INTEGRATION BY SUBSTITUTION

Evaluate the following integrals by substitution.

1. $\int (3x+4)^2 dx$

let : $u = 3x + 4$

$du = 3dx$

$dx = \frac{du}{3}$

$$\int (3x+4)^2 dx = \int u^2 \left(\frac{du}{3}\right)$$

$$= \frac{1}{3}\int u^2 du$$

$$= \frac{1}{3}\left(\frac{u^3}{3}\right) + c$$

$$= \frac{1}{9}u^3 + c$$

$$= \frac{1}{9}(3x+4)^3 + c$$

2. $\int (x^2 - 1)^4 x \, dx$

let : $u = x^2 - 1$

$du = 2x \, dx$

$x \, dx = \frac{du}{2}$

$$\int (x^2 - 1)^4 x dx = \int u^4 \left(\frac{du}{2}\right)$$

$$= \frac{1}{2} \int u^4 du$$

$$= \frac{1}{2} \left(\frac{u^5}{5}\right) + c$$

$$= \frac{1}{10} u^5 + c$$

$$= \frac{1}{10} (x^2 - 1)^5 + c$$

3. $\int sin^5 4x \, cos \, 4x \, dx$

let : $u = \sin 4x$

$$du = \cos 4x(4)dx$$
$$du = 4\cos 4x\, dx$$
$$\cos 4x\, dx = \frac{du}{4}$$
$$\int sin^5 4x\, cos\, 4x\, dx = \int u^5 \left(\frac{du}{4}\right)$$
$$= \frac{1}{4}\int u^5 du$$
$$= \frac{1}{4}\left(\frac{u^6}{6}\right) + c$$
$$= \frac{1}{24} sin^6 4x + c$$

4. $\int (4x^3 + x)\sqrt{4x^2 + 1}\, dx = \int x(4x^2 + 1)\,(4x^2 + 1)^{1/2}\, dx$
$$= \int (4x^2 + 1)^{3/2}\; x\, dx$$

let : $u = 4x^2 + 1$
$$du = 8xdx$$
$$x\, dx = \frac{du}{8}$$
$$\int (4x^3 + x)\sqrt{4x^2 + 1}\, dx = \int u^{3/2}\, \left(\frac{du}{8}\right)$$

$$\int (4x^3 + x)\sqrt{4x^2 + 1}\, dx = \frac{1}{8}\int u^{3/2}\, du$$

$$= \frac{1}{8}\left(\frac{u^{5/2}}{\frac{5}{2}}\right) + c$$

$$= \frac{1}{8}\left(\frac{2}{5}\right)u^{5/2} + c$$

$$= \frac{1}{20}(4x^2 + 1)^{5/2} + c$$

5. $\int \frac{2x^2 - 6x + 4}{x-3}\, dx$

Perform division.

$$\begin{array}{r} 2x + \frac{4}{x-3} \\ x-3 \overline{\big|\; 2x^2 - 6x + 4} \\ \underline{-(2x^2 - 6x)} \\ 4 \end{array}$$

$$\int \frac{2x^2-6x+4}{x-3}\,dx = \int \left(2x + \frac{4}{x-3}\right)\,dx$$

$$= 2\int x\,dx + 4\int \frac{dx}{x-3}$$

let : $u = x - 3$

$$du = dx$$

$$= 2\int x\,dx + 4\int \frac{du}{u}$$

$$= 2\left(\frac{x^2}{2}\right) + 4\ln|x-3| + c$$

$$= x^2 + 4\ln|x-3| + c$$

THANK YOU FOR LISTENING!

REFERENCE :

MATHEMATICAL ANALYSIS

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