BiGaBP の誤差逆伝搬のための理論式

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$$\frac{\partial E}{\partial \Re\left\{\hat{r}_{mk}\right\}} = -\frac{2}{MK_d}\left(\Re\left\{x_{mk}\right\} - \Re\left\{\hat{r}_{mk}\right\}\right)$$

$$\frac{\partial E}{\partial \Im\left\{\hat{r}_{mk}\right\}} = -\frac{2}{MK_d}\left(\Im\left\{x_{mk}\right\} - \Im\left\{\hat{r}_{mk}\right\}\right)$$

$$\frac{\partial E}{\partial \Re\left\{\tilde{y}_{m,nk}(T)\right\}} = \begin{cases} 0 & (1 \leq k \leq K_p) \\ \psi^r_{mk}\dfrac{\Re\left\{\hat{h}_{k,nm}(T)\right\}}{\nu^x_{m,nk}(T)}\dfrac{\partial E}{\partial \Re\left\{\hat{r}_{mk}\right\}} - \psi^r_{mk}\dfrac{\Im\left\{\hat{h}_{k,nm}(T)\right\}}{\nu^x_{m,nk}(T)}\dfrac{\partial E}{\partial \Im\left\{\hat{r}_{mk}\right\}} & (K_p + 1 \leq k \leq K) \end{cases}$$

$$\frac{\partial E}{\partial \Im\left\{\tilde{y}_{m,nk}(T)\right\}} = \begin{cases} 0 & (1 \leq k \leq K_p) \\ \psi^r_{mk}\dfrac{\Im\left\{\hat{h}_{k,nm}(T)\right\}}{\nu^x_{m,nk}(T)}\dfrac{\partial E}{\partial \Re\left\{\hat{r}_{mk}\right\}} + \psi^r_{mk}\dfrac{\Re\left\{\hat{h}_{k,nm}(T)\right\}}{\nu^x_{m,nk}(T)}\dfrac{\partial E}{\partial \Im\left\{\hat{r}_{mk}\right\}} & (K_p + 1 \leq k \leq K) \end{cases}$$

$$\frac{\partial E}{\partial \psi^r_{mk}} = \frac{\Re\left\{\hat{r}_{mk}\right\}}{\psi^r_{mk}}\frac{\partial E}{\partial \Re\left\{\hat{r}_{mk}\right\}} + \frac{\Im\left\{\hat{r}_{mk}\right\}}{\psi^r_{mk}}\frac{\partial E}{\partial \Im\left\{\hat{r}_{mk}\right\}}$$

$$\begin{aligned} \frac{\partial E}{\partial \nu^x_{m,nk}(T)} = &-\psi^r_{mk}\frac{\Re\left\{\hat{h}_{k,nm}(T)\right\}\Re\left\{\tilde{y}_{m,nk}(T)\right\} + \Im\left\{\hat{h}_{k,nm}(T)\right\}\Im\left\{\tilde{y}_{m,nk}(T)\right\}}{\left(\nu^x_{m,nk}(T)\right)^2}\frac{\partial E}{\partial \Re\left\{\hat{r}_{mk}\right\}} \\ &- \psi^r_{mk}\frac{\Re\left\{\hat{h}_{k,nm}(T)\right\}\Im\left\{\tilde{y}_{m,nk}(T)\right\} - \Im\left\{\hat{h}_{k,nm}(T)\right\}\Re\left\{\tilde{y}_{m,nk}(T)\right\}}{\left(\nu^x_{m,nk}(T)\right)^2}\frac{\partial E}{\partial \Im\left\{\hat{r}_{mk}\right\}} \\ &+ \frac{\left|\hat{h}_{k,nm}(T)\right|^2}{\left(\nu^x_{m,nk}(T)\right)^2}\left(\psi^r_{mk}\right)^2\frac{\partial E}{\partial \psi^r_{mk}} \end{aligned}$$

$$\frac{\partial E}{\partial \nu^y_{m,nk}(T)} = \begin{cases} 0 & (1 \leq k \leq K_p) \\ \dfrac{\partial E}{\partial \nu^x_{m,nk}(T)} & (K_p + 1 \leq k \leq K) \end{cases}$$

$$\frac{\partial E}{\partial \psi^x_{n,mk}(T)} = \left\{\left|\hat{h}_{k,nm}(T)\right|^2 + \psi^h_{k,nm}(T)\right\}\sum_{m' \neq m}\frac{\partial E}{\partial \nu^y_{m',nk}(T)}$$

$$\frac{\partial E}{\partial \psi^h_{k,nm}(T)} = \left\{\left|\hat{x}_{n,mk}(T)\right|^2 + \psi^x_{n,mk}(T)\right\}\sum_{m' \neq m}^{M}\frac{\partial E}{\partial \nu^y_{m',nk}(T)} + \begin{cases} 0 & (1 \leq k \leq K_p) \\ E_s\dfrac{\partial E}{\partial \nu^x_{m,nk}(T)} & (K_p + 1 \leq k \leq K) \end{cases}$$

$$\begin{aligned} \frac{\partial E}{\partial \Re\left\{\hat{x}_{n,mk}(T)\right\}} = &-\Re\left\{\hat{h}_{k,nm(T)}\right\}\sum_{m' \neq m}^{M}\frac{\partial E}{\partial \Re\left\{\tilde{y}_{m',nk}(T)\right\}} - \Im\left\{\hat{h}_{k,nm(T)}\right\}\sum_{m' \neq m}^{M}\frac{\partial E}{\partial \Im\left\{\tilde{y}_{m',nk}(T)\right\}} \\ &+ 2\Re\left\{\hat{x}_{n,mk}(T)\right\}\psi^h_{k,nm}(T)\sum_{m' \neq m}^{M}\frac{\partial E}{\partial \nu^y_{m',nk}(T)} \end{aligned}$$

$$\begin{aligned} \frac{\partial E}{\partial \Im\left\{\hat{x}_{n,mk}(T)\right\}} = &\quad \Im\left\{\hat{h}_{k,nm(T)}\right\}\sum_{m' \neq m}^{M}\frac{\partial E}{\partial \Re\left\{\tilde{y}_{m',nk}(T)\right\}} - \Re\left\{\hat{h}_{k,nm(T)}\right\}\sum_{m' \neq m}^{M}\frac{\partial E}{\partial \Im\left\{\tilde{y}_{m',nk}(T)\right\}} \\ &+ 2\Im\left\{\hat{x}_{n,mk}(T)\right\}\psi^h_{k,nm}(T)\sum_{m' \neq m}^{M}\frac{\partial E}{\partial \nu^y_{m',nk}(T)} \end{aligned}$$

$$
\begin{aligned}
\frac{\partial E}{\partial \Re\left\{\hat{h}_{k,nm}(T)\right\}} =& - \Re\left\{\hat{x}_{n,mk}(T)\right\} \sum_{m'\neq m}^{M} \frac{\partial E}{\partial \Re\left\{\tilde{y}_{m',nk}(T)\right\}} - \Im\left\{\hat{x}_{n,mk}(T)\right\} \sum_{m'\neq m}^{M} \frac{\partial E}{\partial \Im\left\{\tilde{y}_{m',nk}(T)\right\}} \\
&+ 2\Re\left\{\hat{h}_{k,nm}(T)\right\} \psi^{x}_{n,mk}(T) \sum_{m'\neq m}^{M} \frac{\partial E}{\partial \nu^{y}_{m',nk}(T)} \\
&+ \begin{cases} 0 & (1 \leq k \leq K_p) \\ \psi^{r}_{mk} \dfrac{\Re\left\{\tilde{y}_{m,nk}(T)\right\}}{\nu^{x}_{m,nk}(T)} \dfrac{\partial E}{\partial \Re\left\{\hat{r}_{mk}\right\}} + \psi^{r}_{mk} \dfrac{\Im\left\{\tilde{y}_{m,nk}(T)\right\}}{\nu^{x}_{m,nk}(T)} \dfrac{\partial E}{\partial \Im\left\{\hat{r}_{mk}\right\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
&+ \begin{cases} 0 & (1 \leq k \leq K_p) \\ -\dfrac{2\Re\left\{\hat{h}_{k,nm}(T)\right\}}{\nu^{x}_{m,nk}(T)} \left(\psi^{r}_{mk}\right)^2 \dfrac{\partial E}{\partial \psi^{r}_{mk}} & (K_p + 1 \leq k \leq K) \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E}{\partial \Im\left\{\hat{h}_{k,nm}(T)\right\}} =& \Im\left\{\hat{x}_{n,mk}(T)\right\} \sum_{m'\neq m}^{M} \frac{\partial E}{\partial \Re\left\{\tilde{y}_{m',nk}(T)\right\}} - \Re\left\{\hat{x}_{n,mk}(T)\right\} \sum_{m'\neq m}^{M} \frac{\partial E}{\partial \Im\left\{\tilde{y}_{m',nk}(T)\right\}} \\
&+ 2\Im\left\{\hat{h}_{k,nm}(T)\right\} \psi^{x}_{n,mk}(T) \sum_{m'\neq m}^{M} \frac{\partial E}{\partial \nu^{y}_{m',nk}(T)} \\
&+ \begin{cases} 0 & (1 \leq k \leq K_p) \\ \psi^{r}_{mk} \dfrac{\Im\left\{\tilde{y}_{m,nk}(T)\right\}}{\nu^{x}_{m,nk}(T)} \dfrac{\partial E}{\partial \Re\left\{\hat{r}_{mk}\right\}} - \psi^{r}_{mk} \dfrac{\Re\left\{\tilde{y}_{m,nk}(T)\right\}}{\nu^{x}_{m,nk}(T)} \dfrac{\partial E}{\partial \Im\left\{\hat{r}_{mk}\right\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
&+ \begin{cases} 0 & (1 \leq k \leq K_p) \\ -\dfrac{2\Im\left\{\hat{h}_{k,nm}(T)\right\}}{\nu^{x}_{m,nk}(T)} \left(\psi^{r}_{mk}\right)^2 \dfrac{\partial E}{\partial \psi^{r}_{mk}} & (K_p + 1 \leq k \leq K) \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E}{\partial \Re\left\{\overline{x}_{n,mk}(t)\right\}} &= \zeta_n^x(t)\frac{\partial E}{\partial \Re\left\{\hat{x}_{n,mk}(t+1)\right\}} - 2\zeta_n^x(t)\Re\left\{\overline{x}_{n,mk}(t+1)\right\}\frac{\partial E}{\partial \psi_{n,mk}^x(t+1)} \\
\frac{\partial E}{\partial \Im\left\{\overline{x}_{n,mk}(t)\right\}} &= \zeta_n^x(t)\frac{\partial E}{\partial \Im\left\{\hat{x}_{n,mk}(t+1)\right\}} - 2\zeta_n^x(t)\Im\left\{\overline{x}_{n,mk}(t+1)\right\}\frac{\partial E}{\partial \psi_{n,mk}^x(t+1)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E}{\partial \Re\left\{\hat{q}_{k,nm}(t)\right\}} &= \zeta_n^h(t)\frac{\phi}{\psi_{k,nm}^q(t)+\phi}\frac{\partial E}{\partial \Re\left\{\hat{h}_{k,nm}(t+1)\right\}} \\
\frac{\partial E}{\partial \Im\left\{\hat{q}_{k,nm}(t)\right\}} &= \zeta_n^h(t)\frac{\phi}{\psi_{k,nm}^q(t)+\phi}\frac{\partial E}{\partial \Im\left\{\hat{h}_{k,nm}(t+1)\right\}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E}{\partial \Re\left\{\hat{r}_{n,mk}(t)\right\}} &= \frac{\gamma(t)}{\cosh^2\left(\frac{\gamma(t)}{c_x}\Re\left\{\hat{r}_{n,mk}(t)\right\}\right)}\frac{\partial E}{\partial \Re\left\{\overline{x}_{n,mk}(t+1)\right\}} \\
\frac{\partial E}{\partial \Im\left\{\hat{r}_{n,mk}(t)\right\}} &= \frac{\gamma(t)}{\cosh^2\left(\frac{\gamma(t)}{c_x}\Im\left\{\hat{r}_{n,mk}(t)\right\}\right)}\frac{\partial E}{\partial \Im\left\{\overline{x}_{n,mk}(t+1)\right\}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E}{\partial \psi_{k,nm}^q(t)} =& \frac{\Re\left\{\hat{q}_{k,nm}(t)\right\}}{\psi_{k,nm}^q(t)}\frac{\partial E}{\partial \Re\left\{\hat{q}_{k,nm}(t)\right\}} + \frac{\Im\left\{\hat{q}_{k,nm}(t)\right\}}{\psi_{k,nm}^q(t)}\frac{\partial E}{\partial \Im\left\{\hat{q}_{k,nm}(t)\right\}} \\
& - \zeta_n^h(t)\frac{\phi\Re\left\{\hat{q}_{k,nm}(t)\right\}}{\left(\psi_{k,nm}^q(t)+\phi\right)^2}\frac{\partial E}{\partial \Re\left\{\hat{h}_{k,nm}(t+1)\right\}} - \zeta_n^h(t)\frac{\phi\Im\left\{\hat{q}_{k,nm}(t)\right\}}{\left(\psi_{k,nm}^q(t)+\phi\right)^2}\frac{\partial E}{\partial \Im\left\{\hat{h}_{k,nm}(t+1)\right\}} \\
& + \zeta_n^h(t)\frac{\phi^2}{\left(\psi_{k,nm}^q(t)+\phi\right)^2}\frac{\partial E}{\partial \psi_{k,nm}^h(t+1)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E}{\partial \nu_{m,nk}^h(t)} =& \begin{cases}
-\alpha(t)\dfrac{\Re\left\{\hat{x}_{n,mk}^*(t)\tilde{y}_{m,nk}(t)\right\}}{\left(\nu_{m,nk}^h(t)\right)^2}\displaystyle\sum_{k'\neq k}^{K}\psi_{k',nm}^q(t)\frac{\partial E}{\partial \Re\left\{\hat{q}_{k',nm}(t)\right\}} & (1\leq k\leq K_p) \\
-\beta(t)\dfrac{\Re\left\{\hat{x}_{n,mk}^*(t)\tilde{y}_{m,nk}(t)\right\}}{\left(\nu_{m,nk}^h(t)\right)^2}\displaystyle\sum_{k'\neq k}^{K}\psi_{k',nm}^q(t)\frac{\partial E}{\partial \Re\left\{\hat{q}_{k',nm}(t)\right\}} & (K_p+1\leq k\leq K)
\end{cases} \\
& + \begin{cases}
-\alpha(t)\dfrac{\Im\left\{\hat{x}_{n,mk}^*(t)\tilde{y}_{m,nk}(t)\right\}}{\left(\nu_{m,nk}^h(t)\right)^2}\displaystyle\sum_{k'\neq k}^{K}\psi_{k',nm}^q(t)\frac{\partial E}{\partial \Im\left\{\hat{q}_{k',nm}(t)\right\}} & (1\leq k\leq K_p) \\
-\beta(t)\dfrac{\Im\left\{\hat{x}_{n,mk}^*(t)\tilde{y}_{m,nk}(t)\right\}}{\left(\nu_{m,nk}^h(t)\right)^2}\displaystyle\sum_{k'\neq k}^{K}\psi_{k',nm}^q(t)\frac{\partial E}{\partial \Im\left\{\hat{q}_{k',nm}(t)\right\}} & (K_p+1\leq k\leq K)
\end{cases} \\
& + \begin{cases}
\alpha(t)\dfrac{\left|\hat{x}_{n,mk}(t)\right|^2}{\left(\nu_{m,nk}^h(t)\right)^2}\displaystyle\sum_{k'\neq k}^{K}\left(\psi_{k',nm}^q(t)\right)^2 & (1\leq k\leq K_p) \\
\beta(t)\dfrac{\left|\hat{x}_{n,mk}(t)\right|^2}{\left(\nu_{m,nk}^h(t)\right)^2}\displaystyle\sum_{k'\neq k}^{K}\left(\psi_{k',nm}^q(t)\right)^2 & (K_p+1\leq k\leq K)
\end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E}{\partial \Re\{\tilde{y}_{m,nk}(t)\}} = &\begin{cases} \alpha(t)\dfrac{\Re\{\hat{x}_{n,mk}(t)\}}{\nu^h_{m,nk}(t)}\displaystyle\sum_{k'\neq k}^{K}\psi^q_{k',nm}(t)\frac{\partial E}{\partial \Re\{\hat{q}_{k',nm}(t)\}} & (1\le k\le K_p) \\ \beta(t)\dfrac{\Re\{\hat{x}_{n,mk}(t)\}}{\nu^h_{m,nk}(t)}\displaystyle\sum_{k'\neq k}^{K}\psi^q_{k',nm}(t)\frac{\partial E}{\partial \Re\{\hat{q}_{k',nm}(t)\}} & (K_p+1\le k\le K) \end{cases} \\
&+\begin{cases} -\alpha(t)\dfrac{\Im\{\hat{x}_{n,mk}(t)\}}{\nu^h_{m,nk}(t)}\displaystyle\sum_{k'\neq k}^{K}\psi^q_{k',nm}(t)\frac{\partial E}{\partial \Im\{\hat{q}_{k',nm}(t)\}} & (1\le k\le K_p) \\ -\beta(t)\dfrac{\Im\{\hat{x}_{n,mk}(t)\}}{\nu^h_{m,nk}(t)}\displaystyle\sum_{k'\neq k}^{K}\psi^q_{k',nm}(t)\frac{\partial E}{\partial \Im\{\hat{q}_{k',nm}(t)\}} & (K_p+1\le k\le K) \end{cases} \\
&+\begin{cases} 0 & (1\le k\le K_p) \\ \dfrac{\Re\left\{\hat{h}_{k,nm}(t)\right\}}{\nu^x_{m,nk}(t)}\displaystyle\sum_{n'\neq n}^{N}\psi^r_{n',mk}(t)\frac{\partial E}{\partial \Re\{\hat{r}_{n',mk}(t)\}} & (K_p+1\le k\le K) \end{cases} \\
&+\begin{cases} 0 & (1\le k\le K_p) \\ -\dfrac{\Im\left\{\hat{h}_{k,nm}(t)\right\}}{\nu^x_{m,nk}(t)}\displaystyle\sum_{n'\neq n}^{N}\psi^r_{n',mk}(t)\frac{\partial E}{\partial \Im\{\hat{r}_{n',mk}(t)\}} & (K_p+1\le k\le K) \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E}{\partial \Im\{\tilde{y}_{m,nk}(t)\}} = &\begin{cases} \alpha(t)\dfrac{\Im\{\hat{x}_{n,mk}(t)\}}{\nu^h_{m,nk}(t)}\displaystyle\sum_{k'\neq k}^{K}\psi^q_{k',nm}(t)\frac{\partial E}{\partial \Re\{\hat{q}_{k',nm}(t)\}} & (1\le k\le K_p) \\ \beta(t)\dfrac{\Im\{\hat{x}_{n,mk}(t)\}}{\nu^h_{m,nk}(t)}\displaystyle\sum_{k'\neq k}^{K}\psi^q_{k',nm}(t)\frac{\partial E}{\partial \Re\{\hat{q}_{k',nm}(t)\}} & (K_p+1\le k\le K) \end{cases} \\
&+\begin{cases} \alpha(t)\dfrac{\Re\{\hat{x}_{n,mk}(t)\}}{\nu^h_{m,nk}(t)}\displaystyle\sum_{k'\neq k}^{K}\psi^q_{k',nm}(t)\frac{\partial E}{\partial \Im\{\hat{q}_{k',nm}(t)\}} & (1\le k\le K_p) \\ \beta(t)\dfrac{\Re\{\hat{x}_{n,mk}(t)\}}{\nu^h_{m,nk}(t)}\displaystyle\sum_{k'\neq k}^{K}\psi^q_{k',nm}(t)\frac{\partial E}{\partial \Im\{\hat{q}_{k',nm}(t)\}} & (K_p+1\le k\le K) \end{cases} \\
&+\begin{cases} 0 & (1\le k\le K_p) \\ \dfrac{\Im\left\{\hat{h}_{k,nm}(t)\right\}}{\nu^x_{m,nk}(t)}\displaystyle\sum_{n'\neq n}^{N}\psi^r_{n',mk}(t)\frac{\partial E}{\partial \Re\{\hat{r}_{n',mk}(t)\}} & (K_p+1\le k\le K) \end{cases} \\
&+\begin{cases} 0 & (1\le k\le K_p) \\ \dfrac{\Re\left\{\hat{h}_{k,nm}(t)\right\}}{\nu^x_{m,nk}(t)}\displaystyle\sum_{n'\neq n}^{N}\psi^r_{n',mk}(t)\frac{\partial E}{\partial \Im\{\hat{r}_{n',mk}(t)\}} & (K_p+1\le k\le K) \end{cases}
\end{aligned}
$$

$$
\frac{\partial E}{\partial \psi^r_{n,mk}(t)} = \frac{\Re\{\hat{r}_{n,mk}(t)\}}{\psi^r_{n,mk}(t)}\frac{\partial E}{\partial \Re\{\hat{r}_{n,mk}(t)\}} + \frac{\Im\{\hat{r}_{n,mk}(t)\}}{\psi^r_{n,mk}(t)}\frac{\partial E}{\partial \Im\{\hat{r}_{n,mk}(t)\}}
$$

$$
\begin{aligned}
\frac{\partial E}{\partial \nu^x_{m,nk}(t)} = &-\frac{\Re\left\{\hat{h}^*_{k,nm}(t)\tilde{y}_{m,nk}(t)\right\}}{\left(\nu^x_{m,nk}(t)\right)^2}\sum_{n'\neq n}^{N}\psi^r_{n',mk}(t)\frac{\partial E}{\partial \Re\{\hat{r}_{n',mk}(t)\}} \\
&-\frac{\Im\left\{\hat{h}^*_{k,nm}(t)\tilde{y}_{m,nk}(t)\right\}}{\left(\nu^x_{m,nk}(t)\right)^2}\sum_{n'\neq n}^{N}\psi^r_{n',mk}(t)\frac{\partial E}{\partial \Im\{\hat{r}_{n',mk}(t)\}} \\
&+\frac{\left|\hat{h}_{k,nm}(t)\right|^2}{\left(\nu^x_{m,nk}(t)\right)^2}\sum_{n'\neq n}^{N}\left(\psi^r_{n',mk}(t)\right)^2\frac{\partial E}{\partial \psi^r_{n',mk}(t)}
\end{aligned}
$$

$$
\frac{\partial E}{\partial \nu^y_{m,nk}(t)} = \frac{\partial E}{\partial \nu^h_{m,nk}(t)} + \begin{cases} 0 & (1\le k\le K_p) \\ \dfrac{\partial E}{\partial \nu^x_{m,nk}(t)} & (K_p+1\le k\le K) \end{cases}
$$

$$\frac{\partial E}{\partial \psi^x_{n,mk}(t)} = \left(\left|\hat{h}_{k,nm}(t)\right|^2 + \psi^h_{k,nm}(t)\right) \sum_{m' \neq m}^{M} \frac{\partial E}{\partial \nu^y_{m',nk}(t)} + \phi \frac{\partial E}{\partial \nu^h_{m,nk}(t)}$$

$$+ \begin{cases} 0 & (1 \leq k \leq K_p) \\ \left(1 - \zeta^h_n(t)\right) \dfrac{\partial E}{\partial \psi^h_{k,nm}(t+1)} & (K_p + 1 \leq k \leq K) \end{cases}$$

$$\frac{\partial E}{\partial \psi^h_{k,nm}(t)} = \left(\left|\hat{x}_{n,mk}(t)\right|^2 \psi^x_{n,mk}(t)\right) \sum_{m' \neq m}^{M} \frac{\partial E}{\partial \nu^y_{m',nk}(t)} + \begin{cases} 0 & (1 \leq k \leq K_p) \\ E_s \dfrac{\partial E}{\partial \Re\left\{\nu^y_{m',nk}(t)\right\}} & (K_p + 1 \leq k \leq K) \end{cases}$$

$$+ \left(1 - \zeta^h_n(t)\right) \frac{\partial E}{\partial \nu^h_{m,nk}(t+1)}$$

$$
\begin{aligned}
\frac{\partial E}{\partial \Re\left\{\hat{x}_{n,mk}(t)\right\}} &= \begin{cases} 0 & (1 \le k \le K_p) \\ \left(1-\zeta_n^x(t)\right) \dfrac{\partial E}{\partial \Re\left\{\hat{x}_{n,mk}(t+1)\right\}} & (K_p + 1 \le k \le K) \end{cases} \\
&- \Re\left\{\hat{h}_{k,nm}(t)\right\} \sum_{m' \neq m}^{M} \frac{\partial E}{\partial \Re\left\{\tilde{y}_{m',nk}(t)\right\}} - \Im\left\{\hat{h}_{k,nm}(t)\right\} \sum_{m' \neq m}^{M} \frac{\partial E}{\partial \Im\left\{\tilde{y}_{m',nk}(t)\right\}} \\
&+ 2\Re\left\{\hat{x}_{n,mk}(t)\right\} \psi_{k,nm}^h(t) \sum_{m' \neq m}^{M} \frac{\partial E}{\partial \nu_{m',nk}^y(t)} \\
&+ \begin{cases} -\alpha(t) \dfrac{2\Re\left\{\hat{x}_{n,mk}(t)\right\}}{\nu_{m,nk}^h(t)} \displaystyle\sum_{k' \neq k}^{K} \left(\psi_{k',nm}^q(t)\right)^2 \frac{\partial E}{\partial \psi_{k',nm}^q(t)} & (1 \le k \le K_p) \\ -\beta(t) \dfrac{2\Re\left\{\hat{x}_{n,mk}(t)\right\}}{\nu_{m,nk}^h(t)} \displaystyle\sum_{k' \neq k}^{K} \left(\psi_{k',nm}^q(t)\right)^2 \frac{\partial E}{\partial \psi_{k',nm}^q(t)} & (K_p + 1 \le k \le K) \end{cases} \\
&+ \begin{cases} \alpha(t) \dfrac{\Re\left\{\tilde{y}_{m,nk}(t)\right\}}{\nu_{m,nk}^h(t)} \displaystyle\sum_{k' \neq k}^{K} \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Re\left\{\hat{q}_{k',nm}(t)\right\}} & (1 \le k \le K_p) \\ \beta(t) \dfrac{\Re\left\{\tilde{y}_{m,nk}(t)\right\}}{\nu_{m,nk}^h(t)} \displaystyle\sum_{k' \neq k}^{K} \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Re\left\{\hat{q}_{k',nm}(t)\right\}} & (K_p + 1 \le k \le K) \end{cases} \\
&+ \begin{cases} \alpha(t) \dfrac{\Im\left\{\tilde{y}_{m,nk}(t)\right\}}{\nu_{m,nk}^h(t)} \displaystyle\sum_{k' \neq k}^{K} \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Im\left\{\hat{q}_{k',nm}(t)\right\}} & (1 \le k \le K_p) \\ \beta(t) \dfrac{\Im\left\{\tilde{y}_{m,nk}(t)\right\}}{\nu_{m,nk}^h(t)} \displaystyle\sum_{k' \neq k}^{K} \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Im\left\{\hat{q}_{k',nm}(t)\right\}} & (K_p + 1 \le k \le K) \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E}{\partial \Im\left\{\hat{x}_{n,mk}(t)\right\}} &= \begin{cases} 0 & (1 \le k \le K_p) \\ \left(1-\zeta_n^x(t)\right) \dfrac{\partial E}{\partial \Im\left\{\hat{x}_{n,mk}(t+1)\right\}} & (K_p + 1 \le k \le K) \end{cases} \\
&+ \Im\left\{\hat{h}_{k,nm}(t)\right\} \sum_{m' \neq m}^{M} \frac{\partial E}{\partial \Re\left\{\tilde{y}_{m',nk}(t)\right\}} - \Re\left\{\hat{h}_{k,nm}(t)\right\} \sum_{m' \neq m}^{M} \frac{\partial E}{\partial \Im\left\{\tilde{y}_{m',nk}(t)\right\}} \\
&+ 2\Im\left\{\hat{x}_{n,mk}(t)\right\} \psi_{k,nm}^h(t) \sum_{m' \neq m}^{M} \frac{\partial E}{\partial \nu_{m',nk}^y(t)} \\
&+ \begin{cases} -\alpha(t) \dfrac{2\Im\left\{\hat{x}_{n,mk}(t)\right\}}{\nu_{m,nk}^h(t)} \displaystyle\sum_{k' \neq k}^{K} \left(\psi_{k',nm}^q(t)\right)^2 \frac{\partial E}{\partial \psi_{k',nm}^q(t)} & (1 \le k \le K_p) \\ -\beta(t) \dfrac{2\Im\left\{\hat{x}_{n,mk}(t)\right\}}{\nu_{m,nk}^h(t)} \displaystyle\sum_{k' \neq k}^{K} \left(\psi_{k',nm}^q(t)\right)^2 \frac{\partial E}{\partial \psi_{k',nm}^q(t)} & (K_p + 1 \le k \le K) \end{cases} \\
&+ \begin{cases} \alpha(t) \dfrac{\Im\left\{\tilde{y}_{m,nk}(t)\right\}}{\nu_{m,nk}^h(t)} \displaystyle\sum_{k' \neq k}^{K} \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Re\left\{\hat{q}_{k',nm}(t)\right\}} & (1 \le k \le K_p) \\ \beta(t) \dfrac{\Im\left\{\tilde{y}_{m,nk}(t)\right\}}{\nu_{m,nk}^h(t)} \displaystyle\sum_{k' \neq k}^{K} \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Re\left\{\hat{q}_{k',nm}(t)\right\}} & (K_p \le k \le K) \end{cases} \\
&+ \begin{cases} -\alpha(t) \dfrac{\Re\left\{\tilde{y}_{m,nk}(t)\right\}}{\nu_{m,nk}^h(t)} \displaystyle\sum_{k' \neq k}^{K} \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Im\left\{\hat{q}_{k',nm}(t)\right\}} & (1 \le k \le K_p) \\ -\beta(t) \dfrac{\Re\left\{\tilde{y}_{m,nk}(t)\right\}}{\nu_{m,nk}^h(t)} \displaystyle\sum_{k' \neq k}^{K} \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Im\left\{\hat{q}_{k',nm}(t)\right\}} & (K_p + 1 \le k \le K) \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E}{\partial \Re\left\{\hat{h}_{k,nm}(t)\right\}} =& - \Re\left\{\hat{x}_{n,mk}(t)\right\} \sum_{m'\neq m}^{M} \frac{\partial E}{\partial \Re\left\{\tilde{y}_{m',nk}(t)\right\}} - \Im\left\{\hat{x}_{n,mk}(t)\right\} \sum_{m'\neq m}^{M} \frac{\partial E}{\partial \Im\left\{\tilde{y}_{m',nk}(t)\right\}} \\
& + 2\Re\left\{\hat{h}_{k,nm}(t)\right\} \psi^{r}_{n,mk}(t) \sum_{m'\neq m}^{M} \frac{\partial E}{\partial \nu^{y}_{m',nk}(t)} + \left(1-\zeta^{h}_{n}(t)\right) \frac{\partial E}{\partial \Re\left\{\hat{h}_{k,nm}(t+1)\right\}} \\
& + \begin{cases} 0 & (1 \leq k \leq K_p) \\ \dfrac{\Re\left\{\tilde{y}_{m,nk}(t)\right\}}{\nu^{x}_{m,nk}(t)} \displaystyle\sum_{n'\neq n}^{N} \psi^{r}_{n',mk}(t) \frac{\partial E}{\partial \Re\left\{\hat{r}_{n',mk}(t)\right\}} & (K_p+1 \leq k \leq K) \end{cases} \\
& + \begin{cases} 0 & (1 \leq k \leq K_p) \\ \dfrac{\Im\left\{\tilde{y}_{m,nk}(t)\right\}}{\nu^{x}_{m,nk}(t)} \displaystyle\sum_{n'\neq n}^{N} \psi^{r}_{n',mk}(t) \frac{\partial E}{\partial \Im\left\{\hat{r}_{n',mk}(t)\right\}} & (K_p+1 \leq k \leq K) \end{cases} \\
& + \begin{cases} 0 & (1 \leq k \leq K_p) \\ -\dfrac{2\Re\left\{\hat{h}_{k,nm}(t)\right\}}{\nu^{x}_{m,nk}(t)} \displaystyle\sum_{n'\neq n}^{N} \left(\psi^{r}_{n',mk}(t)\right)^{2} \frac{\partial E}{\partial \psi^{r}_{n',mk}(t)} & (K_p+1 \leq k \leq K) \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E}{\partial \Im\left\{\hat{h}_{k,nm}(t)\right\}} =& \Im\left\{\hat{x}_{n,mk}(t)\right\} \sum_{m'\neq m}^{M} \frac{\partial E}{\partial \Re\left\{\tilde{y}_{m',nk}(t)\right\}} - \Re\left\{\hat{x}_{n,mk}(t)\right\} \sum_{m'\neq m}^{M} \frac{\partial E}{\partial \Im\left\{\tilde{y}_{m',nk}(t)\right\}} \\
& + 2\Im\left\{\hat{h}_{k,nm}(t)\right\} \psi^{r}_{n,mk}(t) \sum_{m'\neq m}^{M} \frac{\partial E}{\partial \nu^{y}_{m',nk}(t)} + \left(1-\zeta^{h}_{n}(t)\right) \frac{\partial E}{\partial \Im\left\{\hat{h}_{k,nm}(t+1)\right\}} \\
& + \begin{cases} 0 & (1 \leq k \leq K_p) \\ \dfrac{\Im\left\{\tilde{y}_{m,nk}(t)\right\}}{\nu^{x}_{m,nk}(t)} \displaystyle\sum_{n'\neq n}^{N} \psi^{r}_{n',mk}(t) \frac{\partial E}{\partial \Re\left\{\hat{r}_{n',mk}(t)\right\}} & (K_p+1 \leq k \leq K) \end{cases} \\
& + \begin{cases} 0 & (1 \leq k \leq K_p) \\ -\dfrac{\Re\left\{\tilde{y}_{m,nk}(t)\right\}}{\nu^{x}_{m,nk}(t)} \displaystyle\sum_{n'\neq n}^{N} \psi^{r}_{n',mk}(t) \frac{\partial E}{\partial \Im\left\{\hat{r}_{n',mk}(t)\right\}} & (K_p+1 \leq k \leq K) \end{cases} \\
& + \begin{cases} 0 & (1 \leq k \leq K_p) \\ -\dfrac{2\Im\left\{\hat{h}_{k,nm}(t)\right\}}{\nu^{x}_{m,nk}(t)} \displaystyle\sum_{n'\neq n}^{N} \left(\psi^{r}_{n',mk}(t)\right)^{2} \frac{\partial E}{\partial \psi^{r}_{n',mk}(t)} & (K_p+1 \leq k \leq K) \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E}{\partial \zeta^{x}_{n}(t)} = \sum_{k'=K_p+1}^{K} \sum_{m'=1}^{M} \Bigg\{ & \left(\Re\left\{\overline{x}_{n,m'k'}(t+1) - \Re\left\{\hat{x}_{n,m'k'}(t)\right\}\right\}\right) \frac{\partial E}{\partial \Re\left\{\hat{x}_{n,m'k'}(t+1)\right\}} \\
& + \left(\Im\left\{\overline{x}_{n,m'k'}(t+1) - \Im\left\{\hat{x}_{n,m'k'}(t)\right\}\right\}\right) \frac{\partial E}{\partial \Im\left\{\hat{x}_{n,m'k'}(t+1)\right\}} \\
& + \left(E_s - \left|\overline{x}_{n,m'k'}(t+1)\right|^{2} - \psi^{x}_{n,m'k'}(t)\right) \frac{\partial E}{\partial \psi^{x}_{n,m'k'}(t+1)} \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E}{\partial \zeta^{h}_{n}(t)} = \sum_{k'=1}^{K} \sum_{m'=1}^{M} \Bigg\{ & \left(\frac{\phi \Re\left\{\hat{q}_{k',nm'}(t)\right\}}{\psi^{q}_{k',nm'}(t)+\phi} - \Re\left\{\hat{h}_{k',nm'}(t)\right\} \right) \frac{\partial E}{\partial \Re\left\{\hat{h}_{k',nm'}(t+1)\right\}} \\
& + \left(\frac{\phi \Im\left\{\hat{q}_{k',nm'}(t)\right\}}{\psi^{q}_{k',nm'}(t)+\phi} - \Im\left\{\hat{h}_{k',nm'}(t)\right\} \right) \frac{\partial E}{\partial \Im\left\{\hat{h}_{k',nm'}(t+1)\right\}} \\
& + \left(\frac{\phi \psi^{q}_{k',nm'}(t)}{\psi^{q}_{k',nm'}(t)+\phi} - \psi^{h}_{k',nm'}(t) \right) \frac{\partial E}{\partial \psi^{h}_{k',nm'}(t+1)} \Bigg\}
\end{aligned}
$$

$$\frac{\partial E}{\partial \eta_n^x(t)} = \frac{4\exp\left(-4\eta_n^x(t)\right)}{\left(1+\exp\left(-4\eta_n^x(t)\right)\right)^2}\frac{\partial E}{\partial \zeta_n^x(t)}$$

$$\frac{\partial E}{\partial \eta_n^h(t)} = \frac{4\exp\left(-4\eta_n^h(t)\right)}{\left(1+\exp\left(-4\eta_n^h(t)\right)\right)^2}\frac{\partial E}{\partial \zeta_n^h(t)}$$

$$\begin{aligned}\frac{\partial E}{\partial \alpha(t)} = \quad & \sum_{k'=1}^{K_p}\sum_{m'=1}^{M}\sum_{n'=1}^{N}\left\{\psi_{k',nm}^q(t)\sum_{i\neq k'}^{K_p}\frac{\Re\left\{\hat{x}_{n,mi}^*(t)\tilde{y}_{m,ni}(t)\right\}}{\nu_{m,ni}^h(t)}\frac{\partial E}{\partial \Re\left\{\hat{q}_{k',nm}(t)\right\}}\right.\\
&+\psi_{k',nm}^q(t)\sum_{i\neq k'}^{K_p}\frac{\Im\left\{\hat{x}_{n,mi}^*(t)\tilde{y}_{m,ni}(t)\right\}}{\nu_{m,ni}^h(t)}\frac{\partial E}{\partial \Im\left\{\hat{q}_{k',nm}(t)\right\}}\\
&\left.-\left(\psi_{k',nm}^q(t)\right)^2\sum_{i\neq k'}^{K_p}\frac{\left|\hat{x}_{n,mi}(t)\right|^2}{\nu_{m,ni}^h(t)}\frac{\partial E}{\partial \psi_{k',nm}^q(t)}\right\}\\
&+\sum_{k'=K_p+1}^{K}\sum_{m'=1}^{M}\sum_{n'=1}^{N}\left\{\psi_{k',nm}^q(t)\sum_{i=1}^{K_p}\frac{\Re\left\{\hat{x}_{n,mi}^*(t)\tilde{y}_{m,ni}(t)\right\}}{\nu_{m,ni}^h(t)}\frac{\partial E}{\partial \Re\left\{\hat{q}_{k',nm}(t)\right\}}\right.\\
&+\psi_{k',nm}^q(t)\sum_{i=1}^{K_p}\frac{\Im\left\{\hat{x}_{n,mi}^*(t)\tilde{y}_{m,ni}(t)\right\}}{\nu_{m,ni}^h(t)}\frac{\partial E}{\partial \Im\left\{\hat{q}_{k',nm}(t)\right\}}\\
&\left.-\left(\psi_{k',nm}^q(t)\right)^2\sum_{i=1}^{K_p}\frac{\left|\hat{x}_{n,mi}(t)\right|^2}{\nu_{m,ni}^h(t)}\frac{\partial E}{\partial \psi_{k',nm}^q(t)}\right\}\end{aligned}$$

$$\begin{aligned}\frac{\partial E}{\partial \beta(t)} = \quad & \sum_{k'=1}^{K_p}\sum_{m'=1}^{M}\sum_{n'=1}^{N}\left\{\psi_{k',nm}^q(t)\sum_{i=K_p+1}^{K}\frac{\Re\left\{\hat{x}_{n,mi}^*(t)\tilde{y}_{m,ni}(t)\right\}}{\nu_{m,ni}^h(t)}\frac{\partial E}{\partial \Re\left\{\hat{q}_{k',nm}(t)\right\}}\right.\\
&+\psi_{k',nm}^q(t)\sum_{i=K_p+1}^{K}\frac{\Im\left\{\hat{x}_{n,mi}^*(t)\tilde{y}_{m,ni}(t)\right\}}{\nu_{m,ni}^h(t)}\frac{\partial E}{\partial \Im\left\{\hat{q}_{k',nm}(t)\right\}}\\
&\left.-\left(\psi_{k',nm}^q(t)\right)^2\sum_{i=K_p+1}^{K}\frac{\left|\hat{x}_{n,mi}(t)\right|^2}{\nu_{m,ni}^h(t)}\frac{\partial E}{\partial \psi_{k',nm}^q(t)}\right\}\\
&+\sum_{k'=K_p+1}^{K}\sum_{m'=1}^{M}\sum_{n'=1}^{N}\left\{\psi_{k',nm}^q(t)\sum_{i=K_p+1,i\neq k'}^{K}\frac{\Re\left\{\hat{x}_{n,mi}^*(t)\tilde{y}_{m,ni}(t)\right\}}{\nu_{m,ni}^h(t)}\frac{\partial E}{\partial \Re\left\{\hat{q}_{k',nm}(t)\right\}}\right.\\
&+\psi_{k',nm}^q(t)\sum_{i=K_p+1,i\neq k'}^{K}\frac{\Im\left\{\hat{x}_{n,mi}^*(t)\tilde{y}_{m,ni}(t)\right\}}{\nu_{m,ni}^h(t)}\frac{\partial E}{\partial \Im\left\{\hat{q}_{k',nm}(t)\right\}}\\
&\left.-\left(\psi_{k',nm}^q(t)\right)^2\sum_{i=K_p+1,i\neq k'}^{K}\frac{\left|\hat{x}_{n,mi}(t)\right|^2}{\nu_{m,ni}^h(t)}\frac{\partial E}{\partial \psi_{k',nm}^q(t)}\right\}\end{aligned}$$

$$\begin{aligned}\frac{\partial E}{\partial \gamma(t)} = \sum_{k'=K_p+1}^{K}\sum_{m'=1}^{M}\sum_{n'=1}^{N}&\left\{\frac{\Re\left\{\hat{r}_{n',m'k'}(t)\right\}}{\cosh^2\left(\frac{\gamma(t)}{c_x}\Re\left\{\hat{r}_{n',m'k'}(t)\right\}\right)}\frac{\partial E}{\partial \Re\left\{\overline{x}_{m',n'k'}(t)\right\}}\right.\\
&\left.+\frac{\Im\left\{\hat{r}_{n',m'k'}(t)\right\}}{\cosh^2\left(\frac{\gamma(t)}{c_x}\Im\left\{\hat{r}_{n',m'k'}(t)\right\}\right)}\frac{\partial E}{\partial \Im\left\{\overline{x}_{m',n'k'}(t)\right\}}\right\}\end{aligned}$$