

Theoretical equations for backpropagation of BiGaBP

$$\begin{aligned}\frac{\partial E}{\partial \Re\{\hat{r}_{mk}\}} &= -\frac{2}{MK_d} (\Re\{x_{mk}\} - \Re\{\hat{r}_{mk}\}) \\ \frac{\partial E}{\partial \Im\{\hat{r}_{mk}\}} &= -\frac{2}{MK_d} (\Im\{x_{mk}\} - \Im\{\hat{r}_{mk}\})\end{aligned}$$

$$\begin{aligned}\frac{\partial E}{\partial \Re\{\tilde{y}_{m,nk}(T)\}} &= \begin{cases} 0 & (1 \leq k \leq K_p) \\ \psi_{mk}^r \frac{\Re\{\hat{h}_{k,nm}(T)\}}{\nu_{m,nk}^x(T)} \frac{\partial E}{\partial \Re\{\hat{r}_{mk}\}} - \psi_{mk}^r \frac{\Im\{\hat{h}_{k,nm}(T)\}}{\nu_{m,nk}^x(T)} \frac{\partial E}{\partial \Im\{\hat{r}_{mk}\}} & (K_p + 1 \leq k \leq K) \end{cases} \\ \frac{\partial E}{\partial \Im\{\tilde{y}_{m,nk}(T)\}} &= \begin{cases} 0 & (1 \leq k \leq K_p) \\ \psi_{mk}^r \frac{\Im\{\hat{h}_{k,nm}(T)\}}{\nu_{m,nk}^x(T)} \frac{\partial E}{\partial \Re\{\hat{r}_{mk}\}} + \psi_{mk}^r \frac{\Re\{\hat{h}_{k,nm}(T)\}}{\nu_{m,nk}^x(T)} \frac{\partial E}{\partial \Im\{\hat{r}_{mk}\}} & (K_p + 1 \leq k \leq K) \end{cases}\end{aligned}$$

$$\frac{\partial E}{\partial \psi_{mk}^r} = \frac{\Re\{\hat{r}_{mk}\}}{\psi_{mk}^r} \frac{\partial E}{\partial \Re\{\hat{r}_{mk}\}} + \frac{\Im\{\hat{r}_{mk}\}}{\psi_{mk}^r} \frac{\partial E}{\partial \Im\{\hat{r}_{mk}\}}$$

$$\begin{aligned}\frac{\partial E}{\partial \nu_{m,nk}^x(T)} &= -\psi_{mk}^r \frac{\Re\{\hat{h}_{k,nm}(T)\} \Re\{\tilde{y}_{m,nk}(T)\} + \Im\{\hat{h}_{k,nm}(T)\} \Im\{\tilde{y}_{m,nk}(T)\}}{\left(\nu_{m,nk}^x(T)\right)^2} \frac{\partial E}{\partial \Re\{\hat{r}_{mk}\}} \\ &\quad - \psi_{mk}^r \frac{\Re\{\hat{h}_{k,nm}(T)\} \Im\{\tilde{y}_{m,nk}(T)\} - \Im\{\hat{h}_{k,nm}(T)\} \Re\{\tilde{y}_{m,nk}(T)\}}{\left(\nu_{m,nk}^x(T)\right)^2} \frac{\partial E}{\partial \Im\{\hat{r}_{mk}\}} \\ &\quad + \frac{|\hat{h}_{k,nm}(T)|^2}{\left(\nu_{m,nk}^x(T)\right)^2} (\psi_{mk}^r)^2 \frac{\partial E}{\partial \psi_{mk}^r}\end{aligned}$$

$$\frac{\partial E}{\partial \nu_{m,nk}^y(T)} = \begin{cases} 0 & (1 \leq k \leq K_p) \\ \frac{\partial E}{\partial \nu_{m,nk}^x(T)} & (K_p + 1 \leq k \leq K) \end{cases}$$

$$\frac{\partial E}{\partial \psi_{n,mk}^x(T)} = \left\{ |\hat{h}_{k,nm}(T)|^2 + \psi_{k,nm}^h(T) \right\} \sum_{m' \neq m} \frac{\partial E}{\partial \nu_{m',nk}^y(T)}$$

$$\frac{\partial E}{\partial \psi_{k,nm}^h(T)} = \left\{ |\hat{x}_{n,mk}(T)|^2 + \psi_{n,mk}^x(T) \right\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \nu_{m',nk}^y(T)} + \begin{cases} 0 & (1 \leq k \leq K_p) \\ E_s \frac{\partial E}{\partial \nu_{m,nk}^x(T)} & (K_p + 1 \leq k \leq K) \end{cases}$$

$$\begin{aligned}\frac{\partial E}{\partial \Re\{\hat{x}_{n,mk}(T)\}} &= -\Re\{\hat{h}_{k,nm}(T)\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Re\{\tilde{y}_{m',nk}(T)\}} - \Im\{\hat{h}_{k,nm}(T)\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Im\{\tilde{y}_{m',nk}(T)\}} \\ &\quad + 2\Re\{\hat{x}_{n,mk}(T)\} \psi_{k,nm}^h(T) \sum_{m' \neq m}^M \frac{\partial E}{\partial \nu_{m',nk}^y(T)}\end{aligned}$$

$$\begin{aligned}\frac{\partial E}{\partial \Im\{\hat{x}_{n,mk}(T)\}} &= \Im\{\hat{h}_{k,nm}(T)\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Re\{\tilde{y}_{m',nk}(T)\}} - \Re\{\hat{h}_{k,nm}(T)\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Im\{\tilde{y}_{m',nk}(T)\}} \\ &\quad + 2\Im\{\hat{x}_{n,mk}(T)\} \psi_{k,nm}^h(T) \sum_{m' \neq m}^M \frac{\partial E}{\partial \nu_{m',nk}^y(T)}\end{aligned}$$

$$\begin{aligned}
\frac{\partial E}{\partial \Re \left\{ \hat{h}_{k,nm}(T) \right\}} &= -\Re \left\{ \hat{x}_{n,mk}(T) \right\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Re \left\{ \tilde{y}_{m',nk}(T) \right\}} - \Im \left\{ \hat{x}_{n,mk}(T) \right\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Im \left\{ \tilde{y}_{m',nk}(T) \right\}} \\
&\quad + 2\Re \left\{ \hat{h}_{k,nm}(T) \right\} \psi_{n,mk}^x(T) \sum_{m' \neq m}^M \frac{\partial E}{\partial \nu_{m',nk}^y(T)} \\
&\quad + \begin{cases} 0 & (1 \leq k \leq K_p) \\ \psi_{mk}^r \frac{\Re \left\{ \tilde{y}_{m,nk}(T) \right\}}{\nu_{m,nk}^x(T)} \frac{\partial E}{\partial \Re \left\{ \hat{r}_{mk} \right\}} + \psi_{mk}^r \frac{\Im \left\{ \tilde{y}_{m,nk}(T) \right\}}{\nu_{m,nk}^x(T)} \frac{\partial E}{\partial \Im \left\{ \hat{r}_{mk} \right\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
&\quad + \begin{cases} 0 & (1 \leq k \leq K_p) \\ -\frac{2\Re \left\{ \hat{h}_{k,nm}(T) \right\}}{\nu_{m,nk}^x(T)} (\psi_{mk}^r)^2 \frac{\partial E}{\partial \psi_{mk}^r} & (K_p + 1 \leq k \leq K) \end{cases}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E}{\partial \Im \left\{ \hat{h}_{k,nm}(T) \right\}} &= \Im \left\{ \hat{x}_{n,mk}(T) \right\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Re \left\{ \tilde{y}_{m',nk}(T) \right\}} - \Re \left\{ \hat{x}_{n,mk}(T) \right\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Im \left\{ \tilde{y}_{m',nk}(T) \right\}} \\
&\quad + 2\Im \left\{ \hat{h}_{k,nm}(T) \right\} \psi_{n,mk}^x(T) \sum_{m' \neq m}^M \frac{\partial E}{\partial \nu_{m',nk}^y(T)} \\
&\quad + \begin{cases} 0 & (1 \leq k \leq K_p) \\ \psi_{mk}^r \frac{\Im \left\{ \tilde{y}_{m,nk}(T) \right\}}{\nu_{m,nk}^x(T)} \frac{\partial E}{\partial \Re \left\{ \hat{r}_{mk} \right\}} - \psi_{mk}^r \frac{\Re \left\{ \tilde{y}_{m,nk}(T) \right\}}{\nu_{m,nk}^x(T)} \frac{\partial E}{\partial \Im \left\{ \hat{r}_{mk} \right\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
&\quad + \begin{cases} 0 & (1 \leq k \leq K_p) \\ -\frac{2\Im \left\{ \hat{h}_{k,nm}(T) \right\}}{\nu_{m,nk}^x(T)} (\psi_{mk}^r)^2 \frac{\partial E}{\partial \psi_{mk}^r} & (K_p + 1 \leq k \leq K) \end{cases}
\end{aligned}$$

$$\begin{aligned}\frac{\partial E}{\partial \Re \{\bar{x}_{n,mk}(t)\}} &= \zeta_n^x(t) \frac{\partial E}{\partial \Re \{\hat{x}_{n,mk}(t+1)\}} - 2\zeta_n^x(t) \Re \{\bar{x}_{n,mk}(t+1)\} \frac{\partial E}{\partial \psi_{n,mk}^x(t+1)} \\ \frac{\partial E}{\partial \Im \{\bar{x}_{n,mk}(t)\}} &= \zeta_n^x(t) \frac{\partial E}{\partial \Im \{\hat{x}_{n,mk}(t+1)\}} - 2\zeta_n^x(t) \Im \{\bar{x}_{n,mk}(t+1)\} \frac{\partial E}{\partial \psi_{n,mk}^x(t+1)}\end{aligned}$$

$$\begin{aligned}\frac{\partial E}{\partial \Re \{\hat{q}_{k,nm}(t)\}} &= \zeta_n^h(t) \frac{\phi}{\psi_{k,nm}^q(t) + \phi} \frac{\partial E}{\partial \Re \{\hat{h}_{k,nm}(t+1)\}} \\ \frac{\partial E}{\partial \Im \{\hat{q}_{k,nm}(t)\}} &= \zeta_n^h(t) \frac{\phi}{\psi_{k,nm}^q(t) + \phi} \frac{\partial E}{\partial \Im \{\hat{h}_{k,nm}(t+1)\}}\end{aligned}$$

$$\begin{aligned}\frac{\partial E}{\partial \Re \{\hat{r}_{n,mk}(t)\}} &= \frac{\gamma(t)}{\cosh^2 \left(\frac{\gamma(t)}{c_x} \Re \{\hat{r}_{n,mk}(t)\} \right)} \frac{\partial E}{\partial \Re \{\bar{x}_{n,mk}(t+1)\}} \\ \frac{\partial E}{\partial \Im \{\hat{r}_{n,mk}(t)\}} &= \frac{\gamma(t)}{\cosh^2 \left(\frac{\gamma(t)}{c_x} \Im \{\hat{r}_{n,mk}(t)\} \right)} \frac{\partial E}{\partial \Im \{\bar{x}_{n,mk}(t+1)\}}\end{aligned}$$

$$\begin{aligned}\frac{\partial E}{\partial \psi_{k,nm}^q(t)} &= \frac{\Re \{\hat{q}_{k,nm}(t)\}}{\psi_{k,nm}^q(t)} \frac{\partial E}{\partial \Re \{\hat{q}_{k,nm}(t)\}} + \frac{\Im \{\hat{q}_{k,nm}(t)\}}{\psi_{k,nm}^q(t)} \frac{\partial E}{\partial \Im \{\hat{q}_{k,nm}(t)\}} \\ &\quad - \zeta_n^h(t) \frac{\phi \Re \{\hat{q}_{k,nm}(t)\}}{\left(\psi_{k,nm}^q(t) + \phi \right)^2} \frac{\partial E}{\partial \Re \{\hat{h}_{k,nm}(t+1)\}} - \zeta_n^h(t) \frac{\phi \Im \{\hat{q}_{k,nm}(t)\}}{\left(\psi_{k,nm}^q(t) + \phi \right)^2} \frac{\partial E}{\partial \Im \{\hat{h}_{k,nm}(t+1)\}} \\ &\quad + \zeta_n^h(t) \frac{\phi^2}{\left(\psi_{k,nm}^q(t) + \phi \right)^2} \frac{\partial E}{\partial \psi_{k,nm}^h(t+1)}\end{aligned}$$

$$\begin{aligned}\frac{\partial E}{\partial \nu_{m,nk}^h(t)} &= \begin{cases} -\alpha(t) \frac{\Re \{\hat{x}_{n,mk}^*(t) \tilde{y}_{m,nk}(t)\}}{\left(\nu_{m,nk}^h(t) \right)^2} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Re \{\hat{q}_{k',nm}(t)\}} & (1 \leq k \leq K_p) \\ -\beta(t) \frac{\Re \{\hat{x}_{n,mk}^*(t) \tilde{y}_{m,nk}(t)\}}{\left(\nu_{m,nk}^h(t) \right)^2} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Re \{\hat{q}_{k',nm}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases} \\ &\quad + \begin{cases} -\alpha(t) \frac{\Im \{\hat{x}_{n,mk}^*(t) \tilde{y}_{m,nk}(t)\}}{\left(\nu_{m,nk}^h(t) \right)^2} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Im \{\hat{q}_{k',nm}(t)\}} & (1 \leq k \leq K_p) \\ -\beta(t) \frac{\Im \{\hat{x}_{n,mk}^*(t) \tilde{y}_{m,nk}(t)\}}{\left(\nu_{m,nk}^h(t) \right)^2} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Im \{\hat{q}_{k',nm}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases} \\ &\quad + \begin{cases} \alpha(t) \frac{|\hat{x}_{n,mk}(t)|^2}{\left(\nu_{m,nk}^h(t) \right)^2} \sum_{k' \neq k}^K \left(\psi_{k',nm}^q(t) \right)^2 & (1 \leq k \leq K_p) \\ \beta(t) \frac{|\hat{x}_{n,mk}(t)|^2}{\left(\nu_{m,nk}^h(t) \right)^2} \sum_{k' \neq k}^K \left(\psi_{k',nm}^q(t) \right)^2 & (K_p + 1 \leq k \leq K) \end{cases}\end{aligned}$$

$$\begin{aligned}
\frac{\partial E}{\partial \Re \{\tilde{y}_{m,nk}(t)\}} = & \begin{cases} \alpha(t) \frac{\Re \{\hat{x}_{n,mk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Re \{\hat{q}_{k',nm}(t)\}} & (1 \leq k \leq K_p) \\ \beta(t) \frac{\Re \{\hat{x}_{n,mk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Re \{\hat{q}_{k',nm}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
& + \begin{cases} -\alpha(t) \frac{\Im \{\hat{x}_{n,mk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Im \{\hat{q}_{k',nm}(t)\}} & (1 \leq k \leq K_p) \\ -\beta(t) \frac{\Im \{\hat{x}_{n,mk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Im \{\hat{q}_{k',nm}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
& + \begin{cases} 0 & (1 \leq k \leq K_p) \\ \frac{\Re \{\hat{h}_{k,nm}(t)\}}{\nu_{m,nk}^x(t)} \sum_{n' \neq n}^N \psi_{n',mk}^r(t) \frac{\partial E}{\partial \Re \{\hat{r}_{n',mk}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
& + \begin{cases} 0 & (1 \leq k \leq K_p) \\ -\frac{\Im \{\hat{h}_{k,nm}(t)\}}{\nu_{m,nk}^x(t)} \sum_{n' \neq n}^N \psi_{n',mk}^r(t) \frac{\partial E}{\partial \Im \{\hat{r}_{n',mk}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E}{\partial \Im \{\tilde{y}_{m,nk}(t)\}} = & \begin{cases} \alpha(t) \frac{\Im \{\hat{x}_{n,mk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Re \{\hat{q}_{k',nm}(t)\}} & (1 \leq k \leq K_p) \\ \beta(t) \frac{\Im \{\hat{x}_{n,mk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Re \{\hat{q}_{k',nm}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
& + \begin{cases} \alpha(t) \frac{\Re \{\hat{x}_{n,mk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Im \{\hat{q}_{k',nm}(t)\}} & (1 \leq k \leq K_p) \\ \beta(t) \frac{\Re \{\hat{x}_{n,mk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Im \{\hat{q}_{k',nm}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
& + \begin{cases} 0 & (1 \leq k \leq K_p) \\ \frac{\Im \{\hat{h}_{k,nm}(t)\}}{\nu_{m,nk}^x(t)} \sum_{n' \neq n}^N \psi_{n',mk}^r(t) \frac{\partial E}{\partial \Re \{\hat{r}_{n',mk}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
& + \begin{cases} 0 & (1 \leq k \leq K_p) \\ \frac{\Re \{\hat{h}_{k,nm}(t)\}}{\nu_{m,nk}^x(t)} \sum_{n' \neq n}^N \psi_{n',mk}^r(t) \frac{\partial E}{\partial \Im \{\hat{r}_{n',mk}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases}
\end{aligned}$$

$$\frac{\partial E}{\partial \psi_{n,mk}^r(t)} = \frac{\Re \{\hat{r}_{n,mk}(t)\}}{\psi_{n,mk}^r(t)} \frac{\partial E}{\partial \Re \{\hat{r}_{n,mk}(t)\}} + \frac{\Im \{\hat{r}_{n,mk}(t)\}}{\psi_{n,mk}^r(t)} \frac{\partial E}{\partial \Im \{\hat{r}_{n,mk}(t)\}}$$

$$\begin{aligned}
\frac{\partial E}{\partial \nu_{m,nk}^x(t)} = & - \frac{\Re \{\hat{h}_{k,nm}^*(t) \tilde{y}_{m,nk}(t)\}}{\left(\nu_{m,nk}^x(t)\right)^2} \sum_{n' \neq n}^N \psi_{n',mk}^r(t) \frac{\partial E}{\partial \Re \{\hat{r}_{n',mk}(t)\}} \\
& - \frac{\Im \{\hat{h}_{k,nm}^*(t) \tilde{y}_{m,nk}(t)\}}{\left(\nu_{m,nk}^x(t)\right)^2} \sum_{n' \neq n}^N \psi_{n',mk}^r(t) \frac{\partial E}{\partial \Im \{\hat{r}_{n',mk}(t)\}} \\
& + \frac{\left|\hat{h}_{k,nm}(t)\right|^2}{\left(\nu_{m,nk}^x(t)\right)^2} \sum_{n' \neq n}^N \left(\psi_{n',mk}^r(t)\right)^2 \frac{\partial E}{\partial \psi_{n',mk}^r(t)}
\end{aligned}$$

$$\frac{\partial E}{\partial \nu_{m,nk}^y(t)} = \frac{\partial E}{\partial \nu_{m,nk}^h(t)} + \begin{cases} 0 & (1 \leq k \leq K_p) \\ \frac{\partial E}{\partial \nu_{m,nk}^x(t)} & (K_p + 1 \leq k \leq K) \end{cases}$$

$$\begin{aligned} \frac{\partial E}{\partial \psi_{n,mk}^x(t)} &= \left(\left| \hat{h}_{k,nm}(t) \right|^2 + \psi_{k,nm}^h(t) \right) \sum_{m' \neq m}^M \frac{\partial E}{\partial \nu_{m',nk}^y(t)} + \phi \frac{\partial E}{\partial \nu_{m,nk}^h(t)} \\ &+ \begin{cases} 0 & (1 \leq k \leq K_p) \\ (1 - \zeta_n^h(t)) \frac{\partial E}{\partial \psi_{k,nm}^h(t+1)} & (K_p + 1 \leq k \leq K) \end{cases} \end{aligned}$$

$$\begin{aligned} \frac{\partial E}{\partial \psi_{k,nm}^h(t)} &= \left(|\hat{x}_{n,mk}(t)|^2 \psi_{n,mk}^x(t) \right) \sum_{m' \neq m}^M \frac{\partial E}{\partial \nu_{m',nk}^y(t)} + \begin{cases} 0 & (1 \leq k \leq K_p) \\ E_s \frac{\partial E}{\partial \Re \left\{ \nu_{m',nk}^y(t) \right\}} & (K_p + 1 \leq k \leq K) \end{cases} \\ &+ (1 - \zeta_n^h(t)) \frac{\partial E}{\partial \nu_{m,nk}^h(t+1)} \end{aligned}$$

$$\begin{aligned}
\frac{\partial E}{\partial \Re \{\hat{x}_{n,mk}(t)\}} = & \begin{cases} 0 & (1 \leq k \leq K_p) \\ (1 - \zeta_n^x(t)) \frac{\partial E}{\partial \Re \{\hat{x}_{n,mk}(t+1)\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
& - \Re \{\hat{h}_{k,nm}(t)\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Re \{\tilde{y}_{m',nk}(t)\}} - \Im \{\hat{h}_{k,nm}(t)\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Im \{\tilde{y}_{m',nk}(t)\}} \\
& + 2\Re \{\hat{x}_{n,mk}(t)\} \psi_{k,nm}^h(t) \sum_{m' \neq m}^M \frac{\partial E}{\partial \nu_{m',nk}^y(t)} \\
& + \begin{cases} -\alpha(t) \frac{2\Re \{\hat{x}_{n,mk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \left(\psi_{k',nm}^q(t) \right)^2 \frac{\partial E}{\partial \psi_{k',nm}^q(t)} & (1 \leq k \leq K_p) \\ -\beta(t) \frac{2\Re \{\hat{x}_{n,mk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \left(\psi_{k',nm}^q(t) \right)^2 \frac{\partial E}{\partial \psi_{k',nm}^q(t)} & (K_p + 1 \leq k \leq K) \end{cases} \\
& + \begin{cases} \alpha(t) \frac{\Re \{\tilde{y}_{m,nk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Re \{\hat{q}_{k',nm}(t)\}} & (1 \leq k \leq K_p) \\ \beta(t) \frac{\Re \{\tilde{y}_{m,nk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Re \{\hat{q}_{k',nm}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
& + \begin{cases} \alpha(t) \frac{\Im \{\tilde{y}_{m,nk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Im \{\hat{q}_{k',nm}(t)\}} & (1 \leq k \leq K_p) \\ \beta(t) \frac{\Im \{\tilde{y}_{m,nk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Im \{\hat{q}_{k',nm}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E}{\partial \Im \{\hat{x}_{n,mk}(t)\}} = & \begin{cases} 0 & (1 \leq k \leq K_p) \\ (1 - \zeta_n^x(t)) \frac{\partial E}{\partial \Im \{\hat{x}_{n,mk}(t+1)\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
& + \Im \{\hat{h}_{k,nm}(t)\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Re \{\tilde{y}_{m',nk}(t)\}} - \Re \{\hat{h}_{k,nm}(t)\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Im \{\tilde{y}_{m',nk}(t)\}} \\
& + 2\Im \{\hat{x}_{n,mk}(t)\} \psi_{k,nm}^h(t) \sum_{m' \neq m}^M \frac{\partial E}{\partial \nu_{m',nk}^y(t)} \\
& + \begin{cases} -\alpha(t) \frac{2\Im \{\hat{x}_{n,mk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \left(\psi_{k',nm}^q(t) \right)^2 \frac{\partial E}{\partial \psi_{k',nm}^q(t)} & (1 \leq k \leq K_p) \\ -\beta(t) \frac{2\Im \{\hat{x}_{n,mk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \left(\psi_{k',nm}^q(t) \right)^2 \frac{\partial E}{\partial \psi_{k',nm}^q(t)} & (K_p + 1 \leq k \leq K) \end{cases} \\
& + \begin{cases} \alpha(t) \frac{\Im \{\tilde{y}_{m,nk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Re \{\hat{q}_{k',nm}(t)\}} & (1 \leq k \leq K_p) \\ \beta(t) \frac{\Im \{\tilde{y}_{m,nk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Re \{\hat{q}_{k',nm}(t)\}} & (K_p \leq k \leq K) \end{cases} \\
& + \begin{cases} -\alpha(t) \frac{\Re \{\tilde{y}_{m,nk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Im \{\hat{q}_{k',nm}(t)\}} & (1 \leq k \leq K_p) \\ -\beta(t) \frac{\Re \{\tilde{y}_{m,nk}(t)\}}{\nu_{m,nk}^h(t)} \sum_{k' \neq k}^K \psi_{k',nm}^q(t) \frac{\partial E}{\partial \Im \{\hat{q}_{k',nm}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E}{\partial \Re \{\hat{h}_{k,nm}(t)\}} &= -\Re \{\hat{x}_{n,mk}(t)\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Re \{\tilde{y}_{m',nk}(t)\}} - \Im \{\hat{x}_{n,mk}(t)\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Im \{\tilde{y}_{m',nk}(t)\}} \\
&+ 2\Re \{\hat{h}_{k,nm}(t)\} \psi_{n,mk}^r(t) \sum_{m' \neq m}^M \frac{\partial E}{\partial \nu_{m',nk}^y(t)} + (1 - \zeta_n^h(t)) \frac{\partial E}{\partial \Re \{\hat{h}_{k,nm}(t+1)\}} \\
&+ \begin{cases} 0 & (1 \leq k \leq K_p) \\ \frac{\Re \{\tilde{y}_{m,nk}(t)\}}{\nu_{m,nk}^x(t)} \sum_{n' \neq n}^N \psi_{n',mk}^r(t) \frac{\partial E}{\partial \Re \{\hat{r}_{n',mk}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
&+ \begin{cases} 0 & (1 \leq k \leq K_p) \\ \frac{\Im \{\tilde{y}_{m,nk}(t)\}}{\nu_{m,nk}^x(t)} \sum_{n' \neq n}^N \psi_{n',mk}^r(t) \frac{\partial E}{\partial \Im \{\hat{r}_{n',mk}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
&+ \begin{cases} 0 & (1 \leq k \leq K_p) \\ -\frac{2\Re \{\hat{h}_{k,nm}(t)\}}{\nu_{m,nk}^x(t)} \sum_{n' \neq n}^N (\psi_{n',mk}^r(t))^2 \frac{\partial E}{\partial \psi_{n',mk}^r(t)} & (K_p + 1 \leq k \leq K) \end{cases}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E}{\partial \Im \{\hat{h}_{k,nm}(t)\}} &= \Im \{\hat{x}_{n,mk}(t)\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Re \{\tilde{y}_{m',nk}(t)\}} - \Re \{\hat{x}_{n,mk}(t)\} \sum_{m' \neq m}^M \frac{\partial E}{\partial \Im \{\tilde{y}_{m',nk}(t)\}} \\
&+ 2\Im \{\hat{h}_{k,nm}(t)\} \psi_{n,mk}^r(t) \sum_{m' \neq m}^M \frac{\partial E}{\partial \nu_{m',nk}^y(t)} + (1 - \zeta_n^h(t)) \frac{\partial E}{\partial \Im \{\hat{h}_{k,nm}(t+1)\}} \\
&+ \begin{cases} 0 & (1 \leq k \leq K_p) \\ \frac{\Im \{\tilde{y}_{m,nk}(t)\}}{\nu_{m,nk}^x(t)} \sum_{n' \neq n}^N \psi_{n',mk}^r(t) \frac{\partial E}{\partial \Re \{\hat{r}_{n',mk}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
&+ \begin{cases} 0 & (1 \leq k \leq K_p) \\ -\frac{\Re \{\tilde{y}_{m,nk}(t)\}}{\nu_{m,nk}^x(t)} \sum_{n' \neq n}^N \psi_{n',mk}^r(t) \frac{\partial E}{\partial \Im \{\hat{r}_{n',mk}(t)\}} & (K_p + 1 \leq k \leq K) \end{cases} \\
&+ \begin{cases} 0 & (1 \leq k \leq K_p) \\ -\frac{2\Im \{\hat{h}_{k,nm}(t)\}}{\nu_{m,nk}^x(t)} \sum_{n' \neq n}^N (\psi_{n',mk}^r(t))^2 \frac{\partial E}{\partial \psi_{n',mk}^r(t)} & (K_p + 1 \leq k \leq K) \end{cases}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E}{\partial \zeta_n^h(t)} &= \sum_{k'=K_p+1}^K \sum_{m'=1}^M \left\{ (\Re \{\bar{x}_{n,m'k'}(t+1) - \Re \{\hat{x}_{n,m'k'}(t)\}\}) \frac{\partial E}{\partial \Re \{\hat{x}_{n,m'k'}(t+1)\}} \right. \\
&+ (\Im \{\bar{x}_{n,m'k'}(t+1) - \Im \{\hat{x}_{n,m'k'}(t)\}\}) \frac{\partial E}{\partial \Im \{\hat{x}_{n,m'k'}(t+1)\}} \\
&\left. + \left(E_s - |\bar{x}_{n,m'k'}(t+1)|^2 - \psi_{n,m'k'}^x(t) \right) \frac{\partial E}{\partial \psi_{n,m'k'}^x(t+1)} \right\}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E}{\partial \zeta_n^h(t)} &= \sum_{k'=1}^K \sum_{m'=1}^M \left\{ \left(\frac{\phi \Re \{\hat{q}_{k',nm'}(t)\}}{\psi_{k',nm'}^q(t) + \phi} - \Re \{\hat{h}_{k',nm'}(t)\} \right) \frac{\partial E}{\partial \Re \{\hat{h}_{k',nm'}(t+1)\}} \right. \\
&+ \left(\frac{\phi \Im \{\hat{q}_{k',nm'}(t)\}}{\psi_{k',nm'}^q(t) + \phi} - \Im \{\hat{h}_{k',nm'}(t)\} \right) \frac{\partial E}{\partial \Im \{\hat{h}_{k',nm'}(t+1)\}} \\
&\left. + \left(\frac{\phi \psi_{k',nm'}^q(t)}{\psi_{k',nm'}^q(t) + \phi} - \psi_{k',nm'}^h(t) \right) \frac{\partial E}{\partial \psi_{k',nm'}^h(t+1)} \right\}
\end{aligned}$$

$$\frac{\partial E}{\partial \eta_n^x(t)} = \frac{4 \exp(-4\eta_n^x(t))}{(1 + \exp(-4\eta_n^x(t)))^2} \frac{\partial E}{\partial \zeta_n^x(t)}$$

$$\frac{\partial E}{\partial \eta_n^h(t)} = \frac{4 \exp(-4\eta_n^h(t))}{(1 + \exp(-4\eta_n^h(t)))^2} \frac{\partial E}{\partial \zeta_n^h(t)}$$

$$\begin{aligned} \frac{\partial E}{\partial \alpha(t)} = & \sum_{k'=1}^{K_p} \sum_{m'=1}^M \sum_{n'=1}^N \left\{ \psi_{k',nm}^q(t) \sum_{i \neq k'}^{K_p} \frac{\Re \{ \hat{x}_{n,mi}^*(t) \tilde{y}_{m,ni}(t) \}}{\nu_{m,ni}^h(t)} \frac{\partial E}{\partial \Re \{ \hat{q}_{k',nm}(t) \}} \right. \\ & + \psi_{k',nm}^q(t) \sum_{i \neq k'}^{K_p} \frac{\Im \{ \hat{x}_{n,mi}^*(t) \tilde{y}_{m,ni}(t) \}}{\nu_{m,ni}^h(t)} \frac{\partial E}{\partial \Im \{ \hat{q}_{k',nm}(t) \}} \\ & \left. - \left(\psi_{k',nm}^q(t) \right)^2 \sum_{i \neq k'}^{K_p} \frac{|\hat{x}_{n,mi}(t)|^2}{\nu_{m,ni}^h(t)} \frac{\partial E}{\partial \psi_{k',nm}^q(t)} \right\} \\ & + \sum_{k'=K_p+1}^K \sum_{m'=1}^M \sum_{n'=1}^N \left\{ \psi_{k',nm}^q(t) \sum_{i=1}^{K_p} \frac{\Re \{ \hat{x}_{n,mi}^*(t) \tilde{y}_{m,ni}(t) \}}{\nu_{m,ni}^h(t)} \frac{\partial E}{\partial \Re \{ \hat{q}_{k',nm}(t) \}} \right. \\ & + \psi_{k',nm}^q(t) \sum_{i=1}^{K_p} \frac{\Im \{ \hat{x}_{n,mi}^*(t) \tilde{y}_{m,ni}(t) \}}{\nu_{m,ni}^h(t)} \frac{\partial E}{\partial \Im \{ \hat{q}_{k',nm}(t) \}} \\ & \left. - \left(\psi_{k',nm}^q(t) \right)^2 \sum_{i=1}^{K_p} \frac{|\hat{x}_{n,mi}(t)|^2}{\nu_{m,ni}^h(t)} \frac{\partial E}{\partial \psi_{k',nm}^q(t)} \right\} \end{aligned}$$

$$\begin{aligned} \frac{\partial E}{\partial \beta(t)} = & \sum_{k'=1}^{K_p} \sum_{m'=1}^M \sum_{n'=1}^N \left\{ \psi_{k',nm}^q(t) \sum_{i=K_p+1}^K \frac{\Re \{ \hat{x}_{n,mi}^*(t) \tilde{y}_{m,ni}(t) \}}{\nu_{m,ni}^h(t)} \frac{\partial E}{\partial \Re \{ \hat{q}_{k',nm}(t) \}} \right. \\ & + \psi_{k',nm}^q(t) \sum_{i=K_p+1}^K \frac{\Im \{ \hat{x}_{n,mi}^*(t) \tilde{y}_{m,ni}(t) \}}{\nu_{m,ni}^h(t)} \frac{\partial E}{\partial \Im \{ \hat{q}_{k',nm}(t) \}} \\ & \left. - \left(\psi_{k',nm}^q(t) \right)^2 \sum_{i=K_p+1}^K \frac{|\hat{x}_{n,mi}(t)|^2}{\nu_{m,ni}^h(t)} \frac{\partial E}{\partial \psi_{k',nm}^q(t)} \right\} \\ & + \sum_{k'=K_p+1}^K \sum_{m'=1}^M \sum_{n'=1}^N \left\{ \psi_{k',nm}^q(t) \sum_{i=K_p+1, i \neq k'}^K \frac{\Re \{ \hat{x}_{n,mi}^*(t) \tilde{y}_{m,ni}(t) \}}{\nu_{m,ni}^h(t)} \frac{\partial E}{\partial \Re \{ \hat{q}_{k',nm}(t) \}} \right. \\ & + \psi_{k',nm}^q(t) \sum_{i=K_p+1, i \neq k'}^K \frac{\Im \{ \hat{x}_{n,mi}^*(t) \tilde{y}_{m,ni}(t) \}}{\nu_{m,ni}^h(t)} \frac{\partial E}{\partial \Im \{ \hat{q}_{k',nm}(t) \}} \\ & \left. - \left(\psi_{k',nm}^q(t) \right)^2 \sum_{i=K_p+1, i \neq k'}^K \frac{|\hat{x}_{n,mi}(t)|^2}{\nu_{m,ni}^h(t)} \frac{\partial E}{\partial \psi_{k',nm}^q(t)} \right\} \end{aligned}$$

$$\begin{aligned} \frac{\partial E}{\partial \gamma(t)} = & \sum_{k'=K_p+1}^K \sum_{m'=1}^M \sum_{n'=1}^N \left\{ \frac{\Re \{ \hat{r}_{n',m'k'}(t) \}}{\cosh^2 \left(\frac{\gamma(t)}{c_x} \Re \{ \hat{r}_{n',m'k'}(t) \} \right)} \frac{\partial E}{\partial \Re \{ \bar{x}_{m',n'k'}(t) \}} \right. \\ & \left. + \frac{\Im \{ \hat{r}_{n',m'k'}(t) \}}{\cosh^2 \left(\frac{\gamma(t)}{c_x} \Im \{ \hat{r}_{n',m'k'}(t) \} \right)} \frac{\partial E}{\partial \Im \{ \bar{x}_{m',n'k'}(t) \}} \right\} \end{aligned}$$