{15}{17}\right)$.
  2. Approximate: $\cos^{-1}(0.8824) \approx 28^\circ$, but negative, so $\beta \approx 180^\circ - 28^\circ = 152^\circ$ (3rd quadrant).
  3. Adjust with calculator: $\beta \approx 151.8^\circ$ (to 2 decimals).
- **5.2** Solve for x where $0^\circ \leq x \leq 90^\circ$:
  1. **5.2.1** $\tan x = 2.22$:
     - $x = \tan^{-1}(2.22) \approx 65.7^\circ$.
  2. **5.2.2** $\sec(x + 10^\circ) = 5.759$:
     - $\sec\theta = \frac{1}{\cos\theta}$, so $\cos(x + 10^\circ) = \frac{1}{5.759} \approx 0.1736$.
     - $x + 10^\circ = \cos^{-1}(0.1736) \approx 80^\circ$.
     - $x \approx 80^\circ - 10^\circ = 70^\circ$.
  3. **5.2.3** $\frac{\sin x}{0.2} - 1 = 1.24$:

- $\frac{\sin x}{0.2} - 1 = 1.24 \implies \frac{\sin x}{0.2} = 2.24$.
- $\sin x = 2.24 \cdot 0.2 = 0.448$.
- $x = \sin^{-1}(0.448) \approx 26.6°$.

7 Tips to Make Trigonometry Easy

1. **Draw Pictures**: Sketch triangles or graphs.
2. **Use SOH-CAH-TOA**: Label sides to pick the right ratio.
3. **Memorize Angles**: Practice the special angle table.
4. **Use Identities**: Simplify with $\sin^2\theta + \cos^2\theta = 1$.
5. **Check Quadrants**: Use CAST for angles.
6. **Round Carefully**: Follow instructions (e.g., 2 decimals).