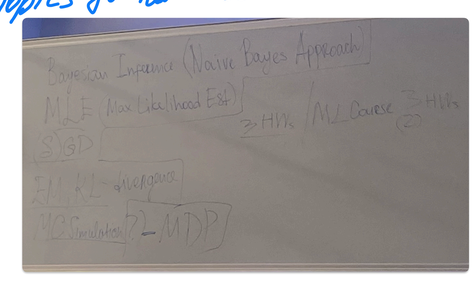
Week 3 Lecture

Topics to have before mid term



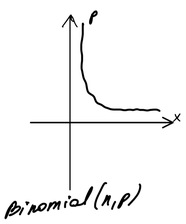
pmf of normal distribution

$X_i \to$ observations

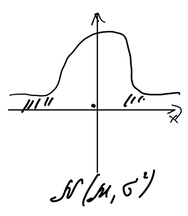
$\mathcal{N}(\mu, \sigma^2) \sim \underbrace{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}_{\text{family of normal distributions}}$

$X \sim \mathcal{N}(\mu, \sigma^2)$

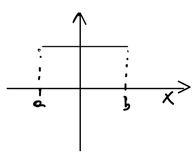
$p(X)$



Binomial(n,p)



$\mathcal{N}(\mu, \sigma^2)$



$x \in [a, b]$

$u(a, b)$

x - dataset

$\mu = E x_i$

$E(X_i - E x_i)^2 = MSE(X_i, E x_i)$

$L \to \max_\theta \quad \theta$

$-L \to \min_\theta$

Concept of statistic

Board Question: Coins

A coin is taken from a box containing three coins, which give heads with probability $p = 1/3, 1/2$, and $2/3$. The mystery coin is tossed 80 times, resulting in 49 heads and 31 tails.

(a) What is the likelihood of this data for each type on coin? Which coin gives the maximum likelihood?

$p^s = 49 \quad p = 31 \quad p_1 = 1/3$

$p_2 = 1/2$

$p_3 = 2/3$

$P_1 = \binom{80}{49} p^{49} \cdot (1-p)^{31} = \binom{80}{49}\left(\frac{1}{3}\right)^{49}\left(1-\frac{1}{3}\right)^{31} = 2^{31} \cdot \left(\frac{1}{3}\right)^{80}$

Week 3 Tutorial

Continuation of the problem

$P(X/p) = C_n^s p^s \cdot q^{n-s} = \binom{80}{49} p^{49} q^{31} = \frac{80!}{49!\,31!} \cdot p^{49}(1-p)^{31}$

$\log \frac{80!}{49!\,31!} + 49 \log p + 31 \log(1-p) = 0$

$\frac{d}{dp} 49 \log p + \frac{d}{dp} 31 \log(1-p) = 0$

$49\left(\frac{1}{p}\right) + 31\left(\frac{1}{1-p}\right) = 0$

$\frac{49}{p} - \frac{31}{1-p} = 0$

$\frac{49}{p} = \frac{31}{1-p}$

$31p = 49(1-p)$

$31p = 49 - 49p$

$80p = 49$

$p = \frac{49}{80}$

Types of distributions:

uniform $U[a, b]$

Normal $N(\mu, \sigma)$

Binomial – Binom(n, p)

Bernoulli: $\to$ Bern(p)

Exponential – Exp$(\lambda) \to \lambda e^{-\lambda x}$

Continuous likelihood

Use the pdf instead of the pmf

Example. Light bulbs

Lifetime of each bulb $\sim \exp(\lambda)$.

Test 5 bulbs and find lifetimes of $x_1, \ldots, x_5$.

(i) Find the likelihood and log likelihood functions.

(ii) Then find the maximum likelihood estimate (MLE) for λ.

answer: *See next slide.*

$P(x|\theta) = \lambda^5 e^{-\lambda(x_1+x_2+x_3+x_4+x_5)}$

$\log\left(\lambda^5 e^{-\lambda(x_1+x_2+x_3+x_4+x_5)}\right) =$

$= \log \lambda^5 - \lambda(x_1+x_2+x_3+x_4+x_5)$

$\lambda = \frac{5}{x_1+x_2+x_3+x_4+x_5}$

$X \sim \exp(\lambda)$ for N observations

$\lambda = \frac{5}{\sum_i^5 x_i}$

$\lambda_{MLE} = \frac{n}{\sum_{i=1}^n x_i}$

$\log L = \log P(x|\theta)$

Log-Likelihood $\to \max_\theta$

1. $\nabla L = ?$
2. $\nabla L \neq 0$
3. $\nabla L \doteq 0$

Problem Set 1

In my town, it's rainy one third of the days. Given that it is rainy, there will be heavy traffic with probability $\frac{1}{2}$, and given that it is not rainy, there will be heavy traffic with probability $\frac{1}{4}$. If it's rainy and there is heavy traffic, I arrive late for work with probability $\frac{1}{2}$. On the other hand, the probability of being late is reduced to $\frac{1}{8}$ if it is not rainy and there is no heavy traffic. In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25. You pick a random day.

a. What is the probability that it's not raining and there is heavy traffic and I am not late?
b. What is the probability that I am late?
c. Given that I arrived late at work, what is the probability that it rained that day?

a) $P(A^{-1} \cap B \cap C) = ?$

b) $P(C) = ?$

c) $P(A/C) = ?$

A = raining $\quad B$ = heavy traffic $\quad C$ = arrive late

$P(A) = \frac{1}{3} \quad P(B/A) = \frac{1}{2} \quad P(C/A \cap B) = 1/2$

$P(B/A^{-1}) = \frac{1}{4} \quad P(C/A^{-1} \cap B^{-1}) = 1/8$

$P(C/A \cap B^{-1}) = 0,25$

$P(C/A^{-1} \cap B) = 0,25$

a

$P(A^{-1} \cap B \cap C) = P(A^{-1} \cap B/C) \cdot P(C) =$

$= \frac{P(C/A^{-1} \cap B) \cdot P(A^{-1} \cap B)}{\cancel{P(C)}} \cdot \cancel{P(C)} =$

$= 0,25 \cdot \frac{2}{12} = \frac{1}{4} \cdot \frac{1}{6} = \frac{1}{24}$

$P(A^{-1} \cap B) = P(A^{-1}/B) \cdot P(B) = \frac{P(B/A^{-1}) \cdot P(A^{-1})}{\cancel{P(B)}} \cdot \cancel{P(B)} = \frac{1}{4} \cdot \frac{2}{3} = \frac{2}{12}$