

INDEFINITE INTEGRALS

Definitions

1. **Differentiation** - is a process of finding the derivative or differential of a given function.

function \rightarrow derivative or differential

2. **Integration** - is the inverse of differentiation. It is the process of finding the function whose derivative or differential is given.

derivative or differential \rightarrow function

Examples :

1.
$$\frac{d}{dx}(x^3) = 3x^2$$

2.
$$\int 3x^2 dx = x^3 + c$$

Basic Integration Formulas

1. $\int du = u + c$
2. $\int (u \pm v) dx = \int u dx \pm \int v dx$
3. $\int a u dx = a \int u dx, \quad a = \text{constant}$
4. $\int u^n dx = \frac{u^{n+1}}{n+1} + c, \quad n \neq -1$
5. $\int \frac{du}{u} = \ln|u| + c$

Examples : Evaluate the following integrals.

1.
$$\begin{aligned} \int (5x^4 - 3x^2 + 6) dx &= \int 5x^4 dx - \int 3x^2 dx + \int 6 dx \\ &= 5 \int x^4 dx - 3 \int x^2 dx + 6 \int dx \\ &= 5 \left(\frac{x^5}{5} \right) + c_1 - 3 \left(\frac{x^3}{3} \right) + c_2 + 6x + c_3 \\ &= x^5 - x^3 + 6x + (c_1 + c_2 + c_3) \end{aligned}$$

$$= x^5 - x^3 + 6x + c$$

$$\begin{aligned} 2. \quad \int (3x + 4)^2 dx &= \int (9x^2 + 24x + 16) dx \\ &= 9 \int x^2 dx + 24 \int x dx + 16 \int dx \\ &= 9 \left(\frac{x^3}{3} \right) + 24 \left(\frac{x^2}{2} \right) + 16x + c \\ &= 3x^3 + 12x^2 + 16x + c \end{aligned}$$

$$\begin{aligned} 3. \quad \int \left(\frac{4}{x^3} + \frac{2}{x} \right) dx &= \int \left(4x^{-3} + \frac{2}{x} \right) dx \\ &= 4 \int x^{-3} dx + 2 \int \frac{dx}{x} \\ &= 4 \left(\frac{x^{-2}}{-2} \right) + 2 \ln|x| + c \\ &= -\frac{2}{x^2} + 2 \ln|x| + c \\ &= 2 \ln|x| - \frac{2}{x^2} + c \end{aligned}$$

$$\begin{aligned}
4. \quad \int \left(\frac{5}{x} - 2\sqrt[3]{x^2} \right) dx &= 5 \int \frac{dx}{x} - 2 \int x^{2/3} dx \\
&= 5 \ln|x| - 2 \left(\frac{x^{5/3}}{\frac{5}{3}} \right) + c \\
&= 5 \ln|x| - 2 \left(\frac{3}{5} \right) x^{5/3} + c \\
&= 5 \ln|x| - \frac{6}{5} x^{5/3} + c
\end{aligned}$$

$$\begin{aligned}
5. \quad \int \left(\frac{x^5 + 3x - 2}{x^3} \right) dx &= \int \left(\frac{x^5}{x^3} + \frac{3x}{x^3} - \frac{2}{x^3} \right) dx \\
&= \int x^2 dx + 3 \int \frac{dx}{x^2} - 2 \int \frac{dx}{x^3} \\
&= \int x^2 dx + 3 \int x^{-2} dx - 2 \int x^{-3} dx \\
&= \frac{x^3}{3} + 3 \left(\frac{x^{-1}}{-1} \right) - 2 \left(\frac{x^{-2}}{-2} \right) + c \\
&= \frac{x^3}{3} - \frac{3}{x} + \frac{1}{x^2} + c
\end{aligned}$$

THANK YOU FOR LISTENING!

REFERENCE :

MATHEMATICAL ANALYSIS

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