Process Control

(CHE 426 Course Notes)

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Chapter 3: Conventional Control Systems and Hardware

Chapter 4: The Control System

Chapter 5: Controller Tuning

Chapter 6: Multivariable Process Control

Chapter 7: Programmable Logic Controllers

Chapter 1

Introduction to Process Control

1.1 Introduction

A control system is required to compensate for the influence of external disturbances such as changes in feed flow rate, feed conditions, product specifications, ambient temperature and so on. The most important task of a control system is to ensure safe operation by monitoring the process conditions and maintaining them within safe operating limits. Within the safe operating conditions, the control system should optimize the process performance under the influence of external disturbances. This involves maintaining product specifications, meeting production targets and making efficient use of raw materials and utilities.

A control mechanism must be used to make changes in the process to reduce or eliminate the negative impact of disturbances. Instruments must be installed to measure the operational performance of a process. These *measured variables* could include temperature, pressure, flow rate, composition, level, pH, density, and particle size. Using the information from the measured variables, other variables need to be manipulated to achieve the control objectives. A control system is then designed, which responds to variations in the measured variables and manipulates variables to control the process.

Let us consider a heat exchanger in which a process stream is heated by condensing steam as shown in Figure 1.1-1. A heat exchanger is a device that allows energy transfer between the hot and the cold streams. Heat exchangers can be classified as indirect contact type and direct contact type. Indirect contact type heat exchangers have no mixing between the hot and cold streams, only energy transfer is allowed. A control valve is installed on the hot or steam line.

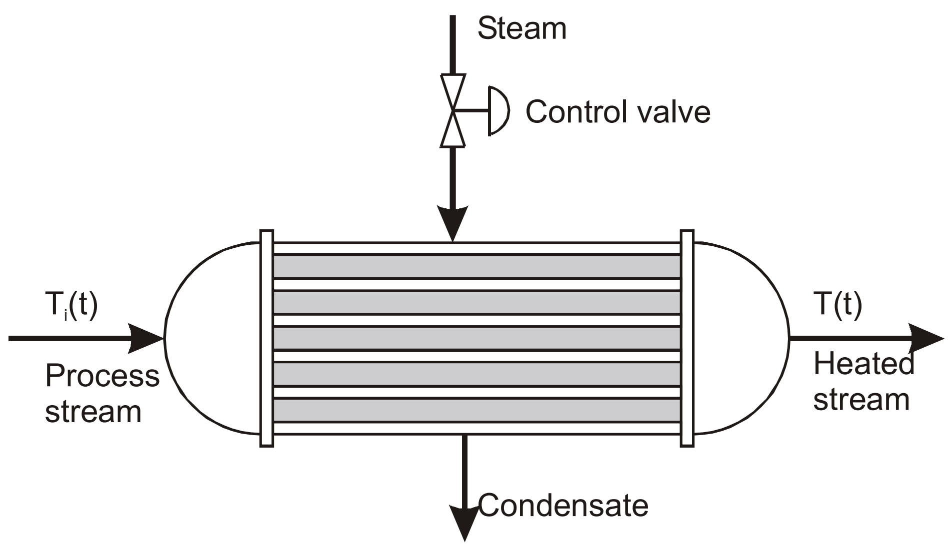


Figure 1.1-1 An indirect contact heat exchanger.

The heat exchanger is used to heat the process fluid from some inlet temperature $T_i(t)$ up to a certain desired outlet temperature $T(t)$. The energy supplied to the process stream comes from the latent heat of condensation of the steam. In this process we want to maintain the outlet process temperature at its desired value regardless of any variation in other variables such as the process stream flow rate or the inlet process temperature. One way to accomplish this objective is by measuring the outlet temperature $T(t)$, comparing it to the desired value call the set point. The outlet temperature is the measured variable. Any deviation from the set point can be corrected by adjusting the control valve to change the steam flow to the heat exchanger. The steam flow rate is the manipulated variable.

A possible control system for the heat exchanger is shown in Figure 1.1-2. This configuration is a basic feedback control loop. A sensor such as a thermocouple, a thermistor, or any resistance temperature device can measure the outlet process stream temperature. This sensor is usually connected to transmitter, which amplifies the output from the sensor and sends it to a controller. The controller compares the signal with the set point and decides the action necessary to maintain the desired temperature. The controller then sends a pneumatic or electrical signal to the final control element, which is the control valve in this case, to adjust the steam flow rate accordingly. The control valves acts as a variable resistance in the steam line since the flow rate depends on the valve stem or plug position. To regulate flow, the flow capacity of the control valve varies from zero when the valve is closed to a maximum when the valve is fully opened. Part of the job of a control engineer is to size control valves for a given service.

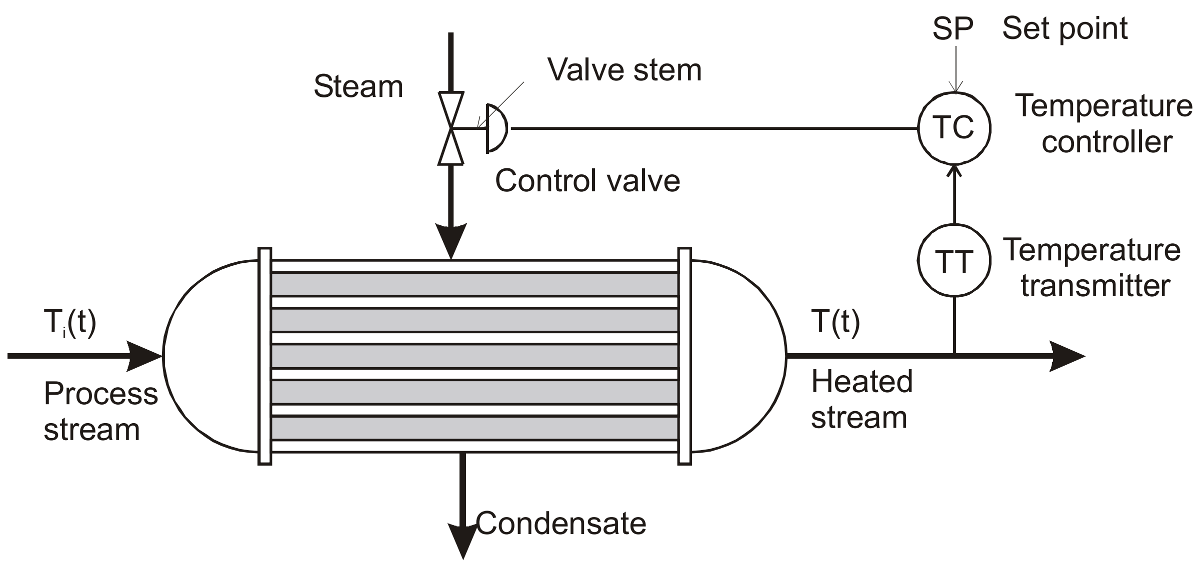


Figure 1.1-2 Heat exchanger control system using control valve.

The sensor, transmitter, and control valve are physically located on the process equipment in the field. The controller is usually located on a panel or in a computer in a control room that is some distance from the process equipment. Wires connect the two locations, carrying current signals from transmitter to the controller and from the controller to the final control element, which is the control valve in Figure 1.1-2.

The control hardware used in chemical and petroleum plants is either analog (pneumatic or electronic) or digital. The analog systems use air-pressure signals (3 to 15 psig) or current/voltage signals (4 to 20 mA, 10 to 50 mA, or 0 to 10 V DC). They are operated by instrument air supplies (25 psig air) or 24 V DC electrical power. Pneumatic systems send air-pressure signals through small tubing. Analog systems use wires.

Since most valves are still actuated by air pressure, current signals are usually converted to an air pressure. An "I/P" (current to pressure) transducer is used to convert 4 to 20 mA signals to 3 to 15 psig signals.

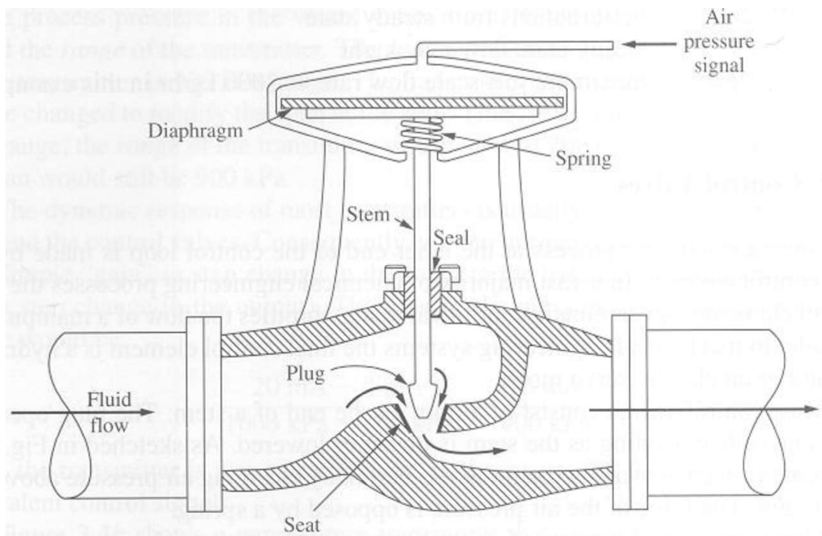


Figure 1.1-3 Typical air-operated control valve.

In chemical engineering processes the final control element is usually an automatic control valve that throttles the flow of a manipulated variable. In mechanical engineering systems the final control element is a hydraulic actuator or an electric servo motor.

Most control valves consist of a plug on the end of a stem as shown in Figure 1.1-3. The plug opens or closes an orifice opening as the stem is raised or lowered. The stem is attached to a diaphragm that is driven by changing air pressure above the diaphragm. The force of the air pressure is opposed by a spring.

A control valve is simply an orifice with a variable area of flow. The volumetric flow rate for incompressible fluid through an orifice is given by

$$Q = C_d A_o \left[\frac{2\Delta P}{\rho(1-\beta^4)}\right]^{1/2} \tag{1.1-1}$$

In this equation ΔP is the pressure drop across the orifice. For a control valve, the flow area and geometric factors, the density of the reference fluid, and the friction loss coefficient are combined into a single coefficient C_v to provide the following formula for the liquid flow through the valve

$$Q = C_v \sqrt{\frac{\Delta P_v}{SG}} = C_v(\rho_w g h_v)^{1/2} \tag{1.1-2}$$

In this equation, ΔP_v is the pressure drop across the valve, SG is the fluid specific gravity, and h_v is the head loss across the valve. The reference fluid for the density is water for liquids and air for gases. Although equation (1.1-2) is similar to equation (1.1-1), the flow coefficient C_v is not dimensionless like the discharge coefficient C_d, but has dimensions of $[L^3][L/M]^{1/2}$.

1.2 Control and Instrumentation

1.2.1 Instruments

Instruments are provided to monitor the key process variables during plant operation. They may be incorporated in automatic control loops, or used for the manual monitoring of the process operation. They may also be part of an automatic computer data logging system. Instruments monitoring critical process variables will be fitted with automatic alarms to alert the operators to critical and hazardous situations.

It is desirable that the process variable to be monitored be measured directly; often, however, this is impractical and some dependent variable, that is easier to measure, is monitored in its place. For example, in the control of distillation columns the continuous, on-line, analysis of the overhead product is desirable but difficult and expensive to achieve reliably, so temperature is often monitored as an indication of composition. The temperature instrument may form part of a control loop, say, reflux flow; with the composition of the overheads checked frequently by sampling and laboratory analysis.

1.2.2 Instrumentation and control objectives

The primary objectives of the designer when specifying instrumentation and control schemes are:

- Safe plant operation:
  - To keep the process variables within known safe operating limits.
  - To detect dangerous situations as they develop and to provide alarms and automatic shut-down systems.
  - To provide interlocks and alarms to prevent dangerous operating procedures.

- Production rate:
  - To achieve the design product output.

- Product quality:
  - To maintain the product composition within the specified quality standards.

- Cost:
  - To operate at the lowest production cost, commensurate with the other objectives.

These are not separate objectives and must be considered together. Product quality, production rate and the cost of production will be dependent on sales requirements. For example, it may be a better strategy to produce a better-quality product at a higher cost. In a typical chemical processing plant these objectives are achieved by a combination of automatic control, manual monitoring and laboratory analysis.

1.2.3 Automatic-control schemes

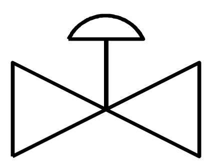
A specialist will usually perform the detailed design and specification of the automatic control schemes for a large project. In this course, only the first step in the specification of the control systems for a process will be considered: the preparation of a preliminary scheme of instrumentation and control, developed from the process flow sheet.

The following procedure can be used when drawing up preliminary P & I diagrams:

1. Identify and draw in those control loops that are obviously needed for steady plant operation, such as: level controls, flow controls, pressure controls, or temperature controls.
2. Identify the key process variables that need to be controlled to achieve the specified product quality. Include control loops using direct measurement of the controlled variable, where possible. If this is not practicable, select a suitable dependent variable.
3. Identify and include those additional control loops required for safe operation, not already covered in steps 1 and 2.
4. It is well worthwhile including additional connections for instruments that may be needed for future trouble-shooting and development, even if the instruments are not installed permanently. This would include: extra thermowells, pressure tapings, orifice flanges, and extra sample points.
5. Decide on the location of sample points.
6. Decide on the need for recorders and the location of the readout points, local or control room. This step would be done in conjunction with steps 1 to 4.
7. Decide on the alarms and interlocks needed; this would be done in conjunction with step 3.

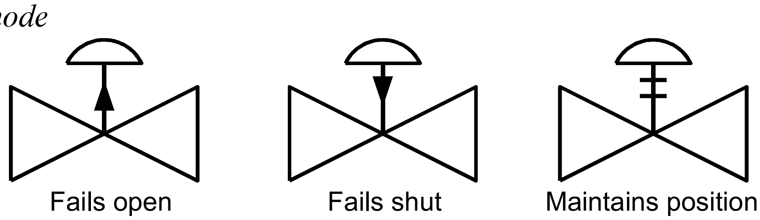
1.2.4 Basic symbols

Control valve



This symbol is used to represent all types of control valve, and both pneumatic and electric actuators.

Failure mode



The direction of the arrow shows the position of the valve on failure of the power supply.

Instruments and controllers

"*Locally mounted*" means that the controller and display is located out on the plant near to the sensing instrument location. Main panel means that they are located on a panel in the control room. Except on small plants, most controllers would be mounted in the control room.

Type of instrument. This is indicated on the circle representing the instrument-controller by a letter code (see Table 1.2-1). The first letter indicates the property measured; for example, F = flow. Subsequent letters indicate the function; for example

I = indicating
RC = recorder controller

The suffixes E and A can be added to indicate emergency action and/or alarm function.

Table 1.2-1. Letter code

Property measured	First letter	Indicating and Controlling	Recording and Controlling
Flow rate	F	FIC	FRC
Level	L	LIC	LRC
Movement or displacement	U	UIC	URC
Pressure	P	PIC	PRC
Quality, analysis or concentration	Q	QIC	QRC
Radiation	R	RIC	RRC
Speed	S	SIC	SRC
Temperature	T	TIC	TRC
Weight, mass or load	W	WIC	WRC

Lines. The instrument connecting lines should be drawn in a manner to distinguish them from the main process lines. Dotted or cross-hatched lines are normally used.

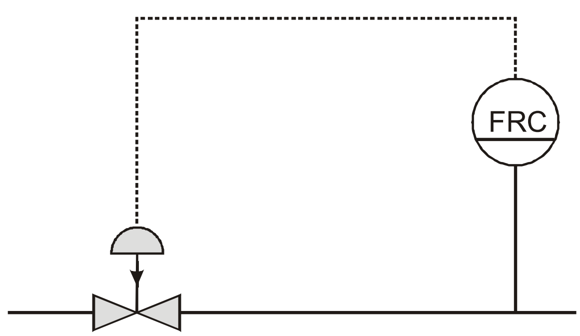


Figure 1.2-1 A typical control loop.

Chapter 2

Integral Transform

2.1 Introduction

Consider pairs of functions related by an expression of the form:

$$\hat{F}(\alpha) = \int_a^b K(\alpha, x) f(x) dx \tag{2.1-1}$$

The function $\hat{F}(\alpha)$ is called the integral transform of $f(x)$ by the kernel $K(\alpha, x)$. The operation may also be considered as mapping a function $f(x)$ in x-space into another function $\hat{F}(\alpha)$ in α-space. Figure 2.1-1 depicts the idea behind the application of integral transform. Certain problems can be solved, if at all, in the original coordinates (space). These problems might be solved relatively easily in the transform coordinates. Then, the inverse transform returns the solution from the transform coordinates to the original system.

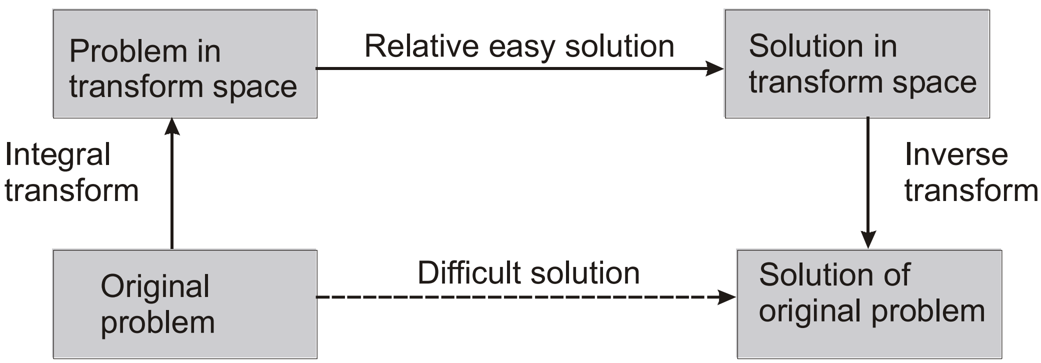


Figure 2.1-1 Application of integral transform.

Two of the most useful of the infinite number of possible transforms are the Laplace transform,

$$\hat{F}(s) = \int_0^\infty e^{-sx} f(x) dx \tag{2.1-2}$$

and the Fourier transform,

$$\hat{F}(k) = \int_{-\infty}^\infty e^{-ikx} f(x) dx \tag{2.1-3}$$

Integral transform can be used to reduce the number of independent variables in a partial differential equation by one. Thus, the one-dimensional heat equation or wave equation can be transformed into an ordinary differential equation in the transformed function $\hat{F}(\alpha)$. An ordinary differential equation becomes an algebraic equation in the transformed domain. It is usually easier to solve the resultant equation in the transform space than it is to solve the original equation.

2.2 The Laplace Transform

Laplace transform is usually used to analyze process dynamic and to design control system. Therefore one of the independent variables is time. The Laplace transform of a function $f(t)$ is defined by

$$\mathscr{L}\{f(t)\} = \widehat{F}(s) = \int_0^\infty e^{-st} f(t)dt \tag{2.2-1}$$

In this equation s is a parameter that may be complex. However we will mostly consider system with real value of s. For the Laplace transform to exist (or the integral (2.2-1) to have a finite value), s must be greater than zero if s is real, or the real part of s must be greater than zero if s is complex. The Laplace transform therefore converts a function of t into a function of s. The limits of integration show that the Laplace transform contains information on the function $f(t)$ for positive time only. This is perfectly acceptable for a physical system since nothing can be done about the past (negative time). Some examples of Laplace transform

$$\mathscr{L}\{t^2\} = \widehat{F}(s) = \int_0^\infty e^{-st} t^2 dt$$

$$\mathscr{L}\{t^2\} = -t^2 \frac{e^{-st}}{s}\Bigg|_0^\infty + \frac{2}{s}\int_0^\infty e^{-st} t dt$$

$$\mathscr{L}\{t^2\} = 0 + \frac{2}{s}\left[-t\frac{e^{-st}}{s}\Bigg|_0^\infty + \frac{1}{s}\int_0^\infty e^{-st} dt\right] = -\frac{2}{s^2}\frac{e^{-st}}{s}\Bigg|_0^\infty$$

$$\mathscr{L}\{t^2\} = \frac{2}{s^3}$$

In general $\mathscr{L}\{t^n\} = \dfrac{n!}{s^{n+1}}$, where n is an integer.

$$\mathscr{L}\{e^{at}\} = \int_0^\infty e^{-st} e^{at} dt = \int_0^\infty e^{-(s-a)t} dt = \frac{1}{s-a}$$

The Matlab command Laplace will take the Laplace transform of a function $f(t)$

```
>> syms s t a
>> laplace(t^2)
ans =
2/s^3
>> laplace(exp(a*t))
ans =
1/(s-a)
```

Example 2.2-1 ---

Determine the Laplace transform of sin ωt

Solution ---

The sine wave is represented in exponential form by

$$\sin \omega t = \frac{e^{i\omega t} - e^{-i\omega t}}{2i}$$

$$\mathscr{L}\{\sin \omega t\} = \int_0^\infty \frac{e^{i\omega t} - e^{-i\omega t}}{2i} e^{at} dt = \frac{1}{2i}\int_0^\infty \left[e^{-(s-i\omega)t} - e^{-(s+i\omega)t}\right] dt$$

$$\mathscr{L}\{\sin \omega t\} = \frac{1}{2i}\left[-\frac{e^{-(s-i\omega)t}}{s-i\omega} + \frac{e^{-(s+i\omega)t}}{s+i\omega}\right]_0^\infty = \frac{1}{2i}\left[-\frac{0-1}{s-i\omega} + \frac{0-1}{s+i\omega}\right]$$

$$\mathscr{L}\{\sin \omega t\} = \frac{1}{2i}\frac{2i\omega}{s^2+\omega^2} = \frac{\omega}{s^2+\omega^2}$$

Matlab can also take the Laplace transform of sine function.

```
>> syms s t a w
>> laplace(sin(w*t))
ans =
w/(s^2+w^2)
```

For the integral $\int_0^\infty e^{-st} f(t)dt$ to exist $f(t)$ should be of *exponential order* as $t \to \infty$ or, in another word, cannot grow faster than an exponent. We say that $f(t)$ is of *exponential order* if there exists a constant α such that

$$\lim_{t \to \infty} e^{-\alpha t} f(t) = \text{finite}$$

For example, the functions 1, 4cos $2t$, $5t$sin $2t$, t^n are all of exponential order because $f(t)e^{-bt} \to 0$ as $t \to \infty$ for any $b > 0$. Function $\exp(t^2)$ is not of exponential order since it diverges much faster than e^{bt} for any value of b.

The above conditions for the Laplace transform to exist are sufficient, but not necessary. The function $t^{-1/2}$ is not of exponential order, because the function is infinite at $t = 0$. However its Laplace transform exits for all $s > 0$. In this case

$$\mathcal{L}\{t^{-1/2}\} = \hat{F}(s) = \int_0^\infty e^{-st}t^{-1/2}dt = \left(\frac{\pi}{\ }\right)^{1/2}$$

2.3 Properties of The Laplace Transform

Linearity

The Laplace transform is a linear operation. This means that if a is a constant, then

$$\mathcal{L}\{af(t)\} = a\mathcal{L}\{f(t)\} = a\hat{F}(s) \tag{2.3-1}$$

The distributive property of addition also follows from the linearity property:

$$\mathcal{L}\{af(t) + bg(t)\} = a\hat{F}(s) + b\hat{G}(s) \tag{2.3-2}$$

These properties can be derived form the definition of Laplace transform.

First Translation Property

$$\text{If } \mathcal{L}\{f(t)\} = \hat{F}(s) \text{ then } \mathcal{L}\{e^{at}f(t)\} = \hat{F}(s - a) \qquad s > a \tag{2.3-3}$$

This property is useful for evaluating transforms of functions that involve exponential functions of time.

Second Translation Property

This property deals with the translation of a function in the time axis, as shown in Figure 2.3-1.

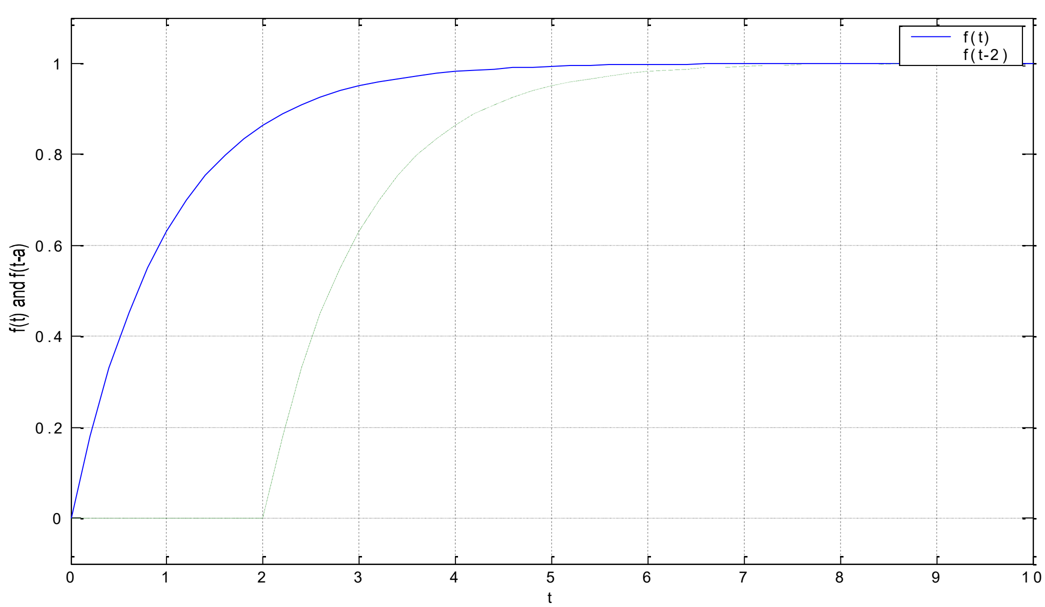


Figure 2.3-1 Function delayed in 2 unit of time.

The translated function is the original function delayed in time as function defined by

$$g(t) = \begin{cases} 0 & 0 \le t < a \\ f(t-a) & a \le t \end{cases} \tag{2.3-4}$$

In this equation $g(t)$ is simply $f(t)$ shifted a units to the right as shown in Figure 2.3-1. The second translation property is given as

$$\mathscr{L}\{f(t-a)\} = e^{-as}\,\hat{F}(s) \tag{2.3-5}$$

This property can be proved as follow

$$\mathscr{L}\{f(t-a)\} = \int_0^\infty e^{-st} f(t-a)dt$$

Let $\tau = t - a$ (or $t = \tau + a$), the above equation becomes

$$\mathscr{L}\{f(t-a)\} = \int_{-a}^\infty e^{-s(\tau+a)} f(\tau)d\tau = \int_0^\infty e^{-s\tau} e^{-as} f(\tau)d\tau$$

$$\mathscr{L}\{f(t-a)\} = e^{-as}\int_0^\infty e^{-s\tau} f(\tau)d\tau = e^{-as}\,\hat{F}(s)$$

$g(t)$ can be written in term of $f(t-a)$ with the use of unit step function $U(t)$ defined by

$$U(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 < t \end{cases}$$

With this definition $g(t) = U(t-a)f(t-a)$, and equation (2.3-5) can be written as

$$\mathscr{L}\{U(t-a)f(t-a)\} = e^{-as}\,\hat{F}(s) \tag{2.3-6}$$

Example 2.3-1[1]. ---

Find the Laplace transform of $e^{-2t}\cos \omega t$

Solution --

We have $\quad \mathscr{L}\{\cos \omega t\} = \int_0^\infty e^{-st} \cos \omega t dt = \dfrac{s}{s^2+\omega^2}$

[1] McQuarrie, D.A., *Mathematical Methods for Scientists and Engineers*, University Science Book, 2003, pg. 813

Replacing s by s + 2 gives

$$\mathcal{L}\{e^{-2t}\cos \omega t\} = \frac{s+2}{(s+2)^2+\omega^2}$$

Example 2.3-2[2]. --

Plot the function defined by $f(t) = 3 - 4(t-1)U(t-1) + 4(t-3)U(t-3)$

Solution ---

The function can be expressed as

$$f(t) = \begin{cases} 3 & 0 \le t < 1 \\ 3 - 4(t-1) = 7 - 4t & 1 \le t < 3 \\ 3 - 4(t-1) + 4(t-3) = -5 & 3 \le t \end{cases}$$

and is plotted in Figure E2.3-2

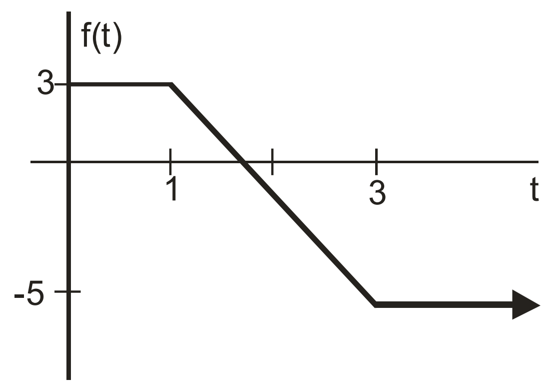


Figure E2.3-2 Plot of $f(t) = 3 - 4(t-1)U(t-1) + 4(t-3)U(t-3)$

Differentiation Property

This property, which establishes a relationship between the Laplace transform of a function and that of its derivatives, is used to transform ordinary differential equations into algebraic equations.

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = \int_0^\infty \frac{df(t)}{dt} e^{-st} dt = s\hat{F}(s) - f(0) \qquad (2.3\text{-}7)$$

This property can be obtained from the definition of Laplace transform as follows

Integrate by parts

[2] McQuarrie, D.A., *Mathematical Methods for Scientists and Engineers*, University Science Book, 2003, pg. 814

$$u = e^{-st} \Rightarrow du = -\,se^{-st}$$

$$dv = \frac{df(t)}{dt}dt \Rightarrow v = f(t)$$

$$\mathscr{L}\left\{\frac{df(t)}{dt}\right\} = e^{-st} f(t)\Big|_0^\infty - \int_0^\infty f(t)(-se^{-st})dt = -f(0) + s\int_0^\infty f(t)e^{-st}dt$$

$$\mathscr{L}\left\{\frac{df(t)}{dt}\right\} = s\,\hat{F}(s) - f(0)$$

The differentiation property can be extended to higher derivatives

$$\mathscr{L}\left\{\frac{d^n f(t)}{dt^n}\right\} = s^n \hat{F}(s) - s^{n-1}f(0) - s^{n-2}\frac{df(t)}{dt}\bigg|_{t=0} - \ldots - \frac{d^{n-1}f(t)}{dt^{n-1}}\bigg|_{t=0} \qquad (2.3\text{-}8)$$

Integration Property

This property establishes the relationship between the Laplace transform of a function and that of its integral.

$$\mathscr{L}\left\{\int_0^t f(t)dt\right\} = \frac{1}{s}\,\hat{F}(s) \qquad (2.3\text{-}9)$$

Final Value Property

This property can be used to find the final, or steady-state, value of a function from its transform. It is also useful in checking the validity of derived transforms. If the limit of $f(t)$ as $t \to \infty$ exits, then it can be found from its Laplace transform as follows:

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s\hat{F}(s) \qquad (2.3\text{-}10)$$

Example 2.3-3[3]. --

Derive the Laplace transform of the differential equation

$$9\,\frac{d^2y(t)}{dt^2} + 6\,\frac{dy(t)}{dt} + y(t) = 2x(t)$$

with initial conditions of zero at steady state: $y(0) = 0$ and $\frac{dy(t)}{dt}\Big|_{t=0} = 0$

[3] Smith and Corripio., *Principles and Practice of Automatic Process Control*, Wiley, 1997, pg. 19

Solution ---

Taking the Laplace transform of each term in the equation yields

$$9\mathscr{L}\left\{\frac{d^2y(t)}{dt^2}\right\} + 6\mathscr{L}\left\{\frac{dy(t)}{dt}\right\} + \mathscr{L}\{y(t)\} = 2\mathscr{L}\{x(t)\}$$

We apply the differentiation property to obtain

$$9s^2Y(s) + 6sY(s) + Y(s) = 2X(s)$$

Solving for $Y(s)$ gives

$$Y(s) = \frac{2}{9s^2 + 6s + 1}X(s)$$

Example 2.3-4[4]. --

Obtain the Laplace transform of the equation $c(t) = U(t - 3)[1 - e^{-(t - 3)/4}]$

Solution ---

Let $c(t) = f(t - 3) = U(t - 3)[1 - e^{-(t - 3)/4}]$ then $f(t) = U(t) - U(t)e^{-t/4}$

$$\mathscr{L}\{f(t)\} = \hat{F}(s) = \frac{1}{s} - \frac{1}{s + 1/4} = \frac{1}{s(4s+1)}$$

Applying the translation property gives

$$\mathscr{L}\{c(t)\} = C(s) = \mathscr{L}\{f(t - 3)\} = e^{-3s}\hat{F}(s)$$

Therefore $\quad C(s) = \dfrac{e^{-3s}}{s(4s+1)}$

We can use the final value property to check the validity of this answer

$$\lim_{t \to \infty} c(t) = \lim_{t \to \infty} U(t - 3)[1 - e^{-(t - 3)/4}] = 1$$

$$\lim_{s \to 0} s\hat{F}(s) = \lim_{s \to 0} s\frac{e^{-3s}}{s(4s+1)} = 1$$ **Check!**

---

[4] Smith and Corripio., *Principles and Practice of Automatic Process Control*, Wiley, 1997, pg. 20

Chapter 2

2.4 The Inversion of Laplace Transforms

Inversion is the opposite operation to taking the Laplace transform. The inversion process is represented by

$$\mathscr{L}^{1}\{\hat{F}(s)\} = f(t) \tag{2.4-1}$$

There are a various methods to invert Laplace transforms. However we can invert a great number of Laplace transform using table and partial fraction. The method of partial fraction is used to reduce the rational function $N(s)/D(s)$ to a sum of simple terms. For this rational function, $N(s)$ and $D(s)$ are polynomials in s with the degree of $N(s)$ being less than the degree of $D(s)$. The general procedure is as follows:

1. To each linear factor of the form $(As + B)^n$ in the denominator there correspond partial fractions

$$\frac{a_1}{As+B} + \frac{a_2}{(As+B)^2} + \ldots + \frac{a_n}{(As+B)^n}$$

2. To each quadratic factor of the form $(As^2 + Bs + C)^n$ in the denominator there correspond partial fractions

$$\frac{a_1 s + b_1}{As^2+Bs+C} + \frac{a_2 s + b_2}{(As^2+Bs+C)^2} + \ldots + \frac{a_n s + b_n}{(As^2+Bs+C)^n}$$

3. The constants *a's* and *b's* are found by making a common denominator of all the partial fractions and comparing the numerators of both sides of the resulting equation.

The followings are example of partial fractions:

Example 2.4-1[5]. --

Expand $\hat{F}(s) = \dfrac{s^2+5s-4}{s^3+s^2-2s}$ in terms of partial fraction.

Solution --

$$\frac{s^2+5s-4}{s^3+s^2-2s} = \frac{a}{s} + \frac{bs+c}{s^2+s-2} = \frac{(a+b)s^2+(a+c)s-2a}{s^3+s^2-2s}$$

Therefore

[5] McQuarrie, D.A., *Mathematical Methods for Scientists and Engineers*, University Science Book, 2003, pg. 825

$$s^2 + 5s - 4 = (a + b)s^2 + (a + c)s - 2a$$

We can equate the coefficients of like powers of s in the numerators on each side to get

$$a + b = 1$$

$$a + c = 5$$

$$-2a = -4$$

Hence $a = 2$, $b = -1$, and $c = 3$. We finally have

$$\frac{s^2 + 5s - 4}{s^3 + s^2 - 2s} = \frac{2}{s} + \frac{-s + 3}{s^2 + s - 2}$$

Example 2.4-2[6]. ---

Expand $\hat{F}(s) = \dfrac{2A^2 s}{s^4 - A^4}$ in terms of partial fraction and find its inversion.

Solution ---

Since $s^4 - A^4 = (s^2 + A^2)(s^2 - A^2) = (s^2 + A^2)(s + A)(s - A)$, we have

$$\frac{2A^2 s}{s^4 - A^4} = \frac{2A^2 s}{(s^2 + A^2)(s + A)(s - A)} = \frac{a}{s + A} + \frac{b}{s - A} + \frac{cs + d}{s^2 + A^2}$$

$$\frac{2A^2 s}{s^4 - A^4} = \frac{a(s - A)(s^2 + A^2) + b(s + A)(s^2 + A^2) + (cs + d)(s^2 - A^2)}{(s^2 + A^2)(s + A)(s - A)}$$

$$2A^2 s = a(s - A)(s^2 + A^2) + b(s + A)(s^2 + A^2) + (cs + d)(s^2 - A^2)$$

The above expression must be valid for any value of s.

Let $s = A \Rightarrow 2A^3 = b(A + A)(A^2 + A^2) \Rightarrow b = 0.5$

Let $s = -A \Rightarrow -2A^3 = a(-A - A)(A^2 + A^2) \Rightarrow a = 0.5$

Equating the coefficient of s^3 : $0 = a + b + c \Rightarrow c = -1$

Equating the coefficient of s^0 : $0 = d \Rightarrow d = 0$

---

[6] McQuarrie, D.A., *Mathematical Methods for Scientists and Engineers*, University Science Book, 2003, pg. 825

$$\frac{2A^2s}{s^4 - A^4} = \frac{1}{2(s+A)} + \frac{1}{2(s-A)} - \frac{s}{s^2+A^2}$$

From the table of Laplace transform

$$\mathscr{L}\{e^{at}\} = \frac{1}{s-a} \text{ and } \mathscr{L}\{\cos at\} = \frac{s}{s^2+a^2}$$

We have

$$f(t) = \frac{1}{2}\mathscr{L}^{1}\left\{\frac{1}{s+a}\right\} + \frac{1}{2}\mathscr{L}^{1}\left\{\frac{1}{s-a}\right\} - \mathscr{L}^{1}\left\{\frac{s}{s^2+a^2}\right\}$$

$$f(t) = \frac{1}{2}e^{-at} + \frac{1}{2}e^{at} - \cos at = \cosh at - \cos at$$

The Matlab statement ilaplace can take the inversion of Laplace transform as shown with the following example

```
>> syms a s t w x
>> ilaplace(s/(s*s+a*a))
ans =
cos((a^2)^(1/2)*t)

>> ilaplace(2*a^2*s/(s^4-a^4))
ans =
2*a^2/(-a^4)^(1/2)*sin(1/2*2^(1/2)*(-a^4)^(1/4)*t)*sinh(1/2*2^(1/2)*(-a^4)^(1/4)*t)
```

Example 2.4-3[7]. --

Find the inverse of $\hat{F}(s) = \dfrac{s+1}{s^2-4s+5}$

Solution --

We first complete the square in the denominator

$$\hat{F}(s) = \frac{s+1}{s^2-4s+5} = \frac{s+1}{s^2-4s+4-4+5} = \frac{s+1}{(s-2)^2+1}$$

$$\hat{F}(s) = \frac{(s-2)+3}{(s-2)^2+1} = \frac{s-2}{(s-2)^2+1} + \frac{3}{(s-2)^2+1}$$

[7] McQuarrie, D.A., *Mathematical Methods for Scientists and Engineers*, University Science Book, 2003, pg. 826

Using the first translation property $\mathscr{L}\{e^{at}f(t)\} = \widehat{F}(s - a)$ and the table of Laplace transform

$$\mathscr{L}\{\sin at\} = \frac{a}{s^2 + a^2} \text{ and } \mathscr{L}\{\cos at\} = \frac{s}{s^2 + a^2}$$

we have $\qquad f(t) = e^{2t}\cos t + 3e^{2t}\sin t$

```
>> syms s
>> ilaplace((s+1)/(s^2-4*s+5))
ans =
exp(2*t)*cos(t)+3*exp(2*t)*sin(t)
```

Example 2.4-4[8]. --

Find the inverse of $\widehat{F}(s) = \dfrac{1 - e^{-3s}}{s^2}$

Solution --

$$\frac{1 - e^{-3s}}{s^2} = \frac{1}{s^2} - \frac{e^{-3s}}{s^2}$$

Using the second translation property $\mathscr{L}\{U(t - a)f(t - a)\} = e^{-as}\widehat{F}(s)$ and the table of Laplace transform $\mathscr{L}\{t\} = \dfrac{1}{s^2}$ we have

$$f(t) = t + (t - 3)U(t - 3)$$

The unit step function $U(t)$ is also called the Heaviside function as shown in the following Matlab example

```
>> syms s
>> ilaplace((1-exp(-3*s))/s^2)
ans =
t-Heaviside(t-3)*t+3*Heaviside(t-3)
```

Example 2.4-5[9]. --

Find the inverse of $\widehat{F}(s) = \dfrac{1}{s^2} + \dfrac{e^{-s}}{s(1 - e^{-s})}$

[8] McQuarrie, D.A., *Mathematical Methods for Scientists and Engineers*, University Science Book, 2003, pg. 827

[9] McQuarrie, D.A., *Mathematical Methods for Scientists and Engineers*, University Science Book, 2003, pg. 828

Solution ---

We can expand $\dfrac{1}{1-e^{-s}}$ as a geometric series to obtain

$$\hat{F}(s) = \frac{1}{s^2} - \frac{e^{-s}}{s}(1 + e^{-s} + e^{-2s} + e^{-3s} + ...)$$

$$\hat{F}(s) = \frac{1}{s^2} - \frac{e^{-s}}{s} - \frac{e^{-2s}}{s} - \frac{e^{-3s}}{s} + ...$$

Using the second translation property $\mathscr{L}\{U(t-a)f(t-a)\} = e^{-as}\hat{F}(s)$ and the table of Laplace transform $\mathscr{L}\{t^n\} = \dfrac{n!}{s^{n+1}}$ we have

$$f(t) = t - U(t-1) - U(t-2) - U(t-3) + ...$$

$$f(t) = \begin{cases} t & 0 < t < 1 \\ t-1 & 1 < t < 2 \\ t-2 & 2 < t < 3 \\ & \vdots \end{cases}$$

Therefore $f(t)$ is a periodic sawtooth function of period 1 as shown in Figure 2.4-1.

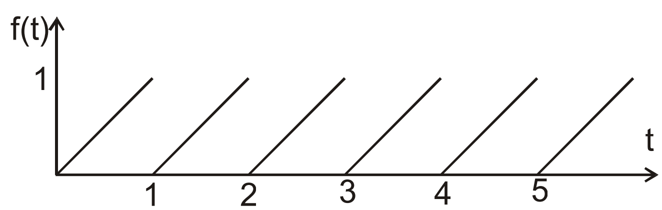


Figure 2.4-1 Plot of $f(t) = t - U(t-1) - U(t-2) - U(t-3) + ...$

Consider a periodic function with period τ so that $f(t-\tau) = f(t)$ for all value of t. The definite integral of a τ-periodic function is the same over any interval of length τ. We will use this property to take the Laplace transform a τ-periodic function.

$$\mathscr{L}\{f(t)\} = \hat{F}(s) = \int_0^\infty e^{-st} f(t)dt$$

$$\hat{F}(s) = \sum_{n=0}^{\infty} \int_{n\tau}^{(n+1)\tau} e^{-st} f(t)dt$$

In this equation each integral is over one period. Now let $t = n\tau + x$, the above equation becomes

$$\hat{F}(s) = \sum_{n=0}^{\infty} e^{-ns\tau} \int_0^{\tau} e^{-sx} f(x)dx = \sum_{n=0}^{\infty} e^{-ns\tau} \int_0^{\tau} e^{-st} f(t)dt$$

The integral is independent of *n* therefore

$$\hat{F}(s) = (1 + e^{-s\tau} + e^{-2s\tau} + e^{-3s\tau} + ...) \int_0^{\tau} e^{-st} f(t)dt$$

Let $\quad S = 1 + e^{-s\tau} + e^{-2s\tau} + e^{-3s\tau} + ...$

$$e^{-s\tau}S = e^{-s\tau} + e^{-2s\tau} + e^{-3s\tau} + ...$$

$$1 + e^{-s\tau}S = 1 + e^{-s\tau} + e^{-2s\tau} + e^{-3s\tau} + ... = S$$

Hence $\quad S = 1 + e^{-s\tau} + e^{-2s\tau} + e^{-3s\tau} + ... = \dfrac{1}{1-e^{-s\tau}}$

$$\hat{F}(s) = \frac{\int_0^{\tau} e^{-st} f(t)dt}{1-e^{-s\tau}}$$

Therefore the Laplace transform of a τ-periodic function will have a factor $\dfrac{1}{1-e^{-s\tau}}$ in the expression.

Convolution Property

If *f* and *g* are piecewise continuous and of exponential order; then

$$\mathscr{L}\{f*g\} = \mathscr{L}\{f\}\mathscr{L}\{g\} \qquad (2.4\text{-}2)$$

We wish to invert the product of two Laplace transforms

$$r(t) = \mathscr{L}^{1}\{\hat{F}(s)\ \hat{}\ (s)\}$$

In this equation

$$\hat{F}(s) = \int_0^{\infty} e^{-st} f(t)dt \quad \text{and} \quad \hat{}\ (s) = \int_0^{\infty} e^{-su} g(u)du$$

We have $\quad \hat{F}(s)\ \hat{}\ (s) = \int_0^{\infty} e^{-su}\hat{F}(s)g(u)du \qquad (2.4\text{-}3)$

From the second translation property

$$\mathscr{L}\{U(t-u)f(t-u)\} = \int_0^{\infty} e^{-st}U(t-u)f(t-u)dt = e^{-us}\hat{F}(s)$$

Substituting $e^{-us}\hat{F}(s)$ into equation (2.4-3) gives

$$\hat{F}(s)\ \hat{\ }(s) = \int_0^\infty e^{-su}\hat{F}(s)g(u)du = \int_0^\infty du\int_0^\infty e^{-st}U(t-u)f(t-u)g(u)dt$$

$$\hat{F}(s)\ \hat{\ }(s) = \int_0^\infty du\int_u^\infty e^{-st}f(t-u)g(u)dt \qquad (2.4\text{-}4)$$

We can interchange the order of integration in equation (2.4-4). Figure 2.4-2a shows the integration over t from u to ∞ followed by the integration over u. Figure 2.4-2b shows the integration over u from 0 to t followed by the integration over t. The two processes cover the same area of the *ut*-diagram.

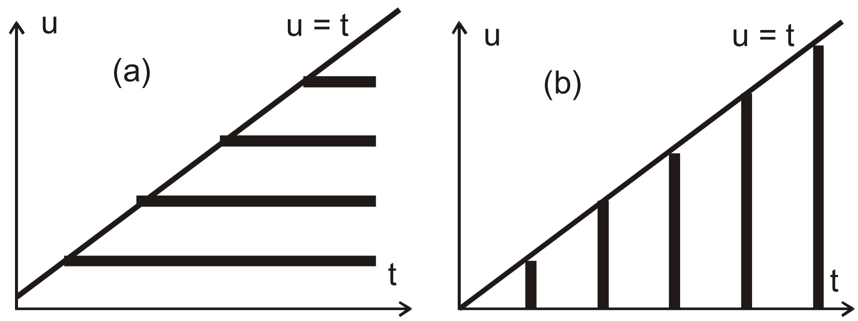


Figure 2.4-2 A depiction of the interchange of the order of integration.

We have

$$\hat{F}(s)\ \hat{\ }(s) = \int_0^\infty e^{-st}dt\int_0^t f(t-u)g(u)du \qquad (2.4\text{-}5)$$

Equation (2.4-5) is just the Laplace transform of $\int_0^t f(t-u)g(u)du$. This type of integral is called *a convolution integral* and is denoted by

$$f*g = \int_0^t f(t-u)g(u)du \qquad (2.4\text{-}6)$$

Therefore

$$\mathscr{L}\{f*g\} = \int_0^\infty e^{-st}dt\int_0^t f(t-u)g(u)du = \hat{F}(s)\ \hat{\ }(s) \qquad (2.4\text{-}7)$$

or

$$f*g = \int_0^t f(t-u)g(u)du = \mathscr{L}^{1}\{\hat{F}(s)\ \hat{\ }(s)\} \qquad (2.4\text{-}8)$$

The convolution operation is commutative; that is, $f*g = g*f$

Example 2.4-6[9]. --

(a) Evaluate $\mathscr{L}\{\int_0^t f(t-u)\sin(u)du\}$

(b) Evaluate $\mathscr{L}^{1}\{\frac{1}{s^2(s^2+4s+5)}\}$ using convolutions.

Solution ---

(a) $$\mathscr{L}\{\int_0^t f(t-u)\sin(u)du\} = \mathscr{L}\{t\}\mathscr{L}\{\sin t\} = \frac{1}{s^2}\frac{1}{s^2+1}$$

(b) We consider the expression $\frac{1}{s^2(s^2+4s+5)}$ as a product of the two Laplace transforms $\frac{1}{s^2}$ and $\frac{1}{s^2+4s+5}$.

We have $\mathscr{L}^{1}\{\frac{1}{s^2}\} = t$

and $\mathscr{L}^{1}\{\frac{1}{s^2+4s+5}\} = \mathscr{L}^{1}\{\frac{1}{(s+2)^2+1}\} = e^{-2t}\sin t$

Therefore $\mathscr{L}^{1}\{\frac{1}{s^2(s^2+4s+5)}\} = t*e^{-2t}\sin t = \int_0^t (t-u)e^{-2u}\sin(u)du$

Note: $t*e^{-2t}\sin t$ denotes the convolution of t and $e^{-2t}\sin t$

The integration in u can be evaluated directly by integration by parts to obtain

$$\mathscr{L}^{1}\{\frac{1}{s^2(s^2+4s+5)}\} = \frac{1}{25}(5t-4) + \frac{4}{25}e^{-2t}\cos t + \frac{3}{25}e^{-2t}\sin t$$

----

[9] Asmar, N., *Partial Differential Equations and Boundary Value Problems*, Prentice Hall, 2000, pg. 390

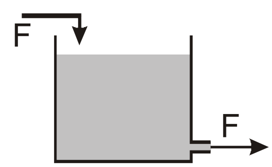
Chapter 2

2.5 Dynamic Response

The dynamic response of a system is its behavior with respect to time. If we are interested in the level of the liquid in the tank then its dynamic response is the value of the liquid level as a function of time. We generally are interested in the dynamic responses of all the variables that define the system: temperature, flow rate, liquid level, concentration, etc. If a first order differential equation describes the system, the dynamic response is called a first-order response. A second order response is the solution of a second order differential equation or a system of two first order ODEs. In the following example we will consider the response of species concentration in a tank due to a step change in the inlet value.

Example 2.5-1. ---

Consider a well-mixed tank with a liquid steady flow rate F of 0.080 m^3/s. The inlet concentration of species A is $C_{\text{A,in}}$ = 1.0 mol/m^3. The tank volume is 2.0 m^3. The system is at steady state when suddenly the inlet concentration increases to 3.0 mol/m^3. Determine the concentration of A, C_{A}, inside the tank as a function of time. The liquid flow rate remains constant at 0.080 m^3/s.



Solution --

Since the liquid volume V in the tank is constant, the species balance is given by

$$V\frac{dC_A}{dt} = F(C_{\text{A,in}} - C_{\text{A}}) \tag{E-1}$$

The species balance for the system at the initial steady state is

$$V\frac{dC_{As}}{dt} = F(C_{\text{As,in}} - C_{\text{As}}) = 0 \tag{E-2}$$

Subtracting equation (E-2) from equation (E-1) gives

$$V\frac{d\left(C_A - C_{As}\right)}{dt} = F[(C_{\text{A,in}} - C_{\text{As,in}}) - (C_{\text{A}} - C_{\text{As}})] \tag{E-3}$$

We now define deviation variables $C_{\text{A}}^{\text{d}} = C_{\text{A}} - C_{\text{As}}$, and $C_{\text{A,in}}^{\text{d}} = C_{\text{A,in}} - C_{\text{As,in}}$. Equation (E-3) becomes

$$V\frac{dC_A^{\,d}}{dt} = F(C_{A,in}^{\,d} - C_A^{\,d}) \tag{E-4}$$

The initial conditions for this equation are: $C_A^d(t = 0) = 0$ and $C_{A,in}^d(t = 0) = 2.0$ mol/m^3. In terms of the time constant $\tau = V/F$, equation (E-4) becomes

$$\frac{dC_A^{\,d}}{dt} + \frac{C_A^{\,d}}{\tau} = \frac{C_{A,in}^{\,d}}{\tau} \tag{E-5}$$

This is a first order linear differential equation with the general form

$$\frac{dC_A^{\,d}}{dt} + \alpha(t)\, C_A^{\,d} = f(t) \tag{E-6}$$

where

$$\alpha(t) = \frac{1}{\tau}$$

$$f(t) = \frac{C_{A,in}^{\,d}}{\tau}$$, $f(t)$ is called the forcing function.

To solve the first order equation, we need to find a function $I(t)$ called the integrating factor. When the integrating factor is multiplied to Eq. (E-6)

$$I(t)\left[\frac{dC_A^{\,d}}{dt} + \alpha(t)\, C_A^{\,d}\right] = I(t)\, f(t) \tag{E-7}$$

The left hand side of Eq. (E-7) will become an exact derivative

$$\frac{d}{dt}\left[I(t)C_A^d\right] = I(t)\, f(t) \tag{E-8}$$

Compare Eq. (E-7) to Eq. (E-8)

$$I(t)\left[\frac{dC_A^{\,d}}{dt} + \alpha(t)\, C_A^{\,d}\right] = \frac{d}{dt}\left[I(t)C_A^d\right]$$

$$I(t)\frac{dC_A^{\,d}}{dt} + I(t)\alpha(t)\, C_A^{\,d} = I(t)\frac{dC_A^{\,d}}{dt} + C_A^{\,d}\frac{dI(t)}{dt}$$

$$I(t)\alpha(t) = \frac{dI(t)}{dt} \;;\text{ or }\; \frac{dI(t)}{I(t)} = \alpha(t)\, dt$$

The integrating factor $I(t)$ is then

$$I(t) = \exp\left[\int \alpha(t)dt\right] \tag{E-9}$$

Equation (E-8) can be integrated to obtain

$$I(t)C_A{}^{d} = \int I(t) f(t) dt \ + C \text{ ; or}$$

$$C_A{}^{d} = \frac{1}{I(t)} \int I(t) f(t) dt \ + \frac{C}{I(t)} \qquad \text{(E-10)}$$

where C is an arbitrary constant of integration.

The integrating factor for Eq. (E-6) is

$$I(t) = \exp\left[\int \alpha(t) dt\right] = \exp\left[\int \frac{1}{\tau} dt\right] = \exp\left(\frac{t}{\tau}\right)$$

The solution is then

$$C_A{}^{d} = \exp\left(-\frac{t}{\tau}\right) \int \exp\left(\frac{t}{\tau}\right) \frac{C_{A,in}^{d}}{\tau} dt \ + C \exp\left(-\frac{t}{\tau}\right)$$

$$C_A{}^{d} = C_{A,in}^{d} \exp\left(-\frac{t}{\tau}\right) \exp\left(\frac{t}{\tau}\right) + C \exp\left(-\frac{t}{\tau}\right)$$

$$C_A{}^{d} = C_{A,in}^{d} \ + C \exp\left(-\frac{t}{\tau}\right)$$

The constant C can be obtained from the initial condition $C_A{}^{d}(t = 0) = 0$

$$0 = C_{A,in}^{d} \ + C \Rightarrow C = - \ C_{A,in}^{d}$$

Hence $$C_A{}^{d} = C_{A,in}^{d} \left[1 - \exp\left(-\frac{t}{\tau}\right)\right] \qquad \text{(E-11)}$$

Equation (E-5) can also be solved using Laplace transform.

$$\mathscr{L}\left\{\frac{dC_A{}^{d}}{dt} + \frac{C_A^{d}}{\tau}\right\} = \mathscr{L}\left\{\frac{C_{A,in}{}^{d}}{\tau}\right\}$$

$$s\,\hat{C}_A^{d}(s) + \ C_A^{d}(t=0) \ + \frac{\cancel{\hat{\ }^{d}_{A}(\ )}}{\tau} = \frac{C_{A,in}{}^{d}}{\tau} \, \frac{1}{s}$$

$$\hat{C}_A^{d}(s) \left(s + \frac{1}{\tau}\right) = \frac{C_{A,in}{}^{d}}{\tau} \, \frac{1}{s} \Rightarrow \hat{C}_A^{d}(s) \ = C_{A,in}{}^{d} \ \frac{1}{s\left(\tau s + 1\right)}$$

$$\hat{C}_A^d(s) = C_{A,in}{}^d\left[\frac{1}{s} - \frac{\tau}{\tau s+1}\right] = C_{A,in}{}^d\left[\frac{1}{s} - \frac{1}{s+\frac{1}{\tau}}\right]$$

Taking the inverse Laplace transform of the above equation we have

$$C_A{}^d = C_{A,in}^d\left[1-\exp\left(-\frac{t}{\tau}\right)\right] \tag{E-11}$$

In this equation, $C_{A,in}^d = C_{A,in} - C_{As,in} = 3 - 1 = 2$ mol/m^3. $\tau = V/F = 2/0.08 = 25$ s. Eq. (E-11) is plotted in Figure E-1. The salt concentration in the tank reaches steady state in about four time constants or 100 seconds.

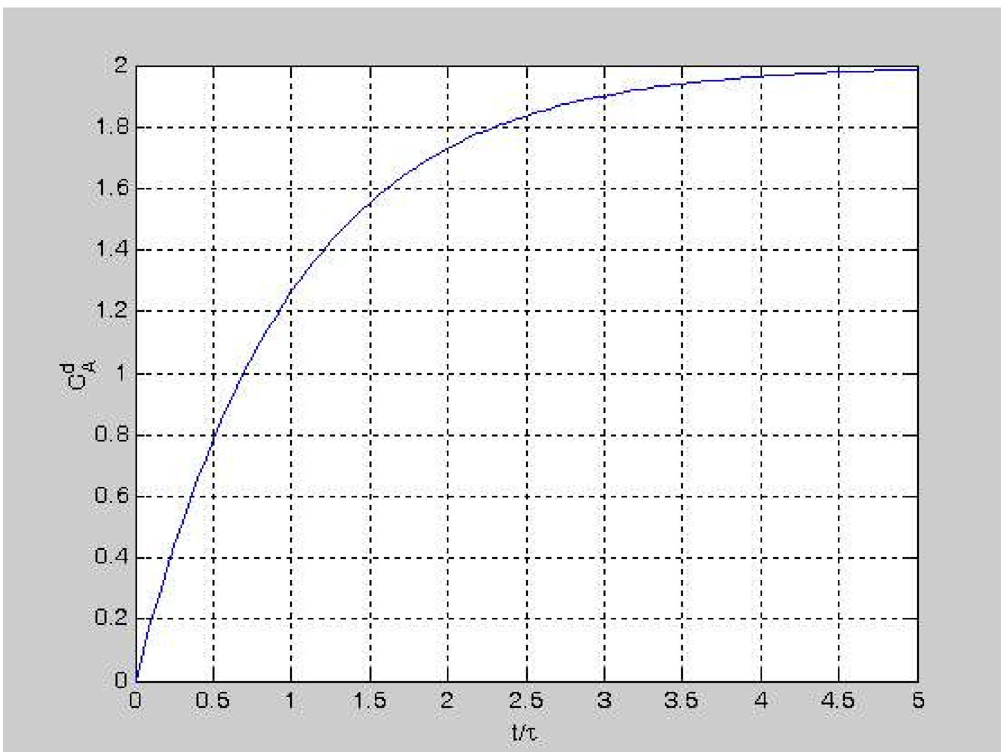
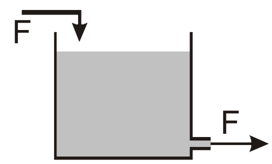


Figure E-1 Deviation of salt concentration inside a well-mixed tank.

2.6 Transfer Function

Consider a well-mixed tank with a liquid steady flow rate F of 0.080 m^3/s. The inlet concentration of species A is $C_{A,in}$ = 1.0 mol/m^3. The tank volume is 2.0 m^3. The system is at steady state when suddenly the inlet concentration changes. Determine the concentration of A, C_A, inside the tank as a function of time. The liquid flow rate remains constant at 0.080 m^3/s.



This problem is similar to example 2.5-1 except the inlet concentration $C_{A,in}$ is now deviated from its steady value by an arbitrary function of time. As before the species balance is given by

$$V\frac{dC_A}{dt} = F(C_{A,in} - C_A) \qquad (2.6\text{-}1)$$

The species balance for the system at the initial steady state is

$$V\frac{dC_{As}}{dt} = F(C_{As,in} - C_{As}) = 0 \qquad (2.6\text{-}2)$$

Subtracting equation (2.6-2) from equation (2.6-1) gives

$$V\frac{d\left(C_A - C_{As}\right)}{dt} = F[(C_{A,in} - C_{As,in}) - (C_A - C_{As})] \qquad (2.6\text{-}3)$$

We now define deviation variables $C_A^d = C_A - C_{As}$, and $C_{A,in}^d = C_{A,in} - C_{As,in}$. Equation (2.6-3) becomes

$$V\frac{dC_A^d}{dt} = F(C_{A,in}^d - C_A^d) \qquad (2.6\text{-}4)$$

At the time equals to zero, $C_A^d(t = 0) = 0$. In terms of the time constant $\tau = V/F$, equation (2.6-4) becomes

$$\frac{dC_A^d}{dt} + \frac{C_A^d}{\tau} = \frac{C_{A,in}^d}{\tau} \qquad (2.6\text{-}5)$$

The variable $C_{A,in}^d$ is called the forcing function or input variable. Equation (E-5) can also be solved using Laplace transform.

$$\mathscr{L}\left\{\frac{dC_A^{\ d}}{dt}+\frac{C_A^d}{\tau}\right\} = \mathscr{L}\left\{\frac{C_{A,in}^{\ \ d}}{\tau}\right\}$$

$C_{A,in}^{d}$ is now a function of time, therefore the Laplace transform of the above equation becomes:

$$s\,\hat{C}_A^d(s) + C_A^d(t=0) + \frac{\hat{C}_A^d(s)}{\tau} = \frac{\hat{C}_{A,in}^{\ \ d}(s)}{\tau}$$

$$\hat{C}_A^d(s)\left(s+\frac{1}{\tau}\right) = \frac{\hat{C}_{A,in}^{\ \ d}(s)}{\tau} \Rightarrow \frac{\hat{C}_A^d(s)}{\hat{C}_{A,in}^{\ \ d}(s)} = \frac{1}{(\tau s+1)} \qquad (2.6\text{-}6)$$

The above expression has the following form

$$\frac{Y(s)}{X(s)} = G(s) \qquad (2.6\text{-}7)$$

$G(s) = \dfrac{1}{(\tau s+1)}$ is called the transfer function of the system. In general the transfer function is the ratio of the response or output variable to the forcing function or input variable both in deviation form and in Laplace domain. The functional relationship contained in a transfer function is often expressed by a block diagram as shown in Figure 2.6-1.

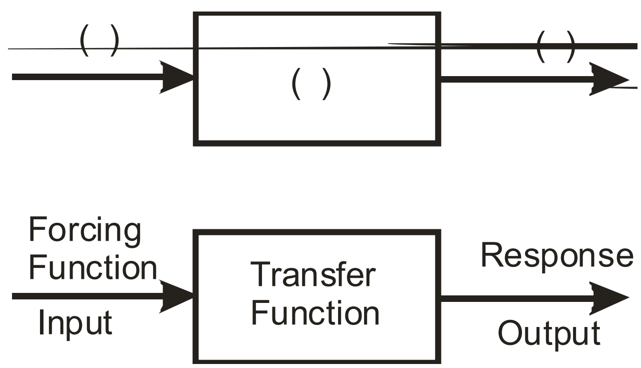


Figure 2.6-1 Block diagram

The transfer function completely described the dynamic characteristics of the system. For a given input variation $X(t)$ and its Laplace transform $X(s)$, the response of the system is simply

$$Y(s) = G(s)\,X(s) \qquad (2.6\text{-}8)$$

In example 2.5-1, $C_{A,in}^{d} = 2U(t) \Rightarrow \hat{C}_{A,in}^{\ \ d}(s) = \frac{2}{s}$. Therefore

$$\hat{C}_A^d(s) = \frac{2}{s}\frac{1}{(\tau s+1)} = 2\left[\frac{1}{s} - \frac{\tau}{\tau s+1}\right] = 2\left[\frac{1}{s} - \frac{1}{s+\frac{1}{\tau}}\right]$$

Taking the inverse Laplace transform of the above equation we have

$$C_A{}^{d} = 2\left[1 - \exp\left(-\frac{t}{\tau}\right)\right]$$

For $C_{A,in}{}^{d} = 2t \Rightarrow \hat{C}_{A,in}{}^{d}(s) = \dfrac{2}{s^2}$. Therefore

$$\hat{C}_A^d(s) = \frac{2}{s^2}\frac{1}{(\tau s+1)} = 2\left[-\frac{\tau}{s} + \frac{1}{s^2} + \frac{\tau^2}{\tau s+1}\right] = 2\left[\frac{1}{s} - \frac{1}{s+\frac{1}{\tau}}\right]$$

$$\hat{C}_A^d(s) = 2\left[-\frac{\tau}{s} + \frac{1}{s^2} + \frac{\tau}{s+\frac{1}{\tau}}\right]$$

Taking the inverse Laplace transform of the above equation we have

$$C_A{}^{d} = 2\left[-\tau + t + \tau\exp\left(-\frac{t}{\tau}\right)\right] = 2\left[t + \tau\left(\exp\left(-\frac{t}{\tau}\right) - 1\right)\right] \qquad (2.6\text{-}9)$$

Equation (2.6-5) describes the behavior of a first order system which has the following general form

$$\tau \frac{}{} + y = K_p x(t) \qquad (2.6\text{-}10a)$$

In this equation, y is the output variable and $x(t)$ is the input forcing function. The initial conditions are

$$y(0) = y_s = K_p x(0) = K_p x_s.$$

At the initial steady state condition we have

$$y_s = K_p x_s \qquad (2.6\text{-}10b)$$

Subtracting Eq. (2.6-10b) from (2.6-10a) yields

$$\tau\frac{d\left(y-y_s\right)}{dt} + y - y_s = K_p\,(x(t) - x_s) \qquad (2.6\text{-}11)$$

Introducing deviation variables $X = x(t) - x_s$ and $Y = y - y_s$, Eq. (2.6-11) becomes

$$\tau\frac{}{} + Y = K_p X \qquad (2.6\text{-}11)$$

The initial conditions become: $Y(0) = 0$ and $X(0) = 0$. Transforming Eq. (2.6-11), we obtain

$$\tau s Y(s) + Y(s) = K_p X(s)$$

Solving for the transfer function $G(s)$ we obtain

$$G(s) = \frac{Y(s)}{X(s)} = \frac{K_p}{\tau s + 1} \qquad (2.6\text{-}12)$$

In this form, τ is the time constant and K_p is the steady-state gain. The steady state gain K_p is the steady-state value that the system attains after being disturbed by a unit-step input $X(s) = 1/s$ so that

$$Y(s) = \frac{K_p}{s\left(\tau s + 1\right)}$$

The ultimate value of $Y(t)$ is

$$\lim_{s\to 0}\,[sY(s)] = \lim_{s\to 0}\frac{K_p}{s\left(\tau s + 1\right)} = K_p$$

Example 2.6-1. ---
Place the following transfer function in standard first-order form, and identify the time constant and steady state gain.

$$\frac{Y(s)}{X(s)} = \frac{2}{s + 1/5}$$

Solution ---

Rearranging to standard form we obtain

$$\frac{Y(s)}{X(s)} = \frac{2}{s + 1/5} = \frac{10}{5s + 1}$$

Therefore the time constant is 5 and the steady-state gain is 10.

Chapter 2

2.7 Block Diagram Algebra

The differential equations in Laplace domain can be represented by block diagrams of transfer functions. All block diagrams are formed by a combination of four basic elements: arrows, summing points, branch points, and blocks. These elements are shown in Figure 2.7-1.

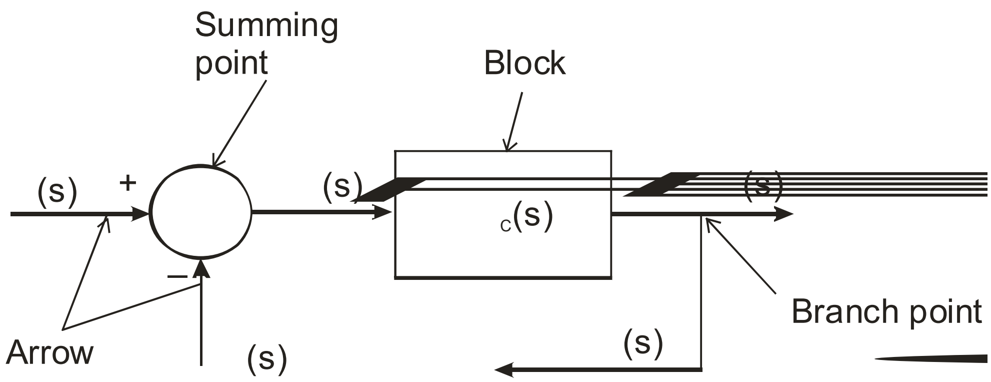


Figure 2.7-1 Elements of a block diagram.

The *arrows* indicate flow of information for process variables or control signals. The *summing points* represent the algebraic summation of the input arrows, $E(s) = R(s) - C(s)$. A *branch point* is the position on an arrow at which the information branches out and goes concurrently to other summing points or blocks. The *blocks* represent the mathematical operation in transfer function form. The arrows and block shown in Figure 2.7-1 represent the mathematical expression

$$M(s) = G_c(s)E(s) = G_c(s)[R(s) - C(s)]$$

Any block diagram can be manipulated algebraically so that a complicated block diagram can be simplified.

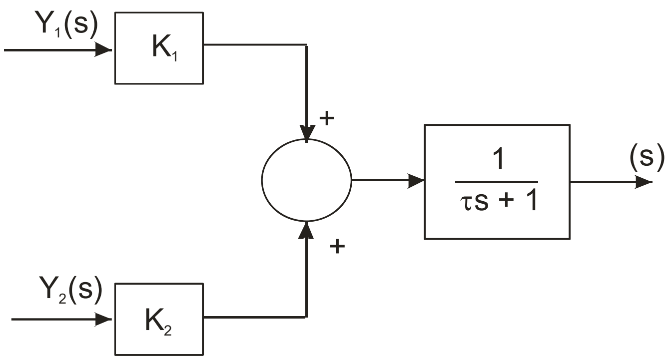
Example 2.7-1. ---

Draw the block diagram depicting the following equation

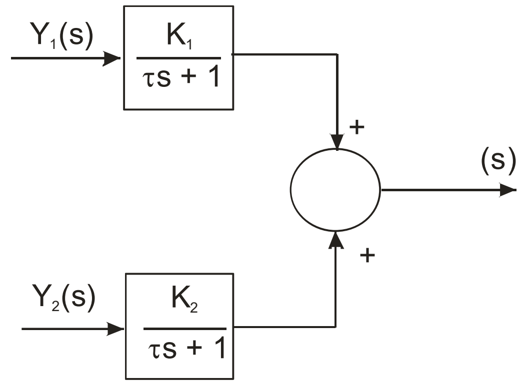
$$Y(s) = \frac{K_1}{\tau s+1}\, Y_1(s) + \frac{K_2}{\tau s+1}\, Y_2(s)$$

Solution --

The equation can be represented by the block diagram (A) or (B). The diagram (B) with fewer blocks is preferred since it is simpler.

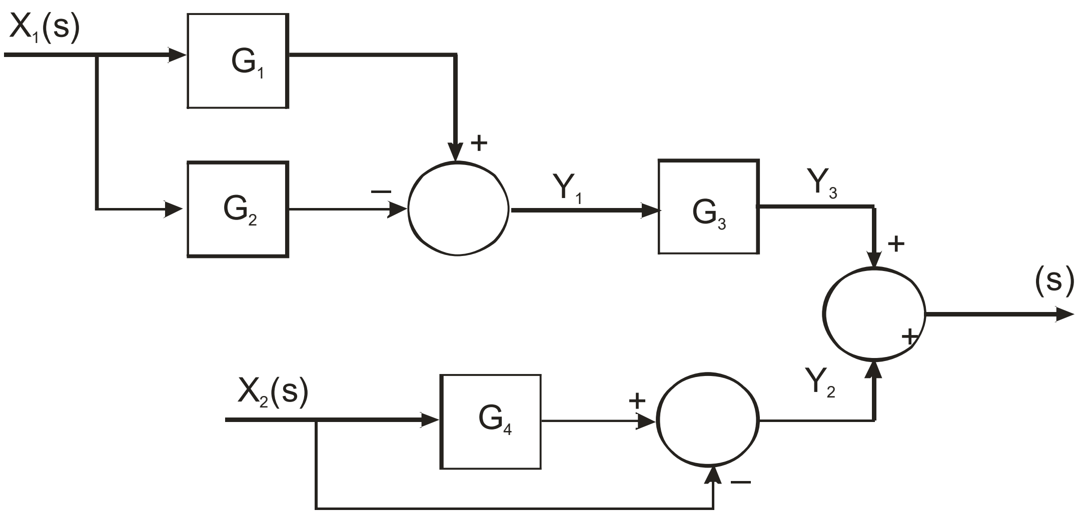


(A)

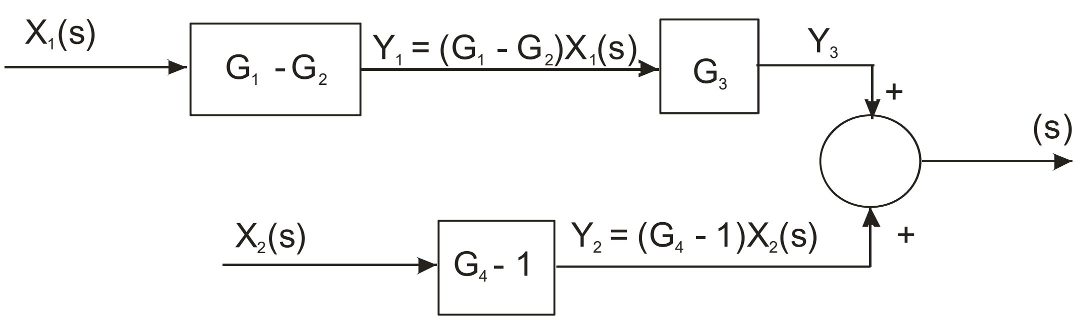


(B)

Example 2.7-2. ---
Determine $Y(s)/X_1(s)$ and $Y(s)/X_2(s)$ from the following block diagram[5].



Solution --
The block diagram can be simplified as follows



Further simplification gives

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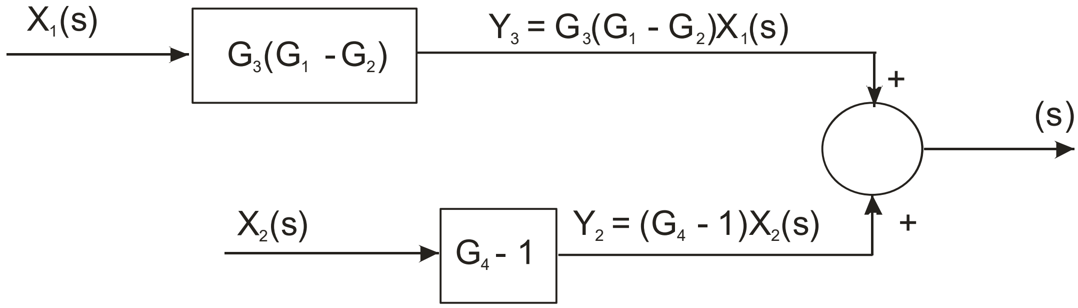
[5] Smith, C.A. & Corripio A. B., Principle and Practice of Automatic Process Control, Wiley, 1997, pg. 99.



From this diagram we have

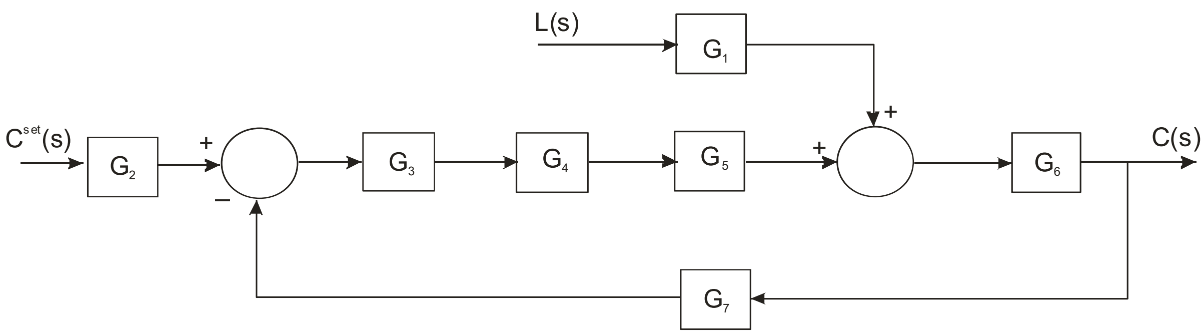
$$Y(s) = Y_3(s) + Y_2(s) = G_3(G_1 - G_2)X_1(s) + (G_4 - 1)X_2(s)$$

Therefore

$$Y(s)/X_1(s) = G_3(G_1 - G_2) \text{ and } Y(s)/X_2(s) = G_4 - 1$$

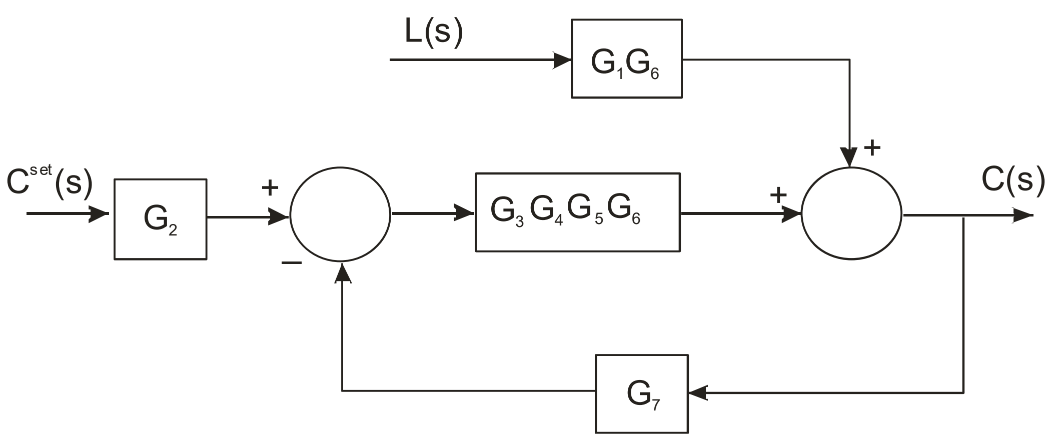
Example 2.7-3. ---

Determine $C(s)/L(s)$ and $C(s)/C^{set}(s)$ from the following block diagram[6] of a typical feedback control system.

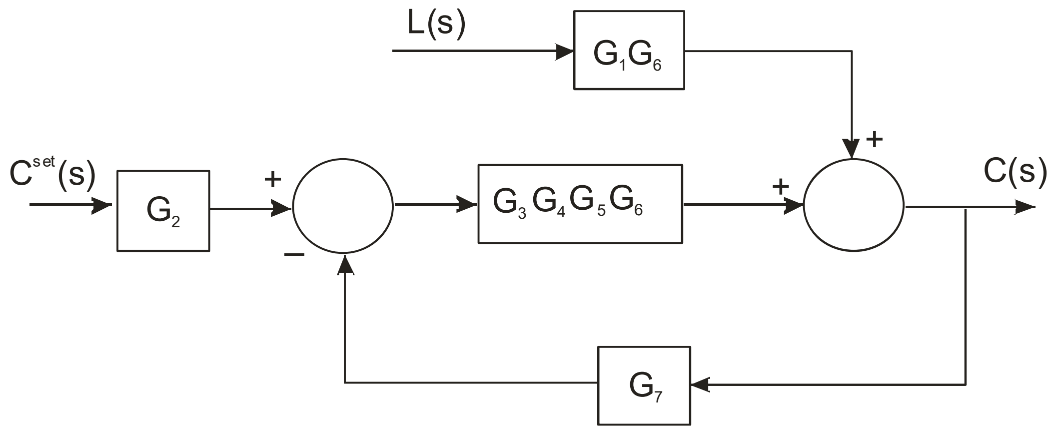


Solution --

The block diagram can be simplified as follows



[6] Smith, C.A. & Corripio A. B., Principle and Practice of Automatic Process Control, Wiley, 1997, pg. 99.



Let $G_c = G_3G_4G_5G_6$ and $G = G_1G_6$ we have

$$C(s) = GL(s) + G_c[G_2C^{set}(s) - G_7C(s)] = GL(s) + G_cG_2C^{set}(s) - G_cG_7C(s)$$

$$C(s)[1 + G_cG_7] = GL(s) + G_cG_2C^{set}(s)$$

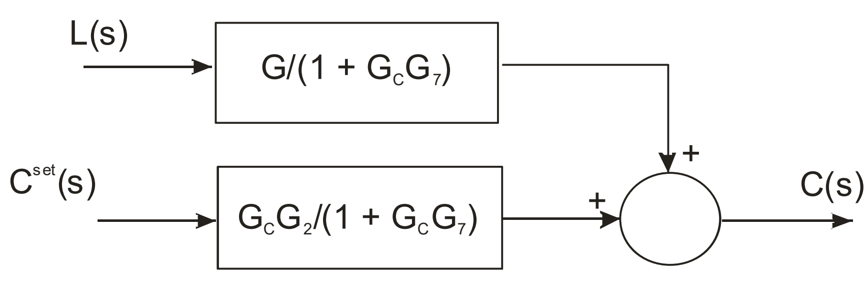
$$C(s) = \frac{G}{1+G_cG_7}L(s) + \frac{G_cG_2}{1+G_cG_7}\ C^{set}(s)$$

Therefore

$$C(s)/L(s) = \frac{G}{1+G_cG_7} = \frac{G_1G_6}{1+G_3G_4G_5G_6G_7}$$

$$C(s)/C^{set}(s) = \frac{G_cG_2}{1+G_cG_7} = \frac{G_2G_3G_4G_5G_6}{1+G_3G_4G_5G_6G_7}$$

The block diagram representation for this system is then



Chapter 3

Conventional Control Systems and Hardware

3.1 Introduction

We will study the basic principles of the control system and the common hardware that is currently used: transmitters, control valves, controllers, etc. The basic concepts of control system structure and control algorithms (type of controllers) remains essentially the same over the years despite the advance in programming and instrumentation hardware[1]. The process control engineer's job is to come up with a control system that will give good, stable, robust performance.

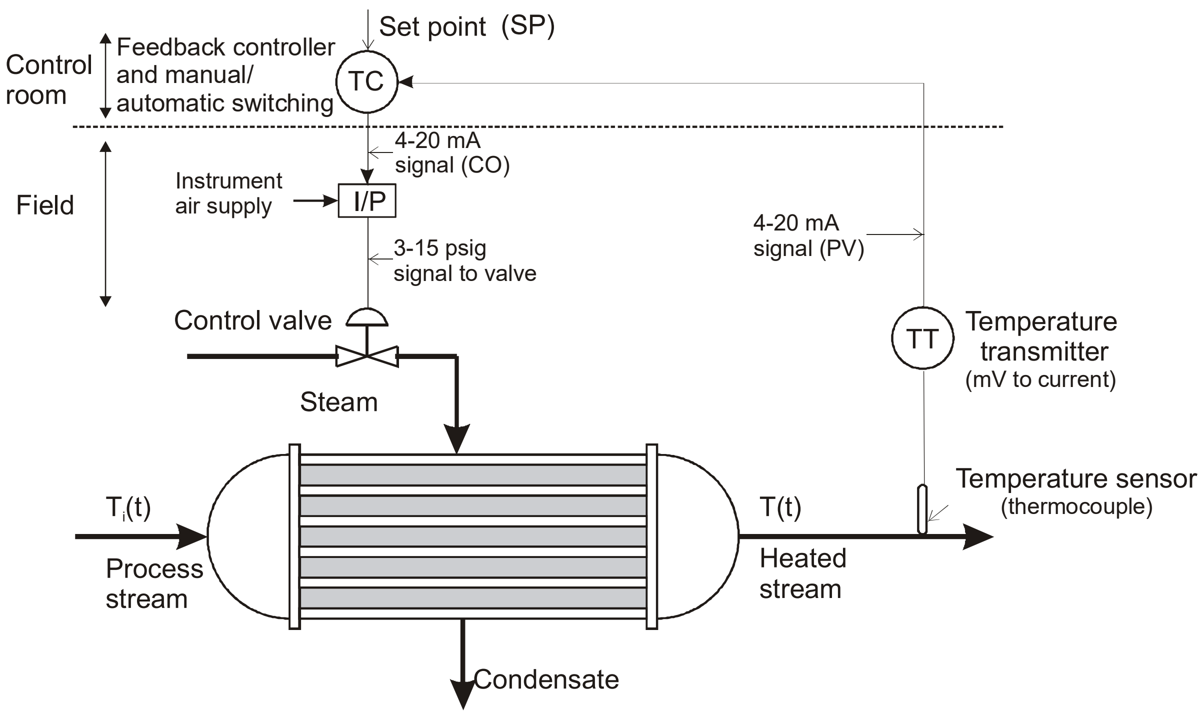


Figure 3.1-1 Heat exchanger control system using control valve.

A basic feedback control loop for a heat exchanger is shown in Figure 3.1-1. A sensor such as a thermocouple, a thermistor, or any resistance temperature device can measure the outlet process stream temperature. This sensor is usually connected to transmitter, which amplifies the output from the sensor and sends it to a controller. The controller compares the signal with the set point and decides the action necessary to maintain the desired temperature. The controller then sends a pneumatic or electrical signal to the final control element, which is the control valve in this case, to adjust the steam flow rate accordingly. The control valves acts as a variable resistance in the steam line since the flow rate depends on the valve stem or plug position. To regulate flow, the flow capacity of the control valve varies from zero when the valve is closed to a maximum when the valve is fully opened. Part of the job of a control engineer is to size control valves for a given service.

[1] Luyben, W. L. and Luyben, M. L., Essentials of Process Control, McGraw Hill, 1997, pg. 68

The sensor, transmitter, and control valve are physically located on the process equipment in the field. The controller is usually located on a panel or in a computer in a control room that is some distance from the process equipment. Wires connect the two locations, carrying current signals from transmitter to the controller and from the controller to the final control element, which is the control valve in Figure 3.1-1.

The control hardware used in chemical and petroleum plants is either analog (pneumatic or electronic) or digital. The analog systems use air-pressure signals (3 to 15 psig) or current/voltage signals (4 to 20 mA, 10 to 50 mA, or 0 to 10 V DC). They are operated by instrument air supplies (25 psig air) or 24 V DC electrical power. Pneumatic systems send air-pressure signals through small tubing. Analog systems use wires.

Since most valves are still actuated by air pressure, current signals are usually converted to an air pressure. An "I/P" (current to pressure) transducer is used to convert 4 to 20 mA signals to 3 to 15 psig signals.

The control valve can be operated manually or automatically using switching hardware or software. The plant operator may want to operate the control valve manually during start-up or under abnormal conditions. All controllers must be able to do the following:

1. Indicate the value of the controlled variable *PV*, which is the signal from the transmitter.
2. Indicate the value of the signal or the controller output *CO* being send to the valve.
3. Indicate the set point signal *SP*.
4. Have a manual/automatic/cascade switch.
5. Have a knob to set the set point when the controller is on automatic.
6. Have a knob to set the signal to the valve when the controller is on manual.

3.2 Sensors

The most common controlled variables are flow rate, temperature, pressure, and level. Devices for measuring other properties, such as composition, pH, density, viscosity, infrared and ultraviolet absorption, and refractive index are available.

3.2-1 Flow

Orifice plate or meter is the most common type of flow rate sensor. Orifice meter is a flat disk with a machine hole at the center. The disk is placed in the process line perpendicular to the fluid motion to create a pressure drop normally in the range of 20 to 200 inches of water. Since the pressure drop across the orifice varies with the square of the flow in turbulent flow, measuring the pressure difference provides a signal that can be related to flow rate. Flow rate can also be measured by turbine meters, sonic flow meters, magnetic flow meters, rotameters, vortex-shedding devices, and pitot tubes.

Turbine meter, shown in Figure 3.2-1, is more expensive but give more accurate measurement. The fluid velocity causes the rotor of the meter to spin. The rotation of the blades is detected by a magnetic pickup coil. The frequency of the pulse generated by the magnetic coil depends on the volumetric flow rate. The pulse is then converted to a 4 to 20 mA signal. The bearings of the turbine meter require clean fluids that have some lubricating properties.

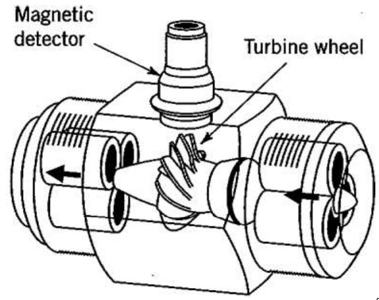


Figure 3.2-1 Turbine flow meter[2].

The operating principle behind the magnetic flow meter is Faraday's law. A voltage is created when a conductive material moves at right angles through a magnetic field. The created voltage is proportional to the intensity of the magnetic field and to the velocity of the fluid. To measure velocity, the intensity of the magnetic field is kept constant so that the voltage is proportional only to the velocity of the fluid. There is no pressure drop through the magnetic flow meter since it does not restrict flow. The magnetic flow meter can be used to measure gravity flow, slurry flows, and flow of fluids close to their vapor pressure. However it cannot be used to measure gases or hydrocarbon liquids since the fluid must have a minimum required conductivity of about 10 μohm/cm^2.

3.2-2 Temperature

Thermocouple is the most common type of temperature sensor. Temperature can also be used to infer other process variable. In distillation column or chemical reactors, temperature can be used to indicate the purity of one of the exit stream or the conversion, respectively. The thermocouple works on the Seebeck effect. An electric current will flow in a circuit of two dissimilar metals if the two junctions are at different temperatures. Figure 3.2-2 is a schematic of a simple circuit. M_1 and M_2 are the two metals, T_H is the temperature being measured, and T_C is the temperature of what is usually known as the cold, or reference, junction. The voltage produced by the Seebeck effect depends on the temperature difference between the two junctions and on the metals used. Iron-constantan thermocouples are commonly used over the 0 to 1300^{o}F temperature range.

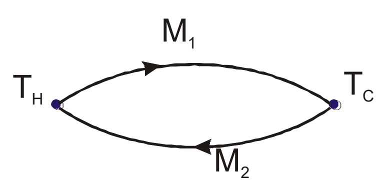


Figure 3.2-2 Simple thermocouple circuit.

Filled-system temperature sensors shown in Figure 3.2-3 are also widely used. Temperature variations cause the expansion or contraction of the fluid in the system, which is sensed by the Bourdon spring and transmitted to an indicator or transmitter. These devices are simple, inherently safe, relatively inexpensive, and reliable.

[2] Smith, C.A. & Corripio A. B., Principle and Practice of Automatic Process Control, Wiley, 1997, pg. 730.

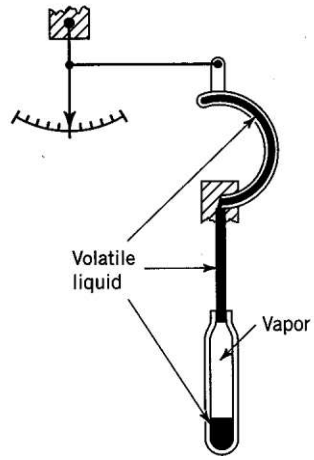


Figure 3.2-3 Typical filled-system thermometer[3].

Resistance thermometers devices (RTDs) are used where accurate temperature or differential temperature is required. These elements are based on the principle that the electrical resistance of pure metals increases with an increase in temperature. The most commonly used metals are platinum, nickel, tungsten, and copper. Figure 3.2-4 is a schematic of a typical RTD.

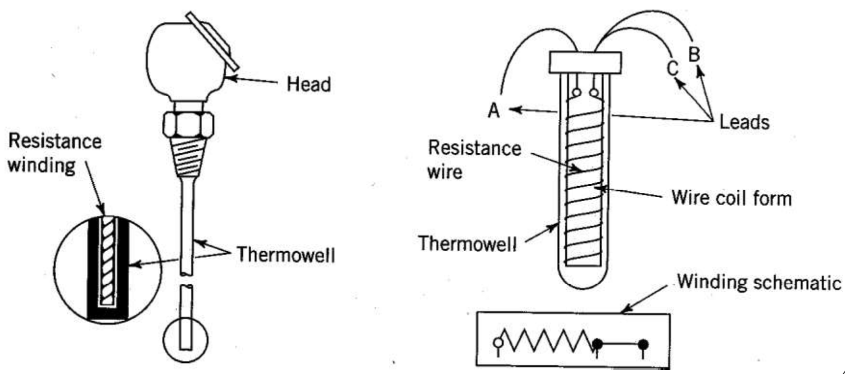


Figure 3.2-4 Schematic of the assembly and components of a RTD[4].

The bimetallic strip thermometer works on the principle that different metals expand with temperature at different rates. Figure 3.2-5 shows a typical bimetallic strip thermometer. The temperature-sensitive element is a composite of two different metals fastened together in to a strip or in a form of a spiral. One metal has a high thermal expansion coefficient (nickel-iron alloy), and the other metal has a low thermal expansion coefficient (invar: 64%Fe, 36% Ni). As the temperature increases, the spiral tends to bend toward the side of the metal with the low thermal coefficient.

[3] Smith, C.A. & Corripio A. B., Principle and Practice of Automatic Process Control, Wiley, 1997, pg. 738.
[4] Smith, C.A. & Corripio A. B., Principle and Practice of Automatic Process Control, Wiley, 1997, pg. 739.

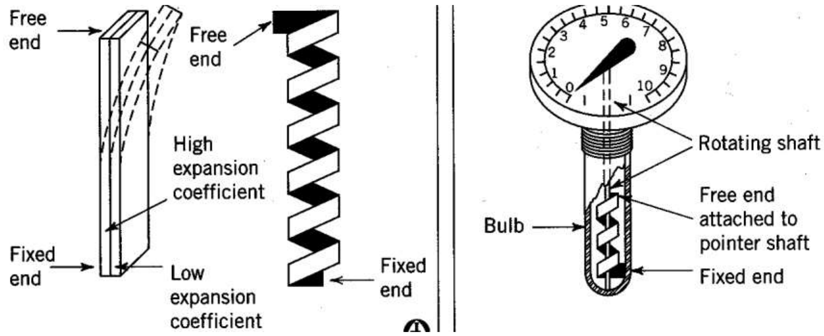


Figure 3.2-5 Bimetallic strip thermometer[5].

3.2-3 Pressure and differential pressure.

The most common pressure sensor is the Bourdon tube shown in Figure 3.2-6. The Bourdon tube is basically a piece of tubing in the form of a horseshoe with one end sealed and the other end connected to the pressure source. The tubing tends to straighten as pressure is applied, and when the pressure is released, the tubing returns to its original form so long as the elastic limit of the material of the tubing was not exceeded. The amount of straightening undergoes is proportional to the applied pressure. If the open end of the tubing is fixed, then the closed end can be connected to a pointer to indicate pressure or to a transmitter to generate a signal.

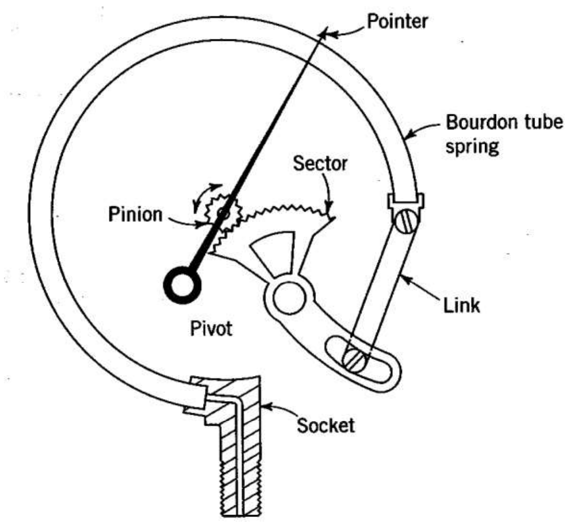


Figure 3.2-6 Simple Bourdon tube[6].

Figure 3.2-7 shows a pressure sensor using diaphragm to measure differential pressure.

[5] Smith, C.A. & Corripio A. B., Principle and Practice of Automatic Process Control, Wiley, 1997, pg. 738.
[6] Smith, C.A. & Corripio A. B., Principle and Practice of Automatic Process Control, Wiley, 1997, pg. 722.

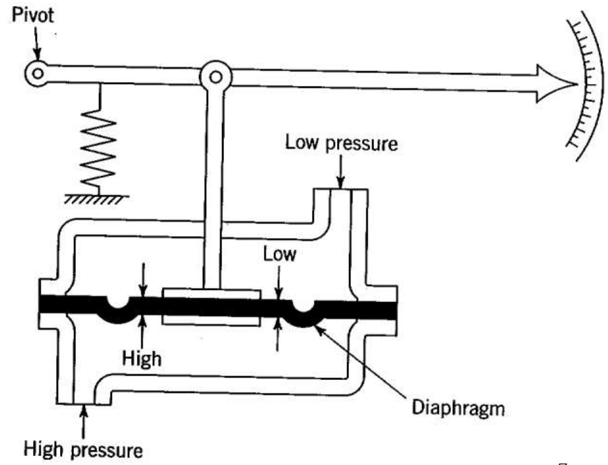


Figure 3.2-7 Differential pressure sensor[7].

3.2-4 Level

The three most common level sensors are the float, air bubbler, and differential pressure sensors. The float sensor is generally installed in an assembly mounted externally to the vessel. It detects the change in buoyant force on a body immersed in the liquid. The force required to keep the float in place, which is proportional to the liquid level, is then converted to a signal by the transmitter.

The bubbler sensor is a hydrostatic pressure sensor. It consists of an air or inert gas pipe immersed in the liquid as shown in Figure 3.2-8. The air or inert gas flow through the pipe is regulated to produce a continuous stream of bubbles. The pressure required to produce this continuous stream is a measure of the hydrostatic head or liquid level.

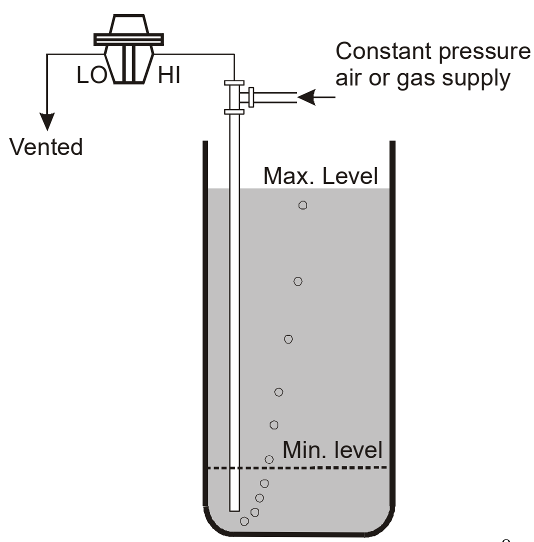


Figure 3.2-8 Air bubbler level sensor[8].

[7] Smith, C.A. & Corripio A. B., Principle and Practice of Automatic Process Control, Wiley, 1997, pg. 727.
[8] Smith, C.A. & Corripio A. B., Principle and Practice of Automatic Process Control, Wiley, 1997, pg. 734.

The differential pressure sensor measures the difference in static pressure between two fixed elevations, one in the vapor above the liquid and the other under the liquid surface. As shown in Figure 3.2-9, the differential pressure between the two level taps is directly related to the liquid level in the vessel. The side the senses the pressure at the bottom of the liquid is referred to as the high-pressure side, and the one that senses the pressure above the liquid level is referred to as the low-pressure side. If the vapors above the liquid are non-condensable, then the low-pressure piping, also known as the dry leg, can be empty. However if the vapors are likely to condense, then we need to account for or prevent condensation of vapor in the dry leg.

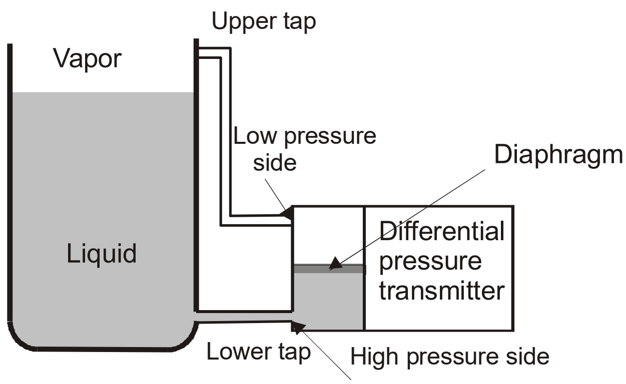


Figure 3.2-9 Differential-pressure level measurment[9].

3.3Transmitters

The transmitters convert the sensor signal (millivolts, mechanical movement, pressure differential, etc.) into a control signal (4 to 20 mA, for example). Consider the pressure transmitter shown in Figure 3.3-1. This transmitter is set up so that its output current signal varies from 4 to 20 mA as the process pressure in the vessel varies from 100 to 1000 kPa gauge. This is called the range of the transmitter. The span of the transmitter is 900 kPa. The zero of the transmitter is 100 kPa. The span and the zero can be changed. If we shifted the zero up to 200 kPa gauge, the range of the transmitter would now be 200 to 1100 kpa gauge while its span would still be 900 kPa.

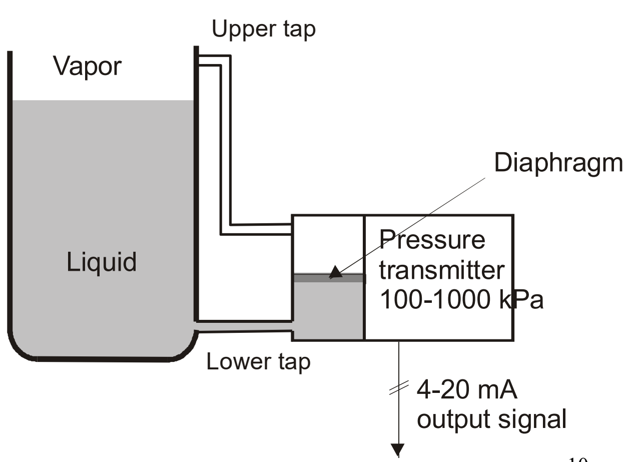


Figure 3.3-1 Typical pressure transmitter[10].

[9] Luyben, W. L. and Luyben, M. L., Essentials of Process Control, McGraw Hill, 1997, pg. 72.

The dynamic response of most transmitters is usually much faster than the responses of the process and the control valves. Therefore, we can normally consider the transmitter as a simple "gain" (a step change in the input to the transmitter gives an instantaneous step change in the output). The gain of the pressure transmitter shown in Figure 3.3-1 is given as

$$\frac{20\text{ mA} - 4\text{ mA}}{1000\text{ kPa} - 100\text{ kPa}} = \frac{16\text{ mA}}{900\text{ kPa}}$$

Thus, the transmitter is just a *transducer* that converts the process variable in kPa to an equivalent control signal in mA.

If an orifice plate is sized to give a pressure drop of 100 in H_2O at a process flow rate of 2000 kg/hr. The ΔP transmitter used with the orifice plate will convert inches of H_2O into milliamperes. The gain of the differential pressure transmitter is then 16 mA/100 in H_2O. However we really want flow rate, not orifice plate pressure drop. Since ΔP is proportional to the square of the flow rate, the relationship between flow rate F (kg/hr) and the transmitter output signal is given by

$$PV = 4 + 16\left(\frac{F}{2000}\right)^2$$

In this equation, PV is the transmitter output signal in mA. The dynamic response of temperature sensors is not much faster than the dynamics of the process and the control valves. The response time of a thermocouple and a heavy thermowell can be 30 seconds or more. If the thermowell is coated with polymer, the response time can be several minutes.

The output signal of a pneumatic transmitter is in pressure at the 15-psig output level. All pneumatic transmitters use a flapper-nozzle arrangement to produce an output signal proportional to the output from the sensor as shown in Figure 3.3-2. The bellows respond to the differential pressure and moves the lever. This moves the flapper towards or away from the nozzle.

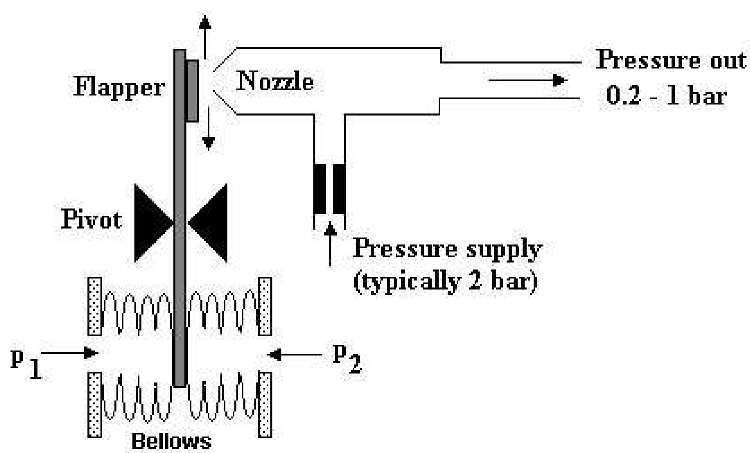


Figure 3.3-2 Flapper-nozzle arrangement.

[10] Luyben, W. L. and Luyben, M. L., Essentials of Process Control, McGraw Hill, 1997, pg. 73.

Chapter 3

3.4 Control Valves

In chemical engineering processes the final control element is usually an automatic control valve that throttles the flow of a manipulated variable. In mechanical engineering systems the final control element is a hydraulic actuator or an electric servo motor.

Most control valves consist of a plug on the end of a stem as shown in Figure 3.4-1. The plug opens or closes an orifice opening as the stem is raised or lowered. The stem is attached to a diaphragm that is driven by changing air pressure above the diaphragm. The force of the air pressure is opposed by a spring.

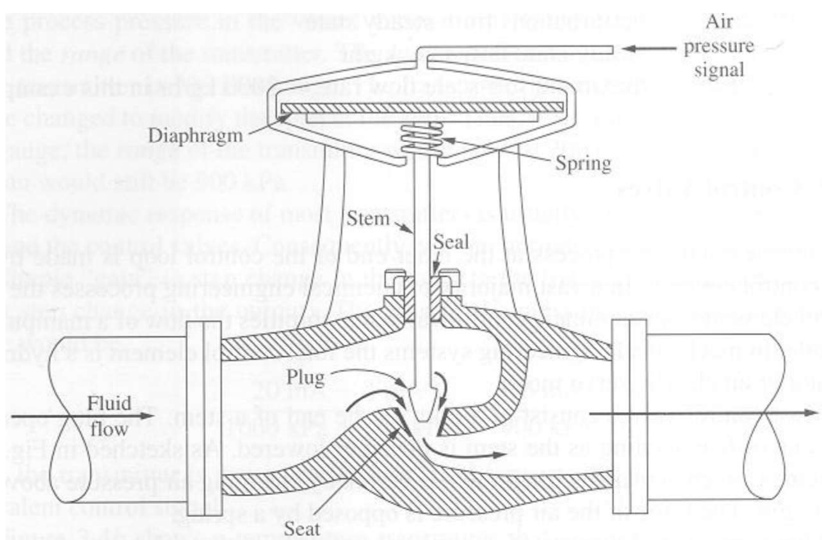


Figure 3.4-1 Typical air-operated control valve.

A control valve is simply an orifice with a variable area of flow. The volumetric flow rate F for incompressible fluid through an orifice is given by

$$F = C_d A_o \left[\frac{2\Delta P}{\rho(1-\beta^4)} \right]^{1/2} \tag{3.4-1}$$

In this equation ΔP is the pressure drop across the orifice. For a control valve, the flow area and geometric factors, the density of the reference fluid, and the friction loss coefficient are combined into a single coefficient C_v to provide the following formula for the liquid flow through the valve

$$F = C_v f_{(x)} \sqrt{\frac{\Delta P_v}{SG}} \tag{3.4-2}$$

In this equation, ΔP_v is the pressure drop across the valve, SG is the fluid specific gravity, and $f_{(x)}$ is the fraction of the total flow area of the valve. The reference fluid for the density is water for liquids and air for gases. Although equation (3.4-2) is similar to equation (3.4-1), the flow coefficient C_v is not dimensionless like the discharge coefficient C_d, but has dimensions of $[L^3][L/M]^{1/2}$.

The sizing of a control valve is a good example of an optimization problem that occurs in designing a plant. Consider the heating process shown in Figure 3.4-2 where liquid with specific gravity of 1 is pumped from a feed tank to a final tank. The flow rate at design conditions is 100 gpm, the pressure, P_0, in the feed tank is atmospheric, the pressure drop over the heat exchanger, ΔP_H, at the design flow rate is 40 psi, and the pressure in the final tank, P_2, is 150 psig. The control valve is half open ($f_{(x)} = 0.5$) at the design flow.

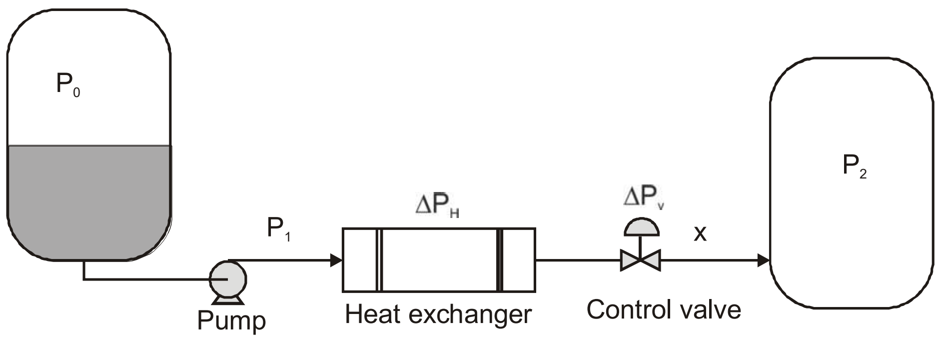


Figure 3.4-2 A simple heating and pumping operation[11].

Without considering control, the process engineer wants to design a system that has a low pressure drop across the control valve. However, the control engineer wants to take a large pressure drop over the valve since the large pressure drop will allow a large range of flow rate through the valve. We will show the calculation for the *turndown ratio* or the ratio of maximum to minimum flow rate through the valve by a numerical example. We size the valve so that it takes a 20-psi pressure drop at the design flow rate of 100 gpm when it is half open.

The valve coefficient C_v can be determine from equation (3.4-2)

$$F = C_v f_{(x)} \sqrt{\frac{\Delta P_v}{SG}} \tag{3.4-2}$$

$$100 = C_v (0.5)\sqrt{20} \Rightarrow C_v = 44.72 \text{ gpm/psi}^{0.5}$$

The pressure drop over the heat exchanger at a given flow rate F is given by

$$\Delta P_H = 40\left(\frac{F}{F_{des}}\right)^2 = 40\left(\frac{F}{100}\right)^2$$

We will assume that the pump curve is flat so that the total pressure drop across the heat exchanger and the valve is constant.

[11] Luyben, W. L. and Luyben, M. L., <u>Essentials of Process Control</u>, McGraw Hill, 1997, pg. 76.

$$\Delta P_T = \Delta P_H + \Delta P_v = 40 + 20 = 60 \text{ psi}$$

At a flow rate F, the pressure drop across the valve is given by

$$\Delta P_v = 60 - 40\left(\frac{F}{100}\right)^2$$

The flow rate is then

$$F = C_v f_{(x)} \sqrt{\frac{\Delta P_v}{SG}} = 44.72 f_{(x)} \sqrt{60 - 40\left(\frac{F}{100}\right)^2}$$

At the maximum flow rate F_{max}, the valve is wide open and $f_{(x)} = 1$.

$$F_{max} = 44.72 \sqrt{60 - 40\left(\frac{F_{max}}{100}\right)^2}$$

The nonlinear equation can be solved by the following Matlab statements

```
>> g=inline('f-44.72*sqrt(60-40*(f/100)^2)','f');
>> fmax=fsolve(g,100,optimset('Display','off'))

fmax =

  115.4697
```

The maximum flow rate through the valve is F_{max} = 115.5 gpm.

The minimum flow rate through the control valve is determined when the valve is about 10 percent open. At a smaller opening the control valve might become unstable, shutting off completely and then popping up partially open. The minimum flow rate F_{min} is then determined from the following equation

$$F_{min} = 0.1*44.72 \sqrt{60 - 40\left(\frac{F_{min}}{100}\right)^2}$$

```
>> g=inline('f-4.472*sqrt(60-40*(f/100)^2)','f');
>> fmin=fsolve(g,100,optimset('Display','off'))

fmin =

   33.3324
```

The minium flow rate through the valve is F_{min} = 33.3 gpm. The turndown ratio for the valve is 115.5/33.3 = 3.46. The following Matlab program lists the calculation of the valve coefficients C_v and turndown ratios for the control valve at a range of pressure drop ΔP_v from 10 to 100 psi.

```
----------------------------------------------------
% Control valve calculation
%
dPv=10:2:100;
nv=length(dPv);fmaxv=zeros(1,nv);fminv=zeros(1,nv);
Cvv=100.0./(0.5*sqrt(dPv));
f=100;fmin=20;
for n=1:nv
   dP=dPv(n);
   Cv=Cvv(n);
   dPt=40+dP;
for i=1:20
   fu=f-Cv*sqrt(dPt-40*(f/100)^2);
   df=1+Cv*80*f/sqrt(dPt-40*(f/100)^2)/100^2;
   er=fu/df;
   f=f-er;
   if abs(er/f)<.00001, break, end
end
for i=1:20
   fu=fmin-.1*Cv*sqrt(dPt-40*(fmin/100)^2);
   df=1+.1*Cv*80*fmin/sqrt(dPt-40*(fmin/100)^2)/100^2;
   er=fu/df;
   fmin=fmin-er;
   if abs(er/fmin)<.00001, break, end
end
fmaxv(n)=f;fminv(n)=fmin;
end
Turndown=fmaxv./fminv;
figure(1); plot(dPv,Turndown)
xlabel('dP(psi)');ylabel('Fmax/Fmin')
grid on
figure(2); plot(dPv,Cvv)
xlabel('dP(psi)');ylabel('Cv')
grid on
```

The valve coefficient C_v is plotted in Figure 3.4-3 as a function of the pressure drop across the valve at the design flow rate of 100 gpm. A smaller value of C_v means a smaller valve.

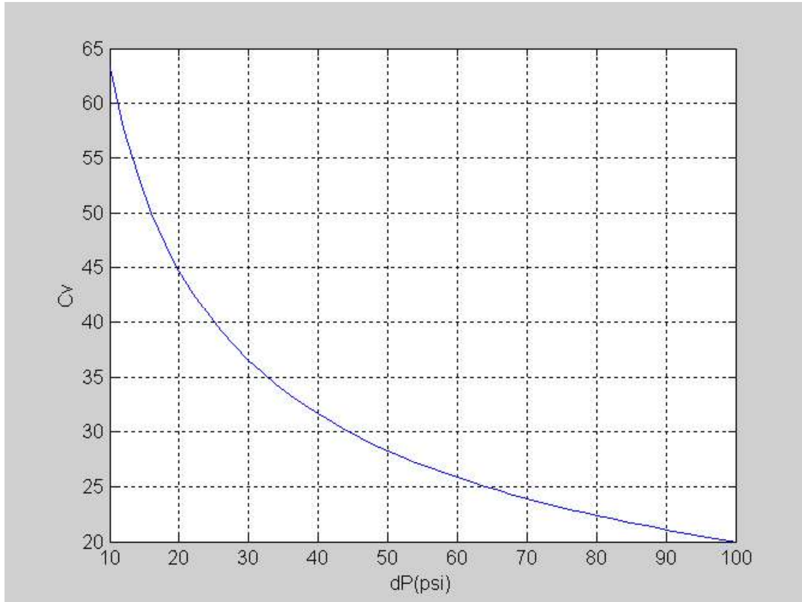


Figure 3.4-3 Valve coefficient at various design pressure drop.

The turndown ratio for the valve is plotted in Figure 3.4-4. As we can see, the turndown ratio becomes larger at higher pressure drop across the valve.

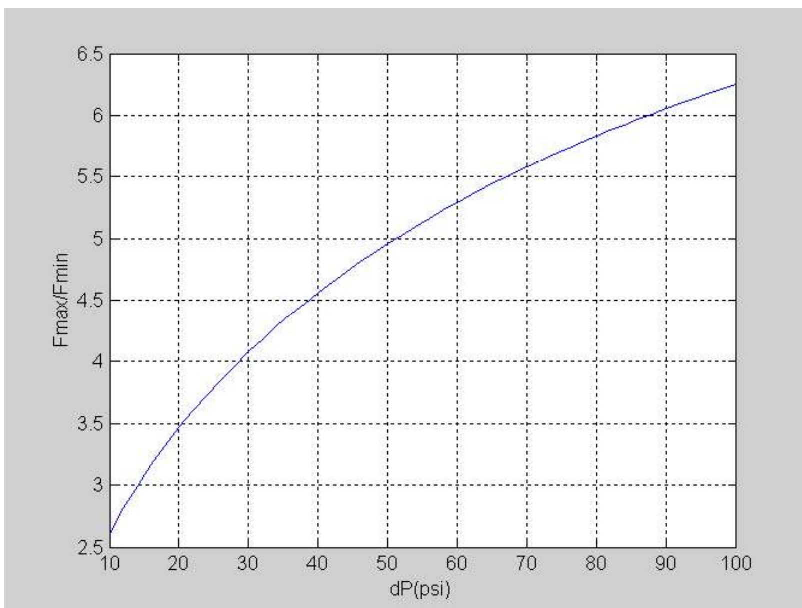


Figure 3.4-4 Turndown ratio at various design pressure drop.

In general, the designer must specify the required maximum flow rate under the worst conditions and the required minimum flow rate. Then the valve flow equations for the maximum and minimum conditions give two equations and two unknowns: the pressure head of the centrifugal pump ΔP_P and the control valve size C_v.

Example 3.4-1[12] --
We want to design a control valve for admitting cooling water to a cooling coil in an exothermic chemical reactor. The normal flow rate is 50 gpm and the minimum flow rate is 25 gpm. To prevent reactor runaways, the valve must be able to provide three times the design flow rate. The pressure drop through the cooling coil is 10 psi at the design flow rate of 50 gpm. The cooling water is pumped from an atmospheric tank. The water leaving the coil runs into a pipe in which the pressure is constant at 2 psig. Size the control valve and the pump.
Solution ---

The pressure drop through the coil depends on the flow rate *F:*

$$\Delta P_c = 10\left(\frac{F}{50}\right)^2$$

The pressure drop over the control valve is the unknown total pressure drop available minus the pressure drop over the coil.

$$\Delta P_v = \Delta P_T - 10\left(\frac{F}{50}\right)^2$$

We use the following equation with SG = 1 to determine the flow rate:

$$F = C_v f_{(x)} \sqrt{\frac{\Delta P_v}{SG}}$$

At the maximum flow rate of 150 gpm, $f_{(x)} = 1$

$$150 = C_v \sqrt{\Delta P_T - 10\left(\frac{150}{50}\right)^2}$$

At the minimum flow rate of 25 gpm, $f_{(x)} = 0.1$

$$25 = 0.1C_v \sqrt{\Delta P_T - 10\left(\frac{25}{50}\right)^2}$$

The two nonlinear equations for the flow rates can be solved by minimizing the following function

$$y = (g_1)^2 + (g_2)^2$$

In this expression, the two functions g_1 and g_2 are the rearranged flow rate equations

[12] Luyben, W. L. and Luyben, M. L., Essentials of Process Control, McGraw Hill, 1997, pg. 79.

$$g_1 = 150 - C_v\sqrt{\Delta P_T - 10\left(\frac{150}{50}\right)^2}$$

$$g_2 = 25 - 0.1C_v\sqrt{\Delta P_T - 10\left(\frac{25}{50}\right)^2}$$

We first create the following function and save as a text file named CvdP.m

```
function y=CvdP(p)
Cv=p(1);dP=p(2);
g1=150-Cv*sqrt(dP-10*(150/50)^2);
g2=25-0.1*Cv*sqrt(dP-10*(25/50)^2);
y=g1*g1+g2*g2;
```

We then use the following Matlab statement

```
>> p=fminsearch('CvdP',[20 100])

p =

  21.3809     139.2187
```

Therefore C_v = 21.38 gpm/psi$^{0.5}$ and ΔP_T = 139.2 psig. The pump head is then $\Delta P_P = \Delta P_T + 2$ = 141.2 psi.

It should be noted that values of C_v and ΔP_T can only be found for a certain range of F_{min} and F_{max}. In another word if you arbitrarily specify F_{min} and F_{max} you might not be able to solve for C_v and ΔP_T.

Control Valve Characteristics

The value of $f_{(x)}$ depends on the valve position and the valve type, which has different seat and different shape of the plug. The function $f_{(x)}$ relating the fraction of total flow area to the fraction of maximum lift or stem position is known as the *inherent valve characteristics*. There are three common valve characteristics: the *quick-opening*, *linear*, and *equal percentage* characteristics. Quick-opening valve with the characteristic shown in Figure 3.4-5 can be used for relief valves and for on-off control system but not for regulating flow. Most of the variation in flow takes place in the lower third of the valve travel and very little flow variation takes place for most of the valve travel.

The two characteristics normally used to regulate flow are the linear and equal percentage characteristics. If constant pressure drop over the valve is assumed and if the stem position is 50 percent open, a linear-trim valve give 50 percent of the maximum flow and an equal-percentage-trim valve gives only 15 percent of the maximum flow.

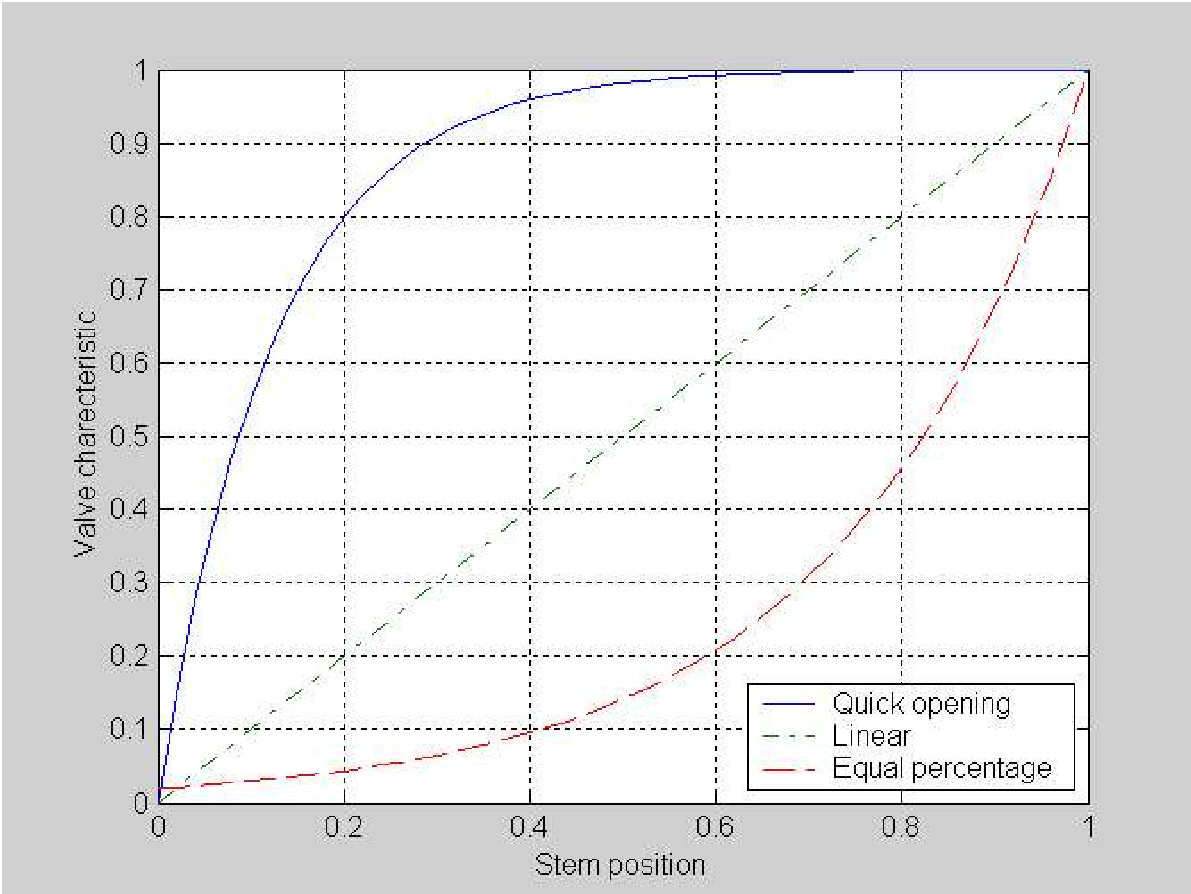


Figure 3.4-5 Inherent control valve characteristics.

The equations for linear and equal percentage characteristics are

Linear: $f_{(x)} = x$

Equal percentage: $f_{(x)} = \alpha^{x-1}$

In the equation for equal-percentage trim the constant α has a value between 20 and 50. Linear-trim valves are normally used when the pressure drop over the control valve is fairly constant and the process is linear so that a linear relationship exists between the controlled variable and the flow rate of the manipulated variable. However, most processes are nonlinear in nature, and many exhibit a decrease in gain with increasing load. For such processes, the equal percentage characteristic, having a gain that increases as the valve opens compensates for the decreasing process gain. From the controller view point, it is the product of the gains of the valve, the process, and the sensor/transmitter, that should remain constant.

The *inherent characteristics* shown in Figure 3.4-5 are those that relate flow to valve position in the situation where the pressure drop over the control valve is constant. The *installed characteristics* are those that result from the variation in the pressure drop over the valve.

Chapter 3

3.5 Sizing of Control Valves

To size a control valve for liquid service, we need the flow rate through the valve, the pressure drop across the valve, and the specific gravity of the liquid. For compressible flow, we also need the inlet pressure and temperature and the average molecular weight of the fluid. The flow coefficient C_v should be obtained from the formula provided by the specific valve manufacture.

Incompressible Flow

The volumetric flow rate for incompressible fluid through an orifice is given by

$$Q = C_d A_o \left[\frac{2\Delta P}{\rho(1-\beta^4)} \right]^{1/2} \tag{3.5-1}$$

In this equation ΔP is the pressure drop across the orifice. For a control valve, the flow area and geometric factors, the density of the reference fluid, and the friction loss coefficient are combined into a single coefficient C_v to provide the following formula for the liquid flow through the valve

$$Q = C_v \sqrt{\frac{\Delta \;_{v}}{\;}} = C_v(\rho_w g h_v)^{1/2} \tag{3.5-2}$$

In this equation, ΔP_v is the pressure drop across the valve, SG is the fluid specific gravity, and h_v is the head loss across the valve. The reference fluid for the density is water for liquids and air for gases. Although equation (3.5-2) is similar to equation (3.5-1), the flow coefficient C_v is not dimensionless like the discharge coefficient C_d, but has dimensions of $[L^3][L/M]^{1/2}$. The normal engineering units of C_v are gpm/(psi)$^{1/2}$. If Q is in gpm and is h_v in ft, equation (3.5-2) becomes

$$Q = 0.658 C_v (h_v)^{1/2} \tag{3.5-3}$$

The value of the flow coefficient C_v is different for each valve and also varies with the valve opening or stem position for a given valve. Figure 3.5-3 shows the flow coefficients for Masoneilan's valves that are provided by the manufacturer from measurements. Different valve plugs are usually available for a given valve, each providing a different flow response or "trim" characteristic when the stem position is changed.

Nominal Trim size	1/4	3/8	½	3/4	1	1.5	2	3	4	6	8	10
Orifice Dia. (in.)	.250	.375	.500	.750	.812	1.250	1.625	2.625	3.500	5.000	6.250	8.000
Valve size (in.)	Reduced Trim			Full Capacity Trim								
3/4	1.7	3.7	6.4	11								
1	1.7	3.7	6.4	11	12							
1½	1.7	3.8	6.6	12	13	25						
2	1.7	3.8	6.7	13	19	26	46					
3				14		31	47	110				
4						32	49	113	195			
6							53	126	208	400		
8								133	224	415	640	
10									233	442	648	1000

Figure 3.5-3 Flow coefficient for Masoneilan's valve Schedule 40.

Example 3.5-1[2] --
A process for transferring oil from a storage tank to a separation tower is shown in Figure E3.5-1. The tank is at atmospheric pressure, and the tower works at 12.7 psia. Nominal oil flow is 700 gpm, its specific gravity is 0.94, and its vapor pressure at the flowing temperature of 90^{o}F is 13.85 psia. The pipe is 8-in. Schedule 40 commercial steel pipe, and the pump efficiency is 75%. Size a valve to control the flow of oil if the frictional pressure drop in the line is found to be 6 psi. Use a pressure drop of 5 psi across the valve and estimate the annual cost if the electricity price is \$0.20/kW-hr and the pump operates 8200 hr per year.

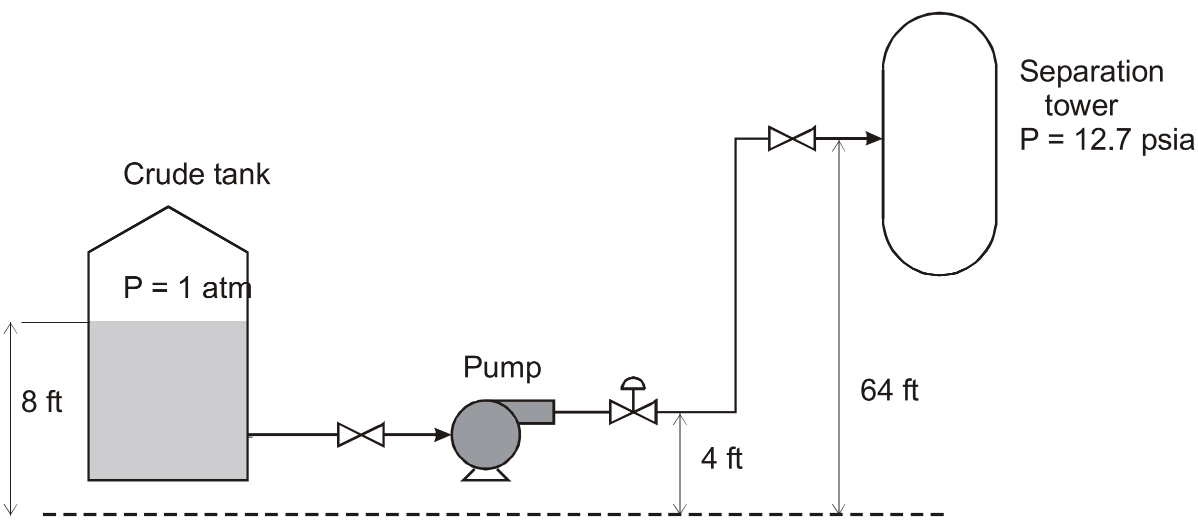


Figure E3.5-1 An oil transferring system

Solution ---

The flow coefficient for the valve can be determined from equation (3.5-2)

[2] Smith and Corripio, Principles and Practice of Automatic Process Control, Wiley, 1997, pg. 209

$$Q = C_v \sqrt{\frac{\Delta P_v}{SG}} \qquad (3.5\text{-}2)$$

Since the nominal flow is 700 gpm, we want 2×700 or 1400 gpm of flow when the pump is fully opened. The maximum valve coefficient is

$$C_v = Q\sqrt{\frac{SG}{\Delta P_v}} = 1400\sqrt{\frac{0.94}{5}} = 607 \text{ gpm/psi}^{1/2}$$

From Figure 3.5-3 an 8-in. Masoneilan's valve Schedule 40 has a C_v of 640 gpm/psi$^{1/2}$. This valve is suitable for the service. Now we need to decide where to place the valve to prevent liquid flashing due to the pressure drop through it. Liquid will flash (vaporize) when the pressure in the line is less than its vapor pressure. A good place for the valve is at the discharge of the pump, where the exit pressure is higher as a result of the hydrostatic pressure due to the 60 ft of elevation plus most of the 6 psi of friction drop. The minimum pressure at the valve exit is

$$P_{\min} = (62.3 \text{ lb/ft}^3)(0.94)(60 \text{ ft})/(144 \text{ in}^2/\text{ft}^2) + 12.7 \text{ psia} = 37.1 \text{ psia.}$$

This pressure is much higher than the vapor pressure of the oil at 13.85 psia.

The annual cost attributable to the 5-psi drop across the valve is

$$\frac{700\text{ gal}}{\text{min}}\ \frac{60\text{ min}}{\text{hr}}\ \frac{1\text{ ft}^3}{7.48\text{ gal}}\ \frac{(5)(144\text{ lb}_f/\text{ft}^2)}{0.75}\ \frac{8{,}200\text{ hr}}{\text{year}}\ \frac{3.766\times10^{-7}\text{ kW}\cdot\text{hr}}{\text{lb}_f\cdot\text{ft}}\ \frac{\$0.20}{\text{kW}\cdot\text{hr}} = \$\,3{,}121/\text{year}$$

To estimate the extra work required by the pump in the above equation we have used the equation

$$Energy = (Power)(time)$$

$$Energy = \left(\frac{Q\Delta P}{\eta_{pump}}\right)(time)$$

It is important to note that a typical process may require several hundred control valves. An optimum control valve sizing also requires the determination of the pressure drop across the valve that might save thousands of dollars on the energy cost annually.

Compressible Flow

Different manufactures have developed different formulas to model the flow of compressible fluids - gases, vapors, and steam - through their control valves. Among several manufactures that produce good valves, including the Crane Company, DeZurik, Foxbora, Fisher Controls, Honeywell, and Masoneilan, we only present the compressible formulas of Masoneilan and Fisher Controls. Their equations and methods are typical of the industry.

The equations for compressible flow derive from the equation for liquids. However the expressions look quite different from the equation for liquids since they contain the unit conversion factors and density corrections for temperature and pressure. The flow coefficient C_v of a valve is the same whether the valve is used for liquid or gas services.

Masoneilan uses the following set of equations.

$$Q_{scfh} = 836 C_v C_f \frac{P_1}{\sqrt{SG \times T_1}} (y - 0.148 y^3) \tag{3.5-3}$$

In this equation Q_{scfh} is the volumetric flow rate in standard cubic feet per hour at the standard conditions of 1 atm and 60°F. C_f is the critical flow factor with a numerical value between 0.6 and 0.95 depending on the valve types. SG is the gas specific gravity with respect to air and is calculated by dividing the molecular weight of the gas by 29, the average molecular weight of air. T_1 is the absolute temperature at the valve inlet in °R. P_1 is the pressure at the valve inlet in psia. y represents the compressibility effects on the flow and is defined by

$$y = \frac{1.63}{{}_f} \sqrt{\frac{\Delta P_v}{P_1}} \tag{3.5-4}$$

For gas or vapor mass flow rate in lb/hr,

$$\cdot = 2.8 C_v C_f P_1 \sqrt{SG \frac{520}{T_1}} (y - 0.148 y^3) \tag{3.5-5}$$

For steam flow in lb/hr,

$$\cdot = 1.83 C_v C_f \frac{P_1}{(1 + 0.0007 T_{SH})} (y - 0.148 y^3) \tag{3.5-6}$$

In this equation T_{SH} is the degree of superheat in °F.

Fisher Controls uses the following set of equations.

$$Q_{scfh} = C_g \sqrt{\frac{520}{SG \times T_1}} P_1 \sin\left[\frac{59.64}{C_1} \sqrt{\frac{\Delta P_1}{P_1}}\right]_{rad} \tag{3.5-7a}$$

$$Q_{\text{scfh}} = C_g \sqrt{\frac{520}{SG \times T_1}}\, P_1 \sin\left[\frac{3417}{C_1}\sqrt{\frac{\Delta P_1}{P_1}}\right]_{\text{deg}} \qquad (3.5\text{-}7b)$$

In equations (3.5-7a) and (3.5-7b) all the symbols are the same as for the Masoneilan formulas except for the two new coefficients C_g and C_1. The coefficient C_g determines the gas flow capacity of the valve, where as the coefficient C_1, defined as C_g / C_v, is functionally the same as the C_f factor in the equations used by Masoneilan. The values of C_1 can be found on Table 10-3, page 318, in Darby's text for various valves. The argument of the sine function must be limited to $\pi/2$ radians or 90 degrees since the flow has reached critical flow conditions and cannot increase above this value without increasing P_1. Let ρ_1 be the density of the gas at P_1 in lb/ft^3, the mass flow rate in lb/hr for steam or vapor at any pressure is given by

$$\dot{} = 1.06 C_g \sqrt{\rho_1 P_1}\, \sin\left[\frac{59.64}{C_1}\sqrt{\frac{\Delta P_1}{P_1}}\right]_{rad} \qquad (3.5\text{-}8a)$$

$$\dot{} = 1.06 C_g \sqrt{\rho_1 P_1}\, \sin\left[\frac{3417}{C_1}\sqrt{\frac{\Delta P_1}{P_1}}\right]_{\text{deg}} \qquad (3.5\text{-}8b)$$

Example 3.5-2[3] --
A 3-in Masoneilan valve with $C_f = 0.9$ and full trim has a capacity factor of 110 gpm/psi$^{1/2}$ when fully opened. The pressure drop across the valve is 10 psi.

(a) Calculate the flow of a liquid solution with density 0.8 g/cm^3.
(b) Calculate the flow of gas with average molecular weight of 35 when the valve inlet conditions are 100 psig and 100°F.
(c) Calculate the flow of the gas from part (b) when the inlet pressure is 5 psig. Calculate the flow both in volumetric and in mass rate units, and compare the results for a 3-in. Fisher Controls valve ($C_v = 120$, $C_g = 4280$, $C_1 = 35.7$).
Solution ---

(a) Calculate the flow of a liquid solution with density 0.8 g/cm^3.

For the liquid solution, the volumetric flow rate is given by

$$Q = C_v \sqrt{\frac{\Delta P_v}{SG}} = 110 \sqrt{\frac{10}{0.8}} = 389 \text{ gpm}$$

The mass flow rate is

$$\dot{} = (389 \text{ gpm})(60 \text{ min/hr})(8.33 \times 0.8 \text{ lb/gal}) = 155{,}600 \text{ lb/hr}$$

[3] Smith and Corripio, Principles and Practice of Automatic Process Control, Wiley, 1997, pg. 206

(b) Calculate the flow of gas with average molecular weight of 35 when the valve inlet conditions are 100 psig and 100°F.

For $C_f = 0.9$, we have

$$y = \frac{1.63}{f}\sqrt{\frac{\Delta P_v}{P_1}} = \frac{1.63}{0.9}\sqrt{\frac{10}{100+14.7}} = 0.535$$

The volumetric flow rate is given by

$$Q_{scfh} = 836 C_v C_f \frac{P_1}{\sqrt{SG \times T_1}}(y - 0.148y^3)$$

$$y - 0.148y^3 = 0.535 - 0.148(0.535)^3 = 0.512$$

$$Q_{scfh} = 836(110)(0.9)\frac{114.7}{\sqrt{(35/29)(560)}}(0.512) = \mathbf{187{,}000\ scfh}$$

The mass flow rate is determined from

$$\dot{} = 2.8 C_v C_f P_1 \sqrt{SG\frac{520}{T_1}}(y - 0.148y^3) \qquad (3.5\text{-}5)$$

$$\dot{} = 2.8(110)(0.9)(114.7)\sqrt{\left(\frac{35}{29}\right)\frac{520}{560}}(0.512) = 17{,}240\ \mathbf{lb/hr}$$

(c) Calculate the flow of the gas from part (b) when the inlet pressure is 5 psig. Calculate the flow both in volumetric and in mass rate units, and compare the results for a 3-in. Fisher Controls valve ($C_v = 120$, $C_g = 4280$, $C_1 = 35.7$).

$$y = \frac{1.63}{f}\sqrt{\frac{\Delta P_v}{P_1}} = \frac{1.63}{0.9}\sqrt{\frac{10}{5+14.7}} = 1.290$$

The volumetric flow rate is given by

$$Q_{scfh} = 836 C_v C_f \frac{P_1}{\sqrt{SG \times T_1}}(y - 0.148y^3)$$

$$y - 0.148y^3 = 1.290 - 0.148(1.290)^3 = 0.972$$

$$Q_{scfh} = 836(110)(0.9)\frac{19.7}{\sqrt{(35/29)(560)}}(0.972) = \mathbf{61{,}000\ scfh}$$

The mass flow rate is determined from

$$\cdot = 2.8 C_v C_f P_1 \sqrt{SG \frac{520}{T_1}} (y - 0.148 y^3) \tag{3.5-5}$$

$$\cdot = 2.8(110)(0.9)(19.7) \sqrt{\left(\frac{35}{29}\right)\frac{520}{560}} (0.972) = \mathbf{5{,}620\ lb/hr}$$

Using formulas from Fisher Controls we have

$$Q_{\mathrm{scfh}} = C_g \sqrt{\frac{520}{SG \times T_1}}\, P_1 \sin\left[\frac{59.64}{C_1}\sqrt{\frac{\Delta P_1}{P_1}}\right]_{rad}$$

$$Q_{\mathrm{scfh}} = 4280 \sqrt{\frac{520}{(35/29)(560)}}\,(19.7)\sin\left[\frac{59.64}{35.7}\sqrt{\frac{10}{19.7}}\right]_{rad} = \mathbf{68{,}700\ scfh}$$

$$\cdot = 1.06 C_g \sqrt{\rho_1 P_1}\, \sin\left[\frac{59.64}{C_1}\sqrt{\frac{\Delta P_1}{P_1}}\right]_{rad}$$

The density is determined from ideal gas law: $\rho_1 = \dfrac{MP_1}{RT_1} = \dfrac{(35)(19.7)}{(10.73)(560)} = 0.115\ \mathrm{lb/ft^3}$

$$\cdot = 1.06(4280)\sqrt{(0.115)(19.7)}\, \sin\left[\frac{59.64}{35.7}\sqrt{\frac{10}{19.7}}\right]_{rad} = \mathbf{6{,}340\ lb/hr}$$

Example 3.5-3[4] ---

A control valve is to regulate the flow of steam into a heat exchanger with a design heat transfer rate of 15 million Btu/hr. The supply steam is saturated at 20 psig. Size the control valve for a pressure drop of 5 psi and 100% overcapacity.

Solution --

At 20 psig, the steam latent heat of condensation is $\Delta H_{\mathrm{evap}} = 930$ Btu/lb, the nominal steam flow is

$$\cdot = \frac{1.5 \times 10^7}{930} = 16{,}130\ \mathrm{lb/hr}$$

The steam is saturated, therefore the degrees of superheat, $T_{SH} = 0$, is zero. For a Masoneilan valve with $C_f = 0.8$, we have

[4] Smith and Corripio, Principles and Practice of Automatic Process Control, Wiley, 1997, pg. 208

$$y = \frac{1.63}{{}_{f}} \sqrt{\frac{\Delta P_v}{P_1}} = \frac{1.63}{0.8} \sqrt{\frac{5}{20 + 14.7}} = 0.773 \qquad (3.5\text{-}4)$$

$$y - 0.148y^3 = 0.773 - 0.148(0.773^3) = 0.705$$

For steam flow in lb/hr,

$$\cdot \; = 1.83 C_v C_f \frac{P_1}{(1 + 0.0007T_{SH})}(y - 0.148y^3) \qquad (3.5\text{-}6)$$

$$16{,}130 = 1.83 C_v (0.8)(34.7)(0.705) \Rightarrow C_v = 450 \text{ gpm/psi}^{1/2}$$

To design for 100% overcapacity, we need a maximum $C_{v,max} = 2C_v = 950$ gpm/psi$^{1/2}$. From Figure 3.5-3, a 10-in. Masoneilan valve, with a coefficient of 1000, is the smallest valve that can provide the service.

Using Fisher Controls equation, we have

$$\cdot \; = 1.06 C_g \sqrt{\rho_1 P_1} \sin\left[\frac{59.64}{C_1}\sqrt{\frac{\Delta P_1}{P_1}}\right]_{rad}$$

Saturated steam at 20 psig, $T_{sat} = 259^{o}$F, $\rho_1 = 0.0833$ lb/ft^3. Let $C_1 = 35$, we have

$$16{,}130 = 1.06 C_g \sqrt{(0.0833)(34.7)} \sin\left[\frac{59.64}{35}\sqrt{\frac{5}{34.7}}\right]_{rad} \Rightarrow C_g = 15{,}000$$

For 100% overcapacity $C_{g,max} = 2(15{,}000) = 30{,}000$. The corresponding C_v is then

$$C_v = C_g/C_1 = 30{,}000/35 = 856 \text{ gpm/psi}^{1/2}$$

We obtain similar value for the valve coefficient C_v from both methods.

Chapter 3

3.6 Controllers

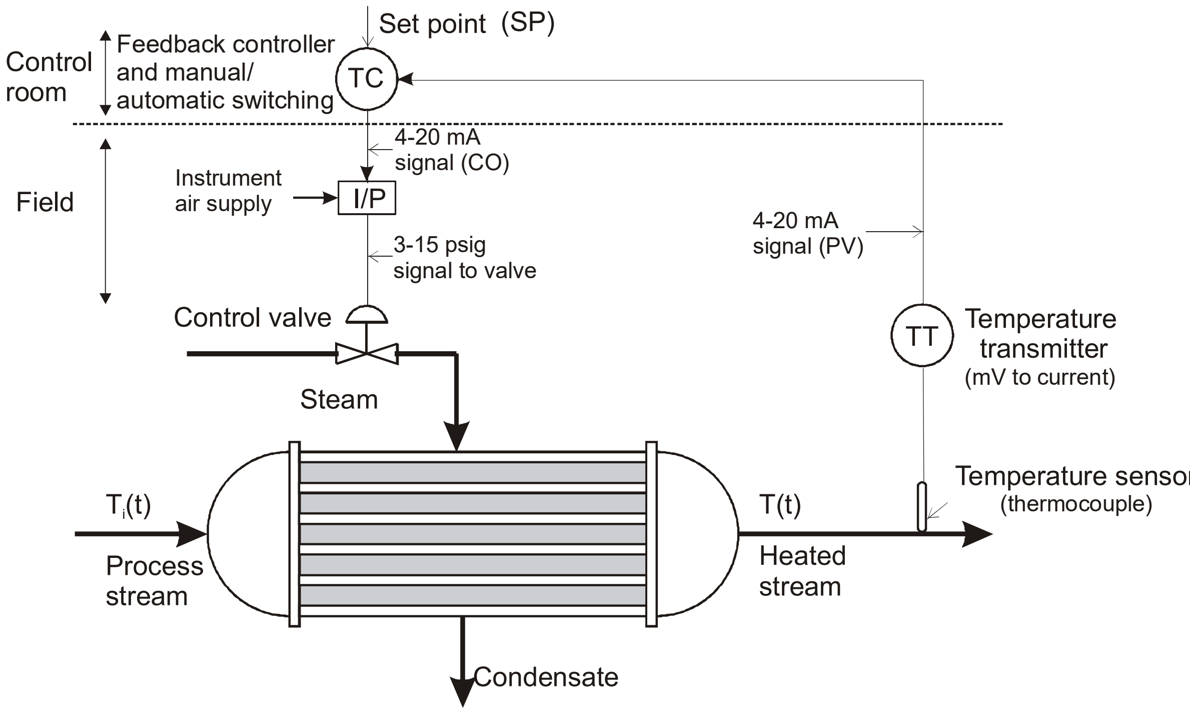


Figure 3.6-1 Heat exchanger control system using control valve.

A basic feedback control loop for a heat exchanger is shown in Figure 3.6-1. A sensor such as a thermocouple, a thermistor, or any resistance temperature device can measure the outlet process stream temperature. This sensor is usually connected to transmitter, which amplifies the output from the sensor and sends it to a controller. The controller compares the signal with the set point and decides the action necessary to maintain the desired temperature. The controller then sends a pneumatic or electrical signal to the final control element, which is the control valve in this case, to adjust the steam flow rate accordingly. The control valves acts as a variable resistance in the steam line since the flow rate depends on the valve stem or plug position. To regulate flow, the flow capacity of the control valve varies from zero when the valve is closed to a maximum when the valve is fully opened.

We will discuss the action of standard commercial controllers when they detect a difference between the desired value of the controlled variable (the set point) and the actual value. This difference, or error signal, is defined as

$$E(t) = SP - PV \qquad (3.6\text{-}1)$$

In this equation, SP is the set point and PV is the actual value of the process variable.

Proportional Controller (P)

The proportional controller is the simplest type of controller we will discuss. A proportional controller changes its output signal, *CO*, in direct proportion to the error signal. The equation that describes its operation is

$$CO = \text{Bias} \pm K_c(SP - PV) \qquad (3.6\text{-}2)$$

The bias signal is a constant and is the output from the controller when the error is zero. K_c is the controller gain. The larger the gain, the more the controller output will change for a given error. For example, if the gain is 1, an error of 10% of scale (a change of 1.6 mA in an analog electronic 4-20 mA system) will change the controller output by 10% of scale. The gain on the controller can be made either positive or negative by setting a switch in an analog controller or by specifying the desired sign in a digital controller. A positive gain results in the controller output decreasing when the process variable increases. This “increase-decrease” action is called a “reverse-acting” controller. To obtain a direct-acting controller, we must either use a negative K_c or reverse the definition of the error, that it, $E(t) = PV - SP$. The correct sign depends on the action of the transmitter (which is usually direct), the action of the valve (air-to-open or air-to-close), and the effect of the manipulated variable on the controlled variable.

Proportional controller is simple to operate since it has only one adjustable parameter, K_c. However using proportional controller alone will not eliminate offset, which is the difference between the set point value and the actual output value at steady state. The reason why offset exists can be proved rigorously. However we will use a simple explanation for offset. Consider the liquid level control system shown in Figure 3.6-2 at steady state with $\bar{F}_i = \bar{F}_o$ = 150 gpm and $\bar{h}$ = 6 ft.

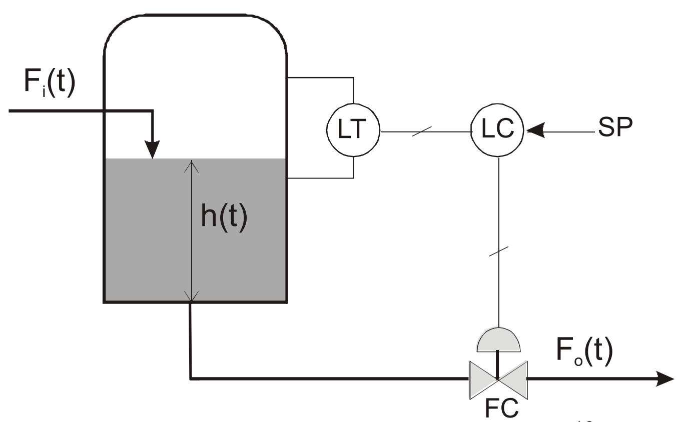


Figure 3.6-2 Liquid level control loop[13].

When the inlet flow increases to 170 gpm, the liquid level increases and the controller in turn increases its output to open the valve and bring the level back down. Using a direct acting proportional controller we have

$$CO = 50\% - K_c(SP - PV) = 50\% - K_c(6 - PV)$$

[13] Smith, C.A. & Corripio A. B., Principle and Practice of Automatic Process Control, Wiley, 1997, pg. 226.

The liquid level increases so that *PV* is greater than 6 ft. For steady state operation, the outlet flow must also be 170 gpm. Let us assume that the signal required to the valve is 60% to deliver 170 gpm. The output *CO* from the controller must be 60%. From the equation $CO = 50\% - K_c(6 - PV)$, the error term $(6 - PV)$ cannot be zero at steady state for *CO* to have any value different than 50%.

Many instrument manufacturers use the term *proportional band* (*PB*) defined by

$$PB = \frac{100}{K_c}$$

Thus a wide (or high) *PB* is a low gain, and a narrow (or low) *PB* is a high gain.

Proportional-Integral Controller (PI)

By adding integral action to a proportional controller, offset can be eliminated completely. Integral action acts on the final control in a manner proportional to the integral of the error over time. A proportional and integral controller, PI controller, can be represented by the following equation

$$CO = \text{Bias} + K_c\, E(t) + \frac{K_c}{\tau_I}\int_0^t E(t)dt \qquad (3.6\text{-}3)$$

In this equation, τ_I is the integral or reset time. To understand the physical meaning of the reset time, let consider the case when a constant error of 1% in magnitude is introduced in the controller at time $t = 0$. At this moment, the controller output is

$$CO = 50\% + K_c\,(1) + \frac{K_c}{\tau_I}\int_0^t (1)dt$$

$$CO = 50\% + K_c + \frac{K_c}{\tau_I}\,t$$

When the error is introduced, the controller output changes immediately by an amount equal to K_c due to the proportional mode. The controller output also increases linearly with time so that when $t = \tau_I$ the output becomes

$$CO = 50\% + K_c + K_c$$

Thus, the integral mode repeats the immediate action taken by the proportional mode in a reset time. More weight will be given to the integral term with smaller value of reset time.

We will now explain why the PI controller removes the offset. Consider the liquid level control system shown in Figure 3.6-2 at steady state with $\overline{F}_i = \overline{F}_o = 150$ gpm and $\overline{h} = 6$ ft. The inlet flow now increases to 170 gpm. When this happens, the liquid level increases so that *PV* is greater than 6 ft. The controller in turn increases its output to open the valve to bring the level back down. Let us assume that the signal required to the valve is 60% to deliver 170 gpm. The output *CO* from the controller must be 60%. Under PI control, as long as the error is present, the controllers keeps changing its output (integrating the error). Once the error goes to zero at time t_f, the controller output is given by

$$CO = 50\% + K_c\,(0) + \frac{K_c}{\tau_I}\int_0^{t_f} E(t)dt \;+\; \frac{K_c}{\tau_I}\int_{t_f}^{t} (0)dt$$

$$CO = 50\% + 0 + 10\% + 0$$

Therefore, with a zero error, the integral term is not zero but rather 10%, which provides the required output of 60%.

Proportional-Integral-Derivative Controller (PID)

The third type of controller action is derivative control. Derivative action will control the final control element in proportion to the derivative of the process error. Its purpose is to anticipate where the process is heading by looking at the time rate of change of the error. Derivate action can be combined with proportional and integral action producing a PID controller. This controller is given by the following equation

$$CO = \text{Bias} + K_c\,E(t) + \frac{K_c}{\tau_I}\int_0^{t} E(t)dt \;+ K_c\tau_D\frac{dE(t)}{dt} \qquad (3.6\text{-}4)$$

In this equation τ_D is the derivative time constant. A PID controller will not have any steady state offset, but a PD controller would. The performance of a PID controller can be optimized by adjusting the parameters K_c, τ_I, and τ_D. Optimizing controllers to give the quickest and most stable response by adjusting the controller parameters is referred to as tuning the controller.

Example 3.6-1[14] --

The temperature of a CSTR is controlled by an electronic (4 to 20 mA) feedback control system containing (1) a 100 to 200°F temperature transmitter, (2) a PI controller with integral time set at 3 minutes and proportional band at 25, and (3) a control valve with linear trim, air-to-open action, and C_v = 4 gpm/psi$^{0.5}$ through which cooling water flows. The pressure drop across the valve is a constant 25 psi. If the steady state controller output is 12 mA, how much cooling water is going through the valve? If a sudden disturbance increases reactor temperature by 5°F, what will be the immediate effect on the controller output signal and the water flow rate?

Solution --

(a) <u>If the steady state controller output is 12 mA, how much cooling water is going through the valve?</u>

We use the following equation with SG = 1 to determine the flow rate:

$$F = C_v f_{(x)} \sqrt{\frac{\Delta P_v}{SG}} = C_v f_{(x)} \sqrt{\Delta P_v}$$

At CO = 12 mA, stem position $x = \dfrac{12-4}{20-4}$ = 0.5. For linear trim $f_{(x)}$ = 0.5. The steady cooling water through the valve is

$$F = (4)(0.5)\sqrt{25} = \mathbf{10\ gpm}$$

(b) <u>If a sudden disturbance increases reactor temperature by 5°F, what will be the immediate effect on the controller output signal and the water flow rate?</u>

A change in 5°F corresponds to a change in current given by

$$\Delta T_m = 5\ ^\circ\text{F}\left(\frac{16\ \text{mA}}{100^\circ\text{F}}\right) = 0.8\ \text{mA}$$

The change in CO is then

$$\Delta CO = K_c \Delta T_m = \left(\frac{100}{25}\right)(0.8) = 3.2\ \text{mA}$$

The CO signal will change from 12 mA to (12 + 3.2) = 15.2 mA. Stem position $x = \dfrac{15.2-4}{20-4}$ = 0.70. For linear trim $f_{(x)}$ = 0.70. The water flow rate will change from 10 gpm to

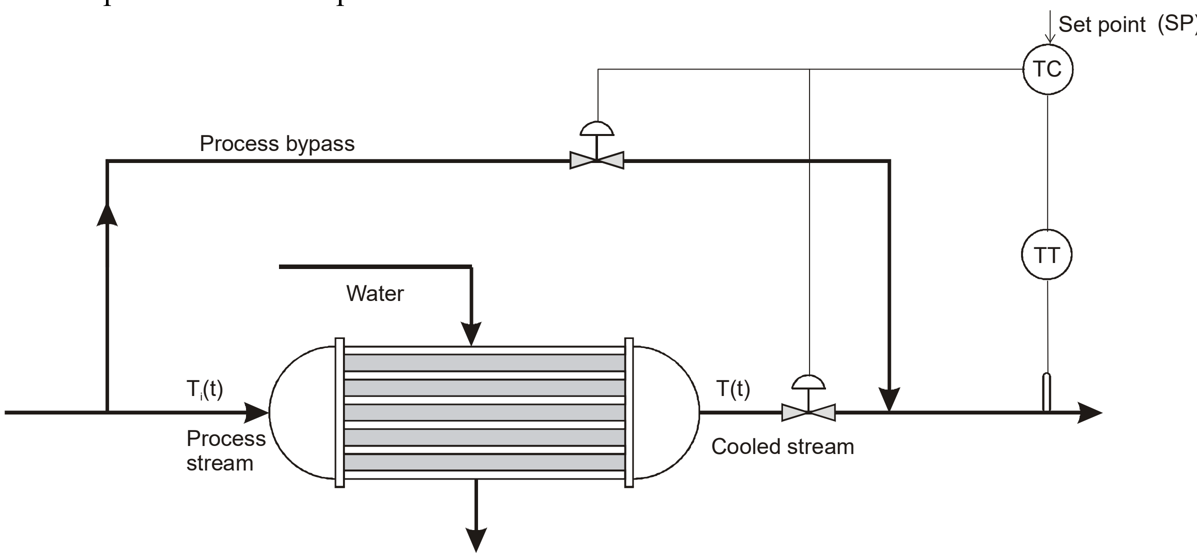
$$F = (4)(0.70)\sqrt{25} = \mathbf{14\ gpm}$$

[14] Luyben, W. L., <u>Process Modeling, Simulation, and Control for Chemical Engineers</u>, McGraw Hill, 1990, pg. 238.

Example 3.6-2[15] --

The bypass cooling system shown is designed so that the total flow of 100 gpm of a liquid with heat capacity of 0.5 Btu/lb·°F is split under the normal conditions, 25 % going around the bypass and 75 % going through the cooler. Process inlet and outlet temperature under these conditions are 250 and 150°F. Inlet and outlet water temperatures are 80 and 120°F. Process side pressure drop the exchanger is 10 psi. The control valves have linear trim and are designed to be half open at design rates with a 15 psi drop over the bypass valve and 5 psi drop over the cooler valve. Liquid density is constant at 62.3 lb/ft^3.

What will the valve positions be if the total process flow is reduced to 25 percent of design and the process outlet temperature is held at 150°F?



Solution ---

We use the following equation with SG = 1 to determine the valve coefficient C_v:

$$F = C_v f_{(x)} \sqrt{\frac{\Delta P_v}{SG}} = C_v f_{(x)} \sqrt{\Delta P_v}$$

For the bypass valve:

$$C_v = \frac{25}{0.5\sqrt{15}} = 12.9 \text{ gpm/psi}^{0.5}$$

For the cooler valve:

$$C_v = \frac{75}{0.5\sqrt{5}} = 67.1 \text{ gpm/psi}^{0.5}$$

The mass flow rate of the process stream is

---

[15] Luyben, W. L., Process Modeling, Simulation, and Control for Chemical Engineers, McGraw Hill, 1990, pg. 239.

$$\dot{n} = (100 \text{ gpm})(60 \text{ min/hr})(62.3 \text{ lb/ft}^3)\left(\frac{\text{ft}^3}{7.48 \text{ gal}}\right) = 50{,}000 \text{ lb/hr}$$

The heat removed by the cooler is given by

$$Q = (50{,}000 \text{ lb/hr})(\ 0.5 \text{ Btu/lb}\cdot{}^{o}\text{F})(250^{o}\text{F} - 150^{o}\text{F}) = 2.5\times10^6 \text{ Btu/hr}$$

The temperature T of the liquid leaving the cooler can be evaluated

$$0.75T + 0.25\times250\ {}^{o}\text{F} = 150^{o}\text{F} \Rightarrow T = 116.7\ {}^{o}\text{F}$$

The performance of the heat exchanger is calculated from the equation

$$Q = UA\Delta T_{\text{lm}}$$

$$\Delta T_{\text{lm}} = \frac{(250-120)-(116.7-80)}{\ln\left(\dfrac{250-120}{116.7-80}\right)} = 73.75^{o}\text{F}$$

$$UA = (2.5\times10^6 \text{ Btu/hr})/\ (73.75^{o}\text{F}) = 33{,}900 \text{ Btu/hr}\cdot{}^{o}\text{F}$$

The mass flow rate of the cooling water is then

$$\dot{n}_c = \frac{2.5\times10^6}{120-80} = 62{,}500 \text{ lb/hr}$$

If the total process flow is reduced to 25 percent of design, the heat transfer rate is then (0.25)(2.5×10^6 Btu/hr) = 0.625×10^6 Btu/hr. The cooling water exit temperature for the new condition is

$$T_{\text{c,out}} = 80 + \frac{0.625\times10^6}{62{,}500} = 90^{o}\text{F}$$

Assuming that UA remains constant, we can evaluate the temperature of the process stream leaving the heat exchanger

$$0.625\times10^6 = 33{,}900\frac{(250-90)-(T-80)}{\ln\left(\dfrac{250-90}{T-80}\right)}$$

The nonlinear equation can be solved by the following Matlab statements

```
>> g=inline('0.625e6-33900*((250-90)-(t-80))/log((250-90)/(t-80))','t');
```

```
>> T=fsolve(g,81,optimset('Display','off'))

T =

  80.0273
```

The flow rate through the bypass F_B is evaluated

$$250\, F_B + (25 - F_B)(80.027) = 25(150) \Rightarrow F_B = 10.29 \text{ gpm.}$$

The flow through the cooler is then 25 – 10.29 = 14.71 gpm. The pressure drop through the cooler is then

$$10\left(\frac{14.71}{75}\right)^2 = 0.385 \text{ psi}$$

Let ΔP_T = total pressure drop at low flow. We have:

For bypass valve: $\Delta P_v = \Delta P_T$

For cooler valve: $\Delta P_v = \Delta P_T - 0.385$

Let $f_{(x)}$ = fraction of full range of controller output, the bypass valve is air to open and the cooler valve is air to close. The faction that bypass valve is open is $f_{(x)}$ and the faction that cooler valve is open is $1 - f_{(x)}$. The flow equation through bypass and cooler valves are then

$$10.29 = 12.9\, f_{(x)}(\Delta P_T)^{0.5}$$

$$14.71 = 67.1(1 - f_{(x)})\, (\Delta P_T - 0.385)^{0.5}$$

We first create the following function and save as a text file named fdP.m

```
function y=fdP(p)
f=p(1);dP=p(2);
g1=10.29-12.9*f*sqrt(dP);
g2=14.71-67.1*(1-f)*sqrt(dP-0.385);
y=g1*g1+g2*g2;
```

We then use the following Matlab statement

```
>> p=fminsearch('fdP',[0.5 1])

p = 0.7475    1.1388
```

Therefore $f_{(x)} = 0.75$ and $\Delta P_T = 1.14$ psi.

Chapter 3

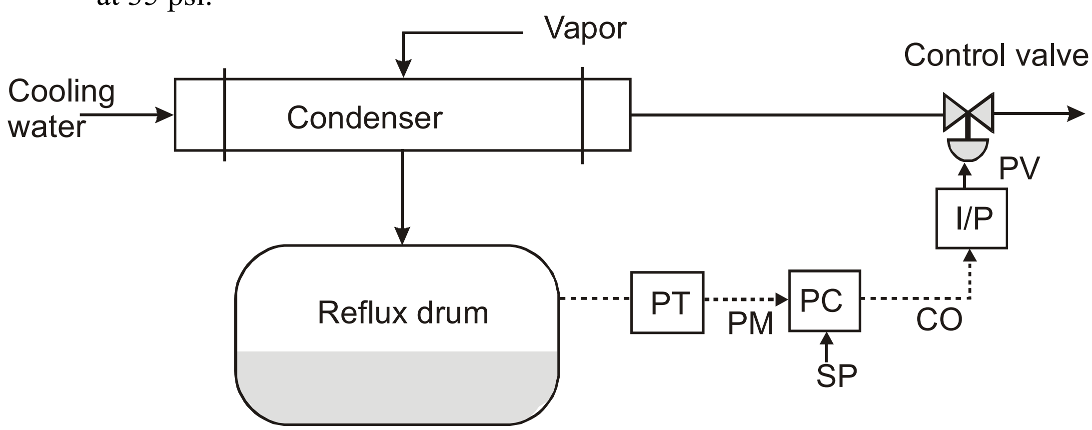
Example 3.6-3.[15]---

The overhead vapor from a depropanizer distillation column is totally condensed in a water-cooled condenser at 120°F and 227 psig. The vapor is 95 mol % propane and 5 mol % isobutene. The vapor design flow rate is 25,500 lb/h and average latent heat of vaporization is 125 Btu/lb.

Cooling water inlet and outlet temperatures are 80 and 105°F, respectively. The condenser heat transfer area is 1000 ft^2. The cooling water pressure drop through the condenser at design rate is 5 psi. A linear-trim control valve is installed in the cooling water line. The pressure drop over the valve is 30 psi at design with the valve half open.

The process pressure is measured by an electronic (4-20 mA) pressure transmitter whose range is 100-300 psig. An analog electronic proportional controller with a gain of 3 is used to control process pressure by manipulating cooling water flow. The electronic signal from the controller (CO) is converted into a pneumatic signal in the I/P transducer.

a) Calculate the cooling water flow rate (gpm) at design conditions. Water density is 62.3 lb/ft^3 and 1 ft^3 = 7.48 gal.
b) Calculate the size coefficient (C_v) of the control valve.
c) Calculate the values of the signals PM, CO, SP, and PV at design conditions.
d) Suppose the process pressure jumps 10 psi. How much will the cooling water flow rate increase? Determine values for PM, CO, SP, and PV at this higher pressure. Assume that the total pressure drop over the condenser and control valve is constant at 35 psi.



Solution ---

a) Calculate the cooling water flow rate (gpm) at design conditions.

$$\left(25,500\frac{\text{lb}}{\text{hr}}\right)\left(\frac{125\text{ Btu}}{\text{lb}}\right)\left(\frac{\text{lb}\cdot{}^{\circ}\text{F}}{\text{Btu}}\right)\left(\frac{1}{25^{\circ}\text{F}}\right)\left(\frac{7.48\text{ gal}}{62.3\text{ lb}}\right)\left(\frac{\text{hr}}{60\text{ min}}\right) = 255\text{ gpm}$$

b) Calculate the size coefficient (C_v) of the control valve.

[15] Luyben, W. L., "Common Plumbing and Control Errors in Plantwide Flowsheets", in Chemical Engineering Education, Vol. 39, Number 3, 2005, pg. 241.

$$F = C_v f_{(x)} \sqrt{\frac{\Delta P_v}{SG}}$$

$$255 = C_v\,(0.5)\sqrt{30} \;\Rightarrow C_v = 93.16 \text{ gpm/psi}^{0.5}$$

c) At design conditions: P = 227 psig, valve 50% open

$$\text{PM} = 4 + 16\left(\frac{227 - 100}{300 - 100}\right) = 14.16 \text{ mA} = \text{SP}$$

$$\text{CO} = 4 + 0.5(16) = 12 \text{ mA}$$

Since the control valve regulates the cooling water flow rate it must be fails open so that the vapor pressure in the reflux drum will not increase to an unsafe value due to power failure. The valve is fully open at PV = 3 psig, CO = 4 mA and fully closed at PV = 15 psig, CO = 20 mA. Using linear interpolation, at CO = 12 mA

$$\frac{\text{PV} - 3}{15 - 3} = \frac{12 - 4}{20 - 4} \Rightarrow \text{PV} = 3 + (12)\,\frac{8}{16} = 9 \text{ psig}$$

d) ΔP = 10 psi $\Rightarrow$ ΔPM = (16/200)(10) = 0.8 mA $\Rightarrow$ PM = 14.96 mA

ΔCO = – 3(0.8) = – 2.4 mA $\Rightarrow$ CO = 12 – 2.4 = 9.6 mA

ΔPV = – 2.4(12/16) = – 1.8 psi $\Rightarrow$ PV = 9 – 1.8 = 7.2 psig

Water flow rate

$$\Delta P_v = 35 - 5\left(\frac{\text{F}}{255}\right)^2, f_{(x)} = (15 - \text{PV})/12 = 0.65$$

$$F = 0.65(93.16)\sqrt{35 - 5\left(\frac{F}{255}\right)^2}$$

```
g=inline('f-.65*93.16*sqrt(35-5*(f/255)^2)','f')

> f1=fsolve(g,300,optimset('Display','off'))

f1 =

  3.1640e+002
```

F = 316 gpm

3.7 Common Errors in Development of Flowsheets

The commercial process simulators permit simulations in which a real chemical plant cannot operate. For example, it allows materials to flow from one unit to another even though the first unit is at a lower pressure than the second. In this section we will show some common errors that students often make in developing flowsheets.

One of the common errors in student flowsheets is not to have any valve in a line connecting process units that are operating at different pressures as shown in Figure 3.7-1. There must be a valve in the line between two vessels at different pressures. The pressure at 10 bar cannot be reduced to 2 bar by just cooling the process stream.

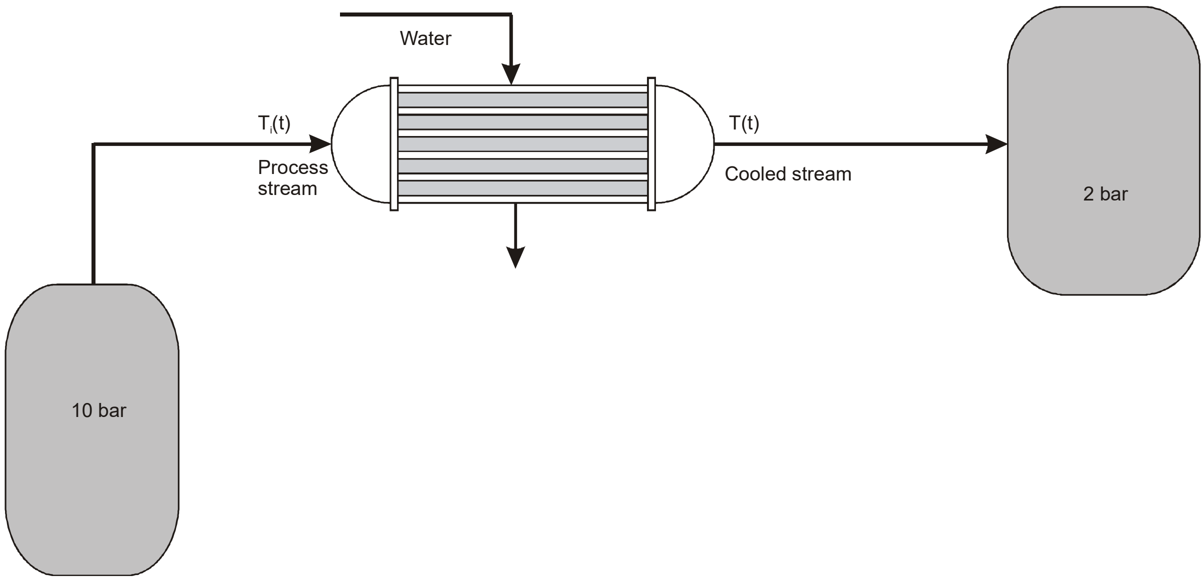


Figure 3.7-1 Missing valve between two vessels at different pressure.

Another common error is two valves in a liquid-filled line as shown in Figure 3.7-2[16]. Since a liquid is essentially incompressible, its flow rate is the same at any point in a liquid-filled line. There should be only one valve in the line that is regulating the flow rate of liquid since two valves cannot function independently. For gas systems, valves can be used in a line at several locations. Since gas in compressible, the instantaneous flow rates through the two valves do not have to be equal as is the case with liquids.

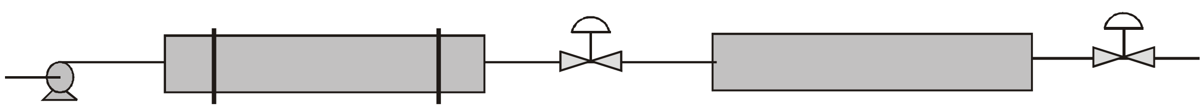


Figure 3.7-2 Two valves in a liquid-filled line is not necessary.

Pumps and compressors are used to raised the pressure of liquid and vapor streams, respectively. Consider a saturated liquid stream leaving from the base of a distillation column

[16] Luyben, W. L., "Common Plumbing and Control Errors in Plantwide Flowsheets", in Chemical Engineering Education, Vol. 39, Number 3, 2005, pg. 202.

as shown in Figure 3.7-3. The base of the column must be located at an elevation high enough to provide adequate pressure at the pump suction to satisfy the net positive suction head (NPSH). The NPSH is required to prevent the formation of vapor in the pump. If a control valve is installed between the column and the pump section, the pressure drop over the valve will reduce the pressure at the pump suction so that the NPSH requirement is not satisfied. Therefore control valves in liquid systems should be located downstream of centrifugal pumps.

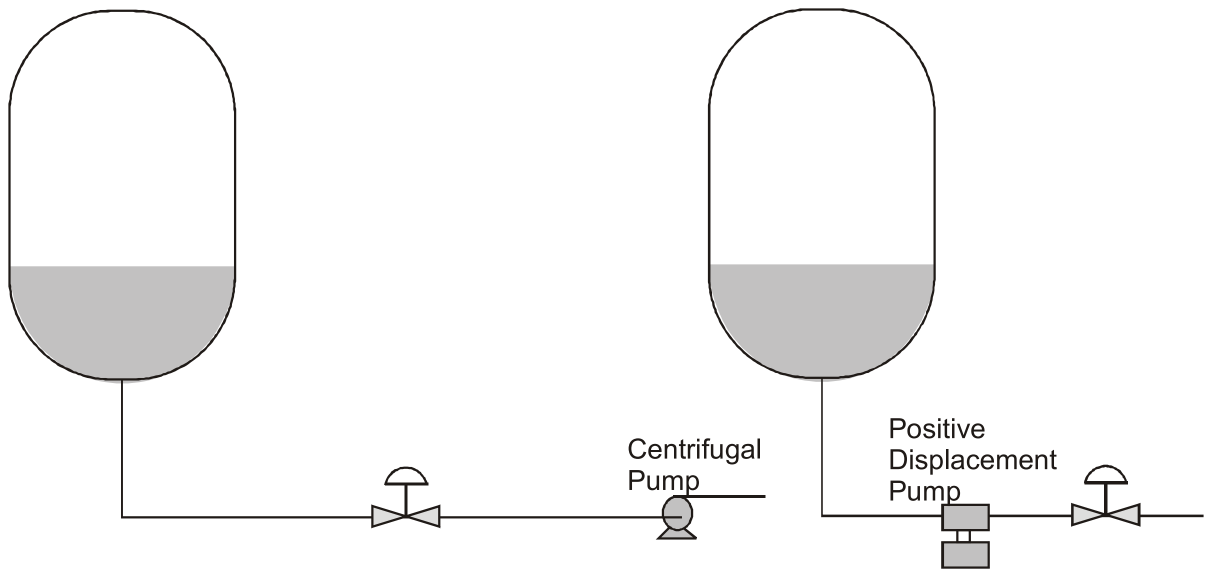


Figure 3.7-3 Forbidden pump plumbing[17].

There should be no valve after positive displacement pumps. The flow rate of the liquid can only be regulated by changing the stroke or speed of the pump or by bypassing liquid from the pump discharge back to some upstream position. Reducing the valve opening in the pump discharge will not change the flow rate of liquid through the pump. It will just increase the pump discharge pressure and raise the power requirement of the motor driving the pump.

The mass flow rate of gas depends on the density of the gas at the compressor suction, so changing the suction pressure will change the mass flow rate. Therefore throttling a valve in the compressor suction can be used to control the gas flow rate. However, throttling a valve in the compressor discharge, as shown in Figure 3.7-4, does not change the gas flow rate. It just increases the compressor discharge pressure and power requirement. The gas flow rate in a compressor system can be regulated by

1) Suction throttling
2) Bypass or spill-back from discharge to suction
3) Variation of compressor speed

Variation of compressor speed is the most energy efficient but requires a variable-speed electric motor or a variable-speed drive, which is typically a steam turbine if high-pressure steam is available in the plant.

[17] Luyben, W. L., "Common Plumbing and Control Errors in Plantwide Flowsheets", in Chemical Engineering Education, Vol. 39, Number 3, 2005, pg. 204.

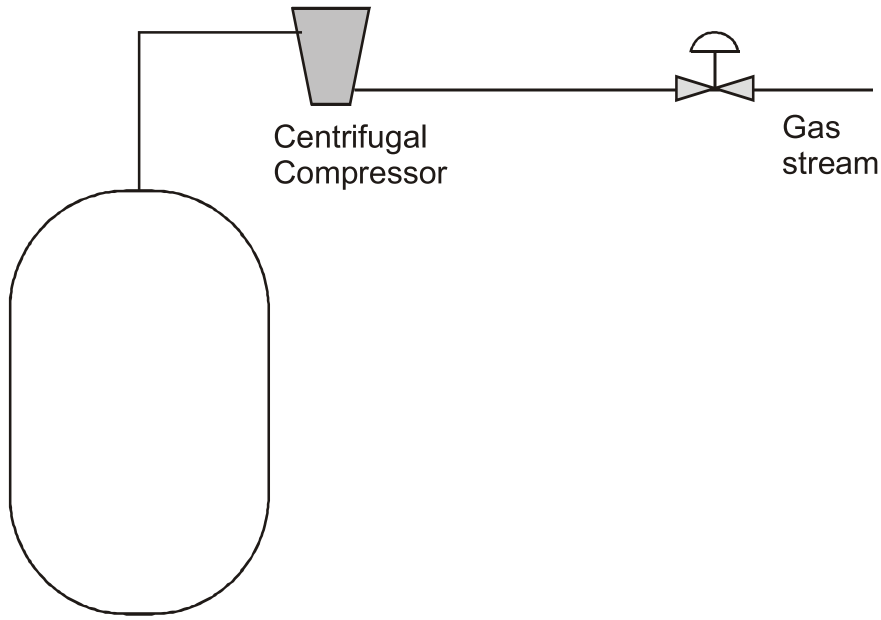


Figure 3.7-4 A valve in the compressor discharge cannot change the gas flow rate.

Liquid levels in vessels and pressure in gas-filled systems must be controlled some way. Liquid levels can be controlled by manipulating a downstream or an upstream valve. There are some exceptions to the requirement for controlling liquid levels. For example, when a solvent is circulating around inside a process and there are no losses of this solvent. There will be a liquid level somewhere in the process that is not controlled as the solvent circulation rate changes.

Gas pressure can be controlled by regulating the flow of gas into or out of the system. Gas pressure can also be controlled by regulating the rate of generation of gas as in a vaporizer, in a distillation column reboiler, or in a boiling exothermic reactor. In some distillation columns where relative volatilities increase with decreasing pressure, then heat removal is maximized to keep pressure as low as possible. In these systems, pressure is not controlled.

Example 3.7-1.[20]--

An engineer from Catastrophic Chemical Company has designed a system in which a positive-displacement pump is used to pump water from an atmosphere tank into a pressurized tank operating at 150 psig. A control valve is installed between the pump discharge and the pressurized tank.

With the pump running at a constant speed and stroke length, 350 gpm of water is pumped when the control valve is wide open and the pump discharge pressure is 200 psig.

If the control valve is pinched back to 50 percent open, what will be the flow rate of water and the pump discharge pressure?

Solution ---

[20] Luyben, W. L., "Common Plumbing and Control Errors in Plantwide Flowsheets", in Chemical Engineering Education, Vol. 39, Number 3, 2005, pg. 251.

The size coefficient (C_v) of the control valve can be determined from

$$F = C_v f_{(x)} \sqrt{\frac{\Delta P_v}{SG}}$$

350 gpm of water is pumped when the control valve is wide open, $f_{(x)} = 1$ and $SG = 1$

$$350 = C_v \sqrt{50} \Rightarrow C_v = 49.5 \text{ gpm/psi}^{0.5}$$

If the control valve is pinched back to 50 percent open, the flow rate of water is still 350 gpm since the pump is positive-displacement. The pressure drop across the valve is then

$$350 = 0.5C_v \sqrt{\Delta \;_v} \Rightarrow \Delta P_v = (2 \times 350/49.5)^2 = 200 \text{ psi}$$

The pump discharge pressure is then 200 + 150 = 350 psig.

Never put a valve on the discharge of a positive displacement in liquid service!

Chapter 4

The Control System

4.1 Introduction

Complete control systems and the fundamental concepts of feedback will be discussed. Consider a stirred-tank heater shown in Figure 4.1-1.A liquid stream at a temperature T_i enters an insulated, well-mixed tank at a constant mass flow rate m. It is desired to maintain the temperature in the tank at T_R by means of a controller. If the measured temperature T_m differs from the desired temperature T_R, the controller changes the heat input through the final control element to reduce the difference or error $\varepsilon = T_R - T_m$. We can use a proportional controller to change the heat input to the tank by an amount that is proportional to the error ε.

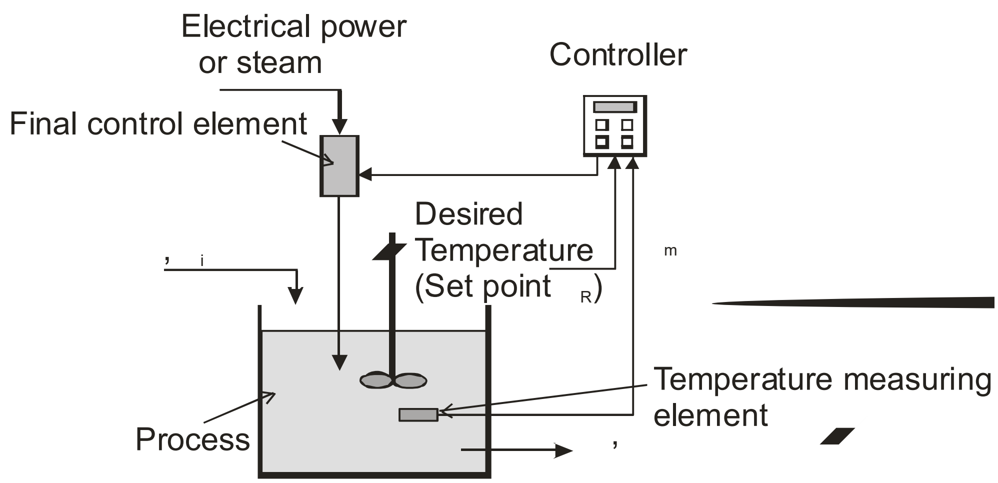


Figure 4.1-1 Control system for a stirred-tank heater[1].

The source of heat input q may be electricity or steam. If an electrical source were used, the final control element might be a variable transformer that changes the heat input by adjusting current to a resistance heating element. If steam were used, the final control element would be a control valve that adjusts the amount of steam supplied to heat the tank. In either case, the output signal from the controller should adjust the heat supplied q in such a way to maintain the liquid temperature in the tank near the set point.

If the liquid temperature in the tank is not controlled we have an open-loop system. Since the tank temperature, or liquid temperature in the tank, is controlled we have a closed-loop system. This control system is a feed-back system because the measured value of the controlled variable is returned or "fed back" to a device called the comparator. In the comparator, the controlled variable is compared with the desired value or set point. An error is generated when there is any difference between the measured variable and the set point.

[1] .R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 165

This error enters a controller, which adjusts the final control element to reduce the error to zero.

The control system shown in Figure 4.1-1 can be represented by a block diagram shown in Figure 4.1-2. In this diagram, the set point indicates the desired value of the controlled variable. The load refers to a change in any variable that may cause the controlled variable of the process to change. For this system, the possible loads are the inlet temperature, the mass flow rate, and the heat loss from the tank. Figure 4.1-2 shows the inlet temperature T_i as the load.

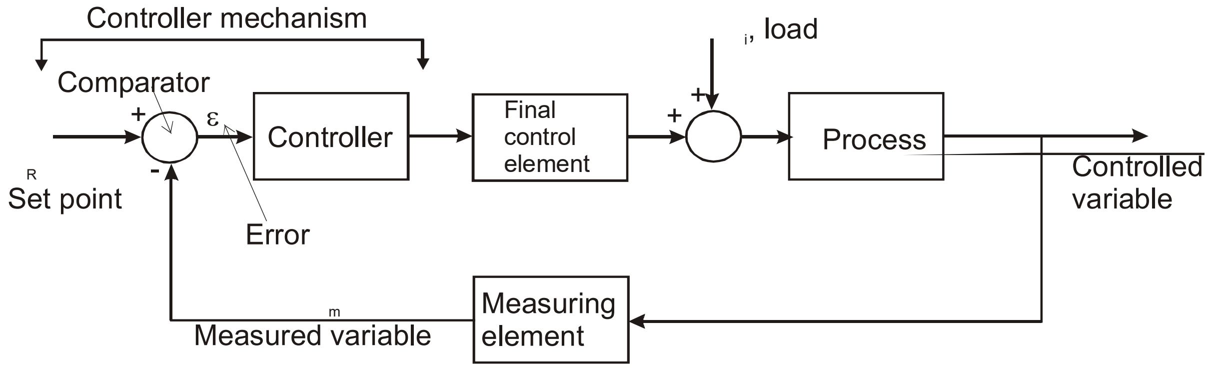


Figure 4.1-2 Block diagram of a simple negative feedback system.

The system shown in Figure 4.1-2 is a negative feedback system where the difference between the set point and the controlled variable is used to reduce the error. Consider the situation where the system is at steady state so that $T = T_m = T_R$ and then the load T_i suddenly increases. T and T_m would start to increase, which would cause the error $\varepsilon = T_R - T_m$ to become negative since T_R is fixed. With proportional control, the decrease in error would cause the controller and final control element to decrease the heat supplied to the tank and would cause T to approach T_R.

There are two types of problem in control system: servo problem and regulatory problem. In servo problem there is no change in load T_i and the set point T_R would be changed accordance with the desired variation in tank temperature. In the regulatory problem there is no change in the set point T_R and the control system would maintain the tank temperature at T_R in spite of changes in load T_i.

4.2 Open Loop Response

An unsteady-state energy balance around the stirred-tank heater shown in Figure 4.1-1 gives

$$\rho C V \frac{dT}{dt} = mC(T_i - T_{ref}) - mC(T - T_{ref}) + q \qquad (4.2\text{-}1)$$

In this equation, C is the heat capacity of the fluid and V is the liquid volume inside the tank. At steady state, Eq. (4.2-1) becomes

$$0 = mC(T_{is} - T_{ref}) - mC(T_s - T_{ref}) + q_s \qquad (4.2\text{-}2)$$

Subtracting Eq. (4.2-2) from Eq. (4.2-1) gives

$$\rho CV \frac{d(T - T_s)}{dt} = mC(T_i - T_{is}) - mC(T - T_s) + q - q_s \tag{4.2-3}$$

Notice that the reference temperature T_{ref} cancels in the subtraction. In terms of the deviation variables T_i^d, T^d, and Q Eq. (4.2-3) becomes

$$\rho CV \frac{dT^d}{dt} = mC(T_i^d - T^d) + Q \tag{4.2-4}$$

The deviation variables are defined as: $T_i^d = T_i - T_{is}$, $T^d = T - T_s$, and $Q = q - q_s$. Taking the Laplace transform of Eq. (4.2-4) gives

$$\rho CVsT^d(s) = mC[T_i^d(s) - T^d(s)] + Q(s) \tag{4.2-5}$$

Rearranging the equation to solve for $T^d(s)$ yields

$$T^d(s)\left(\frac{\rho V}{m}s + 1\right) = T_i^d(s) + \frac{Q(s)}{mC}$$

$$T^d(s) = \frac{1}{\tau s + 1} T_i^d(s) + Q(s)\frac{1/(mC)}{\tau s + 1} \tag{4.2-6}$$

In this equation $\tau = \dfrac{\rho V}{m}$ [min] and the gain for $Q(t)$ is $\dfrac{1}{mC}\left[\dfrac{^\circ\mathrm{C}}{\mathrm{kJ/min}}\right]$. The open loop response for the tank temperature can be presented by the following block diagram.

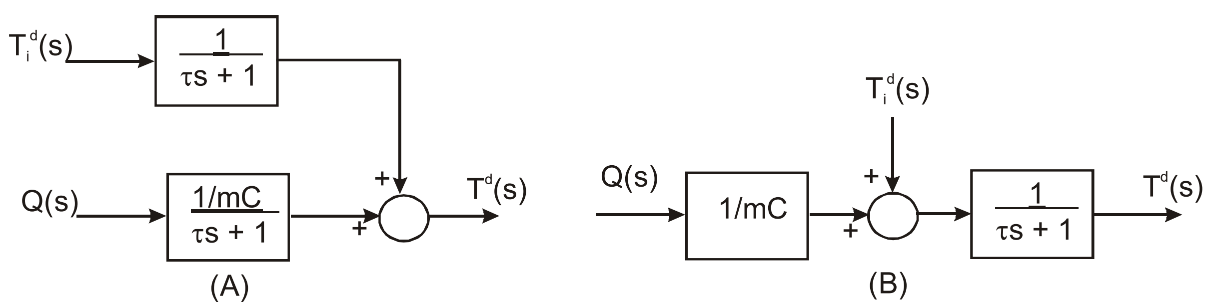


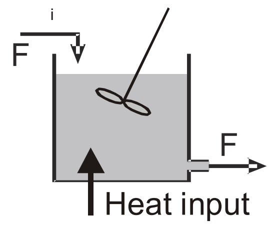
Figure 4.2-1 Two equivalent block arrangements for Eq. (4.2-6).

Example 4.2-1[2]. ---

Consider a well-mixed tank with a water steady flow rate F of 200 L/min. The volume of water in the tank is 1,000 L. The inlet water temperature is 60°C. The system is at steady state with heat input sufficient to heat the outlet water temperature to 80°C.

(a) Determine the response of the outlet tank temperature to a step change in the inlet temperature from 60°C to 70°C

(b) Determine the response of the outlet tank temperature to a step increase in the heat input of 42 kW.



(c) Determine the response of the outlet tank temperature to a simultaneous step change in the inlet temperature from 60°C to 70°C and a step increase in the heat input of 42 kW.

Solution --

The steady-state heat input q_s may be found from the steady-state energy balance,

$$0 = mC(T_{is} - T_{ref}) - mC(T_s - T_{ref}) + q_s$$

$$q_s = mC(T_s - T_{is}) = (200)(4.184)(80 - 60) = 280 \text{ kW}$$

Therefore $\quad Q = q - q_s = q - 280$ kW

The outlet tank temperature is given by Eq. (4.2-6)

$$T^d(s) = \frac{1}{\tau s+1} T_i^d(s) + Q(s)\,\frac{1/(mC)}{\tau s+1}$$

The time constant and steady state gain for the process are evaluated:

$$\tau = \frac{\rho V}{m} = \frac{\rho V}{\rho F} = \frac{V}{F} = \frac{1000}{200} = 5 \text{ min}$$

$$\frac{1}{mC} = \frac{60}{(200)(4.184)} = 0.0717\ ^{\circ}\text{C/kW}$$

(a) Determine the response of the outlet tank temperature to a step change in the inlet temperature from 60°C to 70°C

[2] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 172

For this case $\quad Q = 0 \Rightarrow Q(s) = 0$

$$T_i^d(t) = 70 - 60 = 10 \Rightarrow T_i^d(s) = 10/\text{s}.$$

$$T^d(s) = \frac{1}{\tau s + 1} T_i^d(s) = \frac{1}{5s+1}\frac{10}{\underline{\hspace{1em}}}$$

In the time domain we have

$$T^d(t) = 10(1 - e^{-t/5}) = T - T_s$$

The actual tank outlet temperature is

$$T = T_s + T^d(t) = 80 + 10(1 - e^{-t/5})$$

(b) Determine the response of the outlet tank temperature to a step increase in the heat input of 42 kW.

For this case $\quad T_i^d(t) = 0 \Rightarrow T_i^d(s) = 0$

$$Q = 42 \text{ kW} \Rightarrow Q(s) = 42/s$$

$$T^d(s) = \frac{1}{\tau s + 1} T_i^d(s) + \frac{1/(mC)}{\tau s + 1} Q(s) = \frac{0.0717}{5s+1}\frac{42}{s} = \frac{1}{5s+1}\frac{3}{s}$$

In the time domain we have

$$T^d(t) = 3(1 - e^{-t/5}) = T - T_s$$

The actual tank outlet temperature is

$$T = T_s + T^d(t) = 80 + 3(1 - e^{-t/5})$$

(c) Determine the response of the outlet tank temperature to a simultaneous step change in the inlet temperature from 60°C to 70°C and a step increase in the heat input of 42 kW.

$$T^d(s) = \frac{1}{\tau s + 1} T_i^d(s) + \frac{1/(mC)}{\tau s + 1} Q(s) = \frac{1}{5s+1}\frac{10}{\underline{\hspace{1em}}} + \frac{1}{5s+1}\frac{3}{s}$$

$$T^d(s) = \frac{1}{5s+1}\frac{13}{\underline{\hspace{1em}}}$$

In the time domain we have

$$T^d(t) = 13(1 - e^{-t/5}) = T - T_s$$

The actual tank outlet temperature is

$$T = T_s + T^d(t) = 80 + 13(1 - e^{-t/5})$$

Since the system is linear, the effects are additive. The actual outlet tank temperatures for all cases are plotted in Figure E4.2-1.

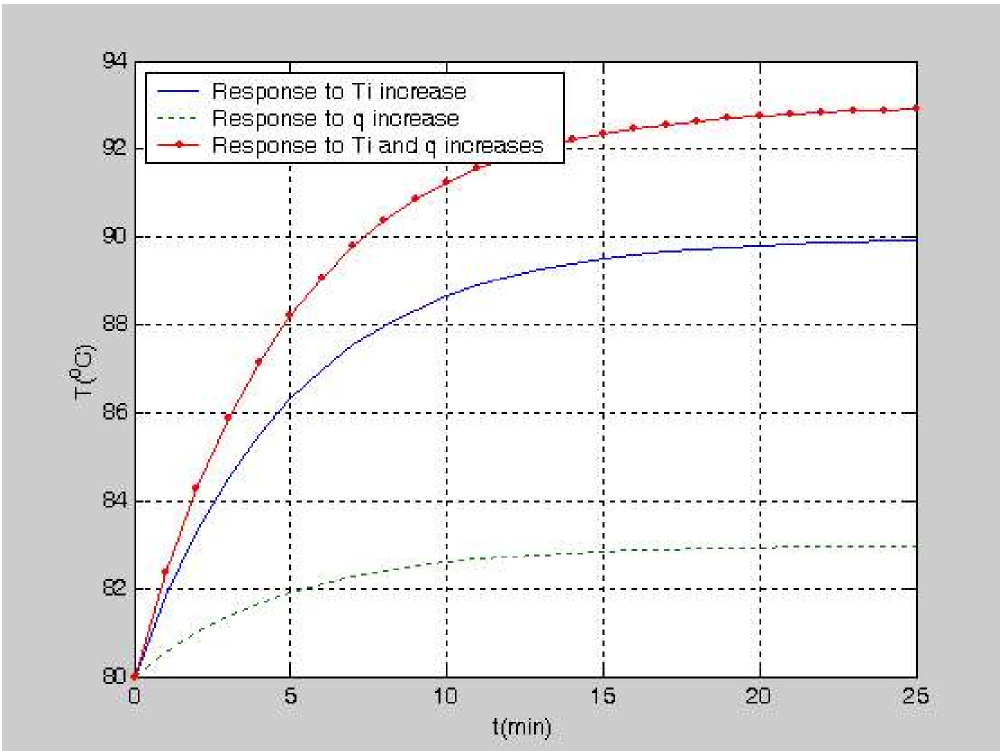
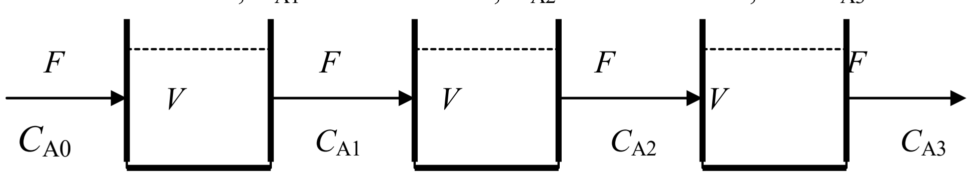


Figure E4.2-1 Outlet tank temperature

Example 4.2-2[3]. ---

Determine the open loop response of three ideal CSTRs in series when the inlet concentration of A, C_{A0}, changes from 0.8 to 1.8 kmol/m^3. The reaction is first order with reaction rate constant k = 0.5 s^{-1}. The tank volume V is 0.10 m^3 and the liquid flow rate F is 0.05 m^3/s. At t = 0, C_{A1} = 0.4 kmol/m^3, C_{A2} = 0.2 kmol/m^3, and C_{A3} = 0.1 kmol/m^3.



Solution --

Material balance for component A over the first CSTR gives

$$\frac{d(C_{A1}V)}{dt} = FC_{A0} - FC_{A1} - k\,C_{A1}V$$

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[3] Luyben, W. L., Process Modeling, Simulation and Control for Chemical Engineers, McGraw Hill, 1997, pg. 119

Dividing the equation by V yields

$$\frac{dC_{A1}}{dt} = (C_{A0} - C_{A1})/\tau - k\, C_{A1} \text{ where } \tau = V/F = 2 \text{ s}$$

Similar balances for tank 2 and tank 3 give

$$\frac{dC_{A2}}{dt} = (C_{A1} - C_{A2})/\tau - k\, C_{A2}$$

$$\frac{dC_{A3}}{dt} = (C_{A2} - C_{A3})/\tau - k\, C_{A3}$$

The above equations may be solved by the following Matlab codes:

```
% Example 4.2-2
tspan=0:0.1:5;
[t,ca]=ode45('cstr3',tspan,[0.4 0.2 0.1]);
plot(t,ca); axis([0 4 0 1]); grid on
xlabel('t(s)');ylabel('C_A(mol/m^3)')
legend('C_{A1}','C_{A2}','C_{A3}',2)

function yca=cstr3(t,ca)
ca1=ca(1);ca2=ca(2);ca3=ca(3);
ca0=1.8; k=0.5;tao=2;
yca(1,1)=(ca0-ca1)/tao-k*ca1;
yca(2,1)=(ca1-ca2)/tao-k*ca2;
yca(3,1)=(ca2-ca3)/tao-k*ca3;
```

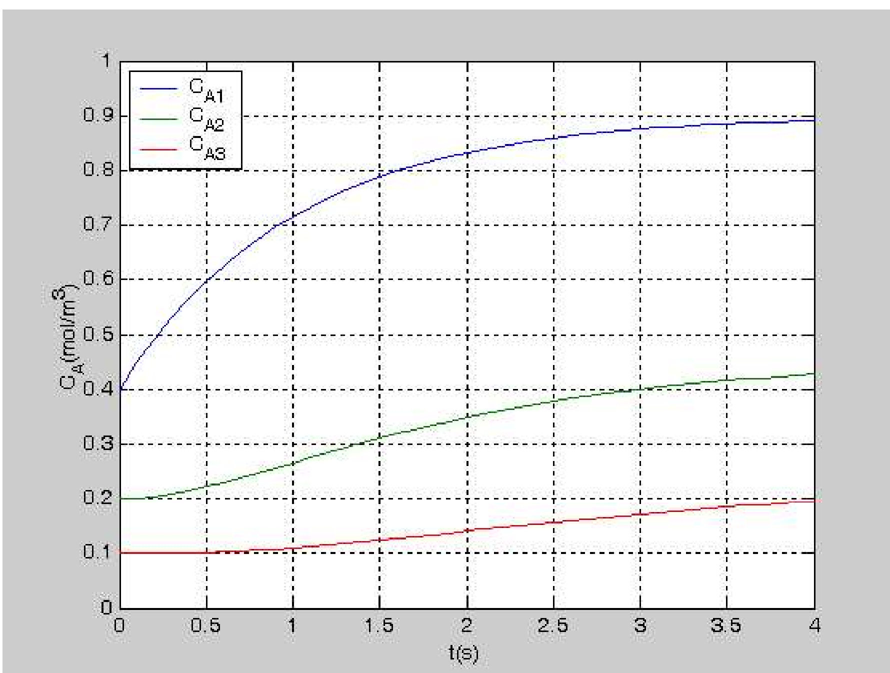

Figure E4.2-2 Outlet concentrations of A from each tank.

Example 4.2-3[4]. --

The temperature sensing element for the stirred-tank in Figure 4.1-1 is a thermocouple. The manufacturer's specifications state that the thermocouple has a response time of 45 s (with the response time defined by the manufacturer as the time required for the thermocouple's reading to be 90 percent complete after a step change). Assuming that the thermocouple behaves as a first-order system, determine the transfer function for the temperature measuring element.

Solution ---

The thermocouple behaves as a first-order system so that

$$\frac{T_m^d(s)}{T^d(s)} = \frac{1}{\tau_m s + 1}$$

For a unit step change, $T^d(s) = \frac{1}{s} \Rightarrow T_m^d(s) = \frac{1}{s}\frac{1}{\tau_m s + 1}$

In the time domain, $T_m^d(t) = [1 - \exp(-t/\tau_m)]$

Since the ultimate value of T_m^d is 1 and the response is 90 percent complete at $t = 45$ s we have

$$0.9 = [1 - \exp(-45/\tau_m)] \Rightarrow -45/\tau_m = \ln(0.1) = -2.3026$$

$$\tau_m = 45/2.3026 = 19.5433 \text{ s} = 0.33 \text{ min}$$

Therefore, the transfer function relating the measured temperature to the actual temperature in the tank is

$$\frac{T_m^d(s)}{T^d(s)} = \frac{1}{0.33s + 1}$$

The block diagram for the stirred-tank heater, including the thermocouple, is shown in Figure 4.2-2.

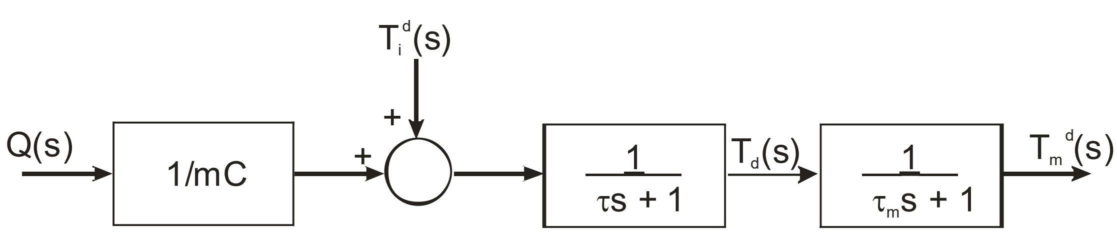

Figure 4.2-2. Block diagram for stirred-tank heater and measuring element.

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[4] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 175

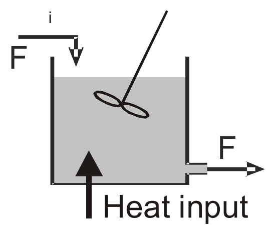
Chapter 4

Example 4.2-4[1]. --

Consider a well-mixed tank with a water steady flow rate F of 200 L/min. The volume of water in the tank is 1,000 L. The inlet water temperature is 60°C. The system is at steady state with heat input sufficient to heat the outlet water temperature to 80°C. Suddenly the inlet temperature experiences a step change from 60°C to 70°C. The thermocouple measuring the tank temperature has a first order transfer function relating the measured temperature T_m^d to the actual temperature $T^d(s)$ in the tank according to

$$\frac{T_m^d(s)}{T^d(s)} = \frac{1}{0.33s+1}$$

Plot the actual tank temperature and the measured temperature as a function of time.



Solution ---

The steady-state heat input q_s may be found from the steady-state energy balance,

$$0 = mC(T_{is} - T_{ref}) - mC(T_s - T_{ref}) + q_s$$

$$q_s = mC(T_s - T_{is}) = (200/60)(4.184)(80 - 60) = 280 \text{ kW}$$

Therefore $\quad Q = q - q_s = q - 280$ kW

The outlet tank temperature is given by Eq. (4.2-6) for the case $Q = 0 \Rightarrow Q(s) = 0$

$$T^d(s) = \frac{1}{\tau s+1} T_i^d(s) + Q(s)\,\frac{1/(mC)}{\tau s+1} = \frac{1}{\tau s+1} T_i^d(s)$$

The time constant and steady state gain for the process are evaluated:

$$\tau = \frac{\rho V}{m} = \frac{\rho V}{\rho F} = \frac{V}{\quad} = \frac{1000}{200} = 5 \text{ min}$$

[1] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 172

$$\frac{1}{mC} = \frac{60}{(200)(4.184)} = 0.0717\ ^{\circ}\text{C/kW}$$

$$T_i^d(t) = 70 - 60 = 10 \Rightarrow T_i^d(s) = 10/\text{s}.$$

$$T^d(s) = \frac{1}{\tau s+1}\,T_i^d(s) = \frac{1}{5s+1}\,\frac{10}{\_\_}$$

In the time domain we have

$$T^d(t) = 10(1 - e^{-t/5}) = T - T_s$$

The measured temperature is given by

$$T_m^d(t) = T^d(t)\,\frac{1}{0.33s+1} = \frac{10}{s(0.33s+1)(5s+1)}$$

$$T_m^d(t) = \frac{10}{\_\_} + \frac{0.71}{s+3.03} - \frac{10.71}{s+0.2}$$

In the time domain we have

$$T_m^d(t) = 10 + 0.71\exp(-3.03t) - 10.71\exp(-0.2t)$$

The actual and measured temperatures are plotted in Figure E-1. From the graph, it is clear that the measured temperature lags behind the actual temperature.

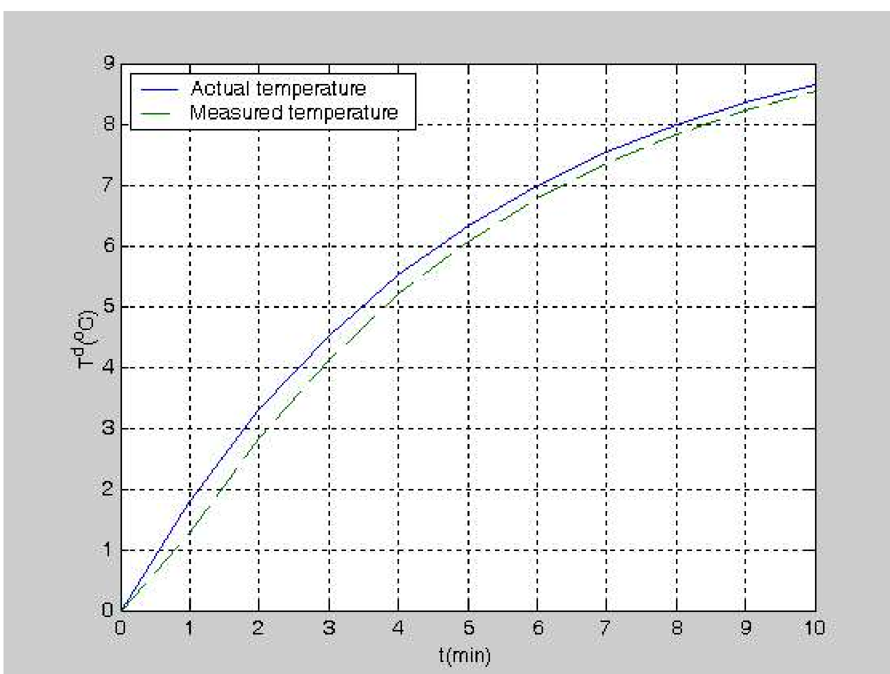


Figure E-1 Response for actual and measured temperatures.

4.3 Closed Loop Response

When the measured temperature is transmitted to the controller to adjust the steam flow rate so that the tank temperature is maintained near the set point we have a closed loop response. The blocks representing the controller and the final control element can be combined into one block. In this way, we only need to consider the overall response between the error in the temperature and the heat input to the tank. For a proportional controller, the heat input to the tank is

$$q = K_C \varepsilon + q_0 \tag{4.3-1}$$

In this equation, $\varepsilon = T_R - T_m$, where T_R = set point temperature, K_C = proportional sensitivity or controller gain, and q_0 is the heat input when $\varepsilon = 0$. q_0 is also called the bias value. At steady state, it is assumed that the set point, the process temperature, and the measured temperature are all equal to one another; thus

$$T_{Rs} = T_s = T_{ms} \tag{4.3-2}$$

The deviation for error is defined as $\varepsilon^d = \varepsilon - \varepsilon_s$ where $\varepsilon_s = T_{Rs} - T_{ms}$. Since $T_{Rs} = T_{ms}$, $\varepsilon_s = 0$. Therefore

$$\varepsilon^d = \varepsilon \tag{4.3-3}$$

The error itself is a deviation variable. At steady state there is no error

$$q_s = K_C \varepsilon_s + q_0 = 0 + q_0.$$

The steady-state output from the controller/heater is termed the bias value. Equation (4.3-1) can now be written in terms of q_s.

$$q = K_C \varepsilon + q_s \Rightarrow Q = q - q_s = K_C \varepsilon \tag{4.3-4}$$

Taking the Laplace transform of Eq. (4.3-4) gives us the proportional controller transfer function

$$Q(s) = K_C \varepsilon(s) \tag{4.3-5}$$

The error ε can be written as

$$\varepsilon = T_R - T_m = T_R^d - T_m^d \tag{4.3-6}$$

In Laplace domain, the error is then

$$\varepsilon(s) = T_R^d(s) - T_m^d(s) \tag{4.3-7}$$

The block diagram for a feed back control loop is shown in Figure 4.3-1. The expressions of the transfer functions are shown in the block diagram of Figure 4.3-2. It should be noted that the deviation variables are displayed in Figure 4.3-2.

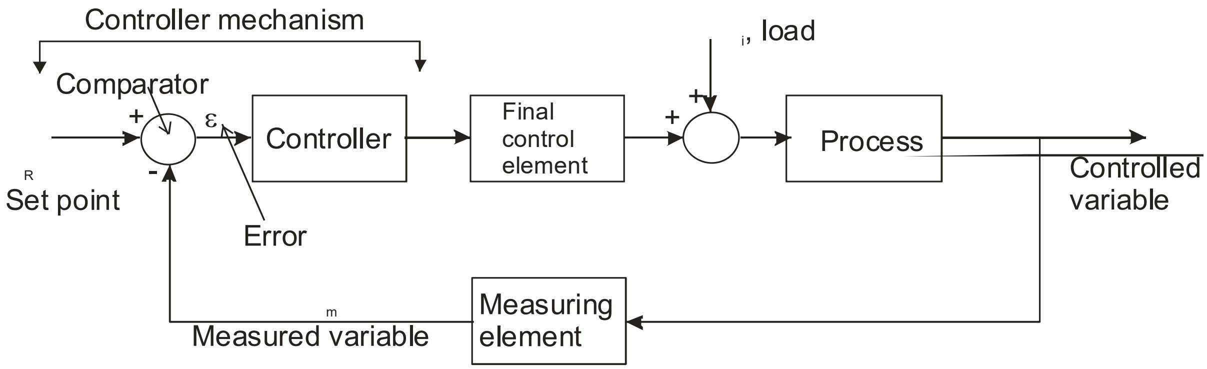


Figure 4.3-1 Block diagram of a simple negative feedback system.

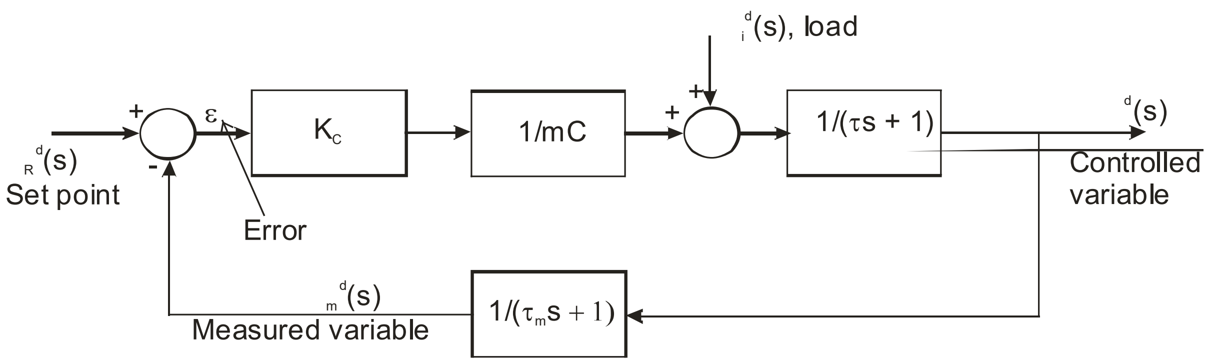


Figure 4.3-2 Block diagram of a negative feedback system with proportional controller.

Using block algebra diagram we obtain the following expression:

$$T^{\mathrm{d}}(s) = \frac{1}{\tau s+1}\,T_{\mathrm{i}}^{\mathrm{d}}(s) + \frac{K_C/(mC)}{\tau s+1}\,T_{\mathrm{R}}^{\mathrm{d}}(s) - \frac{1}{\tau_m s+1}\,\frac{K_C/(mC)}{\tau s+1}\,T^{\mathrm{d}}(s)$$

$$T^{\mathrm{d}}(s)\left[1+\frac{K_C/(mC)}{(\tau_m s+1)(\tau s+1)}\right] = \frac{1}{\tau s+1}\,T_{\mathrm{i}}^{\mathrm{d}}(s) + \frac{K_C/(mC)}{\tau s+1}\,T_{\mathrm{R}}^{\mathrm{d}}(s)$$

$$T^{\mathrm{d}}(s)\left[\frac{mC(\tau_m s+1)(\tau s+1)+K_C}{mC(\tau_m s+1)(\tau s+1)}\right] = \frac{1}{\tau s+1}\,T_{\mathrm{i}}^{\mathrm{d}}(s) + \frac{K_C}{mC(\tau s+1)}\,T_{\mathrm{R}}^{\mathrm{d}}(s)$$

$$T^{\mathrm{d}}(s) = \frac{mC(\tau_m s+1)}{mC(\tau_m s+1)(\tau s+1)+K_C}\,T_{\mathrm{i}}^{\mathrm{d}}(s) + \frac{K_C(\tau_m s+1)}{mC(\tau_m s+1)(\tau s+1)+K_C}\,T_{\mathrm{R}}^{\mathrm{d}}(s)$$

We now consider the response of the tank temperature to a change in the set point of 5°C. The system has a water steady flow rate m of 200 kg/min, process time constant τ = 5 min., measurement time constant τ_{m} = 0.33 min., proportional gain K_{C} = 20.

$$T^{d}(s) = \frac{K_C(\tau_m s+1)}{mC(\tau_m s+1)(\tau s+1)+K_C}\,T_R{}^{d}(s)$$

$$T^{d}(s) = \frac{20(0.33s+1)}{\frac{200}{60}\times 4.184(0.33s+1)(5s+1)+20}\,T_R{}^{d}(s)$$

$$T^{d}(s) = \frac{0.33s+1}{0.6973(0.33s+1)(5s+1)+1}\,\frac{5}{s} = 1.434\frac{1}{s}\,\frac{s+3.0303}{(s+2.6798)(s+0.5505)}$$

Using partial fraction, we obtain

$$T^{d}(s) = 1.434\left[\frac{2.0541}{s}+\frac{0.0614}{s+2.6798}-\frac{2.1155}{s+0.5505}\right]$$

Taking the inverse Laplace transform of this expression give

$$T^{d} = 1.434[2.0541 + 0.0614\exp(-2.6798t) - 2.1155\exp(-0.5505t)]$$

The plot of the response of the tank temperature to a change in the set point of 5°C is shown in Figure 4.3-3.

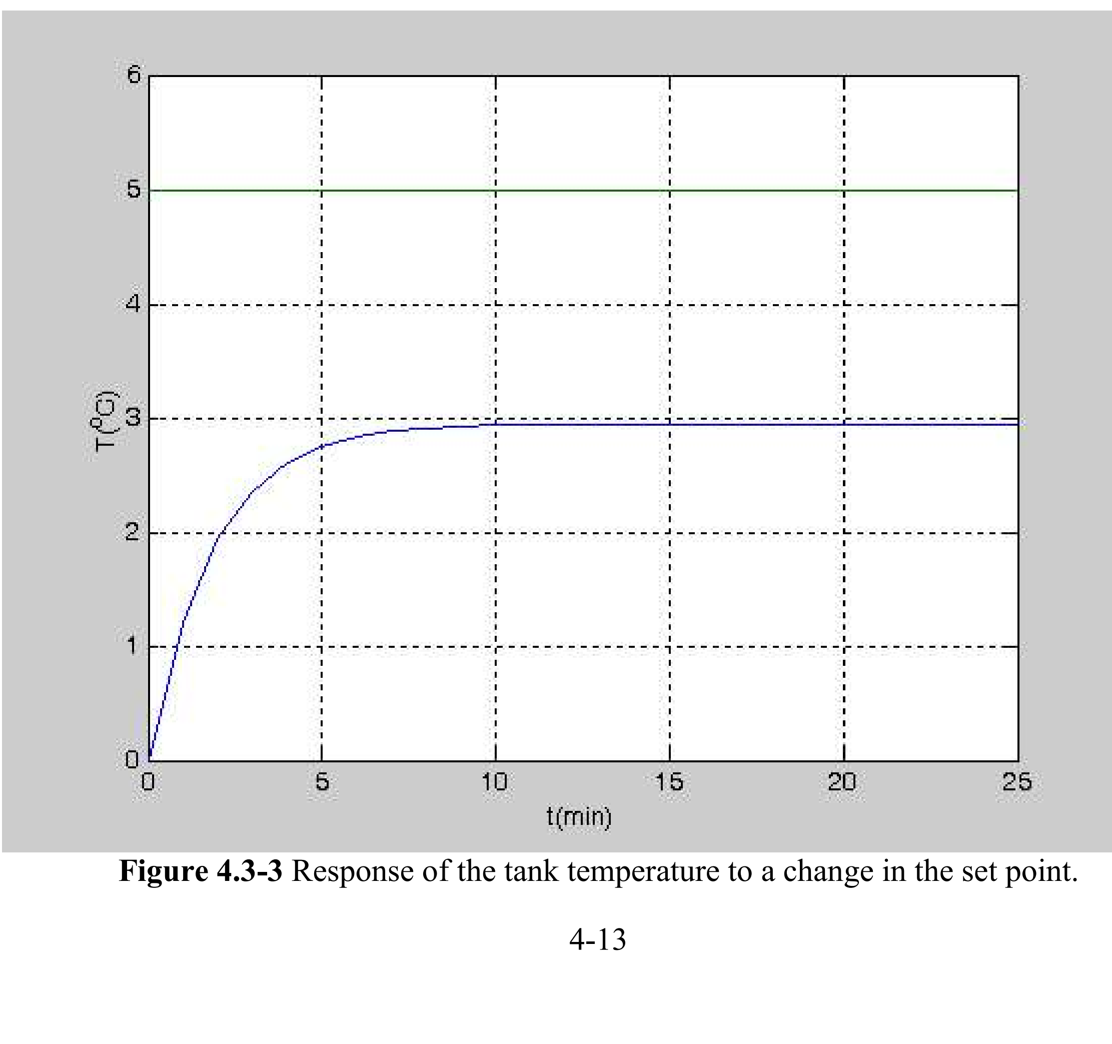


Figure 4.3-3 Response of the tank temperature to a change in the set point.

4.4 Simulink

Simulink is a software package embedded under Matlab for modeling, simulating, and analyzing dynamical systems. This program can be used for linear and nonlinear systems, modeled in continuous time, sampled time, or a hybrid of the two. Systems can have different parts that are sampled or updated at different rates. For modeling, Simulink provides a graphical user interface (GUI) for building models as block diagrams, using click-and-drag mouse operation similar to simulation software such as Hysys or Provision. Using Simulink to model you do not need to directly derive the differential and algebraic equations describing the system. Instead you combine block diagrams to represent physical phenomena and Simulink will solve the underlying differential and algebraic equations for the system.

To start Simulink, you must first start Matlab then click on the Simulink icon on the Matlab toolbar or enter the simulink command at the Matlab prompt. You can run a demo program provided with Simulink models the thermodynamics of a house. To run this demo you enter the thermo command in the Matlab command window. This command starts up Simulink and creates a model shown in Figure 4.4-1.

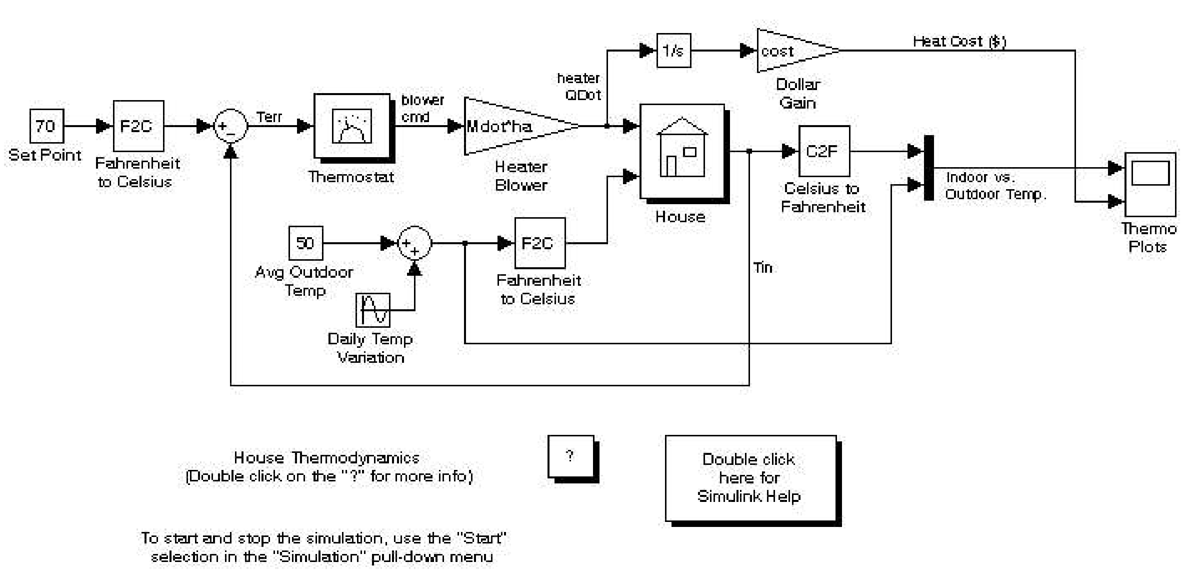


Figure 4.4-1 Thermodynamics model of a house.

To start the simulation, pull down the **Simulation** menu and choose the **Start** command or click the **Start** button on the Simulink toolbar. As the simulation runs, the indoor and outdoor temperatures appear in the Indoor vs. Outdoor Temp plot and the cumulative heating cost appears in the Heat Cost plot. To stop the simulation, choose the **Stop** command from the **Simulation** menu or click the **Pause** button on the toolbar. You can also run the **simulink_tutorial.swf** by double click on it. This program is in the **Distribution folder** of CHE 426. You might have to copy this program to your H: drive before it can execute.

We now use Simulink to obtain the response of the tank temperature to a change in the set point of 5°C. The system has a water steady flow rate m of 200 kg/min, process time constant τ = 5 min., measurement time constant τ_m = 0.33 min., proportional gain K_C = 20. The block diagram in Simulink is shown in Figure 4.4-2.

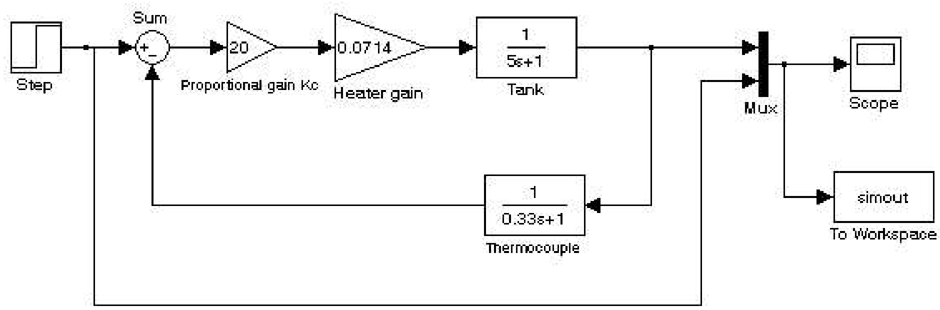


Figure 4.4-2 Simulink model of stirred tank control system.

The heater gain is given by $\frac{1}{mC} = \frac{60}{(200)(4.184)} = 0.0717$ $^oC/kW$. You can write click on each block to input the block parameters. You should notice that the point coming in (between the **Step** and the **Sum** blocks) or out (between the **Tank** and **Mux** blocks) of each line can be achieved by ending a different line on that location. The parameters are listed for some of the blocks:

Step block:

Step time: 0	Initial value: 0	Final value: 5	Sample time: 0

Simout block:

Variable name: simout	Data points: 1000	Decimation: 1	Sample time: 1
Save format: Array			

You can write click a block and flip it using the **Flip block** command from the **Format** menu. Choose the **Simulation parameters** command from the **Simulation** menu and assign 0 for **Start time** and 30 for **Stop time**. With this choice, the *simout* variable will be an array 31×2 with tank temperature and set point temperatures every minute from 0 to 30 minutes. Click on the **Scope** block to see the temperature responses which can be plotted out using the following Matlab codes:

```
>> t=0:30;
>> plot(t,simout)
>> xlabel('t(min)')
>> ylabel('Temperature change(^oC)')
>> legend('Tank temp','Set temp')
```

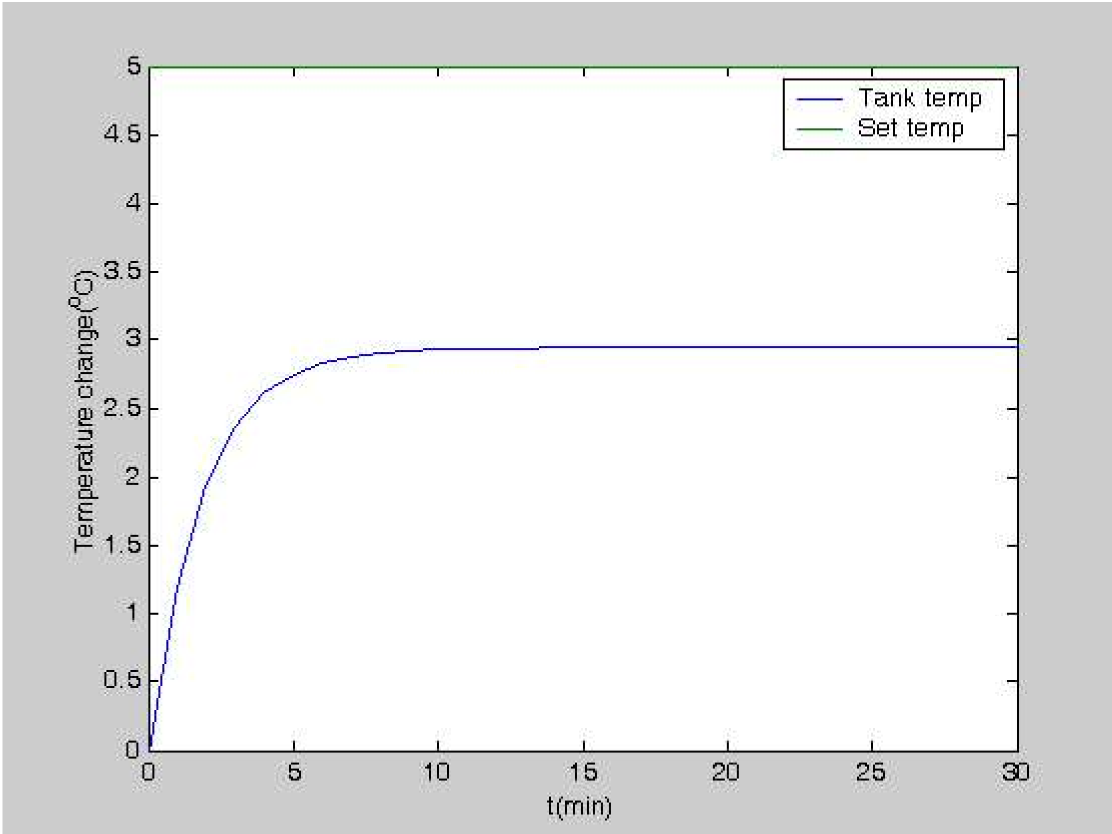


Figure 4.4-3 Response of the tank temperature to a change in the set point.

A control system can be translated to a block diagram that includes the transfer function of the various components. A block diagram is another way of writing the simultaneous differential and algebraic equations that describe the dynamic behavior of the components. The block diagram clearly indicates the relationships among the variables of the systems and the feedback relationship between measured variable and desired variable. A set of equations generally cannot indicate the relationships shown by the block diagram.

Chapter 4

4. 5 Second Order Systems

A second order system can arise from two first-order system in series. Adding a controller to a first order process usually results in a second order system. Some systems are inherently second-order, and they do not result from a series combination of two-first order systems. Second-order systems are described by a second-order differential equation that relates the output variable y to the input variable x (the forcing function) with time as the independent variable. The standard form of a second order differential equation is written as

$$\tau^2 \frac{d^2y}{dt^2} + 2\zeta\tau \frac{dy}{dt} + y = x(t) \qquad (4.5\text{-}1)$$

Notes that it requires two parameters, τ and ζ, to characterize the dynamics of a second-order system in contrast to only one parameter for a first-order system. τ is the time constant and ζ is called the damping coefficient of the system. An inherently second-order system is the response of the manometer reading h to the applied pressure difference $\Delta P = P_1 - P_2$ as shown in Figure 4.5-1. The pressure on both legs of the manometer is initially the same. The length of the fluid column (including both legs) in the manometer is L. At time $t = 0$, a pressure difference is imposed across the legs of the manometer. We will determine the dynamic response of the manometer reading h.

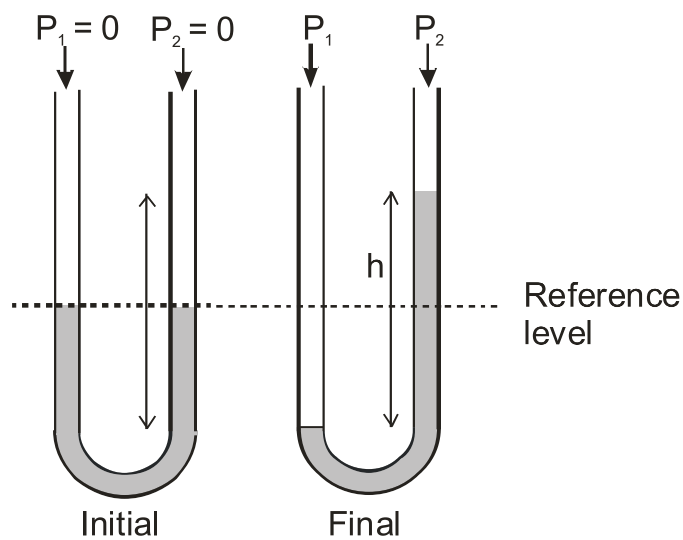


Figure 4.5-1 A manometer system.

We can determine the transfer function between the applied pressure difference ΔP and the manometer reading with the assumption of laminar flow in the manometer and by applying the conservation of momentum to a the manometer fluid. The equation for the conservation of momentum with respect to a control volume (CV) can be written as follows:

$$\left\{\begin{array}{c}\text{rate of accumulation}\\ \text{of momentum in CV}\end{array}\right\} = \left\{\begin{array}{c}\text{rate of momentum}\\ \text{into CV}\end{array}\right\} - \left\{\begin{array}{c}\text{rate of momentum}\\ \text{out of CV}\end{array}\right\} + \left\{\begin{array}{c}\text{sum of forces}\\ \text{acting on CV}\end{array}\right\}$$

$$\text{—}\,(m\ \vec{\ }\ )_{\mathrm{cv}} = \sum_{in}(\dot{m}\vec{V})_i - \sum_{out}(\dot{m}\vec{V})_o + \sum_{on\ \ CV}\vec{F} \tag{4.5-2}$$

The total force acting on the control volume consists both of surface forces and body forces. We choose the control volume to be the manometer fluid then

$$\sum_{in}(\dot{m}\vec{V})_i - \sum_{out}(\dot{m}\vec{V})_o = 0$$

Apply the momentum balance on the control volume.

$$\text{—}\,(m\ \vec{\ }\ )_{\mathrm{cv}} = \sum_{on\ \ CV}\vec{F} = F_{\mathrm{p}} - F_{\mathrm{g}} - F_{\mathrm{f}} \tag{4.5-3}$$

$$\text{—}\,(m\ \vec{\ }\ )_{\mathrm{cv}} = \text{—}\,(A_{\mathrm{m}}L\rho\beta\ \bar{\ }\ )$$

In this equation, $A_{\mathrm{m}} = \pi D^2/4$ = cross-sectional area of manometer, β = correction factor for the velocity due to the fact that the fluid has a parabolic velocity profile = 4/3.

$F_{\mathrm{p}} = (P_1 - P_2)\pi D^2/4$ = net pressure force on the CV

$F_{\mathrm{g}} = \rho g h \pi D^2/4$ = gravitation force on the CV

$F_{\mathrm{f}} = \tau_{\mathrm{w}}(\pi DL)$ = frictional force on the CV

The wall shear stress τ_{w} for laminar flow in a tube is given by

$\tau_{\mathrm{w}} = \dfrac{8\mu\bar{V}}{D}$, where $\bar{\ }$ is the average velocity of the fluid in the tube, which is the velocity of the interface = $0.5dh/dt$ (Note: since the manometer reading h has two interfaces, $dh/dt = 2\,\bar{\ }$). Equation (4.5-3) becomes

$$\rho\frac{\pi D^2}{4}L\left(\frac{4}{3}\right)\frac{1}{2}\frac{d^2h}{dt^2} = (P_1 - P_2)\frac{\pi D^2}{4} - \rho g h\frac{\pi D^2}{4} - \frac{8\mu}{D}\frac{1}{2}\frac{dh}{dt}(\pi DL) \tag{4.5-4}$$

Rearranging Eq. (4.5-4), we obtain

$$\rho\frac{\pi D^2}{4}L\left(\frac{4}{3}\right)\frac{1}{2}\frac{d^2h}{dt^2} + \frac{8\mu}{D}\frac{1}{2}\frac{dh}{dt}(\pi DL) + \rho g h\frac{\pi D^2}{4} = (P_1 - P_2)\frac{\pi D^2}{4}$$

Dividing the equation by $\rho g\pi D^2/4$, we obtain the standard form for a second-order system.

$$\frac{2L}{3g}\frac{d^2h}{dt^2} + \frac{16\mu L}{\rho D^2 g}\frac{dh}{dt} + h = \frac{P_1 - P_2}{\rho g} = \frac{\Delta P}{\rho g} \tag{4.5-5}$$

Comparing Eq.(4.5-5) with the general standard form

$$\tau^2\frac{d^2y}{dt^2} + 2\zeta\tau\frac{dy}{dt} + y = x(t) \tag{4.5-6}$$

We have $y = h$, $\tau^2 = \dfrac{2L}{3g}$, $2\zeta\tau = \dfrac{16\mu L}{\rho D^2 g}$, and $x(t) = \dfrac{\Delta P}{\rho g}$. Solving for τ and ζ, we have

$$\tau^2 = \frac{2L}{3g} \Rightarrow \tau = \left(\frac{2L}{3g}\right)^{1/2}$$

$$\zeta = \frac{8\mu L}{\rho D^2 g\tau} = \frac{8\mu}{\rho D^2}\left(\frac{3L}{2g}\right)^{1/2}$$

If the fluid column is motionless ($dh/dt = 0$) and located at its rest position ($h = 0$), the initial conditions for Eq. (4.5-5) are

$t = 0$, $dh/dt = 0$, and $h = 0$

The Laplace transform of Eq. (4.5-6) becomes

$$\tau^2 s^2 Y(s) + 2\zeta\tau s Y(s) + Y(s) = X(s)$$

The transfer function is then

$$\frac{Y(s)}{X(s)} = \frac{1}{\tau^2 s^2 + 2\zeta\tau s + 1} \tag{4.5-7}$$

Equation (4.5-7) is the standard form of a second-order transfer function. We now discuss the response of a second-order system to the unit step forcing function where $X(s) = 1/s$. For the manometer system, this is equivalent to suddenly applying a pressure difference such that $x(t) = \dfrac{\Delta P}{\rho g} = 1$ across the legs of the manometer at time $t = 0$. Solving for $Y(s)$ we obtain

$$Y(s) = \frac{1}{s}\,\frac{1}{\tau^2 s^2 + 2\zeta\tau s + 1} \tag{4.5-8}$$

The quadratic term can be factored into two linear terms as follows

$$\tau^2 s^2 + 2\zeta\tau s + 1 = (s - s_a)(s - s_b) \tag{4.5-9}$$

In this equation, s_a and s_b are the two roots of equation $\tau^2 s^2 + 2\zeta\tau s + 1 = 0$.

$$s_a = -\frac{\zeta}{\tau} + \frac{\sqrt{\zeta^2 - 1}}{\tau} \text{ and } s_b = -\frac{\zeta}{\tau} - \frac{\sqrt{\zeta^2 - 1}}{\tau}$$

Equation (4.5-8) can now be written as

$$Y(s) = \frac{1}{s}\,\frac{1/\tau^2}{(s - s_a)(s - s_b)} \tag{4.5-10}$$

The response of $Y(t)$ can be found by inverting Eq. (4.5-10). The form of $Y(t)$ depends on the roots s_a and s_b which can be real or complex. There are three cases for the response listed in Table 4.5-1.

Table 4.5-1 Step response of a second-order system

Case	ζ	Nature of roots	Type of response
A	< 1	Complex	Under-damped or oscillatory
B	= 1	Real and equal	Critically damped
C	> 1	Real	Over-damped or non-oscillatory

Case A: Step response for $\zeta < 1$.

The inversion of Eq. (4.5-10) has the form

$$y(t) = C_1 + \exp\left(-\zeta\frac{t}{\tau}\right)\left[C_2\cos\left(\sqrt{1-\zeta^2}\,\frac{t}{\tau}\right) + C_3\sin\left(\sqrt{1-\zeta^2}\,\frac{t}{\tau}\right)\right] \tag{4.5-11}$$

The constant C_1, C_2, and C_3 can be found by partial fractions. By applying the trigonometric identity, we can rearrange Eq. (4.5-11) into the following form

$$y(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}}\,\exp\left(-\zeta\frac{t}{\tau}\right)\,\sin\left[\left(\sqrt{1-\zeta^2}\,\frac{t}{\tau}\right) + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right] \tag{4.5-12}$$

Case B: Step response for $\zeta = 1$.

The roots s_a and s_b are real and equal. The inversion of Eq. (4.5-10) gives the following expression

$$y(t) = 1 - \left(1 + \frac{t}{\tau}\right)\exp\left(-\frac{t}{\tau}\right) \tag{4.5-13}$$

Case C: Step response for $\zeta > 1$.

The inversion of Eq. (4.5-10) gives the following expression

$$y(t) = 1 - \exp\left(-\zeta \frac{t}{\tau}\right) \left[\cosh\left(\sqrt{\zeta^2 - 1}\,\frac{t}{\tau}\right) + \frac{\zeta}{\sqrt{\zeta^2 - 1}} \sinh\left(\sqrt{\zeta^2 - 1}\,\frac{t}{\tau}\right)\right] \qquad (4.5\text{-}14)$$

Figure 4.5-2 shows the nature of the response for various value of the damping coefficient ζ. For $\zeta < 1$ all the response curves are oscillatory in nature and become less oscillatory as ζ is increased. The slop at the origin in Figure 4.5-2 is zero for all values of $\zeta < 1$. The respond of a second order system for $\zeta < 1$ is said to be underdamped. If we step-change the pressure difference across an underdamped manometer, the liquid levels in the two legs will oscillate before stabilizing. The oscillations are characteristic of an underdamped response. The response for $\zeta = 1$ is called critical damping and is the most rapid approach to $y(t) = 1$ without oscillation. For $\zeta > 1$ all the response curves are nonoscillatory and become more sluggish as ζ is increased.

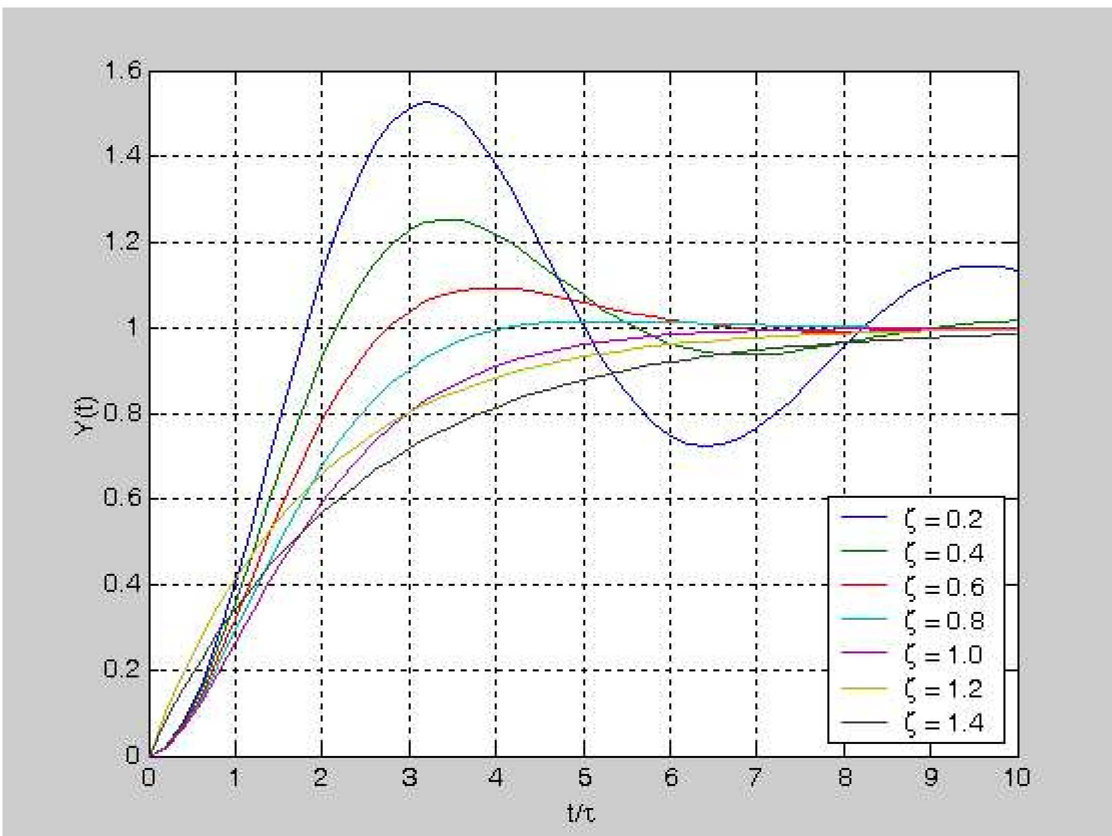


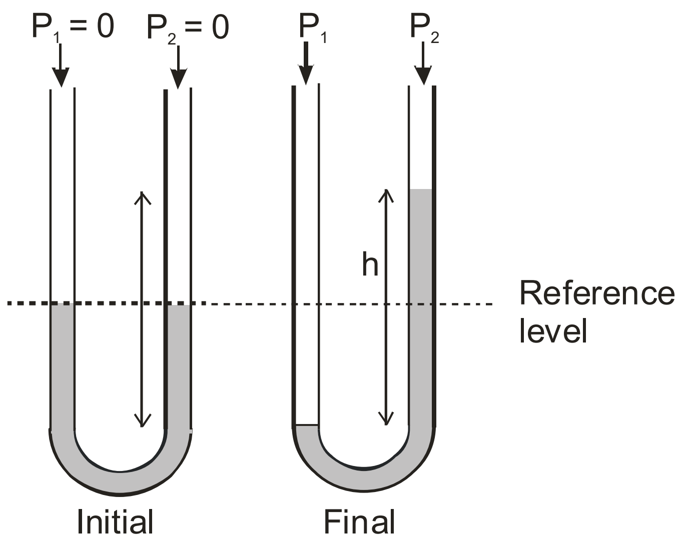
Figure 4.5-2 Response of a second-order system to a unit-step forcing function.

Example 4.5-1[1]. --

Consider a manometer as illustrated below. The manometer is being used to determine the pressure difference between two instrument taps on an air line. The working fluid in the manometer is water. Determine the response of the manometer to a step change in pressure across the legs of the manometer for the cases when the inside tube diameter are 0.11 cm, 0.21 cm, and 0.31 cm.

Data: manometer fluid length, L = 200 cm; g = 980 cm^2/s; μ = 0.01 g/cm·s, ρ = 1.0 g/cm^3.

$$\frac{\Delta P}{\rho g} = \begin{cases} 0 & \text{for } t < 0 \\ 10 \text{ cm for } t \geq 0 \end{cases}$$



Solution --

The response of the manometer reading h to the change in applied pressure is given by the following differential equation

$$\frac{2L}{3g}\frac{d^2h}{dt^2} + \frac{16\mu L}{\rho D^2 g}\frac{dh}{dt} + h = \frac{P_1 - P_2}{\rho g} = \frac{\Delta P}{\rho g}$$

The transfer function for the response is

$$\frac{Y(s)}{X(s)} = \frac{1}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

In this equation, $Y(s) = h(s) - h_s(s)$, $X(s) = \frac{\Delta P}{\rho g} - \left.\frac{\Delta P}{\rho g}\right|_s$, $\tau = \left(\frac{2L}{3g}\right)^{1/2}$, and $\zeta = \frac{8\mu}{\rho D^2}\left(\frac{3L}{2g}\right)^{1/2}$

The numerical values for the time constant τ and the damping coefficient ζ are

$$\tau = \left(\frac{2L}{3g}\right)^{1/2} = \left(\frac{2\times 200}{3\times 980}\right)^{1/2} = 0.369 \text{ s}$$

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[1] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 144

$$\zeta = \frac{8\mu}{\rho D^2}\left(\frac{3L}{2g}\right)^{1/2} = \frac{8\times 0.01}{1.0\times D^2}\left(\frac{3\times 200}{2\times 980}\right)^{1/2} = \frac{0.0443}{D^2}$$

D(cm)	0.11	0.21	0.31
ζ	3.66	1.00	0.46

The response in the Laplace domain is

$$Y(s) = \frac{10}{s}\,\frac{1}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

The coefficients for the second-order response are listed for each diameter in the following table

D(cm)	τ^2	$2\zeta\tau$	Transfer function
0.11	0.136	2.70	$\frac{1}{0.136s^2 + 2.70s + 1}$
0.21	0.136	0.738	$\frac{1}{0.136s^2 + 0.738s + 1}$
0.31	0.136	0.340	$\frac{1}{0.136s^2 + 0.340s + 1}$

The manometer response can be obtained from Simulink with the block diagram shown in Figure E-1.

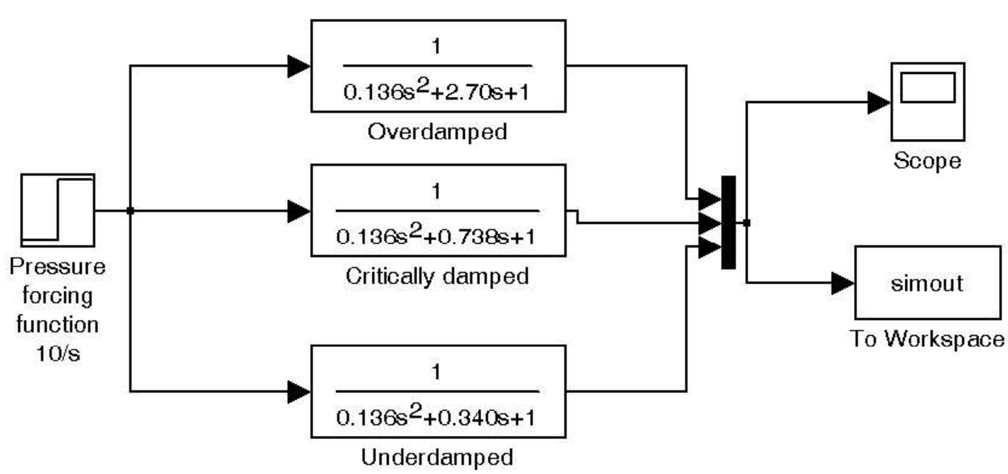

Figure E-1 Simulink block diagram for manometer simulation

Figure E-2 shows the responses result from the Simulink model. The *simout* variable is an array with three column and 51 row at time interval of 0.2 sec.

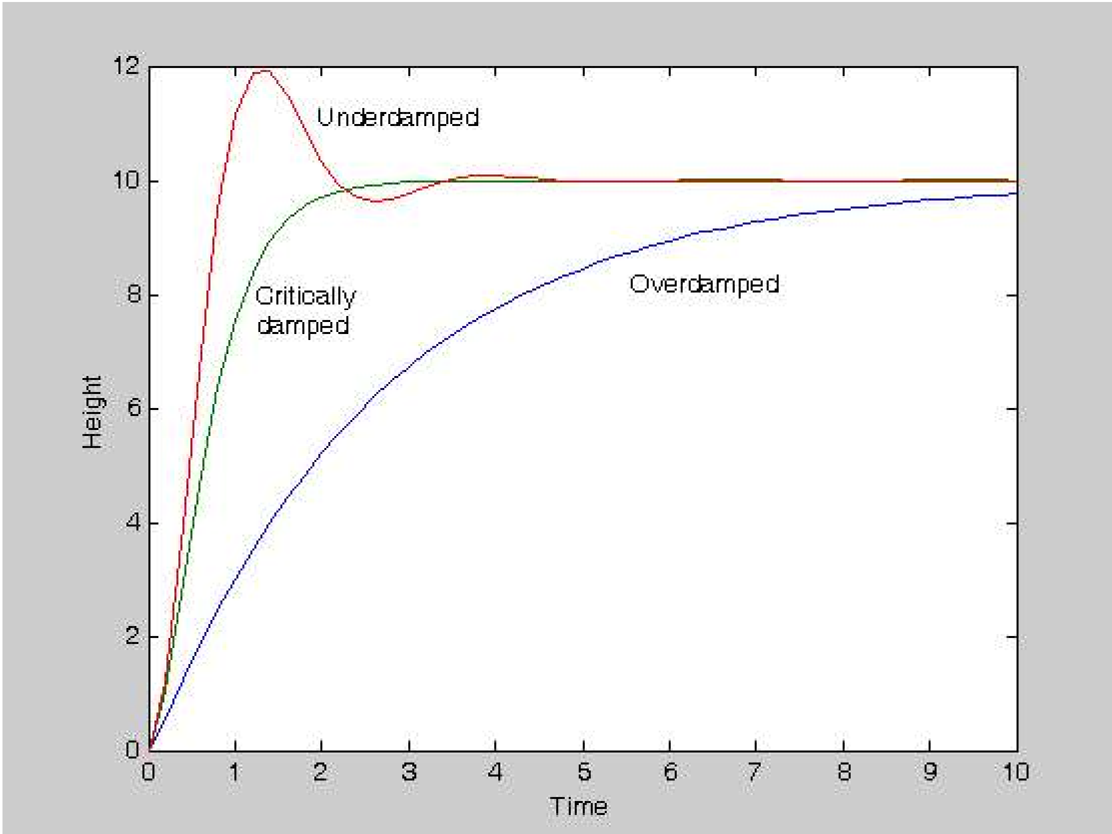

Figure E-2 Manometer response to step input.

Figure **E-2** can also be obtained from plotting the manometer equations in the time domain.

Step response for $\zeta < 1$.

$$y(t)/10 = 1 - \frac{1}{\sqrt{1-\zeta^2}} \exp\left(-\zeta \frac{t}{\tau}\right) \sin\left[\left(\sqrt{1-\zeta^2}\,\frac{t}{\tau}\right) + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right]$$

$$y(t)/10 = 1 - 1.13\exp(-1.25t) \sin\left(2.41t + \tan^{-1} 1.93\right)$$

Step response for $\zeta = 1$.

$$y(t)/10 = 1 - \left(1 + \frac{t}{\tau}\right) \exp\left(-\frac{t}{\tau}\right)$$

$$y(t)/10 = 1 - \left(1 + \frac{t}{0.369}\right) \exp\left(-\frac{t}{0.369}\right)$$

Step response for $\zeta > 1$.

$$y(t)/10 = 1 - \exp\left(-\zeta\frac{t}{\tau}\right)\left[\cosh\left(\sqrt{\zeta^2-1}\frac{t}{\tau}\right)+\frac{\zeta}{\sqrt{\zeta^2-1}}\sinh\left(\sqrt{\zeta^2-1}\frac{t}{\tau}\right)\right]$$

$$y(t)/10 = 1 - \exp(-9.92t)[\cosh(9.45t) + 1.04\sinh(9.45t)]$$

The underdamped response curve to a unit step increase is shown in Figure 4.5-3. We now defined the expressions used to describe the response. These expressions can be derived from the time response given by the following equation.

$$y(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}}\exp\left(-\zeta\frac{t}{\tau}\right)\sin\left[\left(\sqrt{1-\zeta^2}\frac{t}{\tau}\right)+\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right] \quad (4.5\text{-}12)$$

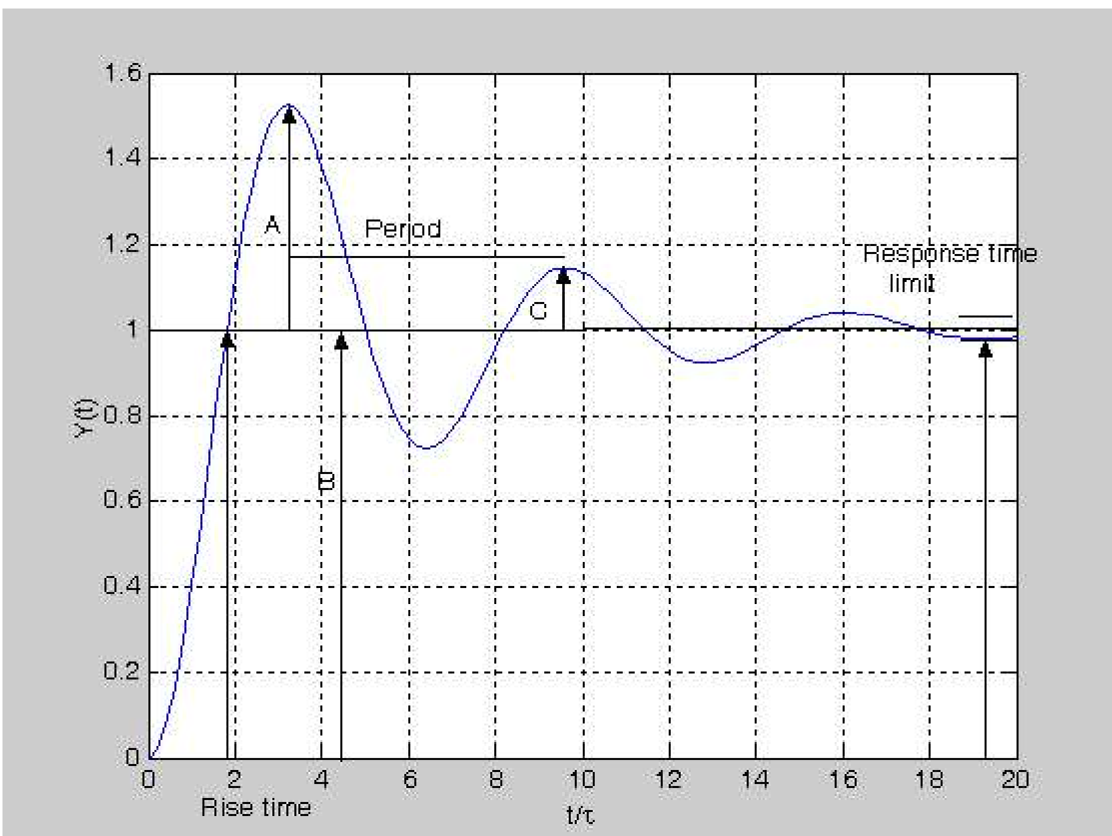

Figure 4.5-3 Underdamped response characteristic.

A) *Overshoot*. Overshoot is a measure of how much the response exceeds the steady state value following a step change and is expressed as the ratio A/B in Figure 4.5-3.

$$\text{Overshoot} = \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right) \tag{4.5-15}$$

B) *Decay ratio*. The decay ratio is defined as the ratio of the sizes of successive peaks and is given by C/A in Figure 4.5-3.

$$\text{Decay ratio} = \exp\left(-\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\right) = (\text{Overshoot})^2 \tag{4.5-16}$$

C) *Rise time*. Rise time t_r is the time required for the response to first reach its steady state value. Rise time increases with increasing ζ.

D) *Response time*. This is the time required for the response to come within $\pm$ 5 percent of its steady state value and remain within this limit. The limits $\pm$ 5 percent are arbitrary, and other limits can be used to define a response time.

E) *Period of oscillation*. The radian frequency is the coefficient of t in the sine term of equation (4.5-12). Thus

$$\text{Radian frequency } \omega = \frac{\sqrt{1-\zeta^2}}{\tau} \tag{4.5-17}$$

The radian frequency is related to the cyclical frequency f by $\omega = 2\pi f$ and the period of oscillation is given by $T = 1/f$.

$$f = \frac{1}{\ } = \frac{1}{2\pi}\frac{\sqrt{1-\zeta^2}}{\tau} \tag{4.5-18}$$

F) *Natural period of oscillation*. This is the period when the damping coefficient is equal to zero. The natural radian frequency is then

$$\omega_n = \frac{1}{\tau} \tag{4.5-19}$$

The corresponding natural cyclical frequency f_n and period T_n are related by the expression

$$f_n = \frac{1}{\ _n} = \frac{1}{2\pi\tau} \tag{4.5-20}$$

The natural frequency is related to the actual frequency by taking the ratio of Eq. (4.5-18) and Eq. (4.5-20)

$$\frac{f}{f_n} = \sqrt{1-\zeta^2} \tag{4.5-21}$$

Chapter 4

4.6 Controller

Figure 4.6-1 shows the control hardware required to control the temperature of a stream leaving a heat exchanger. The hardware consists of the following components listed here along with their respective conversions:

Sensor (temperature-to-voltage)
Transducer/Temperature transmitter (voltage-to-current)
Computer/Controller (current-to-current)
Transducer/Converter (current-to-pressure)
Control valve (pressure-to-flow rate)

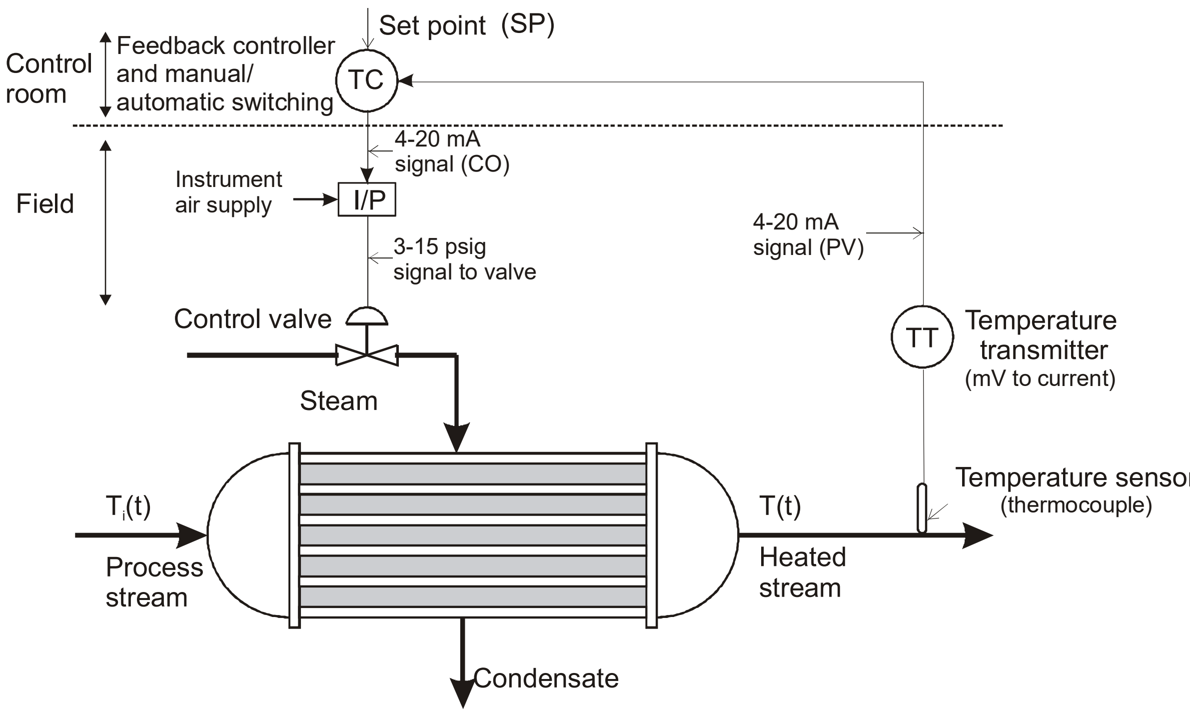

Figure 4.6-1 Heat exchanger control system using control valve.

A thermocouple is used to measure the temperature; the voltage signal from the thermocouple is sent to a temperature transmitter (transducer) which produces a current output in the range of 4 to 20 mA. The output signal is a linear function of the input. The signal from the temperature transmitter enters the controller where it compares the signal with the set point to produce an error signal. The controller converts the error to an output signal in the range of 4 to 20 mA accordance with the controller algorithm which can be on/off or any combination of proportional, derivative, and integral actions. The output of the controller enters the I/P transducer which produces an output in the range of 3 to 15 psig, as a linear function of the input. Finally, the air pressure output of the transducer is sent to the top of the control valve, which adjusts the flow of steam to the heat exchanger. Beside electricity required for the transducer, computer, and controller a source of 20 psig is needed for the I/P transducer.

We now consider the qualitative process of the heat exchanger feedback control system where the outlet temperature of the heat exchanger is equal to the set point at steady state. If the temperature of the cold process stream suddenly decreases the thermocouple will detect a decrease in the outlet temperature and produce a signal to the controller. As soon as the controller detects the drop in temperature, relative to the set point, it will increase the output signal (higher current). This will cause the output pressure from the I/P transducer to increase and to open the valve wider to admit a greater flow of steam. The increase in heat input will eventually increase the output temperature and move it toward the set point. In a well-tuned control system, the response of the temperature will oscillate around the set point before coming to steady state.

In using block diagram to describe a feedback control system, the transmitter, controller, and transducer will usually be lumped into one block as shown in Figure 4.6-2. We need transfer functions for valves and controllers in these block diagrams. These transfer functions are based on ideal devices that can be used to approximate a real process. These approximations are sufficiently good to describe the dynamic behavior of controller mechanisms for ordinary design purposes.

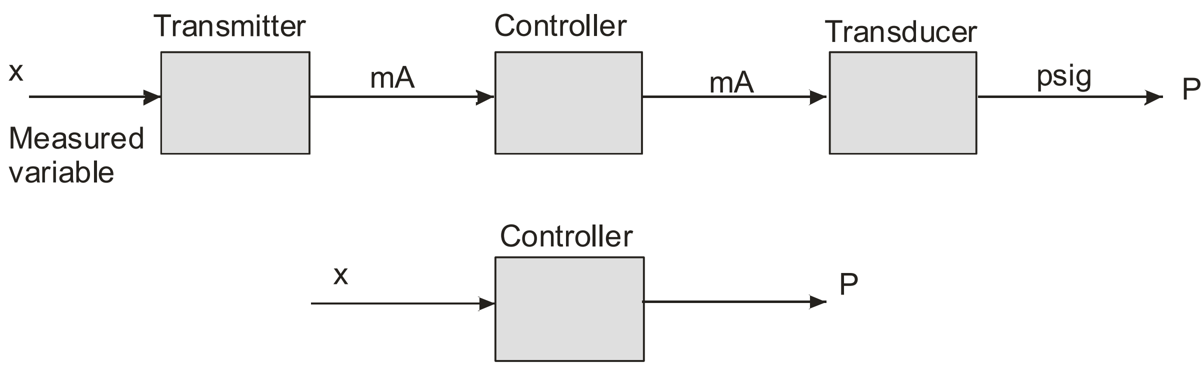

Figure 4.6-2 Equivalent block for transmitter, controller, and transducer.

The relationship between flow and the valve-top pressure for a linear valve can be described by a first-order transfer function; thus

$$\frac{Q(s)}{P(s)} = \frac{K_v}{\tau_v s + 1} \tag{4.6-1}$$

In this equation K_v is the steady-state gain, i.e., the constant of proportionality between the steady-state flow rate and the valve-top pressure, and τ_v is the time constant of the valve. In many practical systems, the time constant of the valve is very small when compared with the time constants of other components of the control system so that the valve contributes negligible dynamic lag to the system. In these cases the transfer function of the valve can be approximated by a constant.

$$\left.\frac{Q(s)}{P(s)}\right|_{\text{fast dynamic}} = K_v \tag{4.6-2}$$

Consider a first-order valve and a first-order process connected in series, as shown in Figure 4.6-3. The process could produce the reactor temperature.

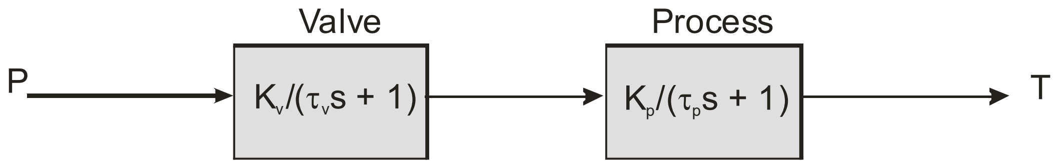

Figure 4.6-3 Block diagram for first-order valve and a first order process.

From the block diagram, the relationship between the air pressure to the valve and the reactor temperature is

$$\frac{T(s)}{P(s)} = \frac{K_v K_p}{(\tau_v s + 1)(\tau_p s + 1)} \tag{4.6-3}$$

For a unit-step change in the valve-top pressure P,

$$T(s) = \frac{1}{s}\frac{K_v K_p}{(\tau_v s + 1)(\tau_p s + 1)} \tag{4.6-4}$$

In the time domain

$$T(t) = K_v K_p \left[1 - \frac{\tau_v \tau_p}{\tau_v - \tau_p}\left(\frac{1}{\tau_p}\exp\left(-\frac{t}{\tau_v}\right) - \frac{1}{\tau_v}\exp\left(-\frac{t}{\tau_p}\right)\right)\right]$$

If the time constant of the valve is much smaller than the time constant of the process ($\tau_v << \tau_p$), the first exponential term is much smaller than the second exponential term and the above equation can be approximated by

$$T(t) = K_v K_p \left[1 - \exp\left(-\frac{t}{\tau_p}\right)\right]$$

This equation can be obtained from the inversion of the following transfer function

$$T(s) = \frac{1}{s}\frac{K_v K_p}{\tau_v s + 1} \tag{4.6-5}$$

Therefore the combination of process and valve is essentially first-order when the time constant of the valve is much smaller than that of the process ($\tau_v << \tau_p$). A typical pneumatic valve has a time constant of the order of 1 s. Many industrial processes behave as first-order systems or a series of first-order systems having time constants that may range from a minute to an hour. For these systems the transfer function of the valve can be taken as a constant K_p.

Proportional Controller (P)

The proportional controller is the simplest type of controller we will discuss. A proportional controller changes its output signal, *CO*, in direct proportion to the error signal. The equation that describes its operation is

$$CO = \text{Bias} + K_c(SP - PV) = \text{Bias} + K_c\varepsilon \qquad (4.6\text{-}6)$$

The bias signal is a constant and is the output from the controller when the error is zero. K_c is the controller gain. The larger the gain, the more the controller output will change for a given error. For example, if the gain is 1, an error of 10% of scale (a change of 1.6 mA in an analog electronic 4-20 mA system) will change the controller output by 10% of scale. The gain on the controller can be made either positive or negative by setting a switch in an analog controller or by specifying the desired sign in a digital controller. A positive gain results in the controller output decreasing when the process variable increases. Many instrument manufacturers use the term *proportional band* (*PB*) defined by

$$PB = \frac{100}{K_c}$$

Thus a wide (or high) *PB* is a low gain, and a narrow (or low) *PB* is a high gain. We now put Eq. (4.6-6) in terms of deviation variable. We assume that the error ε_s is zero at time $t = 0$ so that ε itself is a deviation variable. Thus

$$P(t) = CO - CO_s = K_c\varepsilon(t) \qquad (4.6\text{-}7)$$

Taking the transform of equation (4.6-7) gives the transfer function of an ideal proportional controller.

$$\frac{P(s)}{\varepsilon(s)} = K_c \qquad (4.6\text{-}8)$$

The actual behavior of a proportional controller is shown in Figure 4.6-4. The controller output will saturate (level out) at CO_{max} = 15 psig or 20 mA at the upper end and at CO_{min} = 3 psig or 4 mA at the lower end of the output. The ideal transfer function is just a straight line with slope equal to K_c.

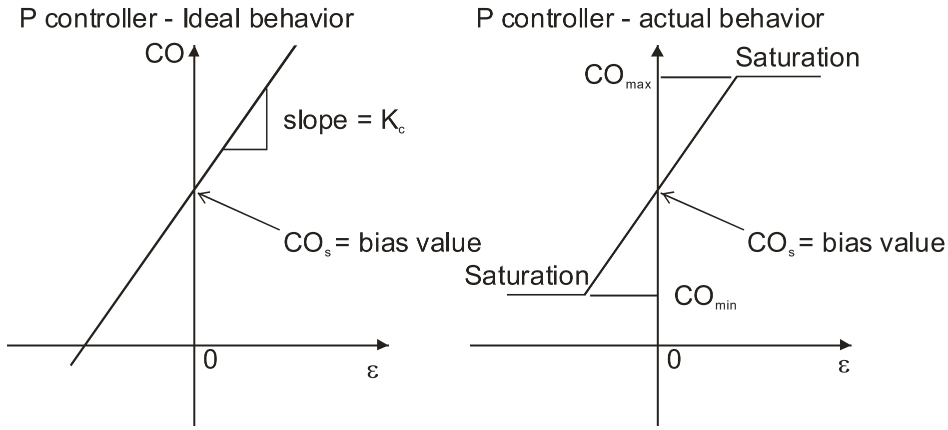

Figure 4.6-4. Proportional controller output as a function of error input to the controller.

Example 4.6-1[1]. --

A pneumatic proportional controller is used to control the cold stream leaving a heat exchanger within the range of 60 to 120°F. The controller gain is adjusted so that the output pressure goes from 3 psig (valve fully closed) to 15 psig (valve fully open) as the measured temperature goes from 71 to 75°F with the set point held constant.

a) Find the controller gain K_c.
b) Find the error in temperature that will cause the control valve to go from fully closed to fully open if the controller gain is 0.4 psi/°F.

Solution ---

a) $$\text{Gain} = \frac{\Delta p}{\Delta \varepsilon} = \frac{15\text{ psig - 3 psig}}{75^\circ\text{F - }71^\circ\text{F}} = 3\text{ psi/}^\circ\text{F}$$

b) Find error in temperature

$$\Delta T = \frac{\Delta p}{\text{gain}} = \frac{12\text{ psi}}{0.4\text{ psi/}^\circ\text{F}} = 30^\circ\text{F}$$

At this level of gain, the valve will be fully open if the error signal reaches 30°F.

On/Off Controller

If the gain K_c is very high, the valve will move from one extreme position to the other when the process deviates only slightly from the set point. This very sensitive action is called on/off action because the valve is either fully open (on) or fully closed (off); i.e., the valve acts as a switch. The thermostat used in a home-heating system is an example of on/off controller. The actual on/off controller has a dead band where the error reaches some finite positive value before the controller "turn on." Conversely, the error must fall to some finite negative value before the controller "turn off." The dead band makes the controller less sensitive to noise and prevents the phenomenon of *chattering*, where the controller will rapidly cycle on and off as the error fluctuates about zero. On/off controller behavior is shown in Figure 4.6-5.

[1] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 193

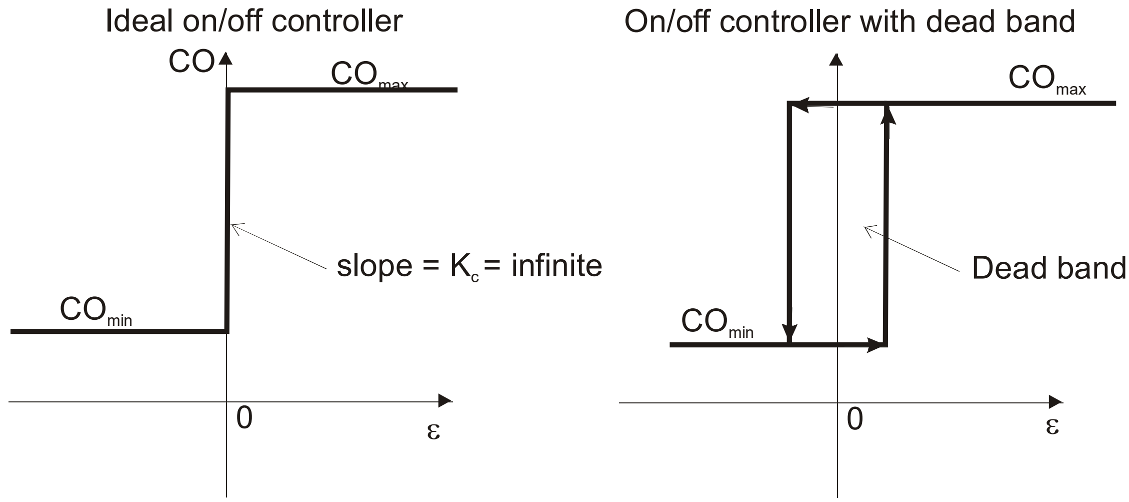

Figure 4.6-5. On/off controller output as a function of error input to the controller.

Proportional-Integral Controller (PI). By adding integral action to a proportional controller, offset can be eliminated completely. Integral action acts on the final control in a manner proportional to the integral of the error over time. A proportional and integral controller, PI controller, can be represented by the following equation

$$CO = \text{Bias} + K_c\,\varepsilon(t) + \frac{K_c}{\tau_I}\int_0^t \varepsilon(t)dt \qquad (4.6\text{-}9)$$

In this equation, τ_I is the integral or reset time. To understand the physical meaning of the reset time, let consider the case when a constant error of 1% in magnitude is introduced in the controller at time $t = 0$. At this moment, the controller output is

$$CO = 50\% + K_c\,(1) + \frac{K_c}{\tau_I}\int_0^t (1)dt$$

$$CO = 50\% + K_c + \frac{K_c}{\tau_I}t$$

When the error is introduced, the controller output changes immediately by an amount equal to K_c due to the proportional mode. The controller output also increases linearly with time so that when $t = \tau_I$ the output becomes

$$CO = 50\% + K_c + K_c$$

Thus, the integral mode repeats the immediate action taken by the proportional mode in a reset time. More weight will be given to the integral term with smaller value of reset time. We again introduce the deviation variable $P(t) = CO - CO_s$ into Eq. (4.6-9) and then take the transform to obtain the transfer function for the proportional integral controller.

$$\frac{P(s)}{\varepsilon(s)} = K_c\left(1 + \frac{1}{\tau_I s}\right) \qquad (4.6\text{-}10)$$

The *reset rate* is defined as the reciprocal of integral time τ_I. The integral adjustment on a controller may be denoted by integral time or reset rate. The calibration of the proportional and integral action can be checked by observing the jump and slope of a step response, as shown in Figure 4.6-6.

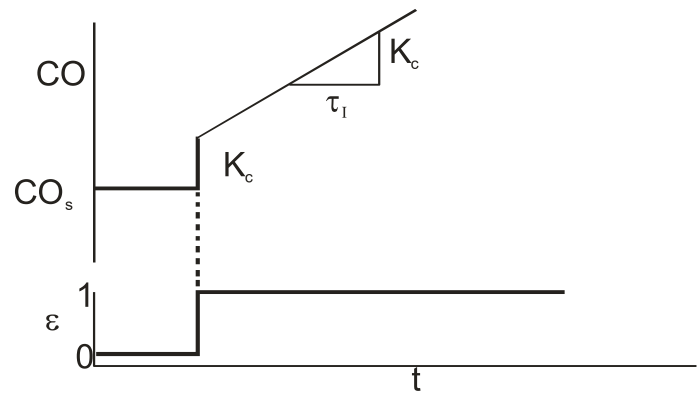

Figure 4.6-6. Response of a PI controller to a unit-step change in error.

Proportional-Derivative Controller (PD). The third type of controller action is derivative control. Derivative action will control the final control element in proportion to the derivative of the process error. Its purpose is to anticipate where the process is heading by looking at the time rate of change of the error. The derivative controller is most active when the error is changing rapidly. It serves to reduce process oscillation. Derivative action is usually combined with proportional action by the following equation

$$CO = \text{Bias} + K_c\, E(t) + K_c \tau_D \frac{dE(t)}{dt} \tag{4.6-11}$$

The transfer function for the PD controller is given by

$$\frac{P(s)}{\varepsilon(s)} = K_c\left(1+\tau_D s\right) \tag{4.6-12}$$

The action of a PD controller for a linear change in error is shown in Figure 4.6-7.

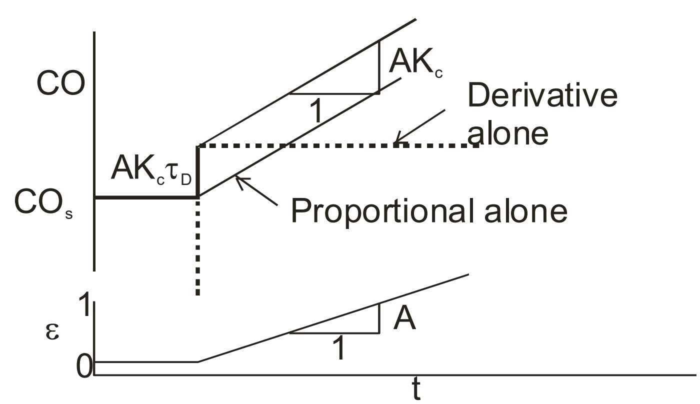

Figure 4.6-7. Response of a PD controller to a linear change in error.

Proportional-Integral-Derivative Controller (PID)

Derivate action can be combined with proportional and integral action producing a PID controller. This controller is given by the following equation

$$CO = \text{Bias} + K_c\, E(t) + \frac{K_c}{\tau_I} \int_0^t E(t)dt\ + K_c \tau_D \frac{dE(t)}{dt} \tag{4.6-13}$$

In this equation τ_D is the derivative time constant. A PID controller will not have any steady state offset, but a PD controller would. The performance of a PID controller can be optimized by adjusting the parameters K_c, τ_I, and τ_D. Optimizing controllers to give the quickest and most stable response by adjusting the controller parameters is referred to as tuning the controller. The transfer function for the PID controller is given by

$$\frac{P(s)}{\varepsilon(s)} = K_c \left(1 + \tau_D s + \frac{1}{\tau_I s} \right) \tag{4.6-14}$$

Chapter 4

4.7 Temperature Control of a Stirred Tank Heater

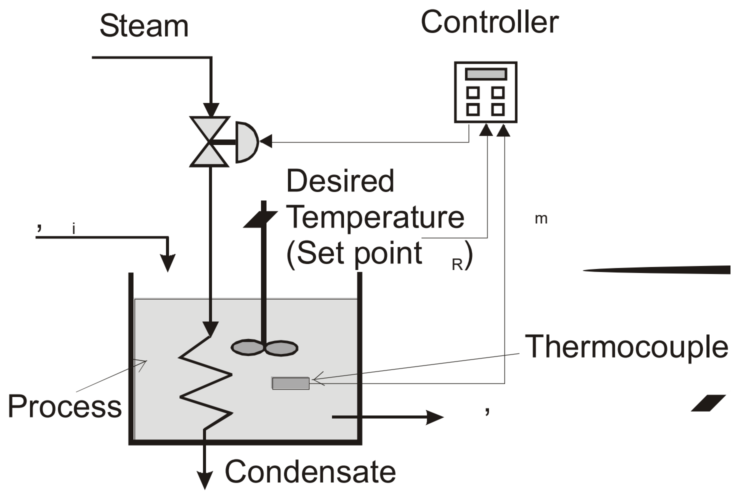

Figure 4.7-1 Temperature control system for a stirred-tank heater[1].

We now discuss a numerical example of stirred tank heater with temperature control of the process shown in Figure 4.7-1. Temperature control is important because a high temperature tends to decompose the product, whereas a low temperature results in incomplete mixing. The tank is heated by steam condensing inside a coil. A PID controller is used to control the temperature in the tank by manipulating the steam valve position. Derive the complete block diagram and the closed-loop transfer function from the following design data.

Process. The feed has a density ρ of 68.0 lb/ft^3 and a heat capacity c_p of 0.80 Btu/lb·^{o}F. The volume V of liquid in the reactor is maintained constant at 120 ft^3. The coil consists of 205 ft of 4-in. schedule 40 steel pipe that weighs 10.8 lb/ft and has a heat capacity of 0.12 Btu/lb·^{o}F and an outside diameter of 4.5 in. The overall heat transfer coefficient U, based on the outside are of the coil, has been estimated as 2.1 Btu/min·ft^2·^{o}F. The saturated steam is available at 30 psia; it can be assumed that its latent heat of condensation λ is constant at 966 Btu/lb. It can also be assumed that the inlet temperature T_i is constant.

Design Conditions. The feed flow F at design condition is 15 ft^3/min, and its temperature T_i is 100^{o}F. The contents of the tank must be maintained at a temperature T of 150^{o}F. Possible disturbances are changes in feed rate and temperature.

Temperature Sensor and Transmitter. The temperature sensor has a calibrated range of 100 to 200^{o}F and a time constant τ_T of 0.75 min.

Control Valve. The control valve is to be designed for 100% overcapacity, and pressure drop variation can be neglected. The valve is an equal percentage valve with a rangeability parameter α of 50, $f_{(x)} = \alpha^{x-1} = 50^{x-1}$. The actuator has a time constant τ_v of 0.20 min.

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[1] Smith, C.A. & Corripio A. B., Principle and Practice of Automatic Process Control, Wiley, 1997, pg. 258.

Process. Making an energy balance on the liquid in the tank, assuming negligible heat losses, perfect mixing, and constant volume and physical properties, we obtain

$$V\rho c_p \frac{dT(t)}{dt} = F(t)\rho c_p T_i + UA[T_s(t) - T(t)] - F(t)\rho c_p T(t) \qquad (4.7\text{-}1)$$

In this equation, $T_s(t)$ is the condensing steam temperature and A is the outside area of the coil for heat transfer. Define the function g to be the right hand side of Eq. (4.7-1)

$$g = F(t)\rho c_p T_i + UA[T_s(t) - T(t)] - F(t)\rho c_p T(t)$$

Using Taylor's series expansion around the steady state values $\bar{T}$, $\bar{F}$, and $\bar{T}_s$ gives

$$g = g\big|_{\bar{T},\bar{T}_s,\bar{F}} + \frac{\partial g}{\partial F}\bigg|_{\bar{T},\bar{T}_s}(F(t) - \bar{F}) + \frac{\partial g}{\partial T_s}\bigg|_{\bar{T},\bar{F}}(T_s(t) - \bar{T}_s) + \frac{\partial g}{\partial T}\bigg|_{\bar{F},\bar{T}_s}(T(t) - \bar{T})$$

We have

$$g\big|_{\bar{T},\bar{T}_s,\bar{F}} = \bar{F}\rho c_p T_i + UA[\bar{T}_s - \bar{T}] - \bar{F}\rho c_p \bar{T}$$

$$\frac{\partial g}{\partial F}\bigg|_{\bar{T},\bar{T}_s} = \rho c_p T_i - \rho c_p \bar{T} = \rho c_p (T_i - \bar{T})$$

$$\frac{\partial g}{\partial T_s}\bigg|_{\bar{T},\bar{F}} = UA$$

$$\frac{\partial g}{\partial T}\bigg|_{\bar{F},\bar{T}_s} = -UA - \bar{F}\rho c_p$$

Therefore

$$g = g\big|_{\bar{T},\bar{T}_s,\bar{F}} + \rho c_p (T_i - \bar{T})(F(t) - \bar{F}) + UA(T_s(t) - \bar{T}_s) - (UA + \bar{F}\rho c_p)(T(t) - \bar{T})$$

In terms of the deviation variable ($T^d(t) = T(t) - \bar{T}$, $T_s^d(t) = T_s(t) - \bar{T}_s$, and $F^d(t) = F(t) - \bar{F}$), we have

$$g = g\big|_{\bar{T},\bar{T}_s,\bar{F}} + \rho c_p (T_i - \bar{T})F^d(t) + UA\, T_s^d(t) - (UA + \bar{F}\rho c_p)T^d(t)$$

Equation (4.7-1) can be written as

$$V\rho c_p \frac{dT(t)}{dt} = g \qquad (4.7\text{-}1)$$

At steady state

$$V\rho c_p \frac{d\overline{T}}{dt} = g\big|_{\overline{T},\overline{T}_s,\overline{F}}$$

$$V\rho c_p \frac{dT^d(t)}{dt} = g - g\big|_{\overline{T},\overline{T}_s,\overline{F}}$$

Equation (4.7-1) becomes

$$V\rho c_p \frac{dT^d(t)}{dt} = \rho c_p(T_i - \overline{\ })F^d(t) + UAT_s^d(t) - (UA + \overline{\ }\rho c_p)T^d(t) \qquad (4.7\text{-}2)$$

Equation (4.7-2) is a linearized equation around the steady state conditions. Making an energy balance on the coil, assuming that the coil metal is at the same temperature as the condensing steam, we obtain

$$C_M \frac{dT_s(t)}{dt} = \lambda w(t) - UA[T_s(t) - T(t)] \qquad (4.7\text{-}3)$$

In this equation, $w(t)$ is the steam flow rate in lb/min and C_M is the heat capacitance of the coil metal in Btu/^{o}F. Since there is no nonlinear term in Eq. (4.7-3), this equation can be expressed in terms of the deviation variable by subtracting it from the following steady energy balance

$$0 = \lambda\overline{w} - UA[\overline{T}_s - \overline{\ }]$$

In terms of the deviation variables, the energy equation for the coil becomes

$$C_M \frac{dT_s^d(t)}{dt} = \lambda w^d(t) - UAT_s^d(t) + UA\,T^d(t) \qquad (4.7\text{-}4)$$

Taking the Laplace transform of Eq. (4.7-2) gives

$$V\rho c_p sT^d(s) = \rho c_p(T_i - \overline{\ })F^d(s) + UAT_s^d(s) - (UA + \rho c_p\overline{\ })T^d(s)$$

Solving for $T^d(s)$ yields

$$T^d(s) = \frac{\rho c_p(T_i - \overline{T})}{V\rho c_p s + UA + \rho c_p\overline{F}}F^d(s) + \frac{UA}{V\rho c_p s + UA + \rho c_p\overline{F}}T_s^d(s)$$

Rearranging the equation we have

$$T^{\mathrm{d}}(s) = \frac{\dfrac{\rho c_{\mathrm{p}}(T_i - \overline{T})}{UA + \rho c_{\mathrm{p}}\overline{F}}}{\dfrac{V\rho c_{\mathrm{p}}}{UA + \rho c_{\mathrm{p}}\overline{F}}s + 1} F^{\mathrm{d}}(s) + \frac{\dfrac{UA}{UA + \rho c_{\mathrm{p}}\overline{F}}}{\dfrac{V\rho c_{\mathrm{p}}}{UA + \rho c_{\mathrm{p}}\overline{F}}s + 1} T_{\mathrm{s}}^{\mathrm{d}}(s)$$

$$T^{\mathrm{d}}(s) = \frac{K_{\mathrm{F}}}{\tau s + 1} F^{\mathrm{d}}(s) + \frac{K_{\mathrm{s}}}{\tau s + 1} T_{\mathrm{s}}^{\mathrm{d}}(s) \tag{4.7-5}$$

In this equation $K_{\mathrm{F}} = \dfrac{\rho c_{\mathrm{p}}(T_i - \overline{T})}{UA + \rho c_{\mathrm{p}}\overline{F}}$, $K_{\mathrm{s}} = \dfrac{UA}{UA + \rho c_{\mathrm{p}}\overline{F}}$, and $\tau = \dfrac{V\rho c_{\mathrm{p}}}{UA + \rho c_{\mathrm{p}}\overline{F}}$

Taking the Laplace transform of Eq. (4.7-4) gives

$$C_{\mathrm{M}}s\, T_{\mathrm{s}}^{\mathrm{d}}(s) = \lambda w^{\mathrm{d}}(s) - UAT_{\mathrm{s}}^{\mathrm{d}}(s) + UA\, T^{\mathrm{d}}(s)$$

Solving for $T_{\mathrm{s}}^{\mathrm{d}}(s)$ yields

$$T_{\mathrm{s}}^{\mathrm{d}}(s) = \frac{\lambda}{C_{\mathrm{M}}s + UA} w^{\mathrm{d}}(s) + \frac{UA}{C_{\mathrm{M}}s + UA} T^{\mathrm{d}}(s)$$

Rearranging the equation we have

$$T_{\mathrm{s}}^{\mathrm{d}}(s) = \frac{\dfrac{\lambda}{UA}}{\dfrac{C_{\mathrm{M}}}{UA}s + 1} w^{\mathrm{d}}(s) + \frac{1}{\dfrac{C_{\mathrm{M}}}{UA}s + 1} T^{\mathrm{d}}(s)$$

$$T_{\mathrm{s}}^{\mathrm{d}}(s) = \frac{K_{\mathrm{w}}}{\tau_{\mathrm{c}}s + 1} w^{\mathrm{d}}(s) + \frac{1}{\tau_{\mathrm{c}}s + 1} T^{\mathrm{d}}(s) \tag{4.7-6}$$

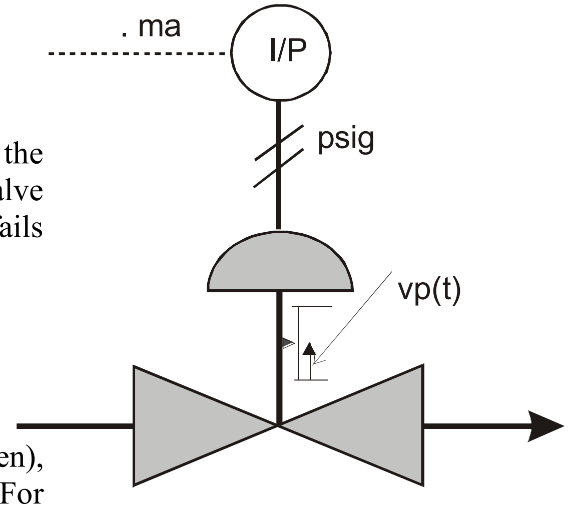
In this equation $K_{\mathrm{w}} = \dfrac{\lambda}{UA}$ and $\tau_{\mathrm{c}} = \dfrac{C_{\mathrm{M}}}{UA}$

Control Valve. The transfer function for an equal percentage valve with constant pressure drop is given by

$$G_{\mathrm{v}}(s) = \frac{w^d(s)}{P(s)} = \frac{K_{\mathrm{v}}}{\tau_{\mathrm{v}}s + 1}$$

In this equation, $P(s)$ is the controller output signal in percent controller output (%CO), and the valve gain is defined as

$$K_v = \frac{dF}{dCO} = \frac{dF}{dC_v}\frac{dC_v}{dvp}\frac{dvp}{dCO},\ \left[\frac{gpm}{\%\text{CO}}\right]$$



The dependence of the valve position is simply the conversion of percent controller output to the faction valve position, but the sign depends on whether the valve fails closed or opened.

$$\frac{dvp}{dCO} = \pm\frac{1}{100},\ \frac{\text{fraction } vp}{\%CO}$$

The plus sign is used if the valve fails closed (air-to-open), the minus sign if the valve fails opened (air-to-closed). For equal percentage valve

$$C_v = C_{v,\max}\alpha^{\text{vp-1}} \Rightarrow \frac{dC_v}{dvp} = (\ln\alpha)C_{v,\max}\alpha^{\text{vp-1}}$$

We use $\frac{dC_v}{dvp}$ at the steady state condition: $\frac{d\bar{C}_v}{dvp} = (\ln\alpha)C_{v,\max}\alpha^{\overline{vp}-1} = (\ln\alpha)\,\bar{C}_v$

The flow rate is given by $F = C_v\left(\frac{\Delta P_v}{\text{SG}}\right)^{1/2} \Rightarrow \frac{dF}{dC_v} = \left(\frac{\Delta P_v}{\text{SG}}\right)^{1/2}$

$$K_v = \frac{dF}{dC_v}\frac{dC_v}{dvp}\frac{dvp}{dCO} = \left(\frac{\Delta P_v}{\text{SG}}\right)^{1/2}(\ln\alpha)\,\bar{C}_v \pm\frac{1}{100} = \pm\bar{C}_v\left(\frac{\Delta P_v}{\text{SG}}\right)^{1/2}\frac{\ln\alpha}{100}$$

$$K_v = \pm\bar{F}\,\frac{\ln\alpha}{100}$$

The gain for liquid flow in mass unit with air-to-open valve is then

$$K_v = \frac{\bar{w}(\ln\alpha)}{100}$$

Sensor/Transmitter. The sensor/transmitter can be represented by a first order transfer function

$$H(s) = \frac{C(s)}{T^{d}(s)} = \frac{K_T}{\tau_T s + 1}$$

In this equation, $C(s)$ is the Laplace transform of the transmitter output signal, %TO, and the transmitter gain (based on a range of 100 to 200^{o}F) is

$$K_T = \frac{100-0}{200-100} = 1.0\ \%TO/^oF$$

The transfer function of the PID controller is

$$G_c(s) = \frac{P(s)}{\varepsilon(s)} = \frac{P(s)}{R(s)-C(s)} = K_c\left(1+\tau_D s+\frac{1}{\tau_I s}\right)$$

In this equation, K_c is the controller gain, τ_I, is the integral time, and τ_D is the derivative time. We now have all the necessary transfer functions for the temperature control loop. The block diagram for this feedback control system is shown in Figure 4.7-2.

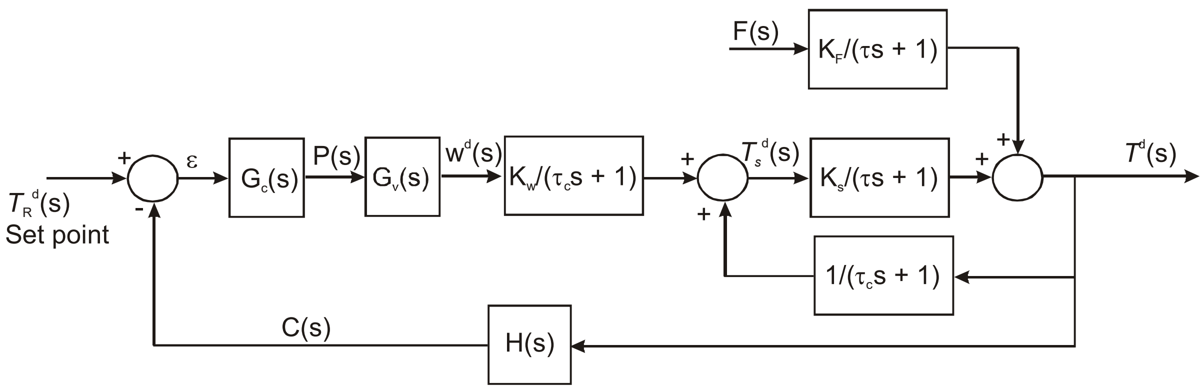


Figure 4.7-2. Block diagram of temperature control loop of stirred tank heater.

The block diagram of Figure 4.7-2 can be simplified to Figure 4.7-3 using rules for block diagram manipulation.

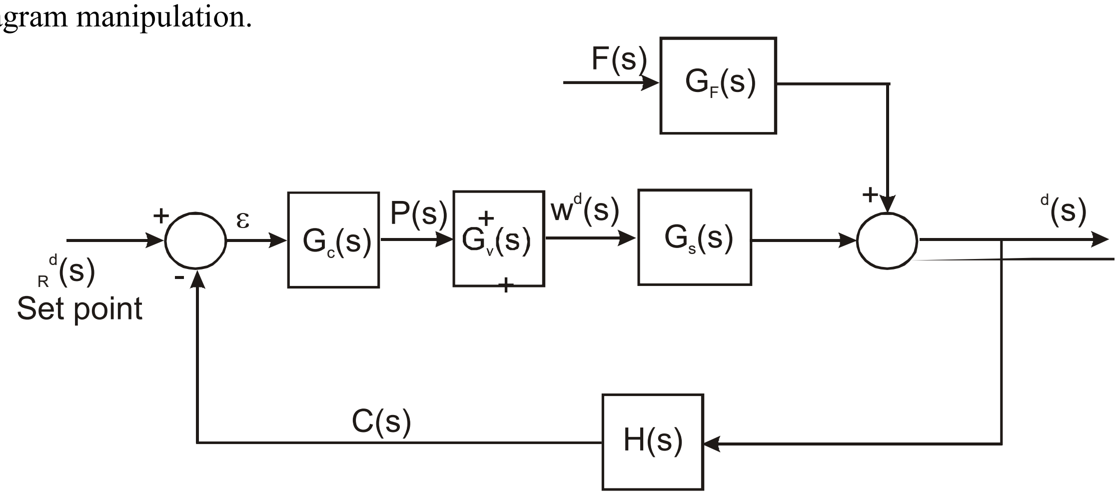


Figure 4.7-3. Simplified block diagram of temperature control loop.

The new transfer functions in the simplified diagram are

$$G_F(s) = \frac{K_F\left(\tau_c s+1\right)}{\left(\tau s+1\right)\left(\tau_c s+1\right)-K_s}$$

$$G_s(s) = \frac{K_w K_s}{(\tau s+1)(\tau_c s+1) - K_s}$$

The closed-loop transfer functions corresponding to the change in set point and flow rate are

$$\frac{T^d(s)}{T_R{}^d(s)} = \frac{G_c(s)G_v(s)G_s(s)}{1+H(s)G_c(s)G_v(s)G_s(s)}$$

$$\frac{{}^d(\ )}{(\ )} = \frac{G_F}{1+H(s)G_c(s)G_v(s)G_s(s)}$$

Table 4.7-1 lists the numerical values of the parameters in the transfer functions, calculated from the data given in the problem statement. The steady state values are the design conditions and are the initial conditions for any disturbances.

Table 4.7-1 Parameters for stirred tank heater.

A	241.5 ft^2	τ	4.93 min
C_M	265.7 Btu/^{o}F	τ_c	0.524 min
K_F	– 2.06 ^{o}F/(ft^3/min)	K_w	1.905 ^{o}F/(lb/min)
K_s	0.383 ^{o}F/ ^{o}F	K_T	1.0 %TO/ ^{o}F
K_v	1.652 (lb/min)/ %CO	τ_v	0.20 min
τ_T	0.75 min		

From the energy equations for the tank and the coil, we can calculate the steady state temperature and steam flow. At steady state, the energy equations for the tank and the coil become

$$0 = \bar{\ }\rho c_p T_i + UA[\bar{T}_s - \bar{\ }] - \bar{\ }\rho c_p \bar{\ }$$

$$0 = \lambda \bar{w} - UA[\bar{T}_s - \bar{\ }]$$

Solving the tank energy equation for the steam temperature gives

$$\bar{T}_s = \frac{(15)(68)(0.80)(150-100)}{(2.1)(241.5)} + 150 = 230^o\text{F}.$$

Solving the coil energy equation for the steam flow gives

$$\bar{w} = \frac{(2.1)(241.5)(230-150)}{966} = 42.2 \text{ lb/min}$$

Substituting the numerical value of the parameters in the transfer functions we have

$$G_v(s) = \frac{w^d(s)}{P(s)} = \frac{K_v}{\tau_v s+1} = \frac{1.652}{0.2s+1}$$

$$H(s) = \frac{C(s)}{T^{\mathrm{d}}(s)} = \frac{K_{\mathrm{T}}}{\tau_{\mathrm{T}}s+1} = \frac{1}{0.75s+1}$$

$$G_{\mathrm{s}}(s) = \frac{K_{\mathrm{w}}K_{\mathrm{s}}}{(\tau s+1)(\tau_{\mathrm{c}}s+1)-K_{\mathrm{s}}} = \frac{(1.905)(0.383)}{(4.93s+1)(0.524s+1)-0.383}$$

$$G_{\mathrm{s}}(s) = \frac{0.7296}{2.5833s^2+5.454s+0.617}$$

$$G_{\mathrm{F}}(s) = \frac{K_F(\tau_{\mathrm{c}}s+1)}{(\tau s+1)(\tau_{\mathrm{c}}s+1)-K_{\mathrm{s}}} = \frac{(-2.06)(0.524s+1)}{(4.93s+1)(0.524s+1)-0.383}$$

$$G_{\mathrm{F}}(s) = \frac{-1.0794s-2.06}{2.5833s^2+5.454s+0.617}$$

Figure 4.7-4 shows the simulink model to simulate the temperature response to a step change in set point. The PID controller in simulink is given as

$$G_{\mathrm{c,simulink}}(s) = P + I/s + Ds$$

The temperature responses of the stirred tank heater for two reset rates of 0.5 min^{-1} and 1.0 min^{-1} are plotted in Figure 4.7-5 and 4.7-6, respectively. Increasing the reset rate will cause the system to be unstable as confirmed by Figure 4.7-7 with a reset rate of 2.0 min^{-1}.

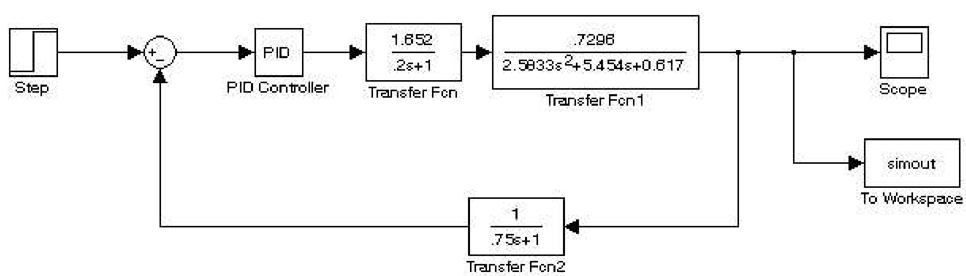


Figure 4.7-4. Simulink model of temperature control loop of stirred tank heater.

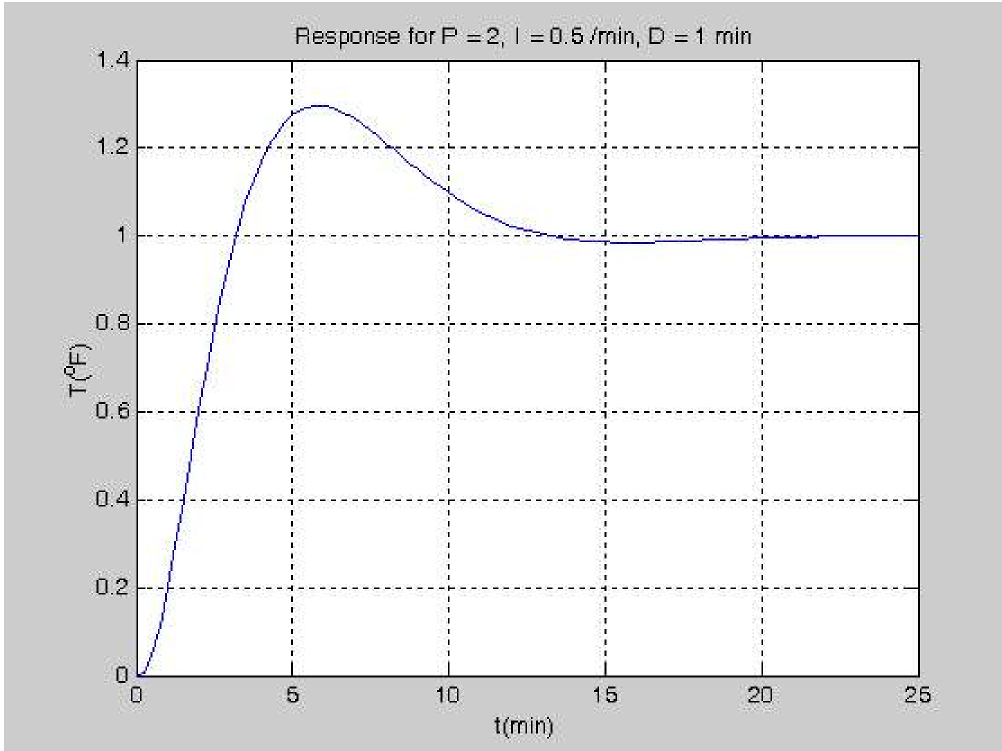


Figure 4.7-5. Temperature response of stirred tank heater to change in set point.

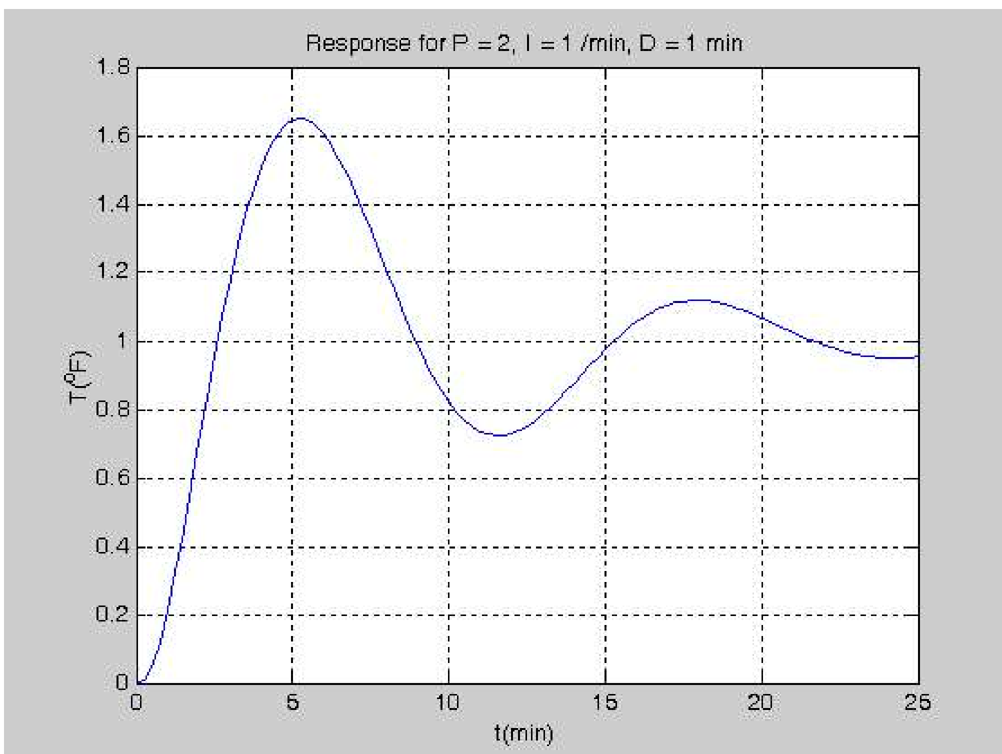


Figure 4.7-6. Temperature response of stirred tank heater to change in set point.

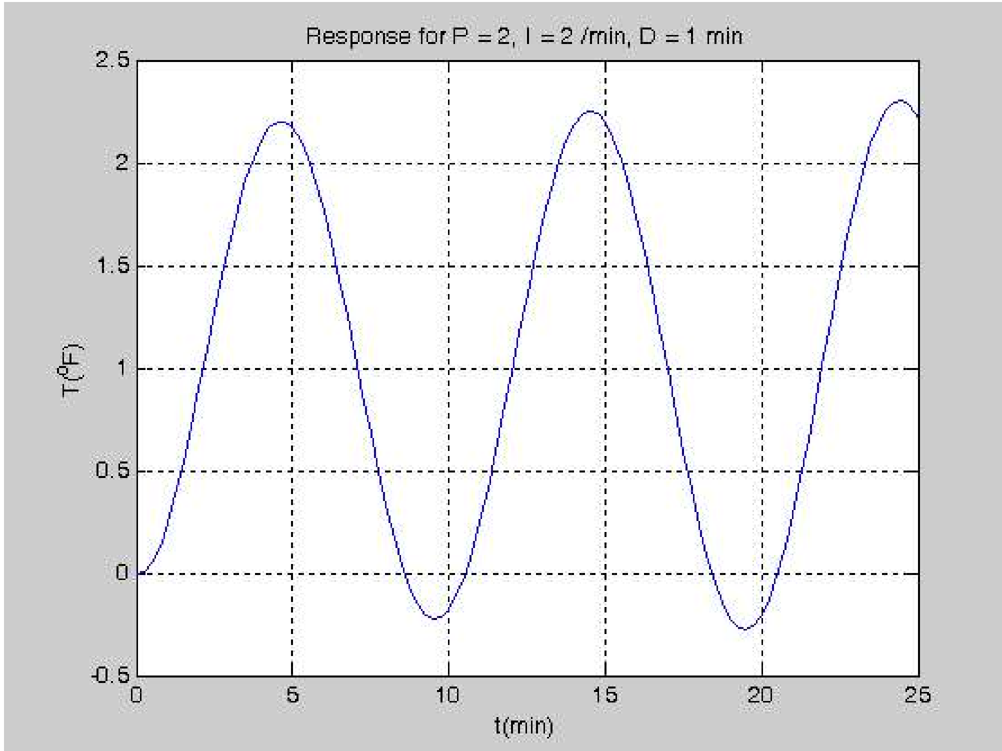


Figure 4.7-7. Unstable response of stirred tank heater to change in set point.

Chapter 4

4.8 Chemical-Reactor Control System

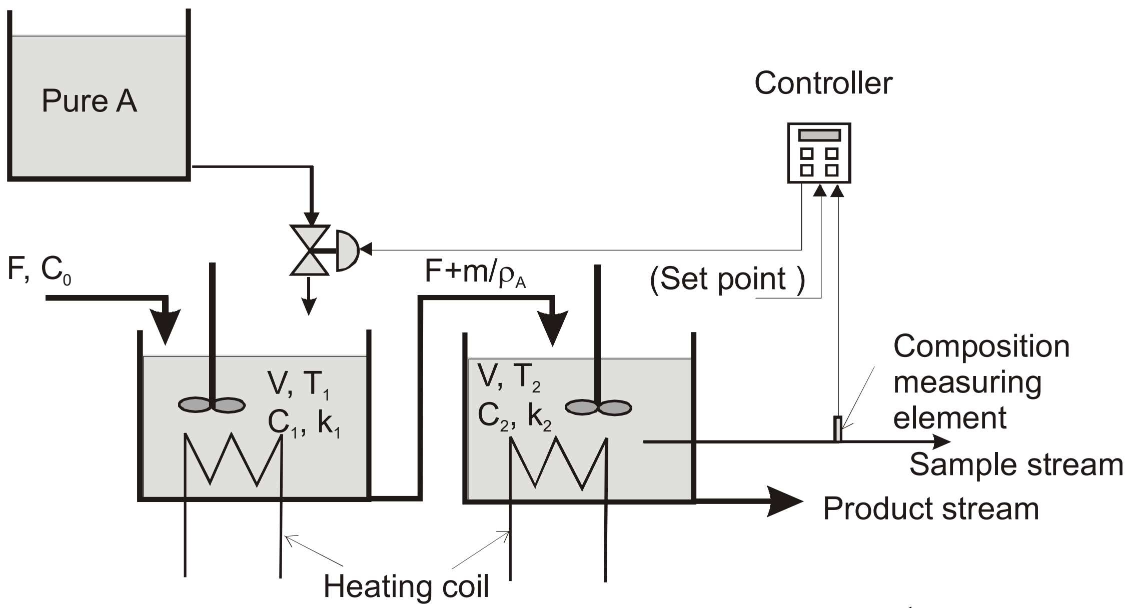


Figure 4.8-1 Control of a stirred-tank chemical reactor[1].

A liquid stream enters tank 1 at a volumetric flow rate *F* in cfm and contains reactant A at a concentration of C_0 [mol A/ft^3]. Reactant A decomposes in the tanks according to the irreversible chemical reaction

$$A \rightarrow B$$

The reaction is first order with reaction rate constant k_1 and k_2 for tank 1 and tank 2 respectively. The reaction is to be carried out in a series of two continuous stirred-tank reactors. The tanks are maintained at different temperatures with tank 2 at a higher temperature. We will neglect any changes in physical properties due to chemical reaction.

The purpose of the control system is to maintain C_2, the concentration of A leaving tank 2 at some desired value in spite of variations in the inlet concentration C_0. This will be accomplished by adding a stream of pure A to tank 1 through a control valve. We wish to simulate the process response to changes in inlet concentration.

Making a material balance on species A around tank 1 gives

$$V\,\frac{dC_1}{dt} = FC_0 + m - \left(F + \frac{m}{\rho_A}\right)\mathrm{C}_1 - k_1\mathrm{C}_1V \tag{4.8-1}$$

---

[1] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 205

In this equation m is the molar flow rate of pure A through control valve, [lbmol/min] and ρ_A is the molar concentration of pure A [lbmol/ft^3]. We assume that the volumetric flow rate of A through the valve m/ρ_A is much less than the inlet flow rate F. Therefore Eq. (4.8-1) becomes

$$V \frac{dC_1}{dt} + (F + k_1V)C_1 = FC_0 + m \tag{4.8-2}$$

Dividing Eq. (4.8-2) by $(F + k_1V)$ yields

$$\frac{V}{F+k_1V}\frac{dC_1}{dt} + C_1 = \frac{F}{F+k_1V}C_0 + \frac{1}{F+k_1V}m$$

$$\tau_1 \frac{dC_1}{dt} + C_1 = \frac{1}{1+k_1\tau}C_0 + \frac{1/F}{1+k_1\tau}m \tag{4.8-3}$$

In this equation τ is the residence time for each tank = V/F and τ_1 is the effective time constant for tank 1 = $\frac{V}{F+k_1V} = \frac{\tau}{1+k_1\tau}$. At steady state, Eq. (4.8-3) becomes

$$C_{1s} = \frac{1}{1+k_1\tau}C_{0s} + \frac{1/F}{1+k_1\tau}m_s \tag{4.8-4}$$

Subtracting Eq. (4.8-4) from Eq. (4.8-3) and introducing the deviation variables

$$C_1^{\mathrm{d}} = C_1 - C_{1s}$$

$$C_0^{\mathrm{d}} = C_0 - C_{0s}$$

$$M^{\mathrm{d}} = m - m_s$$

We obtain

$$\tau_1 \frac{dC_1^d}{dt} + C_1^{\mathrm{d}} = \frac{1}{1+k_1\tau}C_0^{\mathrm{d}} + \frac{1/F}{1+k_1\tau}M^{\mathrm{d}}$$

We will drop the superscript d and consider the variables are deviation variables.

$$\tau_1 \frac{dC_1}{dt} + C_1 = \frac{1}{1+k_1\tau}C_0 + \frac{1/F}{1+k_1\tau}M \tag{4.8-5}$$

Taking the transform of Eq. (4.8-5) yields the transfer function of the first reactor:

$$C_1(s) = \frac{1/(1+k_1\tau)}{\tau_1 s+1}\left[C_0(s) + \frac{1}{F}M(s)\right] \tag{4.8-6}$$

Making a material balance on species A around tank 2 gives

$$V\frac{dC_2}{dt} = FC_1 - FC_2 - k_2C_2V \tag{4.8-7}$$

Rearranging the above equation gives

$$V\frac{dC_2}{dt} + (F + k_2V)C_2 = FC_1 \tag{4.8-8}$$

Dividing Eq. (4.8-3) by (F + k_2V) yields

$$\frac{V}{F+k_2V}\frac{dC_2}{dt} + C_2 = \frac{F}{F+k_2V}C_1$$

$$\tau_2\frac{dC_2}{dt} + C_2 = \frac{1}{1+k_2\tau}C_1 \tag{4.8-9}$$

τ_2 is the effective time constant for tank 2 = $\dfrac{V}{F+k_2V}$ = $\dfrac{\tau}{1+k_2\tau}$ .We have the same form for the deviation variable. Taking the transform of Eq. (4.8-9) gives the transfer function for the second reactor:

$$C_2(s) = \frac{1/(1+k_2\tau)}{\tau_2 s+1}C_1(s) \tag{4.8-10}$$

We can use the following values for the system: Mw_A = 100, ρ_A = 0.8 lbmol/ft^3, C_{0s} = 0.1 lbmol/ft^3, F = 100 cfm, m_s = 1.0 lbmol/min, k_1 = 1/6 min^{-1}, k_2 = 2/3 min^{-1}, V = 300 ft^3, τ = V/F = 3 min. The steady-state concentration of A in tank 1 is given by

$$C_{1s} = \frac{1}{1+k_1\tau}C_{0s} + \frac{1/F}{1+k_1\tau}m_s = \frac{1}{1+\frac{1}{6}\times 3}0.1 + \frac{1/100}{1+0.5}1.0 = 0.0733 \text{ lbmol/ft}^3$$

The steady-state concentration of A in tank 2 is given by

$$C_{2s} = \frac{1}{1+k_2\tau}C_{1s} = \frac{1}{1+\frac{2}{3}\times 3}0.0733 = 0.0244 \text{ lbmol/ft}^3$$

Control valve. The flow of A through the valve varies linearly from 0 to 2 cfm as the valve-top pressure varies from 3 to 15 psig. Neglect the dynamics of the valve, the transfer function for the valve is just a gain. Since m_s/ρ_A = 1.25 cfm, the normal operating pressure on the valve is

$$p_s = 3 + \frac{1.25}{2}(15 - 3) = 10.5 \text{ psig}$$

The gain for the control valve can be written as

$$K_v = \frac{m/\rho_A - 1.25}{p - 10.5} = \frac{2-0}{15-3} = \frac{1}{6} \text{ cfm/psi}$$

Rearranging the above equation yields

$$m - 1.25\rho_A = K_v\rho_A(p - 10.5) \qquad (4.8\text{-}11)$$

In terms of the deviation variables we have

$$M = K_v\rho_A P \text{ , where } M = m - 1.25\rho_A \text{ and } P = p - 10.5$$

Taking the transform of Eq. (4.8-11) gives

$$\frac{M(s)}{P(s)} = K_v\rho_A = \left(\frac{1}{6}\text{cfm/psi}\right)(0.8 \text{ lbmol/ft}^3) = 0.133 \text{ (lbmol/min)/psi}$$

Measuring element. We assume that the measuring element converts the concentration of A to an electronic signal. The output of the measuring element varies from 4 to 20 mA as the concentration of A varies from 0.01 to 0.05 lbmol/ft^3. We will assume that the concentration measuring device is linear and has negligible lag. The gain of the measuring device is therefore

$$K_m = \frac{20-4}{0.05-0.01} = 400 \frac{\text{mA}}{\text{lbmol/ft}^3}$$

The normal signal from the measuring device is

$$\frac{C_{2s} - 0.01}{0.05 - 0.01}(20 - 4) + 4.0 = \frac{0.0244 - 0.01}{0.05 - 0.01}(20 - 4) + 4.0 = 9.76 \text{ mA}$$

The output current from the measuring device is

$$b = 9.76 + K_m(C_2 - 0.0244)$$

In terms of the deviation variables

$$B = K_m C_2^d \text{ , where } B = b - 9.76 \text{ and } C_2^d = C_2 - 0.0244$$

The transfer function for the measuring device is

$$\frac{B(s)}{C_2(s)} = K_m \qquad (4.8\text{-}12)$$

Controller. We will use a proportional controller with a current output signal. The relation between the controller output signal and the error is

$$p = p_s + K_c(C_R - b) = p_s + K_c\varepsilon$$

In this equation C_R = desired current signal (or set point) in mA, K_c is the controller gain, and ε = error = $C_R - b$. The transfer function of the controller is then

$$\frac{P(s)}{\varepsilon(s)} = K_c \tag{4.8-13}$$

Assuming the set point and the signal from the measuring device to be the same when the system is at steady state under normal conditions, the reference value of the set point is

$$C_{Rs} = b_s = 9.76 \text{ mA}$$

The deviation variable for the set point is: $C_R^{\ d} = C_R - C_{Rs}$

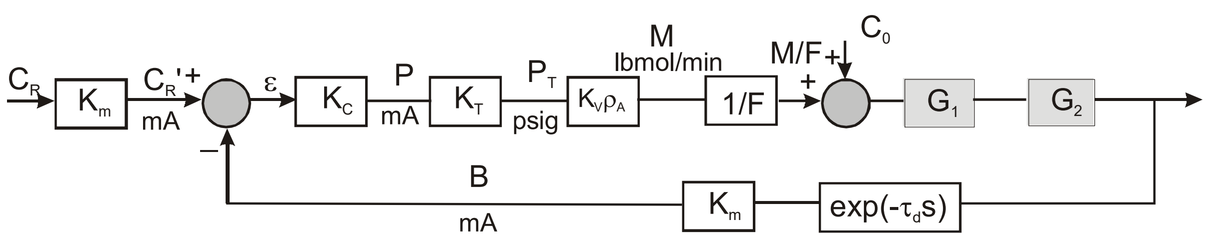
Controller Transducer. The current signal from the controller is converted to pneumatic signal by an I/P transducer with a gain given by

$$\frac{P_T(s)}{P(s)} = K_T = \frac{(15 - 3)\text{ psi}}{(20 - 4)\text{ mA}} = 0.75 \text{ psi/mA}$$

Transportation Lag. A portion of the liquid leaving tank 2 is continuously withdrawn through a sample line at a rate of 0.1 cfm. The sample line has a length of 50 ft and the cross-sectional area of the line is 0.001 ft^2. The transportation lag for the transportation line is

$$\tau_d = \frac{\text{Volume}}{\text{flow rate}} = \frac{50\times 0.001}{0.1} = 0.5 \text{ min}$$

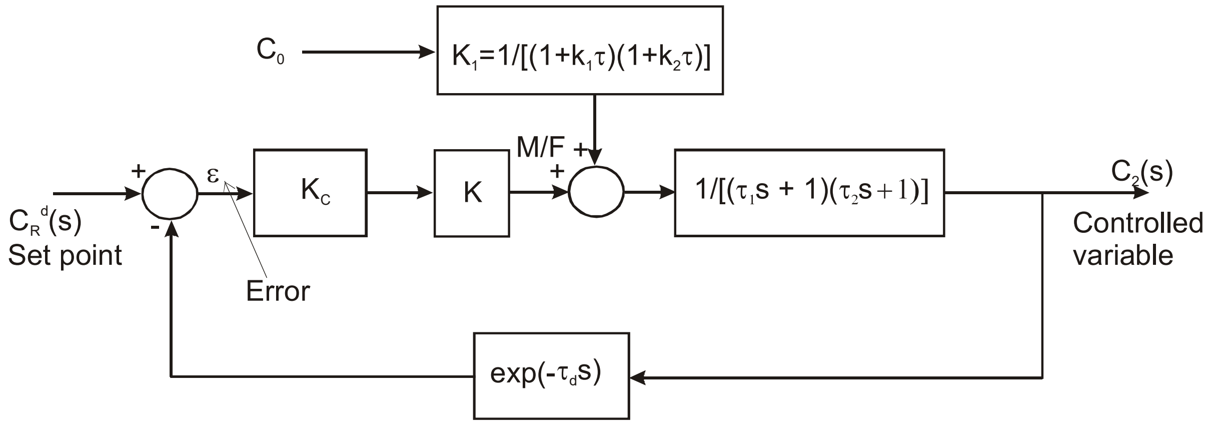
The transfer function for the transportation line is: $\exp(-\tau_d s) = \exp(-0.5s)$. The block diagram for the system can be obtained from the combination of the transfer function of each component.



In this block diagram:

$$G_1 = \frac{1/(1+k_1\tau)}{\tau_1 s+1}, \text{ and } G_2 = \frac{1/(1+k_2\tau)}{\tau_2 s+1}$$

The previous block diagram can be simplified to:



The time constants and the gains of the system can be obtained from the given data: Mw_A = 100, ρ_A = 0.8 lbmol/ft^3, C_{0s} = 0.1 lbmol/ft^3, F = 100 cfm, m_s = 1.0 lbmol/min, k_1 = 1/6 min^{-1}, k_2 = 2/3 min^{-1}, V = 300 ft^3, $\tau = V/F$ = 3 min.

$$\tau_1 = \frac{\tau}{1+k_1\tau} = 3/(1 + 3/6) = 2 \text{ min}$$

$$\tau_2 = \frac{\tau}{1+k_2\tau} = 3/(1 + 3\times 2/3) = 1 \text{ min}$$

$$K_1 = \frac{1}{(1+3/6)(1+2\times 3/3)} = \frac{1}{4.5}$$

$$K = \frac{K_m K_T K_V \rho_A}{F(1+k_1\tau)(1+k_2\tau)} = \frac{400\times 0.75\times (1/6)(0.8)}{100\times 4.5} = 0.089$$

Chapter 5

Controller Tuning

5.1 Introduction

Controller tuning is the adjustment of the controller parameters to achieve satisfactory control. For a PID controller with the transfer function $G_c(s) = K_c\left(1+\tau_D s+\dfrac{1}{\tau_I s}\right)$, we need to select the values for the parameters K_c, τ_I, and τ_D to give the quickest and most stable response. The selection of the controller parameters is essentially an optimization problem in which the designer of the control system attempts to satisfy some criterion to minimize the error and obtain a good control. If the response of the system to a step change in set point or load has minimum overshoot and a one-quarter decay ratio (C/A in Figure 5.1-1), it is considered a good control. Other criteria may include minimum rise time and minimum response time.

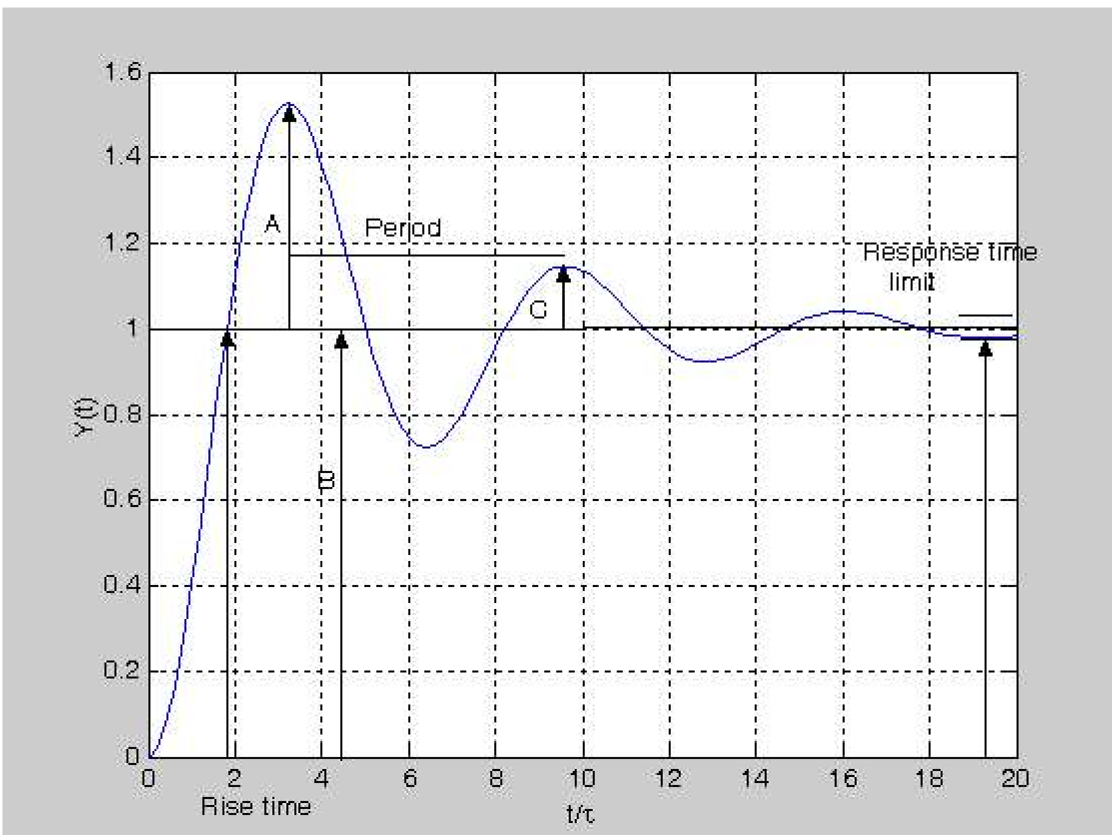


Figure 5.1-1 Underdamped response characteristic.

The common types of control loops are level, flow, temperature, and pressure. The type of controller and the settings used for any one type are very similar from one application to another. For example, most flow control loops use PI controller with low gain or wide proportional band and fast integral action (small integral time τ_I). The following are heuristics that work in most applications[1].

Flow Loops. PI controllers are used in most flow loops. The dynamics of the process are usually very fast so that a wide proportional band setting (PB = 150) or low gain is used to reduce the effect of the noisy signal due to flow turbulence. A low value of integral time (τ_I = 0.1 minute per repeat) is used to get fast set point tracking. This rule-of-thumb would not apply to slow dynamic process such as flow control of condensate-throttled reboilers. Since the vapor flow depends on the rate of condensation, vapor flow can only be varied by a slow process of changing the area for heat transfer in the reboiler.

Level Loops. If level control is used on surge tank, it is relatively unimportant where the level is, as long as it is between some maximum and minimum levels. Therefore, proportional controllers are often used on level loops to give smooth changes in flow rates and to filter out fluctuations in flow rates to downstream units. A PI controller can be used for situations where it is desired to control level tightly, for example, in a reactor where control of residence time is important.

It is simple to tune a P controller for level loop. We could set the bias value at 50 percent of full scale, the set point at 50 percent of full scale, and the proportional band at 50. This means that the control valve for the flow out of the tank will be half open when the tank is half full, wide open when the tank is 75 percent full, and completely shut when the tank is 25 percent full. Changing the proportional band to 100 would mean that the tank would be completely full to have the valve wide open and completely empty to have the valve shut.

Pressure Loops. Pressure loops vary from very tight, fast loops (almost like flow control) to slow averaging loops (almost like level control). An example of a fast pressure loop is the flow of vapor from vessel through a valve throttling where the valve has a direct handle on pressure. An example of a slower pressure loop is the control of vapor pressure by throttling the cooling water flow to a condenser. The water changes the temperature driving force for condensation in the condenser. The vapor pressure changes due to the heat transfer between the cooling water in the shell side and the vapor in the tube side as shown in Figure 5.1-2.

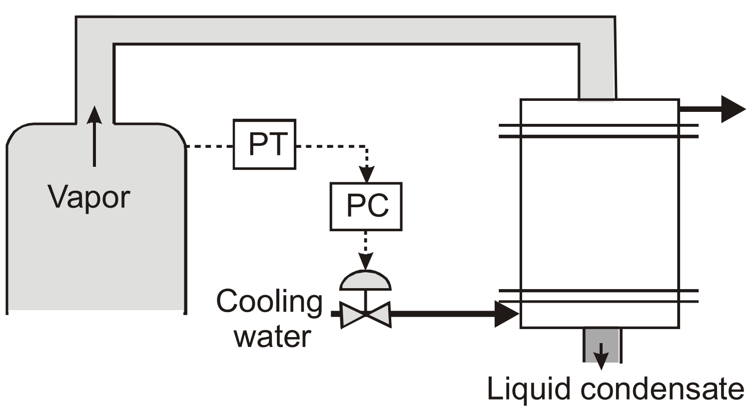


Figure 5.1-2 Pressure control in a slow pressure loop

[1] Luyben, W. L. and Luyben, M. L., Essentials of Process Control, McGraw Hill, 1997, pg. 231

Temperature Loops. PID controllers are often used for temperature control loops when the dynamics are usually moderately slow because of the senor lags and the process heat transfer lags. Proportional band settings are fairly low, depending on temperature transmitter spans and control-valve sizes. The reset time is of the same order as the process time constant. Derivative time is set to about one-fourth the process time constant, depending on the noise in the transmitter signal.

The following trial and error procedure can be followed to tune a controller on line[2].

1. With the controller on manual, eliminate the integral action by setting τ_I at maximum value and eliminate the derivative action by setting τ_D at minimum value.
2. Set the PB at a high value, perhaps 200.
3. Put the controller on automatic.
4. Make a small set point or load change and observed the response of the controlled variable. The gain is low so the response will be sluggish.
5. Double the gain and make another small change in set point or load.
6. Repeating step 5 until the loop becomes very under-damped and oscillatory. The gain at which this occurs is called the *ultimate gain*.
7. Reduce the gain by a factor of 2.
8. Start bringing in the integral action by reducing τ_I by factors of 2, making small disturbances at each value of τ_I to see the effect.
9. Find the value of τ_I that makes the loop very under-damped and set τ_I at twice this value.
10. Start bringing in derivative action by increasing τ_D, making small disturbances at each value of τ_D to see the effect. Find the value of τ_D that gives the tightest control without amplifying the noise in the process measurement signal.
11. Reduce the PB gain by step 10 percent until the desire specification on damping coefficient or overshoot is satisfied.

5.2 Tuning for Minimum Error Integral Criteria

One criterion that can be used for tuning is to specify the closed-loop response to achieve minimum error or deviation of the controlled variable from its set point. The error is a function of time for the duration of the response, so the sum of the error over time must be minimized. This is by definition the integral of the error with time, or the shaded area in the response illustrated in Figure 5.2-1. This criterion reduces the entire response to a single number, or a figure of merit, which can be used to compare different responses that use different sets of controlled parameters. Since the tuning relationships are intended to minimize the integral of the error, their use is referred to as minimum error integral tuning. However, the integral of the error cannot be minimized directly, because a very large negative error would be the minimum. To prevent negative value of the integral, the following formulations can be used.

A) Integral of the square of the error (ISE)

$$\text{ISE} = \int_0^\infty e^2(t)dt \qquad (5.2\text{-}1)$$

[2] Luyben, W. L. and Luyben, M. L., Essentials of Process Control, McGraw Hill, 1997, pg. 234

In this equation $e(t)$ is the usual error (i.e., set point – control variable). The response that has large errors, which usually occur at the beginning of the response, will have large value of ISE.

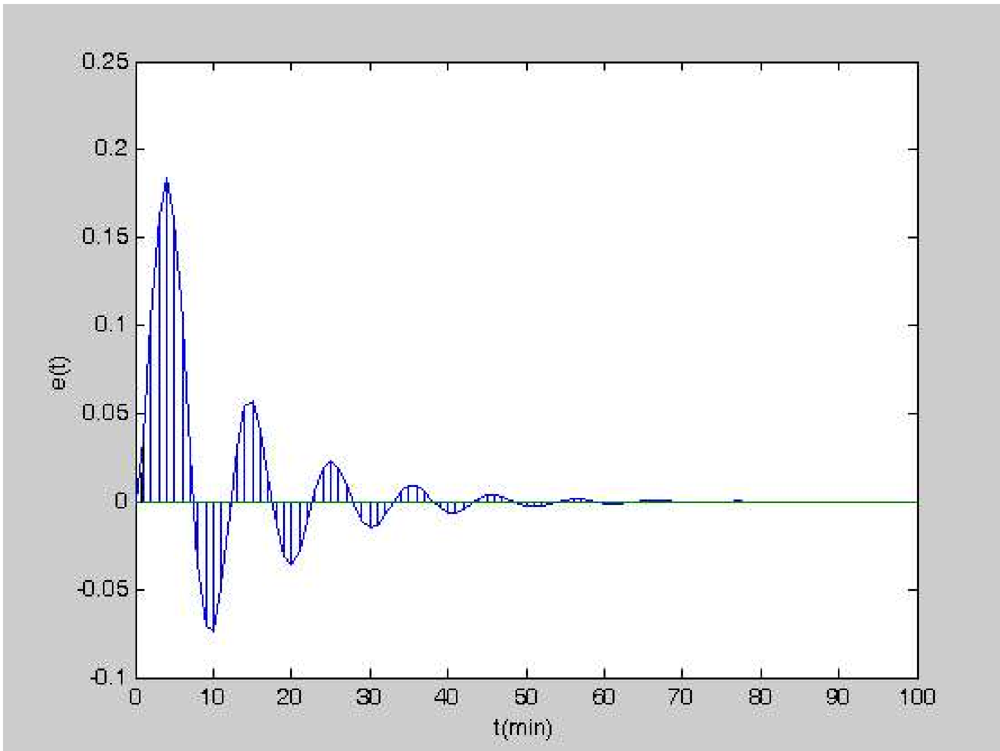


Figure 5.2-1 Definition of error integral for disturbance.

B) Integral of the absolute value of error (IAE).

$$\text{IAE} = \int_0^\infty |e(t)|\,dt \tag{5.2-2}$$

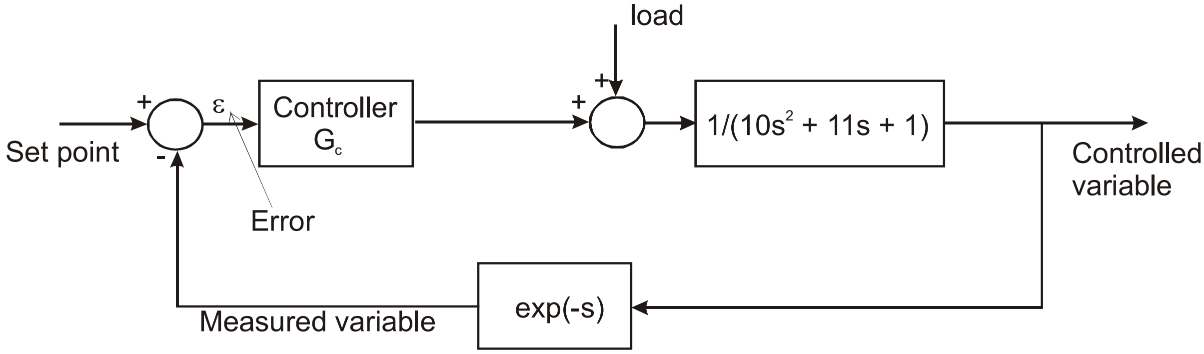
The IAE treat all errors (large and small) in a uniform manner.

C) Integral of time-weighted absolute error (ITAE).

$$\text{ITAE} = \int_0^\infty t\,|e(t)|\,dt \tag{5.2-3}$$

A response that has errors that persist for a long time will have large value of ITAE. The ISE figure of merit is often used in optimal control theory because it can be used more easily in mathematical operations such as differentiation than the figure of merits, which use the absolute value of error.

Example 5.2-1[3]. --
For the control system shown, determine the ISE, ITAE, and IAE for a unit-step load disturbance. Let the controller be a PI controller with $K_c = 6$ and $\tau_I = 4$.



Solution ---
Simulink can be used to simulate this process and calculate the values from the ISE, ITAE, and IAE. The Simulink model is shown in Figure E-1. The transport delay is set for the time delay of 1. The simout variable has sample time of 0.5. The format of the PID controller block from Simulink is $G_{c,\text{Simulink}}(s) = K + I/s + Ds$. Compare with the format $G_c(s) = K_c \left(1 + \tau_D s + \dfrac{1}{\tau_I s}\right)$ we have $K_c = K$, $K_c/\tau_I = I$, and $K_c\tau_D = D$. Therefore $I = K_c/\tau_I = 6/4 = 1.5$.

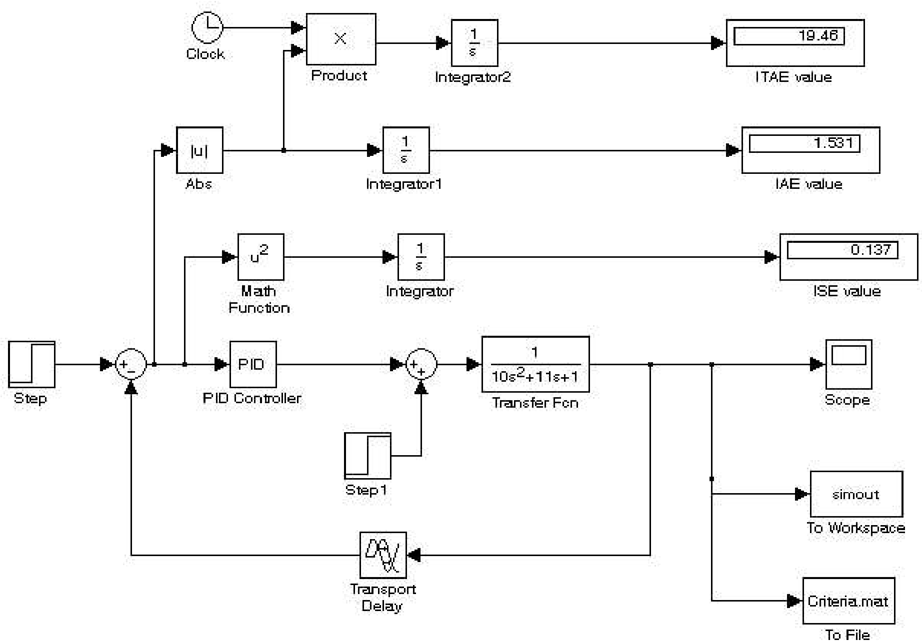


Figure E-1 Simulink model for ISE, ITAE, and IAE.

The response of this process is shown in Figure 5.2-1. From the figures of merit various settings of K_c and τ_I can be compared to obtain the lowest value for the errors. We can use the function fminsearch to vary the controller parameters to find the minimum figure of merit of interest.

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[3] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 394

Figure E-2 shows the Simulink block diagram saved as PItune.mdl.

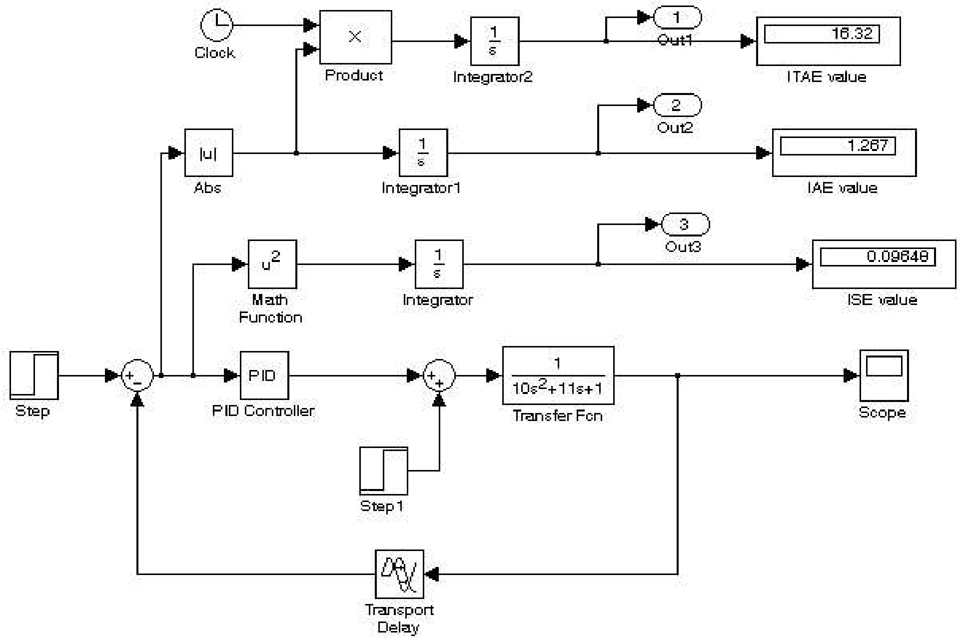
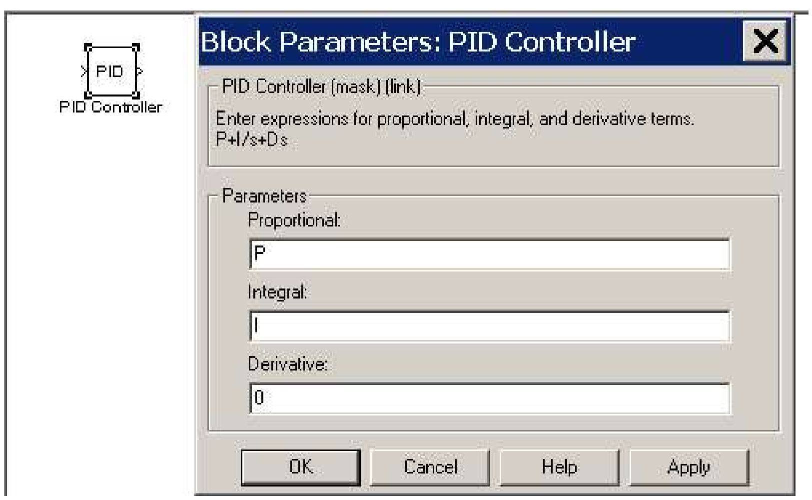


Figure E-2 Simulink model for example 5.2-1

The Out1, Out2, and Out3 blocks are obtained from the Signals & Systemd of the Simulink menu. For the Simulink model of example 5.2-1, the block parameters for the PID controllers must be set to the variables *P* and *I* as shown below:



The following program (meritscore .m) and 3 functions are used to minimize the errors according to the ITAE, IAE, and ISE criteria.

```
global P I
[zn,fval,exitflag]=fminsearch('ITAE',[5.8 1],optimset('Display','off','MaxIter',100));
Kc=zn(1);Taoi=Kc/zn(2);
fprintf('Minimum ITAE, Kc = %8.2f, TaoI = %8.2f\n',Kc,Taoi)
[zn,fval,exitflag]=fminsearch('IAE',[5.8 1],optimset('Display','off','MaxIter',100));
Kc=zn(1);Taoi=Kc/zn(2);
fprintf('Minimum IAE, Kc = %8.2f, TaoI = %8.2f\n',Kc,Taoi)
```

```
[zn,fval,exitflag]=fminsearch('ISE',[5.8 1],optimset('Display','off','MaxIter',100));
Kc=zn(1);Taoi=Kc/zn(2);
fprintf('Minimum ISE, Kc = %8.2f, TaoI = %8.2f\n',Kc,Taoi)


function merit_score=ITAE(PI);
global P I
P=PI(1);
I=PI(2);
[t,x,y]=sim('PItune',50);
merit_score=max(y(:,1));


function merit_score=IAE(PI);
global P I
P=PI(1);
I=PI(2);
[t,x,y]=sim('PItune',50);
merit_score=max(y(:,2));


function merit_score=ISE(PI);
global P I
P=PI(1);
I=PI(2);
[t,x,y]=sim('PItune',50);
merit_score=max(y(:,3));


>> meritscore
Minimum ITAE, Kc =     5.48, TaoI =     6.02
Minimum IAE, Kc =     6.98, TaoI =     6.91
Minimum ISE, Kc =     8.99, TaoI =     8.08
```

5.3 Tuning Rules

Ziegler-Nichols (Z-N) Rules. Ziegler and Nichols developed a closed-loop tuning method with the controller remains in the loop as an active controller in automatic mode. This method uses the ultimate gain K_u and ultimate period P_u obtained from a closed-loop test of the actual process. The value of gain and the period of oscillation that correspond to the sustained oscillation are the ultimate gain K_{cu} and the ultimate period P_u. We could use the following procedure for a real process to determine the Ziegler - Nichols controller settings.

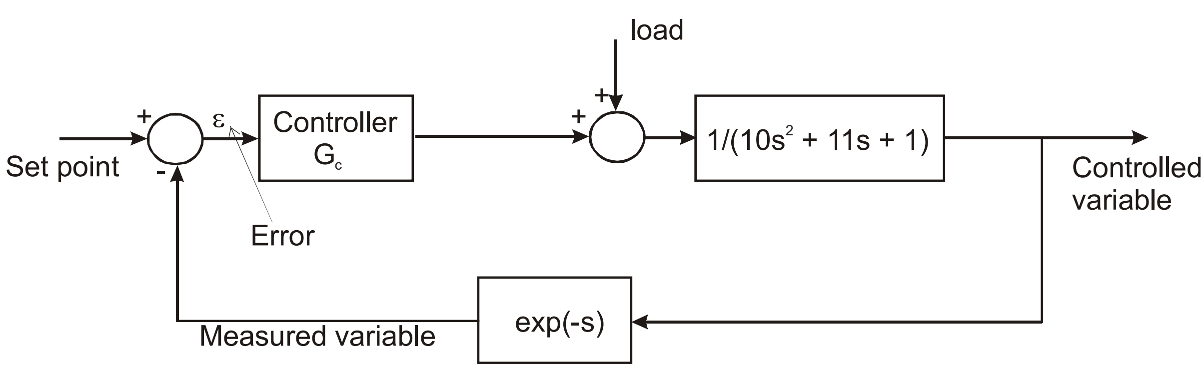
1. After the process reaches steady state at the normal level of operation, eliminate the integral and derivative modes of the controller, leaving only proportional control. On some PID controllers, this requires setting τ_I at maximum minutes per repeat and setting τ_D at minimum minutes. On computer-based controllers, the integral and derivative modes can be removed completely from the controller.
2. Select a low value of proportional gain K_c, disturb the system, and observe the transient response. Obtain the ultimate gain K_u and ultimate period P_u by increasing the gain in small steps until the response first exhibits a sustained oscillation.
3. From the values of K_u and P_u obtained in step 2, use the Ziegler – Nichols rules given in Table 5.3-1 to determine the controller settings (K_c, τ_I, and τ_D).

Table 5.3-1 Ziegler - Nichols controller settings.

Type of control	$G_c(s)$	K_c	τ_I	τ_D
P	K_c	$0.5K_u$		
PI	$K_c\left(1+\dfrac{1}{\tau_I s}\right)$	$0.45K_u$	$\dfrac{P_u}{1.2}$	
PID	$K_c\left(1+\tau_D s+\dfrac{1}{\tau_I s}\right)$	$0.6K_u$	$\dfrac{P_u}{2}$	$\dfrac{P_u}{8}$

There are variations in the tuning rules given in Table 5.3-1 even though the same approach of using K_u and P_u to obtain controller parameters is used. The Z-N settings should be considered as only approximate settings for satisfactory control.

Example 5.3-1[4]. ---
Determine the ultimate gain K_u and ultimate period P_u for the control system shown below.



Solution --

[4] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 394

For this process we remove for the integral and derivative actions by setting the *D* and *I* parameters of the PID block from Simulink to zero ($I = 0$ and $D = 0$). The response for a unit load is a sustained oscillation when $K = 13$ as shown in Figure E-1. Therefore the ultimate gain is 13.

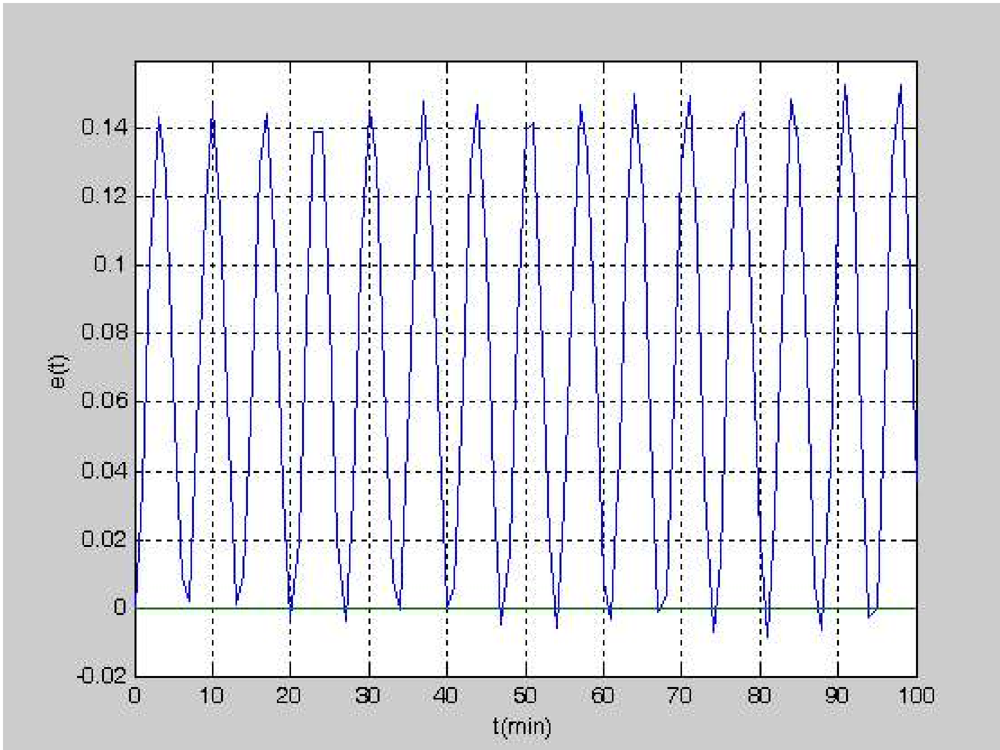


Figure E-1 Sustained oscillation for a unit load disturbance.

The ultimate period is 7 minutes from Figure E-1 and E-2.

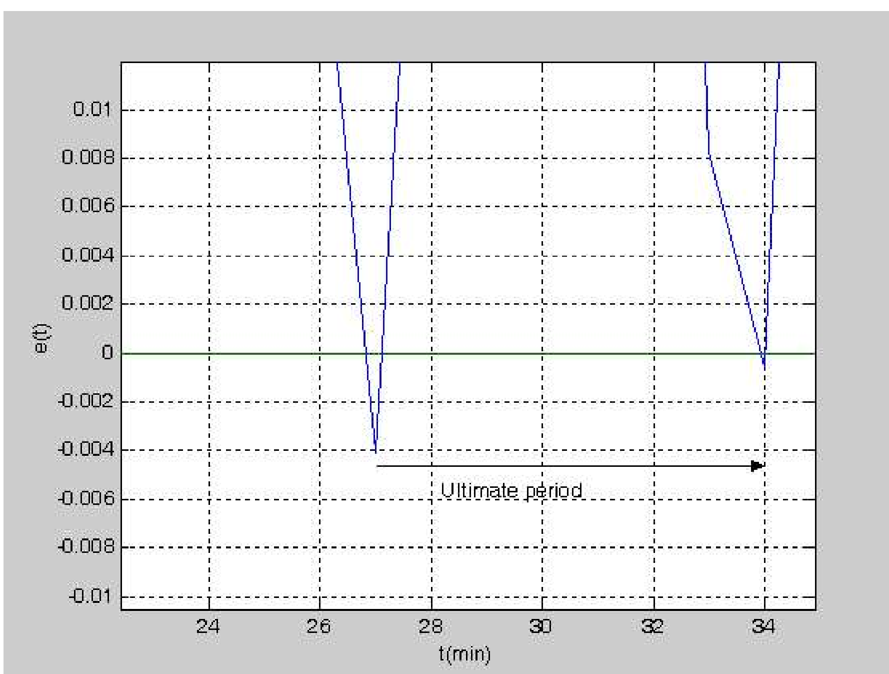


Figure E-1 Ultimate period.

Cohen and Coon (C-C) Rules. Cohen and Coon proposed an *open-loop method*, in which the control action is removed from the controller by placing it in manual mode and an open-loop transient is induced by a step change in the signal to the valve. Figure 5.3-1 shows a typical control loop in which the control action is removed and the response to a step change to the valve is recorded. The response to the system is called the process reaction curve which normally exhibits an S shape as shown in Figure 5.3-2.

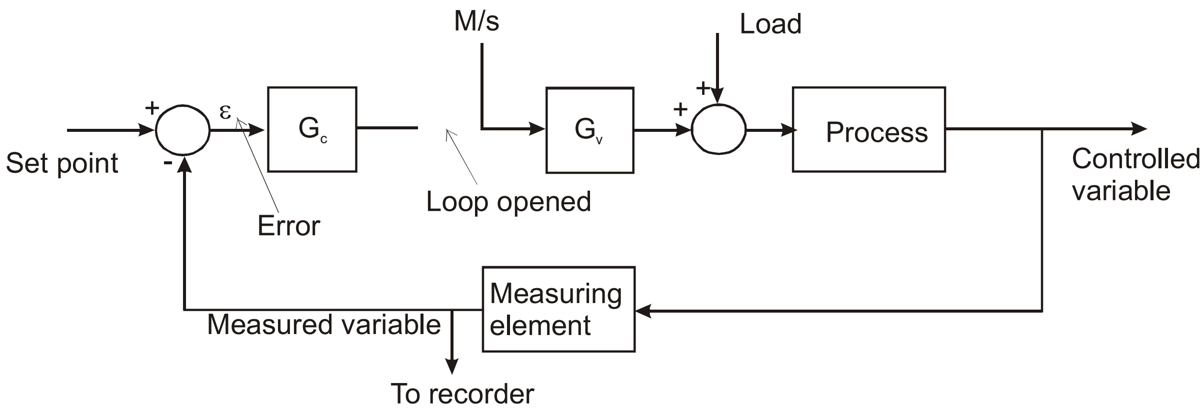


Figure 5.3-1 Open-loop for measurement of a process reaction curve.

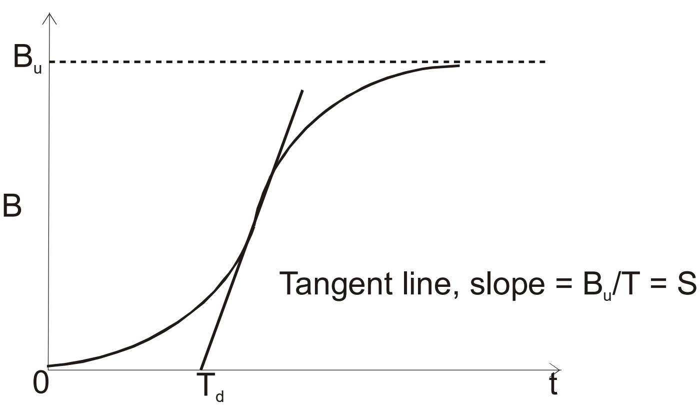


Figure 5.3-2 A typical process reaction curve.

The C-C method can be summarized in the following steps:

1. Switch the controller to manual after the process reaches steady state at the normal level of operation. The controller output will usually remain at the same value after switching as it had before switching.
2. Obtain the process reaction curve with the controller in manual mode by introducing a step disturbance with magnitude M to the valve and recording the response.
3. Draw a tangent line to the curve at the inflection point, as shown in Figure 5.3-2. The time intercept T_d is the apparent transport lag; the apparent first order time constant is given by

$$T = \frac{B_u}{S} \tag{5.3-1}$$

In this equation B_u is the ultimate value of the measured variable B at large t and S is the slope of the tangent line. The steady-state gain that relates B to M is given by

$$K_p = \frac{B_u}{M} \qquad (5.3\text{-}2)$$

4. Use the values of K_p, T, and T_d from step 3 to determine the controller settings. The C-C rules are listed in Table 5.3-2.

Table 5.3-2 Cohen-Coon controller settings

Type of control	K_c	τ_D	τ_I
P	$\frac{1}{K_p}\frac{T}{T_d}\left(1+\frac{T_d}{3T}\right)$		
PI	$\frac{1}{K_p}\frac{T}{T_d}\left(\frac{9}{10}+\frac{T_d}{12T}\right)$		$T_d\frac{30+3T_d/T}{9+20T_d/T}$
PD	$\frac{1}{K_p}\frac{T}{T_d}\left(\frac{5}{4}+\frac{T_d}{6T}\right)$	$T_d\frac{6-2T_d/T}{22+3T_d/T}$	
PID	$\frac{1}{K_p}\frac{T}{T_d}\left(\frac{4}{3}+\frac{T_d}{4T}\right)$	$T_d\frac{4}{11+2T_d/T}$	$T_d\frac{32+6T_d/T}{13+8T_d/T}$

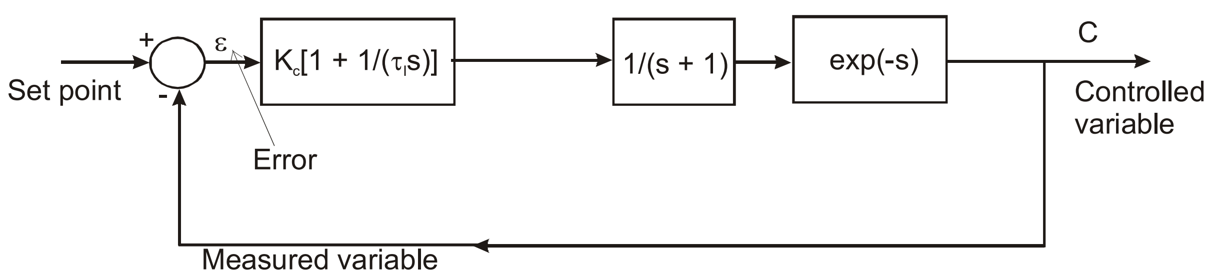
The Cohen-Coon settings were obtained by using a first-order with transport lag model for the process. The expression for this model is

$$G_p(s) = \frac{K_p\exp(-T_d s)}{Ts+1} \qquad (5.3\text{-}3)$$

The controller settings are then derived from Eq. (5.3-3) with the requirements that the response have ¼ decay ratio, minimum offset, and minimum area under the load-response curve.

Example 5.3-2[5]. ---

Determine the Z-N and the C-C settings for the PI control system shown below.



Solution ---

We could apply the Bode criterion for frequency response to obtain the ultimate gain K_u and ultimate period P_u for this control system. Instead we use the Simulink model shown in Figure E-1 with exp(–s) corresponding to a transport lag with time delay = 1. The format of the PID controller block from Simulink is $G_{c,\text{Simulink}}(s) = K + I/s + Ds$. Without integral and derivative actions, $I = 0$ and $D = 0$.

[5] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 404

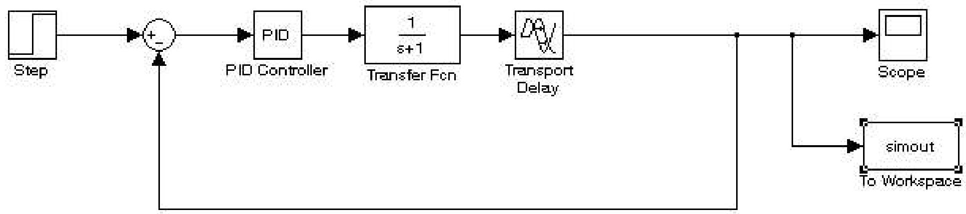


Figure E-1 Simulink model to determine K_u and P_u.

A sustained oscillation is obtained when $K_u = 2.4$. From Figure E-2, the ultimate period P_u = 3.5.

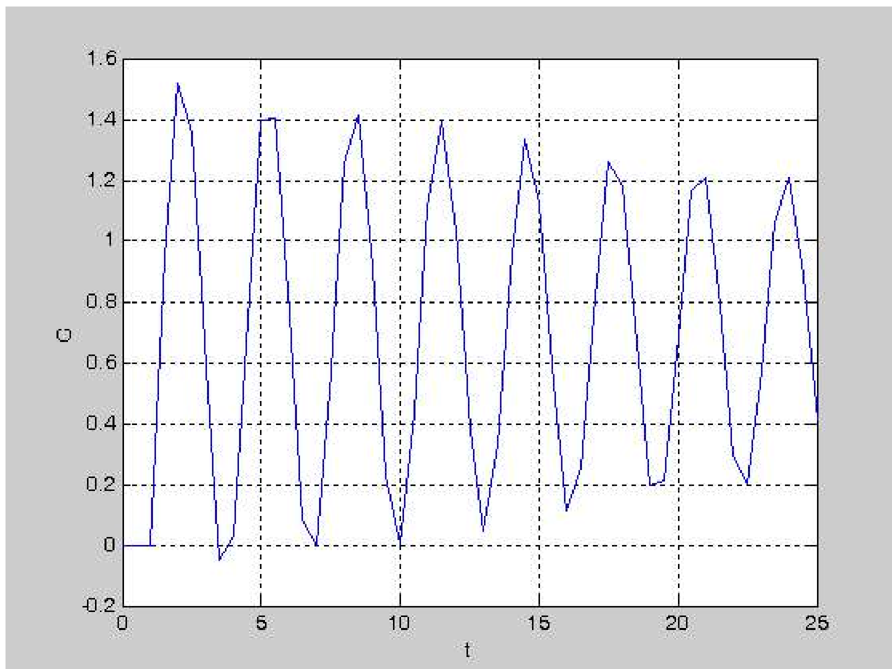


Figure E-2 Closed loop response to change in set point

Applying the Z-N rules for PI control gives

$$K_c = 0.45K_u = (0.45)(2.4) = \mathbf{1.08}$$

$$\tau_I = \frac{P_u}{1.2} = \frac{3.5}{1.2} = \mathbf{2.92}$$

For C-C rules, the transfer function is in the form $G_p(s) = \dfrac{K_p \exp(-T_d s)}{Ts+1}$. Therefore $T = 1$, T_d = 1, and $K_p = 1$.

$$K_c = \frac{1}{K_p}\frac{T}{T_d}\left(\frac{9}{10}+\frac{T_d}{12T}\right) = \frac{1}{1}\left(\frac{9}{10}+\frac{1}{12}\right) = \mathbf{0.983}$$

$$\tau_I = T_d\frac{30+3T_d/T}{9+20T_d/T} = \frac{30+3}{9+20} = \mathbf{1.14}$$

Chapter 5

5.4 Frequency Response

Frequency response is the measurement of an output signal in response to a cyclical input such as a sine forcing function. Frequency response is a valuable tool in the analysis and design of control systems. We will study a graphical technique to obtain the frequency response of linear system.

Consider a first-order system with transfer function

$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{\tau s + 1} \tag{5.4-1}$$

Replacing s by $i\omega$ gives

$$G(i\omega) = \frac{1}{i\omega\tau + 1}$$

The complex number in the denominator is usually eliminated by multiplying the expression by the conjugate of $i\omega\tau + 1$.

$$G(i\omega) = \frac{(-i\omega\tau + 1)}{(i\omega\tau + 1)(-i\omega\tau + 1)} = \frac{1}{1+\omega^2\tau^2} - \frac{\omega\tau}{1+\omega^2\tau^2} \tag{5.4-2}$$

A complex number in rectangular coordinate form ($a + ib$) can be converted to polar form $R\exp(i\phi)$ by the following relationships

$$R = \text{magnitude} = \sqrt{a^2 + b^2}$$

$$\phi = \text{angle} = \tan^{-1}\frac{b}{a}$$

The polar form is shown in Figure 5.4-1. The components of the transfer function in polar coordinates are

$$R = \frac{1}{\sqrt{1+\omega^2\tau^2}} \text{ and } \phi = \tan^{-1}(-\omega\tau)$$

For an input forcing function $X(t) = A\sin(\omega t)$, the transform of this function is

$$X(s) = \frac{A\omega}{s^2 + \omega^2}$$

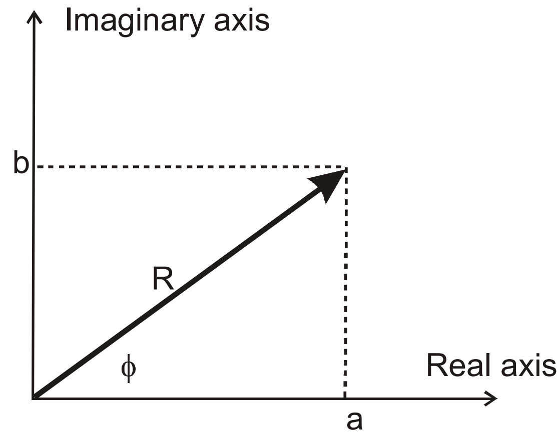


Figure 5.4-1 Coordinates for complex number.

The response or output function is then

$$Y(s) = \frac{A\omega}{s^2+\omega^2}\,\frac{1}{\tau s+1} = \frac{A\omega}{s^2+\omega^2}\,\frac{1/\tau}{s+1/\tau}$$

Taking the inverse transform, the output function in the time domain is

$$Y(t) = \frac{A\omega\tau}{1+\omega^2\tau^2}\exp(-t/\tau) + \frac{A}{\sqrt{1+\omega^2\tau^2}}\sin(\omega t + \phi) \tag{5.4-3}$$

In this equation $\phi = \tan^{-1}(-\omega\tau)$. As $t \to \infty$, the first term on the right side of Eq. (5.4-3) approaches zero and leaves only the steady state solution

$$Y(t)|_s = \frac{A}{\sqrt{1+\omega^2\tau^2}}\sin(\omega t + \phi) \tag{5.4-4}$$

After sufficient time elapses, the response of a first-order system to a sinusoidal input of frequency ω is also a sinusoid of frequency ω with the ratio of the amplitude of the response to that of the input is $1/\sqrt{1+\omega^2\tau^2}$. The steady-state sinusoidal response is shown in Figure 5.4-2. The phase difference between output and input is $\phi = \tan^{-1}(-\omega\tau)$. Therefore, the frequency response of a first order system with transfer function $G(s)$ has the following properties:

$$\text{Amplitude ratio (AR)} = \frac{\text{output amplitude}}{\text{input amplitude}} = |\,G(i\omega)| = R$$

Phase angle = ϕ = angle of $G(i\omega)$ shown in Figure 5.4-1.

To obtain the amplitude ratio AR and phase angle, we substitutes $i\omega$ for s in the transfer function and then finds the magnitude and angle of the resulting complex number, respectively.

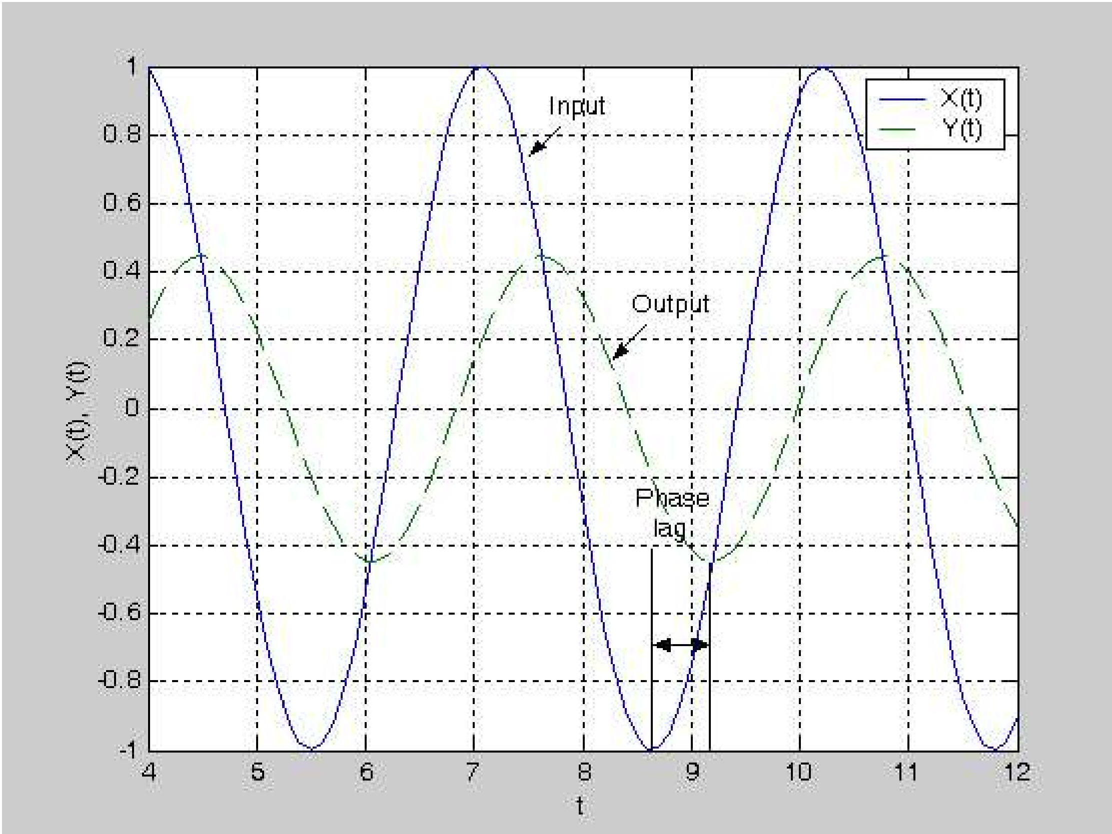


Figure 5.4-2 Steady state sinusoidal response.

Example 5.4-1[1]. --
Find the frequency response of a thermocouple with first order transfer function $G(s)$ to a sinusoidal variation in input temperature. The input temperature has a frequency of $10/\pi$ cycles/min and amplitude of 1^oF.

$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{0.1s+1}$$

Use Simulink to verify the results and to plot the process output and input.

Solution ---

$$\omega = (10 \text{ cycles}/\ \pi \text{ min})(2\ \pi \text{ rad/cycle}) = 20 \text{ rad/min.}$$

Let $s = i\omega = 20i \Rightarrow G(i\omega) = \dfrac{1}{2i+1} = \dfrac{(-2i+1)}{(2i+1)(-2i+1)} = \dfrac{1}{5} - \dfrac{2i}{5}$

$$R = \text{magnitude} = \sqrt{a^2+b^2} = \left(\frac{1}{25}+\frac{4}{25}\right)^{1/2} = \frac{1}{\sqrt{5}} = 0.447$$

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[1] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 290

$$\phi = \text{angle} = \tan^{-1}\frac{b}{a} = \tan^{-1}(-2) = -\,1.1071 \text{ rad} = -\,63.435^{\circ}$$

The simulink model is listed in Figure E-1 and the frequency response is plotted for both the input and output functions. From the graph, the phase lag is

$$\phi = (0.055 \text{ min})(20 \text{ rad/min}) = 1.11 \text{ rad}$$

The amplitude ratio is 0.45/1 = 0.45

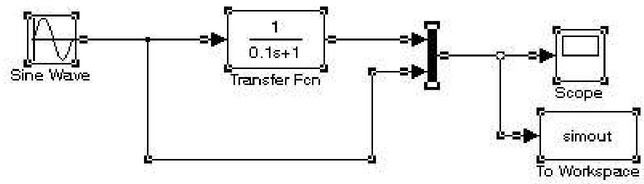

Figure E-1 Simulink model for thermocouple response

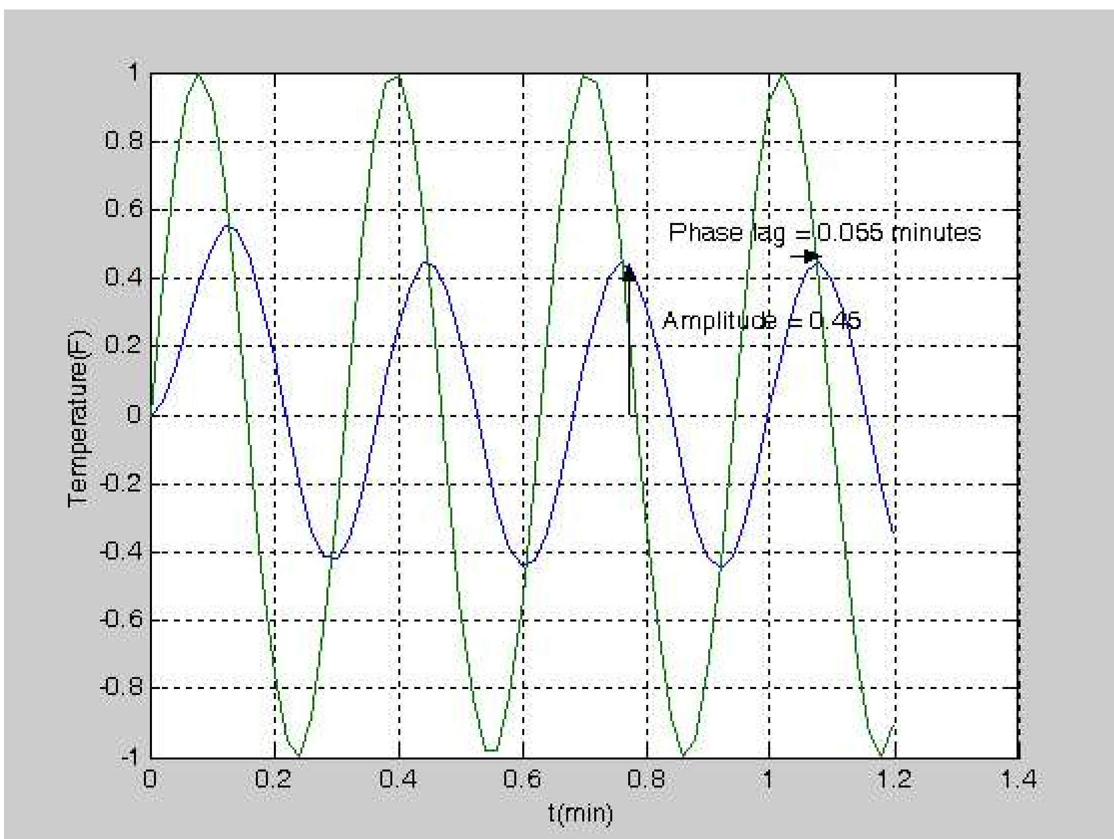

Figure 5.4-2 Steady state sinusoidal response.

Example 5.4-2[2]. --
Find the frequency response of a system with the general second-order transfer function, and compare the results with the response of the second-order system to a sinusoidal function.

$$Y(t) = \frac{A}{\sqrt{\left[1-(\omega t)^2\right]^2 + \left(2\zeta\omega\tau\right)^2}} \sin(\omega t + \phi), \text{ where } \phi = -\tan^{-1}\frac{2\zeta\omega\tau}{1-(\omega\tau)^2}$$

If τ = 1 and ζ = 0.8 and the system is disturbed with a sine wave input of 3 sin(0.5t), determine the form of the response after the transients have decayed and steady state oscillations are established.

Solution ---

The transfer function for a second order system is

$$G(s) = \frac{1}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

Putting $s = i\omega$ yields

$$G(i\omega) = \frac{1}{1-\tau^2\omega^2 + 2i\zeta\omega\tau} = \frac{1}{\left(1-\tau^2\omega^2 + 2i\zeta\omega\tau\right)}\frac{1-\tau^2\omega^2 - 2i\zeta\omega\tau}{\left(1-\tau^2\omega^2 - 2i\zeta\omega\tau\right)}$$

$$G(i\omega) = \frac{1-\tau^2\omega^2 - i2\zeta\omega\tau}{\left[1-(\omega\tau)^2\right]^2 + \left(2\zeta\omega\tau\right)^2}$$

$$R = \text{magnitude} = \sqrt{a^2+b^2} = \sqrt{\frac{\left[1-(\omega\tau)^2\right]^2 + \left(2\zeta\omega\tau\right)^2}{\left\{\left[1-(\omega\tau)^2\right]^2 + \left(2\zeta\omega\tau\right)^2\right\}^2}}$$

$$R = \text{magnitude} = \frac{1}{\sqrt{\left[1-(\omega\tau)^2\right]^2 + \left(2\zeta\omega\tau\right)^2}}$$

$$\phi = \text{angle} = \tan^{-1}\frac{b}{a} = -\tan^{-1}\frac{2\zeta\omega\tau}{1-(\omega\tau)^2}$$

For τ = 1 and ζ = 0.8 we have

$$R = \left\{\left[1-(\omega\tau)^2\right]^2 + \left(2\zeta\omega\tau\right)^2\right\}^{-0.5} = \left\{\left[1-(0.5)^2\right]^2 + \left(2\times0.8\times0.5\right)^2\right\}^{-0.5} = 0.91192$$

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[2] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 290

$$\phi = -\tan^{-1}\frac{2\zeta\omega\tau}{1-(\omega\tau)^2} = -\tan^{-1}\frac{2\times 0.8\times 0.5}{1-0.5^2} = -0.81765$$

The form of the steady-state response is then

$$Y(t) = 3(0.91192)\sin(0.5t - 0.81765)$$

The above function is plotted on Figure E-2 with the response obtained from Simulink. The Simulink model is listed in Figure e-1.

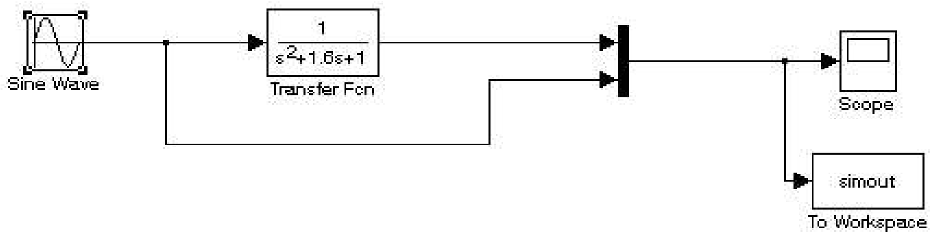


Figure E-1 Simulink model of the second order process

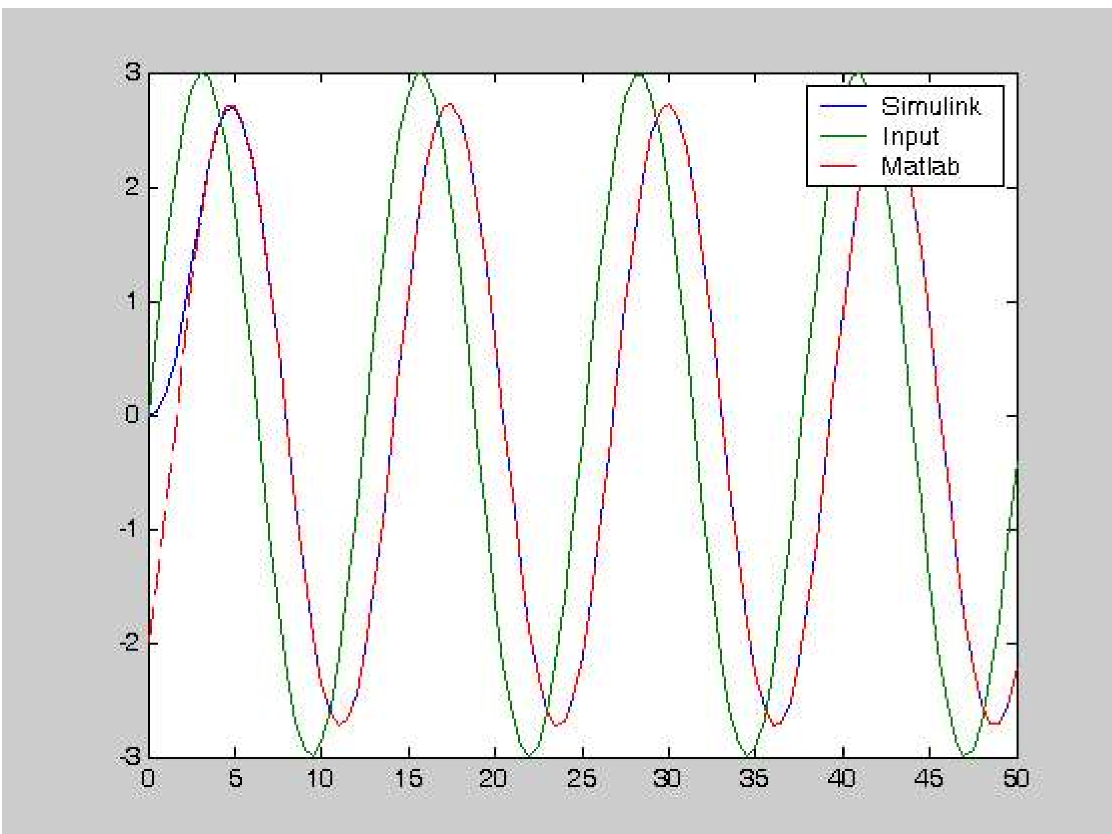
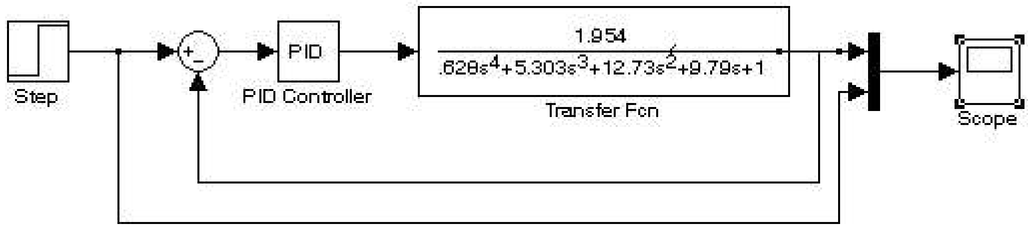


Figure E-2 Matlab and Simulink reponse of the second order process

Example 5.4-3[3]. --
Obtain the ultimate gain and period for the control loop shown below.



Solution --

Let $Y(s)$ = the output function and $X(s)$ = input forcing function, then

$$Y(s) = G_cG_p\,X(s) - G_cG_p\,Y(s) \qquad \text{(E-1)}$$

In this equation, $G_c = K_c$ = proportional controller gain, and

$$G_p = \frac{1.954}{(0.2s+1)(8.34s+1)(0.502s+1)(0.75s+1)}$$

$$G_p = \frac{1.954}{0.628s^4+5.303s^3+12.73s^2+9.79s+1}$$

From Eq. (E-1), the transfer function for the closed loop is

$$\frac{Y(s)}{X(s)} = \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)}$$

The characteristic equation is obtained by setting the denominator of the loop equation to zero.

$$1 + G_cG_p = 0$$

$$1 + \frac{1.954K_c}{0.628s^4+5.303s^3+12.73s^2+9.79s+1} = 0$$

$$0.628s^4 + 5.303s^3 + 12.73s^2 + 9.79s + 1 + 1.954K_c = 0$$

The ultimate frequency can be obtained by substituting $s = i\omega$ into the characteristic equation.

$$0.628\omega^4 - i5.303\omega^3 - 12.73\omega^2 + i9.79\omega + 1 + 1.954K_c = 0$$

[3] Smith, C.A. & Corripio A. B., Principle and Practice of Automatic Process Control, Wiley, 2006, pg. 216

Solve for the ultimate frequency ω_u by setting the imaginary part to zero and for the ultimate gain K_{cu} by setting the real part to zero.

$$(0.628\omega^4 - 12.73\omega^2 + 1 + 1.954K_c) + i(-5.303\omega^3 + 9.79\omega) = 0 + i0$$

$$\omega_u = (9.79/5.303)^{0.5} = 1.359 \text{ rad/s}$$

$$K_{cu} = \frac{-0.628(1.359)^4 + 12.73(1.359)^2 - 1}{1.954} = 10.4$$

The ultimate periods is $P_u = 2\pi/1.359 = 4.6$ s. The response from Simulink to a step change in the forcing function gives a sustained oscillation at the value $K_{cu} = 10.4$. From Figure E-1, the ultimate period is 4.6 (10.9 – 6.3)

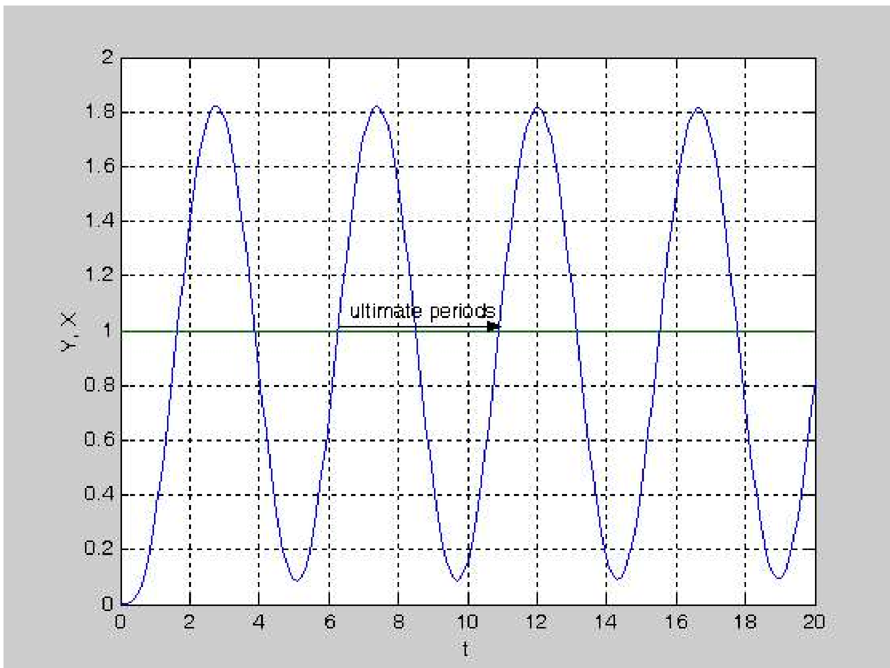


Figure E-1 System response at the ultimate frequency.

Example 5.4-4[4]. ---

For the control system shown in Figure E5.4-4a, determine the controller settings for a PI controller using the Z-N method and the C-C method. The process reaction curve must be modeled by a first-order process with transportation lag shown in Figure E5.4-4b.

[4] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 406

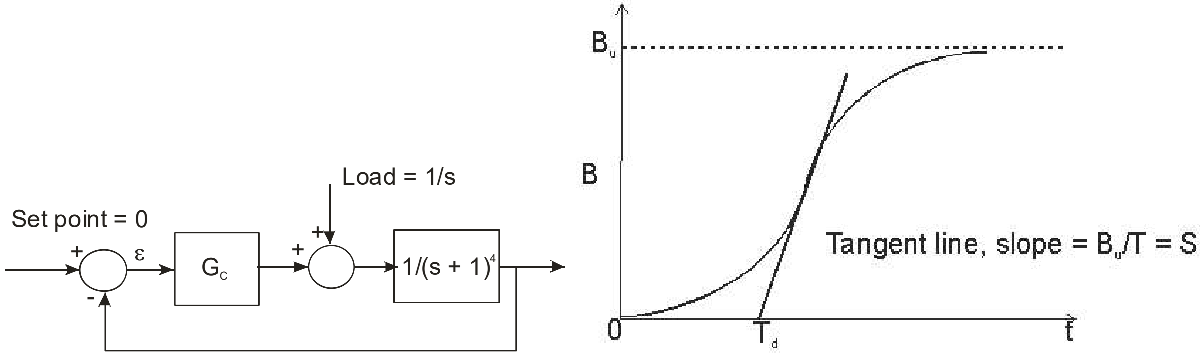


Figure 5.5-4 (a) Process for example 5.4-4, (b) Process reaction curve

Solution --

(a) C-C method:

A unit response of the controlled variable C is given by

$$C(s) = \frac{1}{s(s+1)^4} = \frac{1}{\ } + \frac{A_2}{(s+1)^4} + \frac{A_3}{(s+1)^3} + \frac{A_4}{(s+1)^2} + \frac{A_5}{s+1}$$

$$1 \equiv A_1(s+1)^4 + A_2 s + A_3 s(s+1) + A_4 s(s+1)^2 + A_5 s(s+1)^3$$

Hence $A_1 = 1$, $A_2 = -1$

$s = 1 \Rightarrow 1 = 16 - 1 + 2A_3 + 4A_4 + 8A_5 \Rightarrow \quad A_3 + 2A_4 + 4A_5 = -7$

$s = 2 \Rightarrow 1 = 81 - 2 + 6A_3 + 18A_4 + 54A_5 \Rightarrow \quad 3A_3 + 9A_4 + 27A_5 = -39$

$s = -2 \Rightarrow 1 = 1 + 2 + 2A_3 - 2A_4 + 2A_5 \Rightarrow \quad A_3 - A_4 + A_5 = -1$

We have $A_3 = -1$, $A_4 = -1$, and $A_5 = -1$

$$C(s) = \frac{1}{s(s+1)^4} = \frac{1}{\ } - \frac{1}{(s+1)^4} - \frac{1}{(s+1)^3} - \frac{1}{(s+1)^2} - \frac{1}{s+1}$$

Therefore $C(t) = 1 - \left(\frac{t^3}{6} + \frac{t^2}{2} + t + 1\right)e^{-t}$

Taking the first and second derivatives of the above expression

$$\frac{\ }{\ } = \left(\frac{t^2}{2} + t + 1\right)e^{-t} + \left(\frac{t^3}{6} + \frac{t^2}{2} + t + 1\right)e^{-t} = \frac{^3}{6}e^{-t}$$

$$\frac{^2}{_2} = \frac{1}{6}e^{-t}\left(3t^2 - t^3\right)$$

Setting the second derivative to zero provides the location of the inflection point

$$e^{-t}\left(3t^2 - t^3\right) = 0 \Rightarrow t = 3$$

Therefore

$$C(3) = 1 - \left(\frac{3^3}{6}+\frac{3^2}{2}+3+1\right)e^{-3} = 0.3528$$

$$\left.\frac{dC}{dt}\right|_{t=3} = \frac{3^3}{6}\ e^{-3} = 0.2240 = \frac{C(3)-0}{3-T_d} = \frac{0.3528-0}{3-T_d}$$

The transport lag is then $T_d = 1.425$

Solving for T gives

$$T = \frac{B_u}{S} = \frac{1}{0.224} = 4.4635$$

For C-C rules, we have

$$K_c = \frac{1}{K_p}\frac{T}{T_d}\left(\frac{9}{10}+\frac{T_d}{12T}\right) = \frac{4.4635}{1.425}\left(\frac{9}{10}+\frac{1.425}{12\times 4.4635}\right) = \mathbf{2.90}$$

$$\tau_I = T_d\frac{30+3T_d/T}{9+20T_d/T} = 1.425\frac{30+3(1.425/4.4635)}{9+20(1.425/4.4635)} = \mathbf{2.87}$$

(b) Z-N method

The transfer function for the closed loop is

$$\frac{C(s)}{L(s)} = \frac{G_p(s)}{1+G_c(s)G_p(s)}$$

The characteristic equation is obtained by setting the denominator of the loop equation to zero.

$$1 + G_cG_p = 0$$

Only the proportional controller gain is used to determine the ultimate frequency

$$1 + \frac{K_c}{s^4+4s^3+6s^2+4s+1} = 0$$

$$s^4 + 4s^3 + 6s^2 + 4s + 1 + K_c = 0$$

The ultimate frequency can be obtained by substituting $s = i\omega$ into the characteristic equation.

$$\omega^4 - i4\omega^3 - 6\omega^2 + i4\omega + 1 + K_c = 0$$

Solve for the ultimate frequency ω_u by setting the imaginary part to zero and for the ultimate gain K_{cu} by setting the real part to zero.

$$(\omega^4 - 6\omega^2 + 1 + K_c) + i(-4\omega^3 + 4\omega) = 0 + i0$$

$$\omega_u = 1 \text{ rad/s}$$

$$K_{cu} = 6\omega^2 - \omega^4 - 1 = 4$$

The ultimate periods is $P_u = 2\pi/1 = 2\pi$

Applying the Z-N rules for PI control gives

$$K_c = 0.45K_{cu} = (0.45)(4) = \mathbf{1.8}$$

$$\tau_I = \frac{P_u}{1.2} = \frac{2\pi}{1.2} = \mathbf{5.236}$$

The Simulink model for this system using C-C and Z-N values for controller is shown in Figure E-1. The responses of the system using both methods are shown in Figure E-2. The settings obtained from C-C method gives an unstable response. A better method to fit the process reaction curve to a first order plus dead-time is needed to produce a stable response.

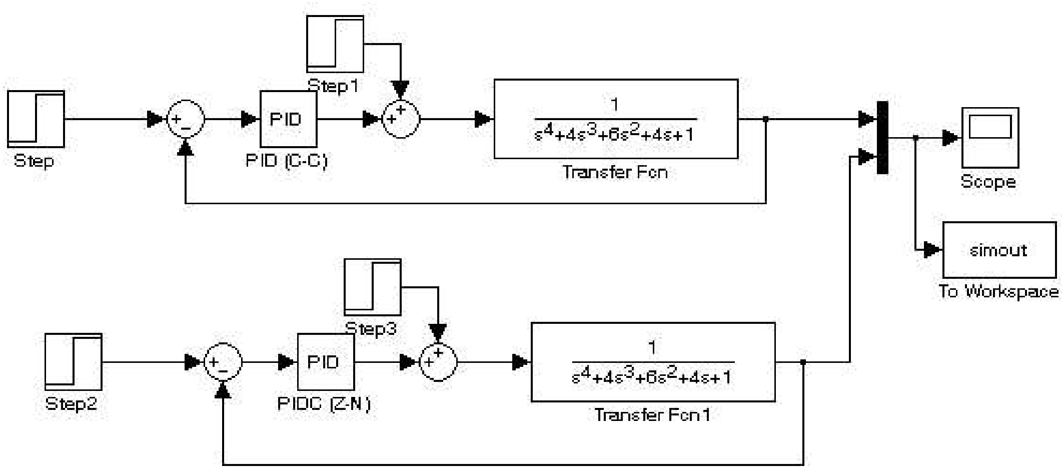


Figure E-1 Simulink model using C-C and Z-N tuning methods.

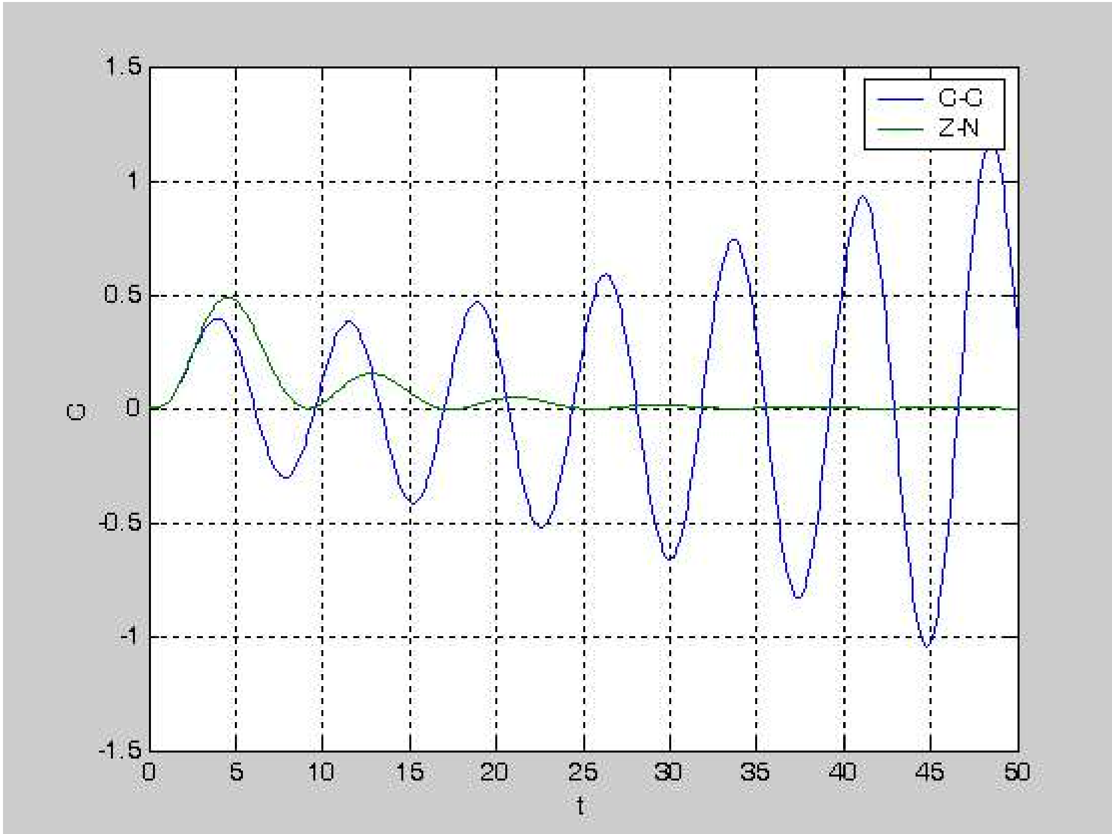


Figure E-2 Comparison of the responses produced by C-C and Z-N methods.

Chapter 6

Multivariable Process Control

6.1 Introduction

Up to this point, we have only considered processes with a single control objective or controlled variable. However real processes will usually have multiple control objectives where more than one variable must be controlled. For processes in which the control objectives do not interact with each other we can still consider each control objective separate from the others. In this chapter we will study control system for processes in which the various control objectives interact with each other. These systems are called multivariable control systems or multiple-input, multiple-output (MIMO) control systems. The response and stability of the multivariable system can be quite different from that of its constituent loops taken separately.

In the blending tank shown in Figure 6.1-1a[1], it is necessary to control both the flow and composition of the outlet stream by manipulating the flow of each of the two inlet streams. Let consider an example in which the tank blends a solution containing 10 weight % salt with a concentrated solution containing 35 weight % salt, to produce 100 lb/h of a solution containing 20 weight % salt at design conditions.

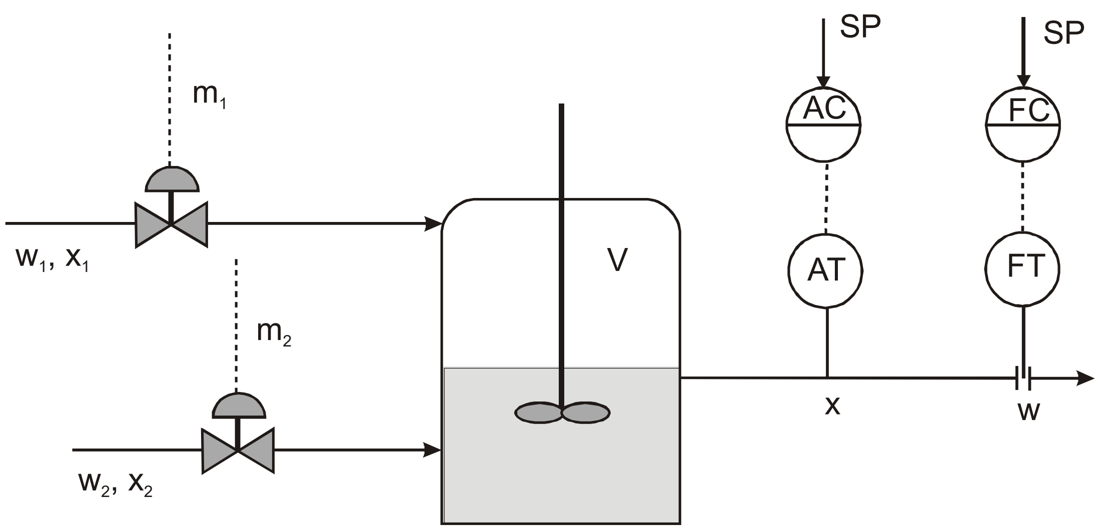


Figure 6.1-1a Example of multivariable control system: blending tank.

From steady-state balances on the total mass and mass of salt around the tank we have

$$w = w_1 + w_2$$

$$wx = w_1x_1 + w_2x_2$$

[1] Smith, C.A. & Corripio A. B., Principle and Practice of Automatic Process Control, Wiley, 2006, pg. 410.

In these equations, w is the stream flow in lb/h, x is the mass fraction of salt in each stream and the subscripts denote the two inlet streams. The required inlet flow rates, $w_{1,s}$ and $w_{2,s}$, at the design conditions can be determined from

$$100 = w_{1,s} + w_{2,s}$$

$$100 \times 0.2 = 0.1 w_{1,s} + 0.35 w_{2,s}$$

The solutions are: $w_{1,s} = 60$ lb/h and $w_{2,s} = 40$ lb/h. We will consider only the composition control loop and assume that the flow of concentrated stream 2 is the manipulated variable. To obtain the gain of the loop for this system, we consider an increase of 2.0 lb/h in w_2 to 42.0 lb/h. The mass fraction in the exit stream is then

$$x = (0.1 \times 60 + 0.35 \times 42)/102 = 0.20294$$

The steady state gain of the flow of stream 2 on the mass percent of salt in the product stream is then

$$K_{x2} = \frac{20.294 - 20.0}{42.0 - 40.0} = 0.1471 \text{ \% salt/(lb/h)}$$

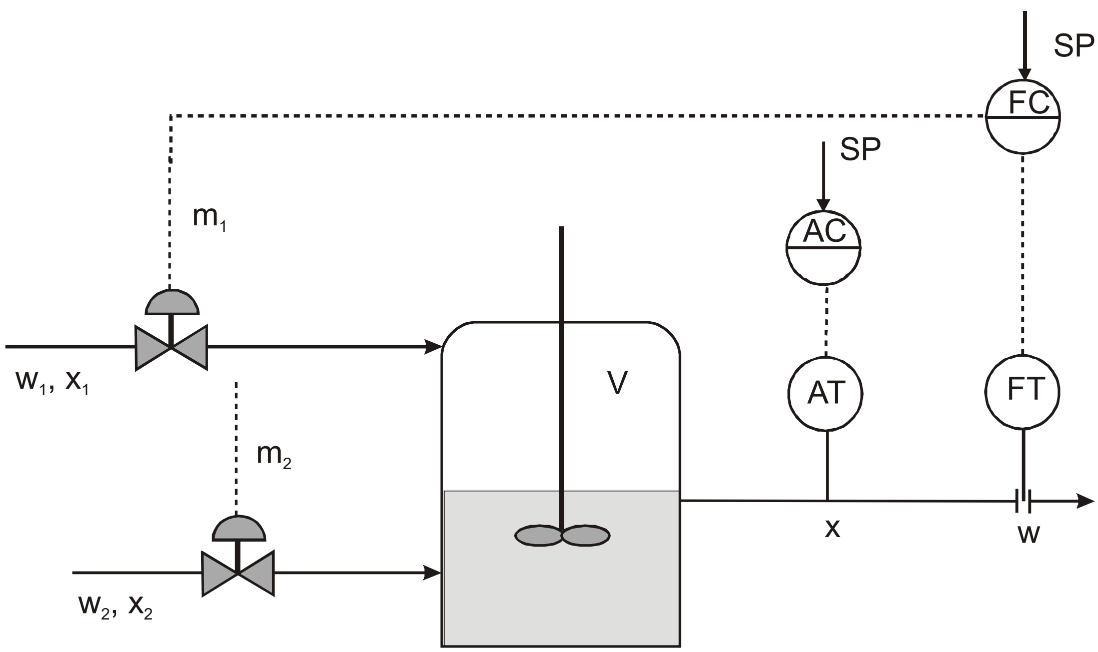


Figure 6.1-1b Bending tank with product flow controller manipulating stream 1.

Consider the control scheme shown in Figure 6.1-1b in which the product flow is controlled by manipulating the flow of the dilute inlet stream, w_1. When the flow of stream 2 is increased from 40.0 to 42.0 lb/h, the product flow controller decreases the flow of the dilute stream to 58.0 lb/h, to keep the product flow at 100.0 lb/h. The product concentration is now

$$x = (0.1 \times 58 + 0.35 \times 42)/100 = 0.2050$$

The closed-loop gain of the product composition loop becomes

$$K_{x2,closed} = \frac{20.5 - 20.0}{42.0 - 40.0} = 0.250 \text{ \% salt/(lb/h)}$$

The percentage increase in the gain of the product composition loop is then

$$100 \times \frac{K_{x2,close} - K_{x2}}{K_{x2}} = 100 \times \frac{0.250 - 0.1471}{0.1471} = 70\%$$

The increase in the gain of the product composition loop is caused by interaction with the flow controller. K_{x2} is the open-loop gain which is the composition loop gain when the flow loop is open.

For system with loop interaction, we need to consider (1) the effect of interaction on the response of the feedback loops, (2) the level of interaction between the loops and the best way to pair the controlled and manipulated variables to reduce the effect of interaction, and (3) the possibility of eliminating or reducing the loop interaction through the design of an appropriate control system.

In the blending tank example, closing the flow loop causes a change in the product composition that is in the same direction as the original change. This is known as *positive interaction* in which produces an increase in the loop gain when the other loop is closed. The interaction is said to be *negative* when the gain of a loop decreases or changes sign when the other loop is closed. The negative interaction causes a change in the controlled variable that is in the opposite direction to the original change.

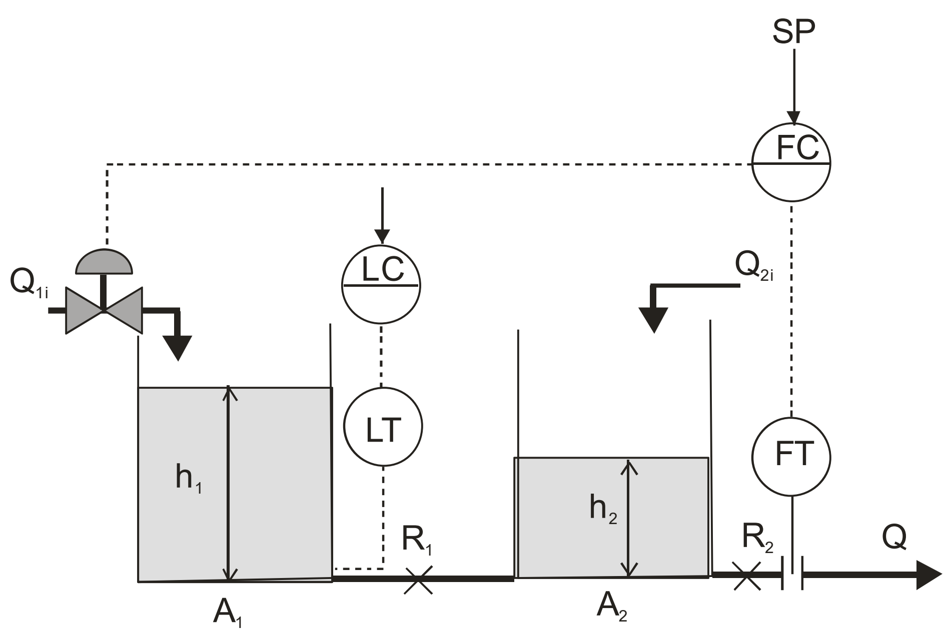


Figure 6.1-2 Multivariable control system: level process

The two tanks system shown in Figure 6.1-2 is an example of a negative interaction loop. The system is initially at steady state with q_{1i} = 8 cfm and q_{2i} = 4 cfm. The following data apply to the tanks: A_1 = 1 ft^2, A_2 = 1.25 ft^2, R_1 = 1 ft/cfm, and R_2 = 0.8 ft/cfm. At the design conditions:

$$q_s = q_{1is} + q_{2is} = 8 + 4 = 12 \text{ cfm}$$

Tank 1: $q_{1i} - q_{1s} = 0$, where $q_{1s} = (h_{1s} - h_{2s})/R_1$

Tank 2: $q_{1s} + q_{2is} - q_{2s} = 0$, where $q_{2s} = h_{2s}/R_2$

Substituting the numerical values we have

$$8 - h_{1s} + h_{2s} = 0$$

$$h_{1s} - h_{2s} + 4 - h_{2s}/0.8 = 0 \Rightarrow h_{2s} = 0.8\times 12 = 9.6 \text{ ft and } h_{1s} = 17.6 \text{ ft}$$

When the flow q_{2i} is increased from 4 to 6 cfm, the final open loop steady state value for h_1 is 19.2 ft ($0.8\times 14 + 8 = 19.2$). The steady state gain is then

$$K_{h1} = \frac{19.2 - 17.6}{6 - 4} = 0.8 \text{ ft/cfm}$$

For the closed loop when the flow q_{2i} is increased from 4 to 6 cfm, the product flow controller decreases the flow of stream 1 from 8 to 6 cfm to keep the product flow at 12 cfm. The final value for h_1 is 15.6 ft ($0.8\times 12 + 6 = 15.6$). The gain of the closed loop is now

$$K_{h1,\text{closed}} = \frac{15.6 - 17.6}{6 - 4} = -\,1.0 \text{ ft/cfm}$$

The change in the sign of the gain occurs when the interaction change is greater than the original change. Because of this change in the sign of the gain, negative interaction can be a more severe problem than positive interaction.

6.2 Control of Interacting Systems

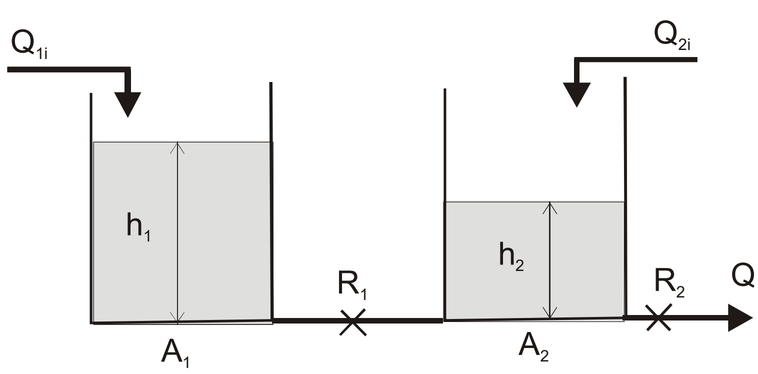


Figure 6.2-1 Multivariable system: level process

Now consider the two tanks system shown in Figure 6.2-1 in which there are two inputs (q_{1i} and q_{2i}) and two outputs (h_1 and h_2). A change in q_{1i} or q_{2i} alone will affect both outputs (h_1 and h_2). The interaction between inputs and outputs is presented by the block diagram of Figure 6.2-2.

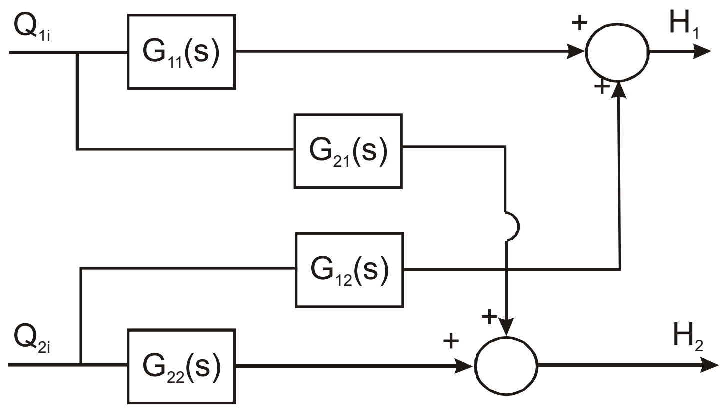


Figure 6.2-2 Block diagram for the two tanks system.

In the block diagram, the transfer functions show how the change in one of the inputs affects both of the outputs. If a change occurs in only Q_{1i}, the responses of H_1 and H_2 are

$$H_1(s) = G_{11}(s)Q_{1i}(s)$$

$$H_2(s) = G_{21}(s)Q_{1i}(s)$$

If the tanks were non-interacting, $G_{12} = 0$ so that a change in flow to tank 2 would not affect H_1. A single loop will not be sufficient to control both H_1 and H_2. We will discuss the control of a 2×2 system shown in Figure 6.2-3. Similar procedure can be extended to system with more than two pairs of inputs and outputs. The control objective for system shown in Figure 6.2-3 is to control C_1 and C_2 independently, in spite of changes in M_1 and M_2 or other load variables not shown.

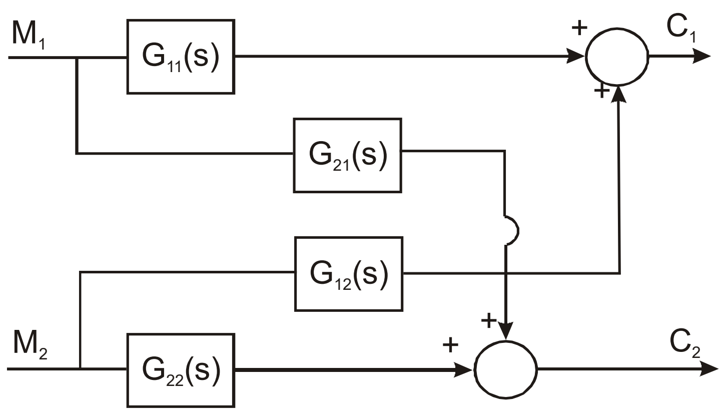


Figure 6.2-3 Multivariable system with two pairs of inputs and outputs.

Two control loops are added to the open system with the block diagram shown in Figure 6.2-4. The controller, the valve, and the measurement element are added to each loop. We want to maintain control of C_1 and C_2. Because of loop interactions in the system, a change in R_1 will also cause C_2 to vary through the transfer function G_{21}. Both outputs C_1 and C_2 will change if either input R_1 or R_2 changes alone. If G_{21} and G_{12} provide weak interaction, the two-controller system shown in Figure 6.2-4 will give satisfactory control. If $G_{21} = G_{12}$, there is no interaction and the two control loops are independent from each other.

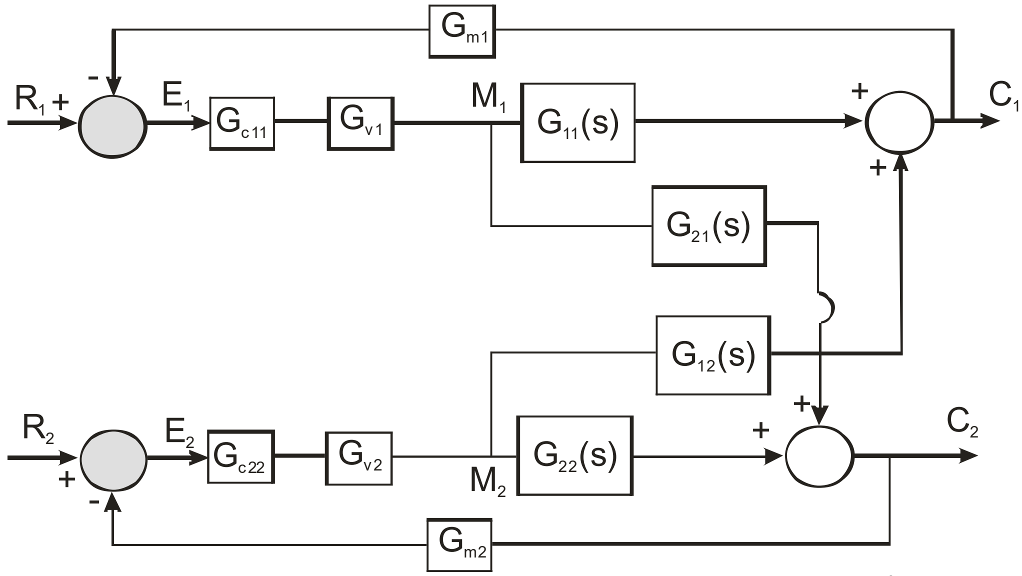


Figure 6.2-4 Multiloop control system with two controllers[2].

Figure 6.2-5 shows a control system with four controllers: two primary controllers (G_{c11} and G_{c22}) and two cross-controllers (G_{c12} and G_{c21}). In principle, these cross-controllers can be designed to eliminate interaction. We will obtain the responses C_1 and C_2 of the system to the change in inputs.

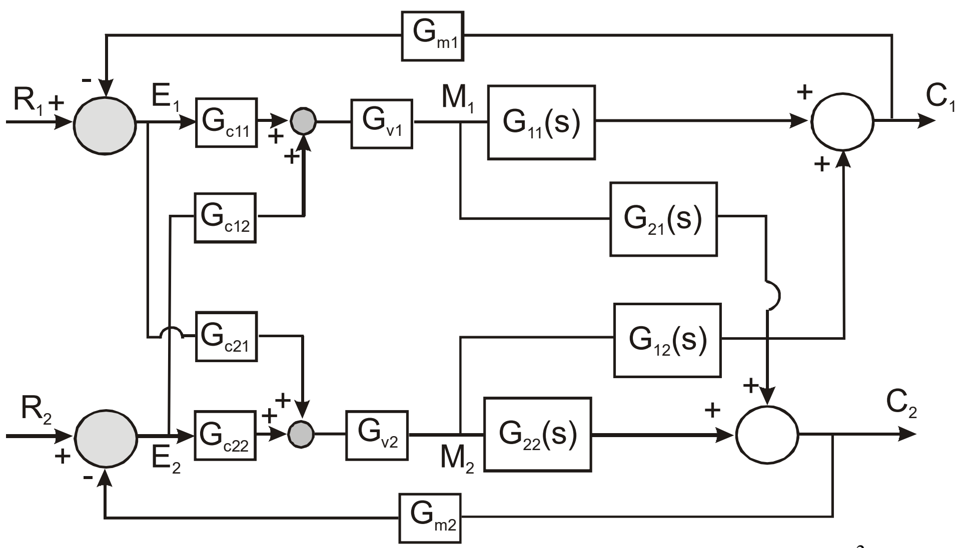


Figure 6.2-5 Multiloop control system with four controllers[3].

From the block diagram of Figure 6.2-5 we have

$$C_1 = G_{11}M_1 + G_{12}M_2 \tag{6.2-1}$$

$$C_2 = G_{21}M_1 + G_{22}M_2 \tag{6.2-2}$$

[2] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 514

[3] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 515

The above equations can be written in matrix form:

$$\mathbf{C} = \mathbf{G_p M} \tag{6.2-3}$$

In this equation

$$\mathbf{C} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix},\ \mathbf{G_p} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}, \text{ and } \mathbf{M} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}$$

The relation between **M** and $\mathbf{E} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$ can also be obtained from the block diagram

$$M_1 = G_{v1}G_{c11}E_1 + G_{v1}G_{c12}E_2 \tag{6.2-4}$$

$$M_2 = G_{v2}G_{c21}E_1 + G_{v2}G_{c22}E_2 \tag{6.2-5}$$

In matrix form

$$\mathbf{M} = \mathbf{G_v G_c E} \tag{6.2-6}$$

Where $\mathbf{G_v} = \begin{bmatrix} G_{v1} & 0 \\ 0 & G_{v2} \end{bmatrix}$ and $\mathbf{G_c} = \begin{bmatrix} G_{c11} & G_{c12} \\ G_{c21} & G_{c22} \end{bmatrix}$

From the diagram we also have

$$E_1 = R_1 - G_{m1}C_1 \tag{6.2-7}$$

$$E_2 = R_2 - G_{m2}C_2 \tag{6.2-8}$$

In matrix form

$$\mathbf{E} = \mathbf{R} - \mathbf{G_m C} \tag{6.2-9}$$

The measuring element matrix $\mathbf{G_m}$ is defined as $\mathbf{G_m} = \begin{bmatrix} G_{m1} & 0 \\ 0 & G_{m2} \end{bmatrix}$, and $\mathbf{R} = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$

From Eqs. (6.2-3) and (6.2-6) we have

$$C = \mathbf{G_p G_v G_c E} = \mathbf{G_o E}, \text{ where } \mathbf{G_o} = \mathbf{G_p G_v G_c}$$

Therefore

$$C = \mathbf{G_o E} = \mathbf{G_o(R - G_m C)} = \mathbf{G_o R - G_o G_m C}$$

$$C[\mathbf{I} + \mathbf{G_o G_m}] = \mathbf{G_o R}$$

$$C = [\mathbf{I} + \mathbf{G_oG_m}]^{-1}\mathbf{G_oR} \qquad (6.2\text{-}10)$$

The closed loop behavior given by Eq. (6.2-10) is similar to the closed-loop response of a SISO system, which may be written as

$$C(s) = \frac{G_o(s)}{1 + G_o(s)G_m(s)} R(s) \qquad (6.2\text{-}11)$$

The matrix term $[\mathbf{I} + \mathbf{G_oG_m}]^{-1}$ is equivalent to the scalar term $1/[1 + G_o(s)G_m(s)]$. A matrix block diagram for Eq. (6.2-10) is shown in the bottom part of Figure 6.2-6. This diagram is equivalent to the control system with two primary controllers and two cross-controllers shown in the top part of Figure 6.2-6. The double line indicates that more than one variable is being transmitted. Each block contains a matrix of transfer functions that relates an output vector to an input vector. The diagram can be simplified further by replacing the three blocks $\mathbf{G_pG_vG_c}$ with block $\mathbf{G_o}$. If there is no loop interaction between **C** and **R,** that is R_1 affects only C_1 and R_2 affects only C_2, the off-diagonal element of $[\mathbf{I} + \mathbf{G_oG_m}]^{-1}\mathbf{G_o}$ must be zero. Since **I** and $\mathbf{G_m}$ are diagonal, $[\mathbf{I} + \mathbf{G_oG_m}]^{-1}\mathbf{G_o}$ will be diagonal if $\mathbf{G_o}$ is diagonal.

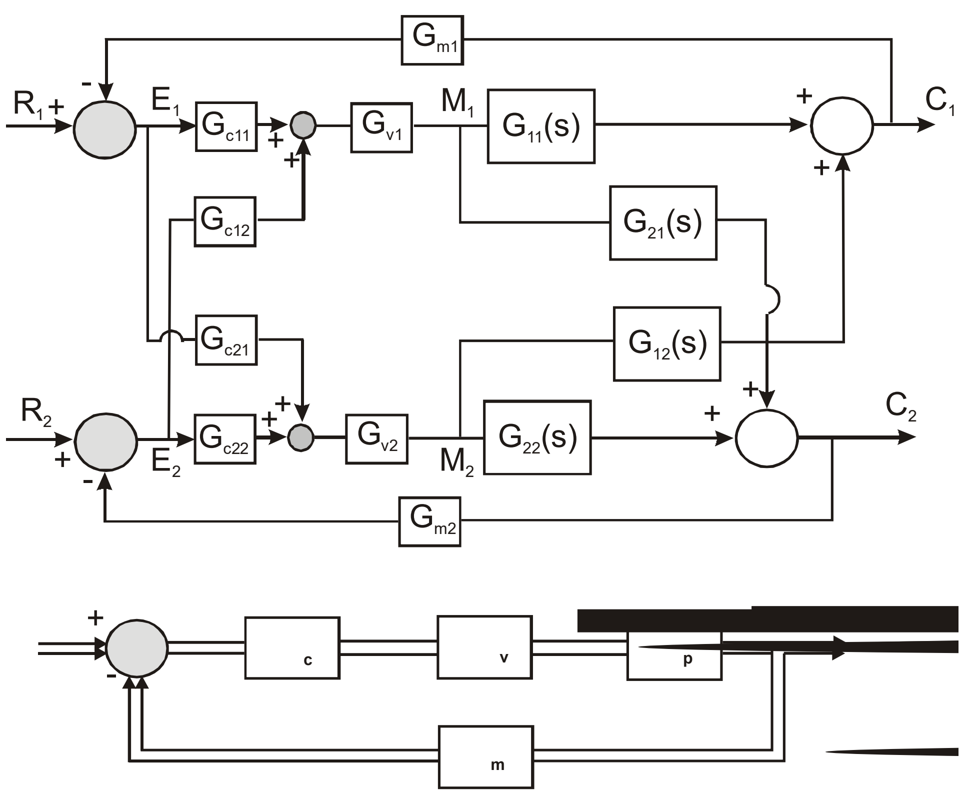


Figure 6.2-6 Block diagram for MIMO control system in terms of matrix blocks[4].

$$\mathbf{G_o} = \mathbf{G_pG_vG_c}$$

[4] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 516

$$\mathbf{G_o} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} G_{v1} & 0 \\ 0 & G_{v2} \end{bmatrix} \mathbf{G_c}$$

$$\mathbf{G_o} = \begin{bmatrix} G_{11}G_{v1} & G_{12}G_{v2} \\ G_{21}G_{v1} & G_{22}G_{v2} \end{bmatrix} \mathbf{G_c} = \begin{bmatrix} G_{11}G_{v1} & G_{12}G_{v2} \\ G_{21}G_{v1} & G_{22}G_{v2} \end{bmatrix} \begin{bmatrix} G_{c11} & G_{c12} \\ G_{c21} & G_{c22} \end{bmatrix}$$

$$\mathbf{G_o} = \begin{bmatrix} G_{11}G_{v1}G_{c11} + G_{12}G_{v2}G_{c21} & G_{11}G_{v1}G_{c12} + G_{12}G_{v2}G_{c22} \\ G_{21}G_{v1}G_{c11} + G_{22}G_{v2}G_{c21} & G_{21}G_{v1}G_{c12} + G_{22}G_{v2}G_{c22} \end{bmatrix} \qquad (6.2\text{-}12)$$

For $\mathbf{G_o}$ to be diagonal, the off-diagonal element of $\mathbf{G_o}$ must be zero

$$G_{11}G_{v1}G_{c12} + G_{12}G_{v2}G_{c22} = 0 \Rightarrow G_{c12} = -\frac{G_{12}G_{v2}G_{c22}}{G_{11}G_{v1}} \qquad (6.2\text{-}13)$$

$$G_{21}G_{v1}G_{c11} + G_{22}G_{v2}G_{c21} = 0 \Rightarrow G_{c21} = -\frac{G_{21}G_{v1}G_{c11}}{G_{22}G_{v2}} \qquad (6.2\text{-}14)$$

Matrix Inverse. The inverse of a square matrix **A** is written as $\mathbf{A}^{-1}$ and

$$\mathbf{A}\,\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

If $\mathbf{AB} = \mathbf{I}$ then $\mathbf{B} = \mathbf{A}^{-1}$ or $\mathbf{A} = \mathbf{B}^{-1}$

Consider a set of equations of the form

$$\mathbf{AX} = \mathbf{D} \Rightarrow \mathbf{A}^{-1}\mathbf{AX} = \mathbf{A}^{-1}\mathbf{D} \Rightarrow \mathbf{IX} = \mathbf{X} = \mathbf{A}^{-1}\mathbf{D}$$

Cofactor matrix: a cofactor matrix is constructed by replacing each element of a square matrix by its cofactor.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \\ 1 & -3 & 5 \end{bmatrix}$$

The cofactor matrix $\mathbf{A}^c$ is

$$\mathbf{A}^c = \begin{bmatrix} A_{11}^c & A_{12}^c & A_{13}^c \\ A_{21}^c & A_{22}^c & A_{23}^c \\ A_{31}^c & A_{32}^c & A_{33}^c \end{bmatrix}, \text{ where } A_{11}^c = \begin{vmatrix} 4 & -2 \\ -3 & 5 \end{vmatrix} = 14,\ A_{12}^c = -\begin{vmatrix} 0 & -2 \\ 1 & 5 \end{vmatrix} = -2$$

Adjoint matrix. Adjoint matrix is the transpose of the cofactor matrix.

$$\mathbf{A}^{\mathrm{a}} = (\mathbf{A}^{\mathrm{c}})^{\mathrm{T}} = \begin{bmatrix} A_{11}^{\;c} & A_{21}^{\;c} & A_{31}^{\;c} \\ A_{12}^{\;c} & A_{22}^{\;c} & A_{32}^{\;c} \\ A_{13}^{\;c} & A_{23}^{\;c} & A_{33}^{\;c} \end{bmatrix}$$

The inverse is then $\mathbf{A}^{-1} = \mathbf{A}^{\mathrm{a}}/|\mathrm{A}|$. This formula can be verified by a direct substitution.

$$\mathbf{A}\,\mathbf{A}^{-1} = \mathbf{A}\,\mathbf{A}^{\mathrm{a}}/|\mathrm{A}| = \frac{1}{D}\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} A_{11}^{\;c} & A_{21}^{\;c} & A_{31}^{\;c} \\ A_{12}^{\;c} & A_{22}^{\;c} & A_{23}^{\;c} \\ A_{13}^{\;c} & A_{32}^{\;c} & A_{33}^{\;c} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

where

$$b_{11} = a_{11}\,A_{11}^{\mathrm{c}} + a_{12}\,A_{12}^{\mathrm{c}} + a_{13}\,A_{13}^{\mathrm{c}} = D$$

$$b_{12} = a_{11}\,A_{21}^{\mathrm{c}} + a_{22}\,A_{12}^{\mathrm{c}} + a_{23}\,A_{13}^{\mathrm{c}} = 0$$

Similar calculations show that the diagonal elements b_{ii} are equal to D and the off-diagonal elements are equal to zero. Therefore

$$\mathbf{A}\,\mathbf{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathbf{I}$$

Example 6.2-1[5]. --

Determine $[s\mathbf{I} - \mathbf{A}]^{-1}$, given $\mathbf{A} = \begin{bmatrix} -3 & 2 \\ 4 & -5 \end{bmatrix}$

Solution --

Let $\mathbf{B} = [s\mathbf{I} - \mathbf{A}] = \begin{bmatrix} s+3 & -2 \\ -4 & s+5 \end{bmatrix} \Rightarrow \mathbf{B}^{\mathbf{c}} = \begin{bmatrix} s+5 & 4 \\ 2 & s+3 \end{bmatrix} \Rightarrow \mathbf{B}^{\mathbf{a}} = \begin{bmatrix} s+5 & 2 \\ 4 & s+3 \end{bmatrix}$

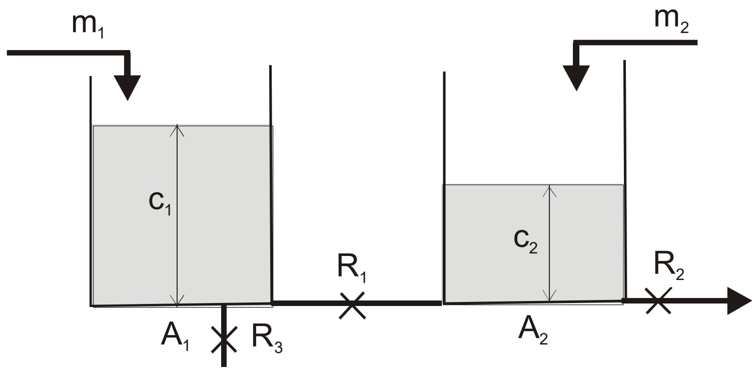
$$\mathbf{B}^{-1} = [s\mathbf{I} + \mathbf{A}]^{-1} = \mathbf{B}^{\mathbf{a}}/|\mathbf{B}| = \frac{\begin{bmatrix} s+5 & 2 \\ 4 & s+3 \end{bmatrix}}{(s+5)(s+3)-8} = \frac{\begin{bmatrix} s+5 & 2 \\ 4 & s+3 \end{bmatrix}}{(s+1)(s+7)}$$

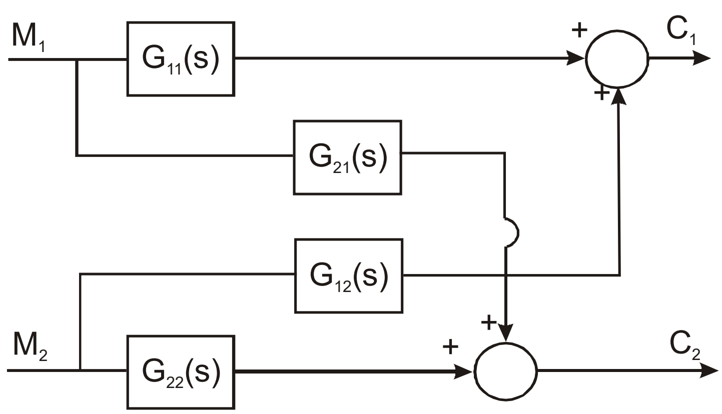
[5] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3nd edition, 2008, pg. 518

Chapter 6

Example 6.2-2[1]. --

Consider the two-tank, interacting liquid-level system shown below and its block diagram. Determine the transfer functions G_{11}, G_{21}, G_{12}, and G_{22}. Data: $A_1 = 1$ ft^2, $A_2 = 0.5$ ft^2, $R_1 = 0.5$ ft/cfm, $R_2 = 2$ ft/cfm, and $R_3 = 1$ ft/cfm.





Solution --

Assume constant density, we have the following equations for tank 1 and tank 2:

$$A_1 \frac{dc_1}{dt} = m_1 - (c_1 - c_2)/R_1 - c_1/R_3$$

$$A_2 \frac{dc_2}{dt} = m_2 + (c_1 - c_2)/R_1 - c_2/R_2$$

Substituting the numerical values into the above equations yields

$$\frac{dc_1}{dt} = m_1 - (c_1 - c_2)/0.5 - c_1 = m_1 - 3c_1 + 2c_2$$

---

[1] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 517

$$0.5\frac{dc_2}{dt} = m_2 + (c_1 - c_2)/\,0.5 - c_2/2 \Rightarrow \frac{dc_2}{dt} = 2m_2 + 4c_1 - 5c_2$$

The two differential equations describing the system are

$$\frac{dc_1}{dt} = m_1 - 3c_1 + 2c_2$$

$$\frac{dc_2}{dt} = 2m_2 + 4c_1 - 5c_2$$

These equations can be written in matrix form as

$$\frac{d\mathbf{c}}{dt} = \mathbf{Ac} + \mathbf{Dm} \qquad \text{(E-1)}$$

In this equation, $\mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$, $\mathbf{A} = \begin{bmatrix} -3 & 2 \\ 4 & -5 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, and $\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}$

Taking the Laplace transform of Eq. (E-1) gives

$$s\mathbf{C}(s) = \mathbf{AC}(s) + \mathbf{DM}(s) \Rightarrow [s\mathbf{I} - \mathbf{A}]\mathbf{C} = \mathbf{DM}$$

$$\mathbf{C} = [s\mathbf{I} - \mathbf{A}]^{-1}\mathbf{DM}$$

The transfer function for the process is then

$$\mathbf{G_p} = [s\mathbf{I} - \mathbf{A}]^{-1}\mathbf{D}$$

Let $\mathbf{B} = [s\mathbf{I} - \mathbf{A}] = \begin{bmatrix} s+3 & -2 \\ -4 & s+5 \end{bmatrix} \Rightarrow \mathbf{B^c} = \begin{bmatrix} s+5 & 4 \\ 2 & s+3 \end{bmatrix} \Rightarrow \mathbf{B^a} = \begin{bmatrix} s+5 & 2 \\ 4 & s+3 \end{bmatrix}$

$$\mathbf{B}^{-1} = [s\mathbf{I} + \mathbf{A}]^{-1} = \mathbf{B^a}/|\mathbf{B}| = \frac{\begin{bmatrix} s+5 & 2 \\ 4 & s+3 \end{bmatrix}}{(s+5)(s+3)-8} = \frac{\begin{bmatrix} s+5 & 2 \\ 4 & s+3 \end{bmatrix}}{(s+1)(s+7)}$$

$$\mathbf{G_p} = [s\mathbf{I} - \mathbf{A}]^{-1}\mathbf{D} = \frac{\begin{bmatrix} s+5 & 2 \\ 4 & s+3 \end{bmatrix}}{(s+1)(s+7)}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \frac{\begin{bmatrix} s+5 & 4 \\ 4 & 2(s+3) \end{bmatrix}}{(s+1)(s+7)}$$

Therefore

$$G_{11} = \frac{s+5}{(s+1)(s+7)} \qquad G_{12} = \frac{4}{(s+1)(s+7)}$$

$$G_{21} = \frac{4}{(s+1)(s+7)} \qquad G_{22} = \frac{2(s+3)}{(s+1)(s+7)}$$

The diagonal elements of $\mathbf{G_p}(s)$ relate c_1 to m_1 and c_2 to m_2. These transfer functions will produce a second-order response to a step change in input that has a finite slope at the origin because of the dynamic of the numerator term that has the form $s + \beta$. The off-diagonal elements have second-order transfer functions without numerator dynamics. For these cases the step response will be second-order with zero slope at the origin. Figure E-1 depicts the Simulink model for the two-tank system. Figures E-2 show the response of c_1 and c_2 to step change in tank 1 only and Figures E-3 shows the response of c_1 and c_2 to step change in tank 2 only.

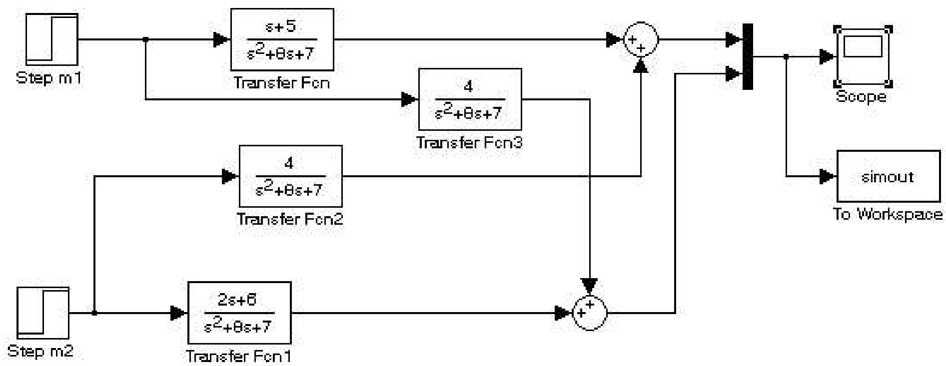


Figure E-1 Simulink model for the two-tank system.

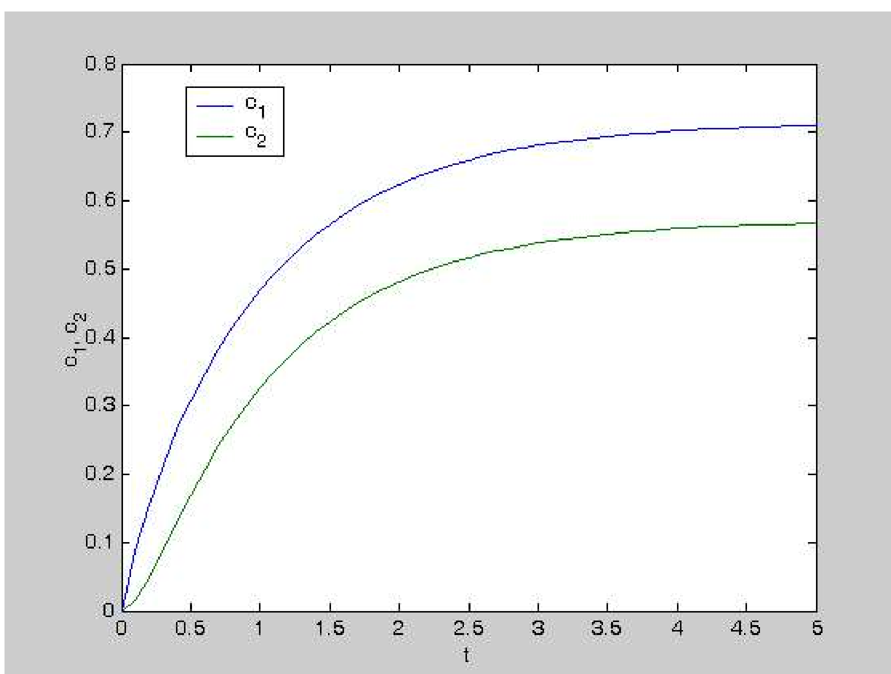


Figure E-2 Response for step change in tank 1, $M_1 = 1/s$, $M_2 = 0$

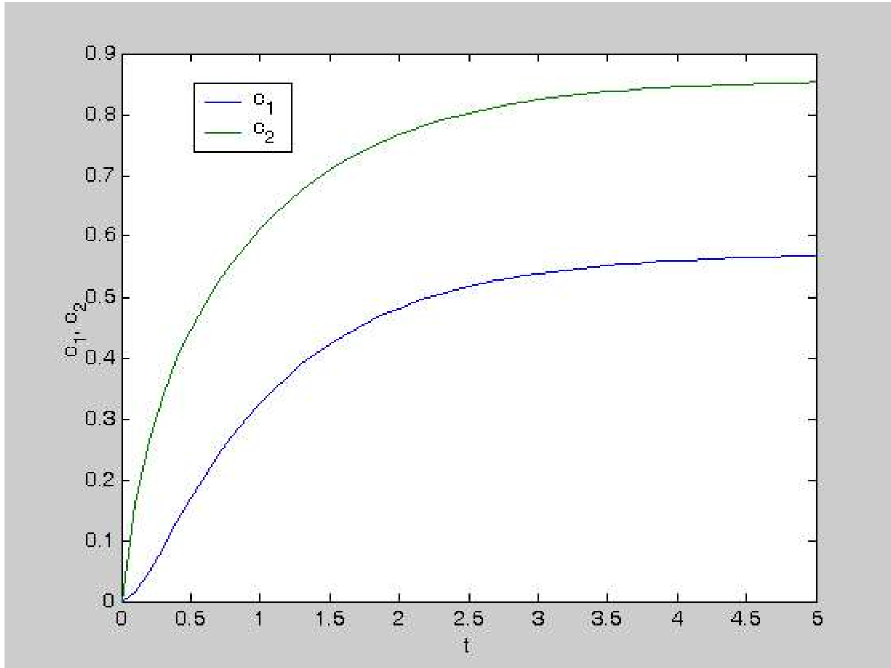
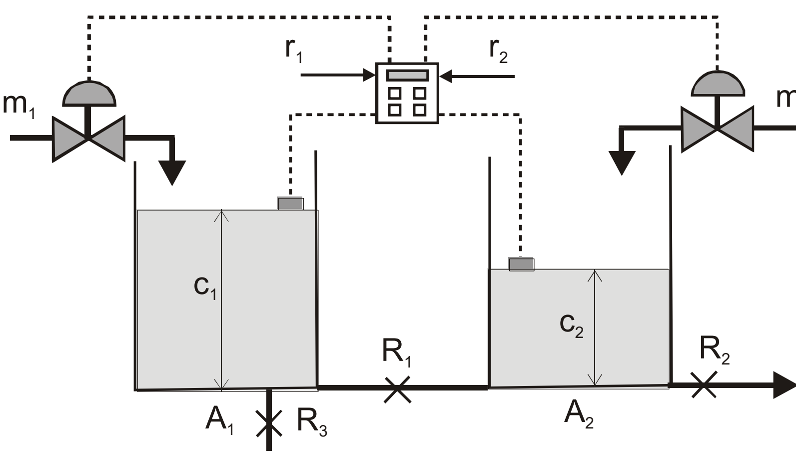


Figure E-3 Response for step change in tank 2, $M_1 = 0$, $M_2 = 1/s$

Example 6.2-3[2]. --

Consider the two-tank, interacting liquid-level system shown below and its block diagram. Determine the controller transfer function matrix $\mathbf{G_c}$ needed to eliminate interaction. The primary controllers are to be proportional; i.e. $G_{c11} = K_1$ and $G_{c22} = K_2$. Data: $A_1 = 1$ ft^2, $A_2 = 0.5$ ft^2, $R_1 = 0.5$ ft/cfm, $R_2 = 2$ ft/cfm, and $R_3 = 1$ ft/cfm. $\mathbf{G_v}$ **and** $\mathbf{G_m}$ are unit diagonal matrices.

$$\mathbf{G_p} = \frac{\begin{bmatrix} s+5 & 4 \\ 4 & 2(s+3) \end{bmatrix}}{(s+1)(s+7)}$$



Solution ---

[2] D.R. Coughanowr and S. LeBlanc, Process Systems Analysis and Control, McGraw-Hill, 3rd edition, 2008, pg. 519

$$G_{11} = \frac{s+5}{(s+1)(s+7)} \qquad G_{12} = \frac{4}{(s+1)(s+7)}$$

$$G_{21} = \frac{4}{(s+1)(s+7)} \qquad G_{22} = \frac{2(s+3)}{(s+1)(s+7)}$$

To eliminate loop interaction the off-diagonal element of $\mathbf{G_o}$ must be zero

$$\mathbf{G_o} = \begin{bmatrix} G_{11}G_{v1}G_{c11} + G_{12}G_{v2}G_{c21} & G_{11}G_{v1}G_{c12} + G_{12}G_{v2}G_{c22} \\ G_{21}G_{v1}G_{c11} + G_{22}G_{v2}G_{c21} & G_{21}G_{v1}G_{c12} + G_{22}G_{v2}G_{c22} \end{bmatrix}$$

Therefore

$$G_{11}G_{v1}G_{c12} + G_{12}G_{v2}G_{c22} = 0 \Rightarrow G_{c12} = -\frac{G_{12}G_{v2}G_{c22}}{G_{11}G_{v1}}$$

$$G_{21}G_{v1}G_{c11} + G_{22}G_{v2}G_{c21} = 0 \Rightarrow G_{c21} = -\frac{G_{21}G_{v1}G_{c11}}{G_{22}G_{v2}}$$

$\mathbf{G_v}$ is a unit diagonal matrix. $G_{v1} = G_{v2} = 1$

$$G_{c12} = -\frac{G_{12}G_{c22}}{G_{11}} = -\frac{4}{(s+1)(s+7)} K_2 \frac{(s+1)(s+7)}{s+5} = -\frac{4K_2}{s+5}$$

$$G_{c21} = -\frac{G_{21}G_{c11}}{G_{22}} = -\frac{4}{(s+1)(s+7)} K_1 \frac{(s+1)(s+7)}{2(s+3)} = -\frac{2K_1}{s+3}$$

The matrix $\mathbf{G_o}$ becomes

$$\mathbf{G_o} = \begin{bmatrix} G_{11}G_{c11} + G_{12}G_{c21} & 0 \\ 0 & G_{21}G_{c12} + G_{22}G_{c22} \end{bmatrix}$$

$$G_{11}G_{c11} + G_{12}G_{c21} = \frac{s+5}{(s+1)(s+7)} K_1 - \frac{4}{(s+1)(s+7)} \frac{2K_1}{s+3} = \frac{(s+5)(s+3)-8}{(s+1)(s+7)(s+3)} K_1$$

$$G_{11}G_{c11} + G_{12}G_{c21} = \frac{K_1}{s+3}$$

$$G_{21}G_{c12} + G_{22}G_{c22} = -\frac{4}{(s+1)(s+7)} \frac{4K_2}{s+5} + \frac{2(s+3)}{(s+1)(s+7)} K_2 = \frac{2(s+5)(s+3)-16}{(s+1)(s+7)(s+5)} K_2$$

$$G_{21}G_{c12} + G_{22}G_{c22} = \frac{2K_2}{s+5}$$

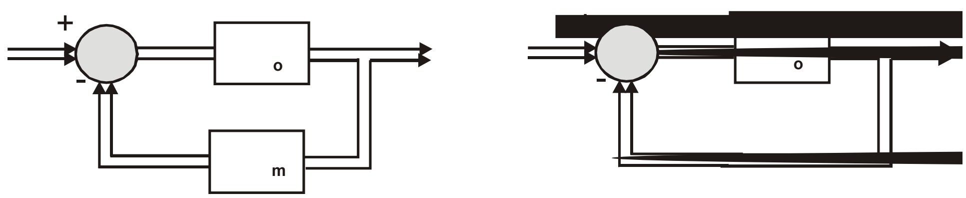
Hence the transfer function for the decoupled system is

$$\mathbf{G_o} = \begin{bmatrix} \dfrac{K_1}{s+3} & 0 \\ 0 & \dfrac{2K_2}{s+5} \end{bmatrix}$$

The measurement matrix $\mathbf{G_m}$ is a unit diagonal matrix.

$$\mathbf{G_m} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

The block diagram for this decoupled system is shown in Figure E-1A. Since $\mathbf{G_m}$ is a unit diagonal matrix, the block diagram is simplified to the unity feedback system of Figure E-1B.



(A) (B)

Figure E-1 Block diagrams for decoupled system.

From Figure E-1 we have

$$\mathbf{C} = \mathbf{G_oE} = \mathbf{G_o(R} - \mathrm{C}) = \mathbf{G_oR} - \mathbf{G_o}\mathrm{C}$$

or

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} G_{o11} & 0 \\ 0 & G_{o22} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} - \begin{bmatrix} G_{o11} & 0 \\ 0 & G_{o22} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

From the above expression we obtain

$$C_1 = G_{o11}R_1 - G_{o11}C_1$$

$$C_2 = G_{o22}R_2 - G_{o11}C_2$$

Solving for $C_1(s)$ gives

$$C_1(s) = \frac{G_{o11}(s)}{1 + G_{o11}(s)}\ R_1(s)$$

Inserting $G_{o11} = \dfrac{K_1}{s+3}$ into the above expression yields

$$C_1(s) = \frac{K_1/(s+3)}{1+K_1/(s+3)}\,R_1(s)$$

Similarly, $$C_2(s) = \frac{G_{o22}(s)}{1+G_{o22}(s)}\,R_2(s),\quad G_{o22} = \frac{2K_2}{s+5},\ \text{and}$$

$$C_2(s) = \frac{2K_2/(s+5)}{1+2K_2/(s+5)}\,R_2(s)$$

The Simulink model for the system with just the primary proportional controller is shown in Figure E-2. The response for a step change in r_1 is shown in Figure E-3 with $G_{c11} = K_1 = 4$ and $G_{c22} = K_2 = 4$. For this case with no cross controllers, a unit step change in r_1 causes both c_1 and c_2 to change.

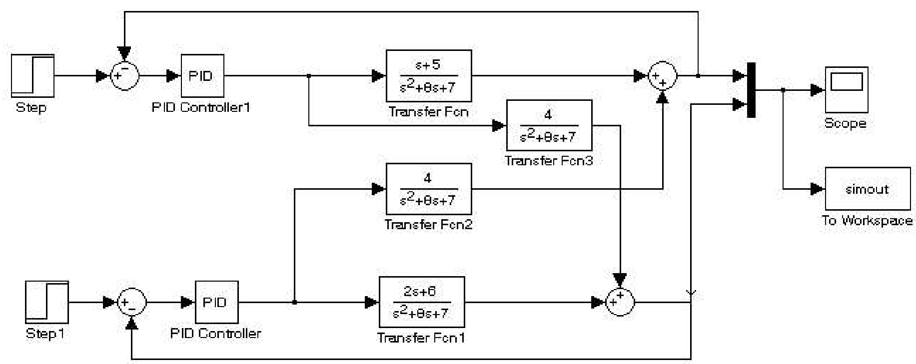


Figure E-2 Simulink model for system with primary controllers.

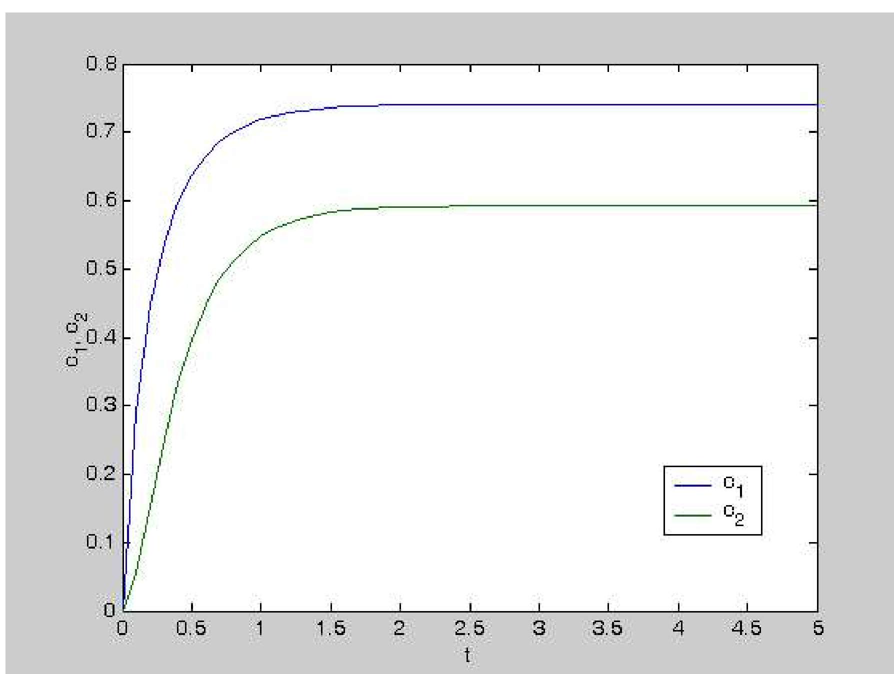


Figure E-3 Response for system with primary controllers.

The Simulink model for the system with both the primary and cross-controllers is shown in Figure E-4. The response for a step change in r_1 is shown in Figure E-5 with $G_{c11} = K_1 = 4$ and $G_{c22} = K_2 = 4$. For this case a change in r_1 does not affect c_2 as predicted by the decoupled system.

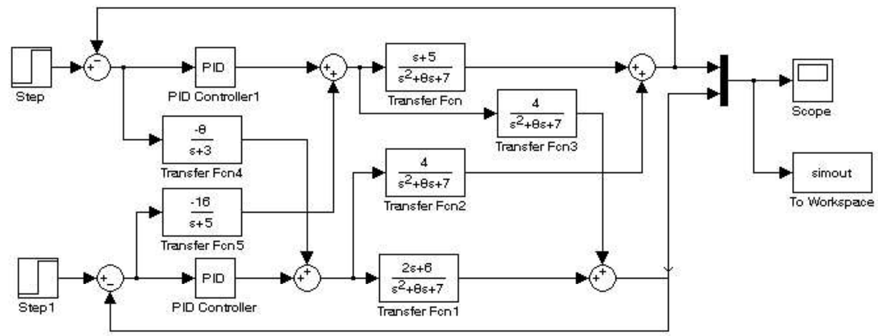


Figure E-4 Simulink model for system with primary and cross controllers.

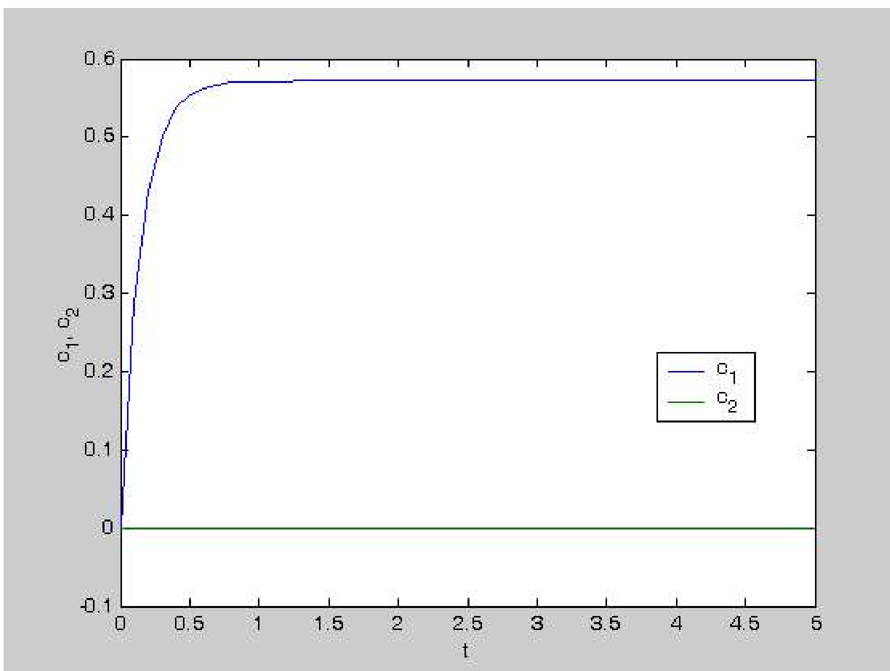


Figure E-5 Response for system with primary and cross controllers.

For the decoupled system we can just use the equation obtained from setting the off-diagonal elements to zero.

$$C_1(s) = \frac{K_1/(s+3)}{1+K_1/(s+3)} R_1(s)$$

The Simulink model for the above equation is shown in Figure E-6 which is equivalent to system with primary and cross controllers. The response for this model, shown in Figure E-7, should be the same as the response obtained from the system with primary and cross controllers.

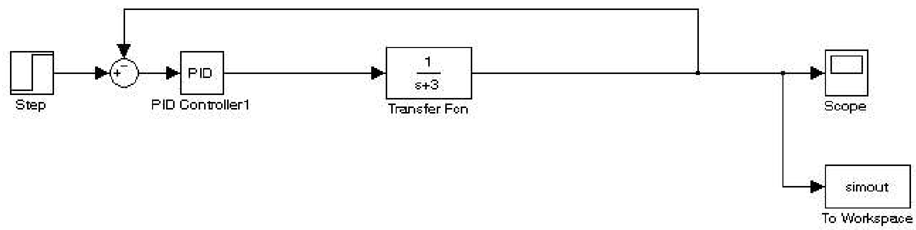


Figure E-6 Simulink model for decoupled system.

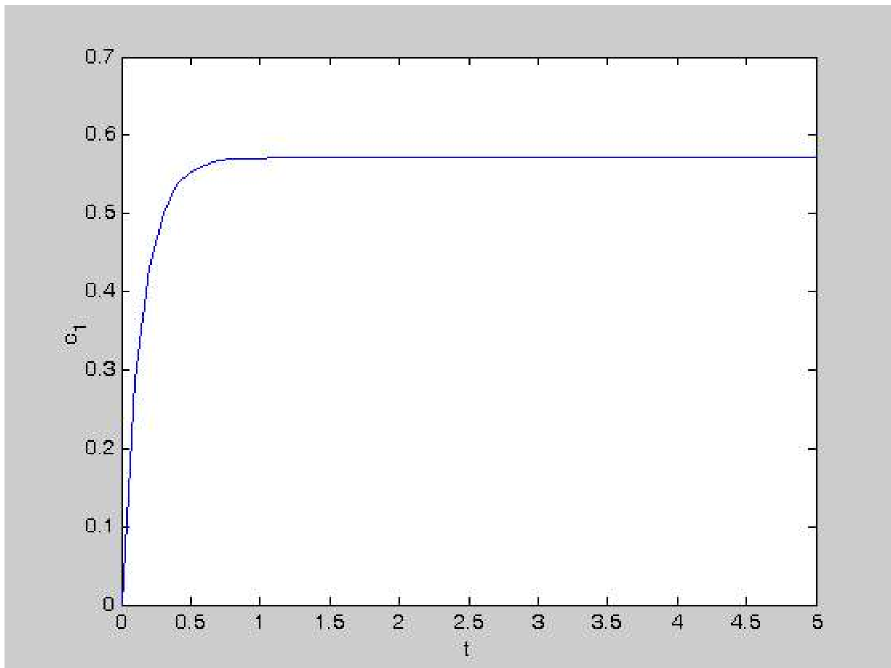


Figure E-7 Response for decoupled system.

The offset can be eliminated with PI controllers.

$$G_{c11} = K_1\frac{s+1}{s} \text{ and } G_{c22} = K_2\frac{s+1}{s}$$

The response for the system with just primary controllers is shown in Figure E-8 for a step change in r_1. The offset is eliminated for c_1.

The cross-controllers can be obtained from the following expressions:

$$G_{11}G_{v1}G_{c12} + G_{12}G_{v2}G_{c22} = 0 \Rightarrow G_{c12} = -\frac{G_{12}G_{v2}G_{c22}}{G_{11}G_{v1}}$$

$$G_{21}G_{v1}G_{c11} + G_{22}G_{v2}G_{c21} = 0 \Rightarrow G_{c21} = -\frac{G_{21}G_{v1}G_{c11}}{G_{22}G_{v2}}$$

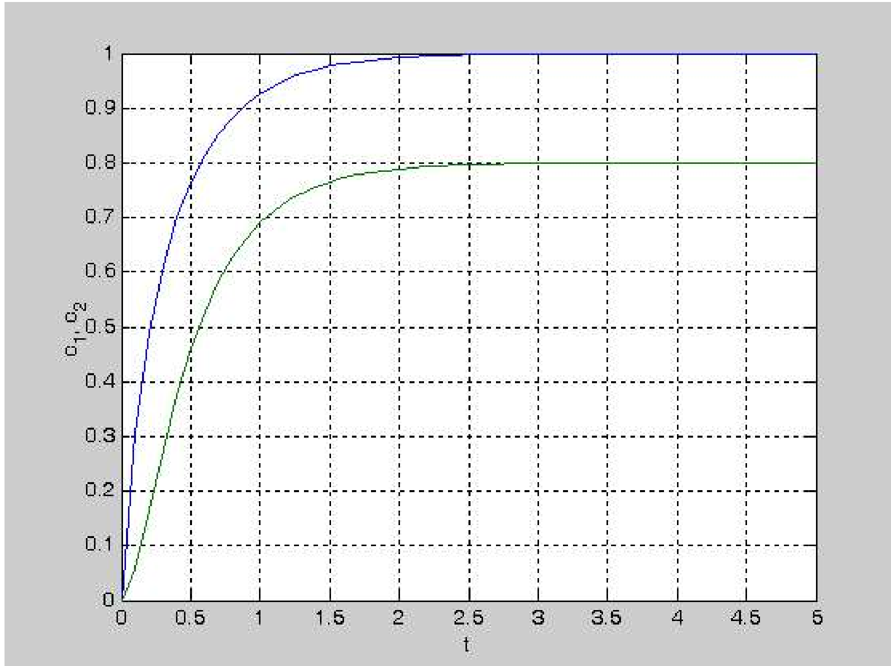


Figure E-8 Response for system with primary PI controllers.

$\mathbf{G_v}$ is a unit diagonal matrix. $G_{v1} = G_{v2} = 1$

$$G_{c12} = -\frac{G_{12}G_{c22}}{G_{11}} = -\frac{4}{(s+1)(s+7)}K_2\frac{s+1}{s}\,\frac{(s+1)(s+7)}{s+5} = -\,\frac{4K_2(s+1)}{s(s+5)}$$

$$G_{c21} = -\frac{G_{21}G_{c11}}{G_{22}} = -\frac{4}{(s+1)(s+7)}K_1\frac{s+1}{s}\,\frac{(s+1)(s+7)}{2(s+3)} = -\,\frac{2K_1(s+1)}{s(s+3)}$$

The Simulink model with cross-controllers is shown in Figure E-9. The response for a step change in r_1 is shown in Figure E-10 with $G_{c11} = K_1\,\frac{s+1}{s} = 4\frac{s+1}{s}$ and $G_{c22} = K_2\,\frac{s+1}{s} = 4\,\frac{s+1}{s}$. For this case a change in r_1 does not affect c_2.

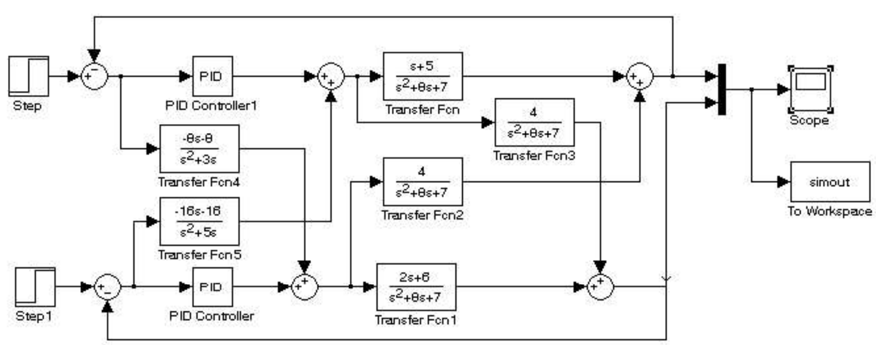


Figure E-9 Simulink model for system with primary and cross PI controllers.

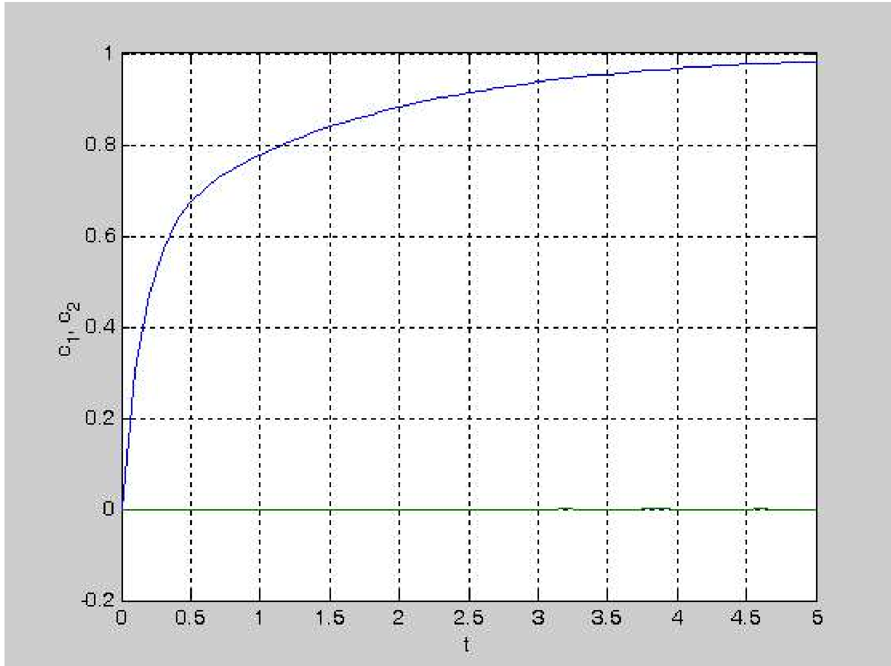


Figure E-10 Response for system with primary and cross PI controllers.

The decoupled model can also be derived from the following matrix $\mathbf{G_o}$

$$\mathbf{G_o} = \begin{bmatrix} G_{11}G_{c11} + G_{12}G_{c21} & 0 \\ 0 & G_{21}G_{c12} + G_{22}G_{c22} \end{bmatrix}$$

$$G_{11}G_{c11} + G_{12}G_{c21} = \frac{s+5}{(s+1)(s+7)} K_1 \frac{s+1}{s} - \frac{4}{(s+1)(s+7)} \frac{2K_1(s+1)}{s(s+3)}$$

$$G_{11}G_{c11} + G_{12}G_{c21} = \frac{(s+5)(s+3)-8}{s(s+7)(s+3)} K_1 = \frac{(s+1)}{s(s+3)} K_1$$

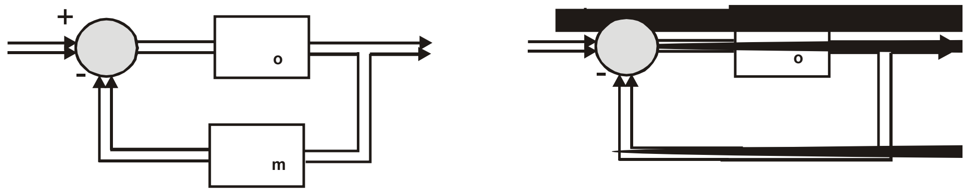
$$G_{21}G_{c12} + G_{22}G_{c22} = -\frac{4}{(s+1)(s+7)} \frac{4K_2(s+1)}{s(s+5)} + \frac{2(s+3)}{(s+1)(s+7)} K_2 \frac{s+1}{s}$$

$$G_{21}G_{c12} + G_{22}G_{c22} = \frac{2(s+5)(s+3)-16}{s(s+7)(s+5)} K_2 = \frac{2(s+1)}{s(s+5)} K_2$$

Hence the transfer function for the decoupled system is

$$\mathbf{G_o} = \begin{bmatrix} \dfrac{(s+1)K_1}{s(s+3)} & 0 \\ 0 & \dfrac{2(s+1)K_2}{s(s+5)} \end{bmatrix}$$

The block diagram for this decoupled system is shown in Figure E-1A. Since $\mathbf{G_m}$ is a unit diagonal matrix, the block diagram is simplified to the unity feedback system of Figure E-1B.



(A) (B)

Figure E-1 Block diagrams for decoupled system.

From Figure E-1 we have

$$\mathbf{C} = \mathbf{G_o E} = \mathbf{G_o(R} - C) = \mathbf{G_o R} - \mathbf{G_o}C$$

or

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} G_{o11} & 0 \\ 0 & G_{o22} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} - \begin{bmatrix} G_{o11} & 0 \\ 0 & G_{o22} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

From the above expression we obtain

$$C_1 = G_{o11}R_1 - G_{o11}C_1 \text{ and } C_2 = G_{o22}R_2 - G_{o11}C_2$$

Solving for $C_1(s)$ gives

$$C_1(s) = \frac{G_{o11}(s)}{1+G_{o11}(s)}\ R_1(s)$$

Inserting $G_{o11} = \dfrac{(s+1)}{s(s+3)}\ K_1$ into the above expression yields

$$C_1(s) = \frac{K_1(s+1)/[s(s+3)]}{1+K_1(s+1)/[s(s+3)]}\ R_1(s)$$

Similarly, $\qquad C_2(s) = \dfrac{G_{o22}(s)}{1+G_{o22}(s)}\ R_2(s)$, $G_{o22} = \dfrac{2(s+1)}{s(s+5)} K_2$, and

$$C_2(s) = \frac{2K_2(s+1)/[s(s+5)]}{1+2K_2(s+1)/[s(s+5)]}\ R_2(s)$$

The decoupled model is shown in Figure E-6 with PID = $4\dfrac{s+1}{s}$

Chapter 7

Programmable Logic Controllers

7.1 Introduction

"A programmable logic controller (PLC) or programmable controller is a digital computer used for automation of electromechanical processes, such as control of machinery on factory assembly lines, amusement rides, or lighting fixtures. PLCs are used in many industries and machines. Unlike general-purpose computers, the PLC is designed for multiple inputs and output arrangements, extended temperature ranges, immunity to electrical noise, and resistance to vibration and impact. Programs to control machine operation are typically stored in battery-backed or non-volatile memory. A PLC is an example of a real time system since output results must be produced in response to input conditions within a bounded time, otherwise unintended operation will result.[1]"

The PLC mainly consists of a CPU, memory areas, and appropriate circuits to receive input/output data. PLC circuitry is divided into three main regions separated by isolation boundaries, shown in Figure 7.1-1. Electrical isolation provides safety, so that a fault in one area does not damage another. Isolation boundaries protect the operator interface and the operator from power input faults or field wiring faults.

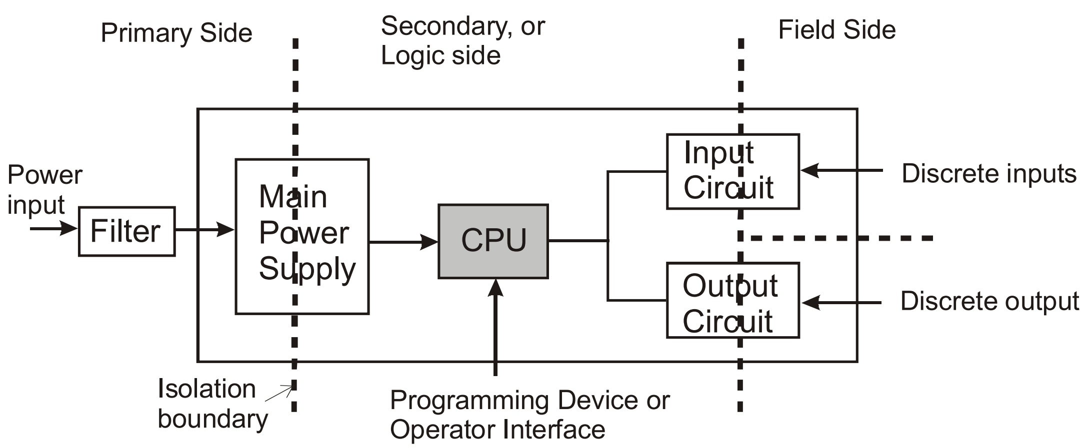


Figure 7.1-1 A PLC's internal configuration and interface

A typical PLC consists of four separate yet interlinked components. These are

1) An input/output section, which connects the PLC to the outside world (device with its sensors, solenoid valves and switches, lamps, heaters and electric motors).
2) A central processing unit (CPU), which is micro-processor-based.
3) A programming device, which may be a hand-held programming console, a special PLC desk-type programmer, similar to a lap-top computer, or a PC with monitor.
4) A power supply (usually 24 V DC) to power input sensors and output signals leading to lamps, motors, heaters and solenoids on the fluid power valves.

[1] http://en.wikipedia.org/wiki/Programmable_logic_controller (2/7/2010)

PLCs operate by monitoring input signals from such sources as push-button switches, proximity sensors, heat sensors, liquid level sensors, limit switches, and pressure and flow rate sensors. "Switches are commonly employed as input devices to indicate the presence or absence of a particular condition in a system or process that is being monitored and/or controlled. In motorized electromechanical systems, limit switches provide the function of making and breaking electrical contacts and consequently electrical circuits. A limit switch is configured to detect when a system's element has moved to a certain position. A system operation is triggered when a limit switch is tripped. Limit switches are widely used in various industrial applications, and they can detect a limit of movement of an article and passage of an article by displacement of an actuating part such as a pivotally supported arm or a linear plunger. The limit switches are designed to control the movement of a mechanical part. Limit switches are typically utilized in industrial control applications to automatically monitor and indicate whether the travel limits of a particular device have been exceeded."[2]

When binary logic changes are detected from the input signals, the PLCs reacts through a user-programmed logic switching network and produces appropriate output signals. These output signals may then be used to operate external loads and switching functions of the attached control system. The programmed "logic" network of the PLCs replaces the previously "hard-wired" network. This logic network may be altered as required by deleting, inserting, or changing certain sections in the PLC's program. These programs can be entered with ladder diagram or graphic logic symbols through a computer with monitor. Manufacturers of programmable logic controllers generally also provide associated ladder logic programming systems. Typically, the ladder logic languages from two manufacturers will not be completely compatible. Even different models of programmable controllers within the same family may have different ladder notation such that programs cannot be seamlessly interchanged between models.

A ladder diagram consists of two vertical lines called bus bars or power rails, and horizontal lines, called rungs or logic lines. The rungs contain one or several normally open contacts, and normally closed contacts, and at the right hand end, immediately before the right hand bus bar, an instruction (or output coil) for action. Figure 7.1-2 shows a normally open contact, a normally closed contact, and a rung indicating when the normally open contacts of both switches close, electricity is able to flow to the motor. The "contacts" may refer to physical inputs to the programmable controller from physical devices such as pushbuttons and limit switches via an integrated or external input module, or may represent the status of internal storage bits which may be generated elsewhere in the program.

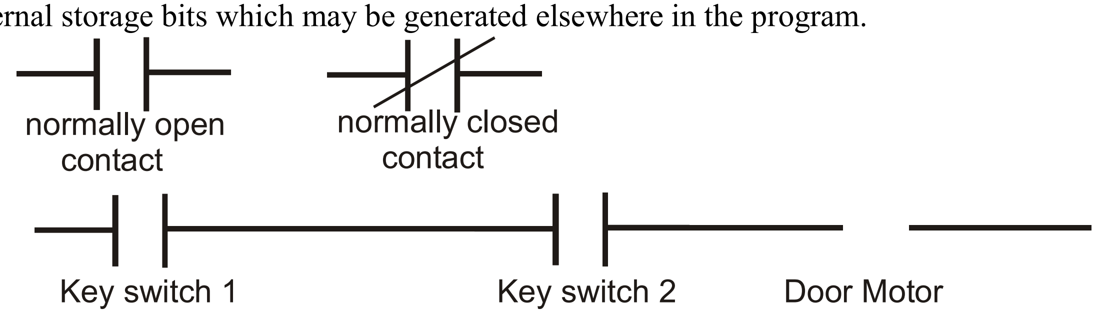


Figure 7.1-2 Ladder diagram circuit.

[2] http://www.electronics-manufacturers.com/products/relays-switches/limit-switch/(2/15/2010)

Each rung of ladder language typically has one coil at the far right. Some manufacturers may allow more than one output coil on a rung.

`--( )--` a regular coil, energized whenever its rung is closed

`--(\)--` a "not" coil, energized whenever its rung is open

`--[ ]--` A regular contact, closed whenever its corresponding coil is energized

`--[\]--` A "not" contact, open whenever its corresponding coil is energized

The "coil" (output of a rung) may represent a physical output which operates some device connected to the programmable controller, or may represent an internal storage bit for use elsewhere in the program.

The logic lines are set of connections between contacts and actuators or coils. If a path can be traced between the left side of the rung and the output, through closed contacts, the rung is true and the output coil storage bit is true. If no path can be traced, then the output is false and the "coil" by analogy to electromechanical relays is considered "de-energized". When logically “true”, the instruction on the right side of the rung is carried out. Figure 7.1-3 shows an example of a ladder logic program to turn on a lamp after a push button has turned on and off 10 times.

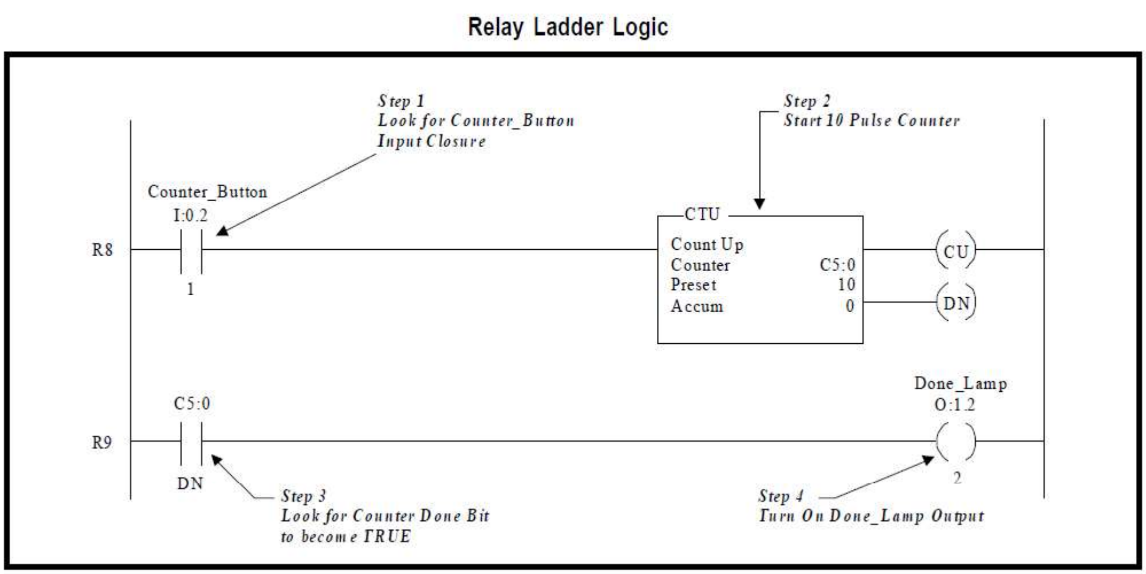


Figure 7.1-3 OptoControl Counter to count the button pushes[3].

[3] http://www.telebaud.com/PDF/rllwp.pdf (2/15/2010)

7.2 A PLC Experiment

We will discuss a simple PLC experiment developed by Dr. Mingheng Li and his students. The materials in this section were provided by Ashley Cheney. In this experiment an electrical heater is turned on whenever the water temperature in a tank falls below a set point temperature T_{set}. The experimental set up consists of a DL05 Micro PLC (Figure 7.2-1), a 20-L water tank with stirrer (Figure 7.2-2), a thermocouple type K, an electrical heater, two relays, and power supplies.

Figure 7.2-1 DL05 Micro PLC

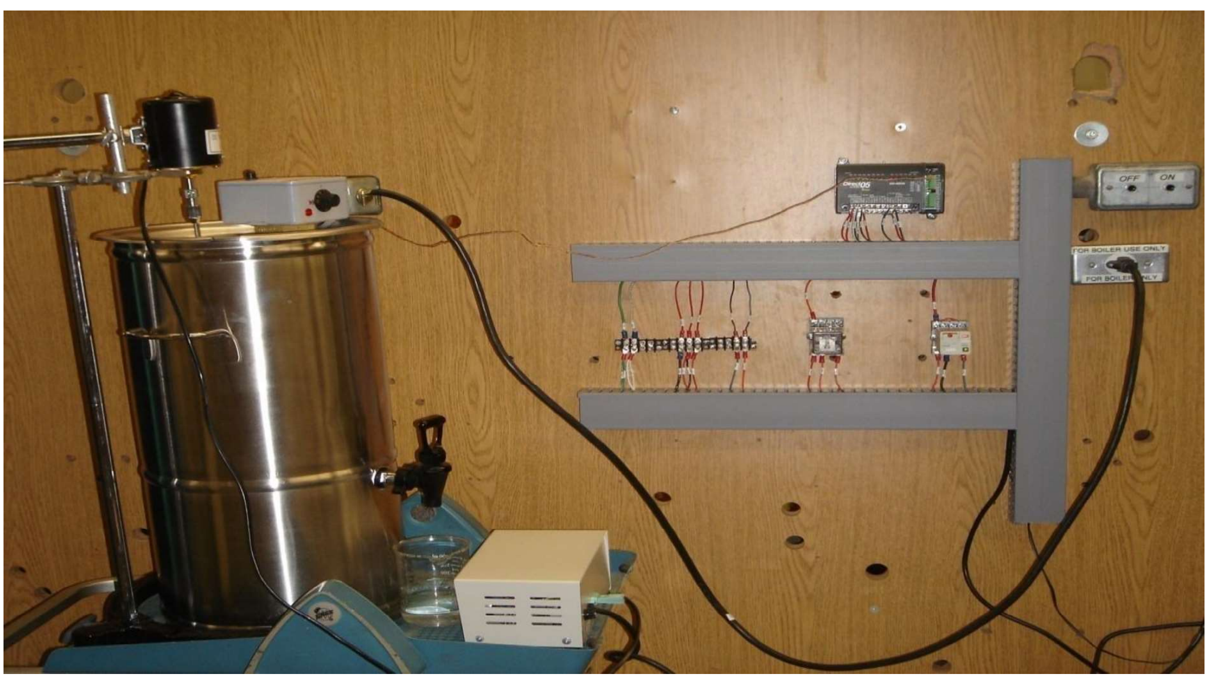


Figure 7.2-2 Water tank heater setup.

In this experiment, relay 1 is used to control the On/Off switch and relay 2 is used to control the electrical heater. When the On switch is pushed, the circuit through relay 1 is closed (line

2-7 in Figure 7.2-3) and remains closed until the Off switch is pushed.

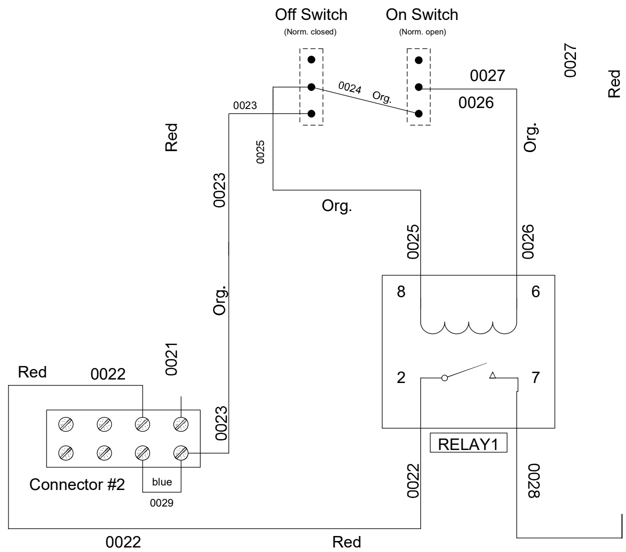


Figure 7.2-3 Relay 1 circuit.

The entire wiring system is shown in Figure 7.2-4[4]. The following description of the diagram was prepared by Ashley Cheney.

"**(6.2) Start/Stop push buttons**

The Start push button (On Switch) is a normally open circuit and is wired in series to the stop button (Off Switch). The circuitry of the start button is wired to the PLC into the X0 input and out to the stop button. The stop button is then essentially connected to the CO common. The purpose of the common connectors is to essentially close the loop and is organized with three relays per common. Notice that the loop is not directly wired to the CO common; it is wired to the AC Live [AC(L)] input connector. The PLC has an external power connection; such that if wired correctly the AC Live [AC(L)] or the AC Neutral [AC(N)] can act as an input or output power connection for an electrically closed loop within a system. In this case, the stop button is wired to the AC(L), which is internally connected to the AC(N). The wiring connection then goes from AC(N) to the input CO common, enforcing an electrically closed loop. Once the start push button is pressed the circuits for both the switch and the relay 1 are closed, which energizes Relay 1 (refer to section 6.3). Immediately after the start button is released the circuit for the On/Off switch opens. The stop push button is normally closed; such that once the stop button is pressed the circuit opens and breaks the entire loop (circuits for both the switch and the relay 1) to shut down the system.

(6.3) Relay 1

Relay 1 is wired in parallel to the start push button and is normally open. The relay consists of two circuits; one is a magnetic coil and the other is high power driver. Once the start button is pressed (the loop is closed) the magnetic coil in Relay 1 is energized and the high power driver closes. Once the high power driver is closed the entire Relay is continuously energized and wired to the PLC into AC Live [AC(L)] and to the output Y0 where Y0 is energized. Notice, again that AC(L) is used as an input connector, which is (as previously mentioned) internally connected to AC(N). The loop is then wired from AC(N) to

[4] Ashley Cheney, Senior Project, Cal Poly Pomona 2010

the output common of YO which is noted as C2. When the relay is energized its purpose is to create a circuit loop that is normally closed, so after the start button is released the circuitry does not break and the system is still able to run without the need of holding down the start push switch. When the output Y0 is energized Y1 becomes energized, which is wired to Relay 2 (refer to section 6.4). The common C2 is wired to the AC neutral [AC(N)] to close the relay loop for both outputs Y0 and Y1."[5]

"**(6.4) Relay 2**

Relay 2 also consists of two circuits; one is a magnetic coil and the other is high power driver. The magnetic coil is wired to the PLC through AC live [AC(L)] and the output Y1. Y1 is energized when Y0 is energized from Relay 1 (refer to section 6.3). When Y1 is energized the magnetic coil closes the high power driver which controls the heating element. Relay 2 is normally opened, but when energized the loop closes and turns on the heating element. The high power driver in Relay 2 is wired to the AC live [AC(L)] output on the PLC and to the live power supply adapter of the heating element (refer to section 6.5).

(6.5) Heating Element

The heating element is energized through Relay 2 (refer to section 6.4). The PLC is programmed in such a way for the heating element to turn on and off within a certain temperature range specified by the operator. If the temperature is equal to or higher than the maximum set point, relay 2 opens and breaks the loop to shut off the heater. If the temperature is equal to or lower than the minimum set point, relay 2 closes and completes the loop to turn the heater back on.

(6.6) Four-channel Thermal Module (F0-04THM)

The Four-channel Thermal Module is wired with a type K thermocouple to Ch1. The thermocouple monitors the temperature of the water in the 20 L tank. A thermocouple is two metals welded together at the tip junction that provide a mA voltage proportional to temperature and is detected and translated into a temperature reading per a calibration curve in the PLC. There are a variety of different types of thermocouples. To indicate the specific type of thermocouple is by color code (refer to the document types of thermocouples). For the type of thermocouple used for this system is a type K, as mentioned, which is color coded in yellow for positive and red for negative. Another approach was executed within the system to measure the temperature, not only through direct heating, but also through indirect heating. (Refer to the Overview Process System document) Two controlled systems were monitored or "Temperature Controlled"."[5]

[5] Ashley Cheney, Senior Project, Cal Poly Pomona 2010.

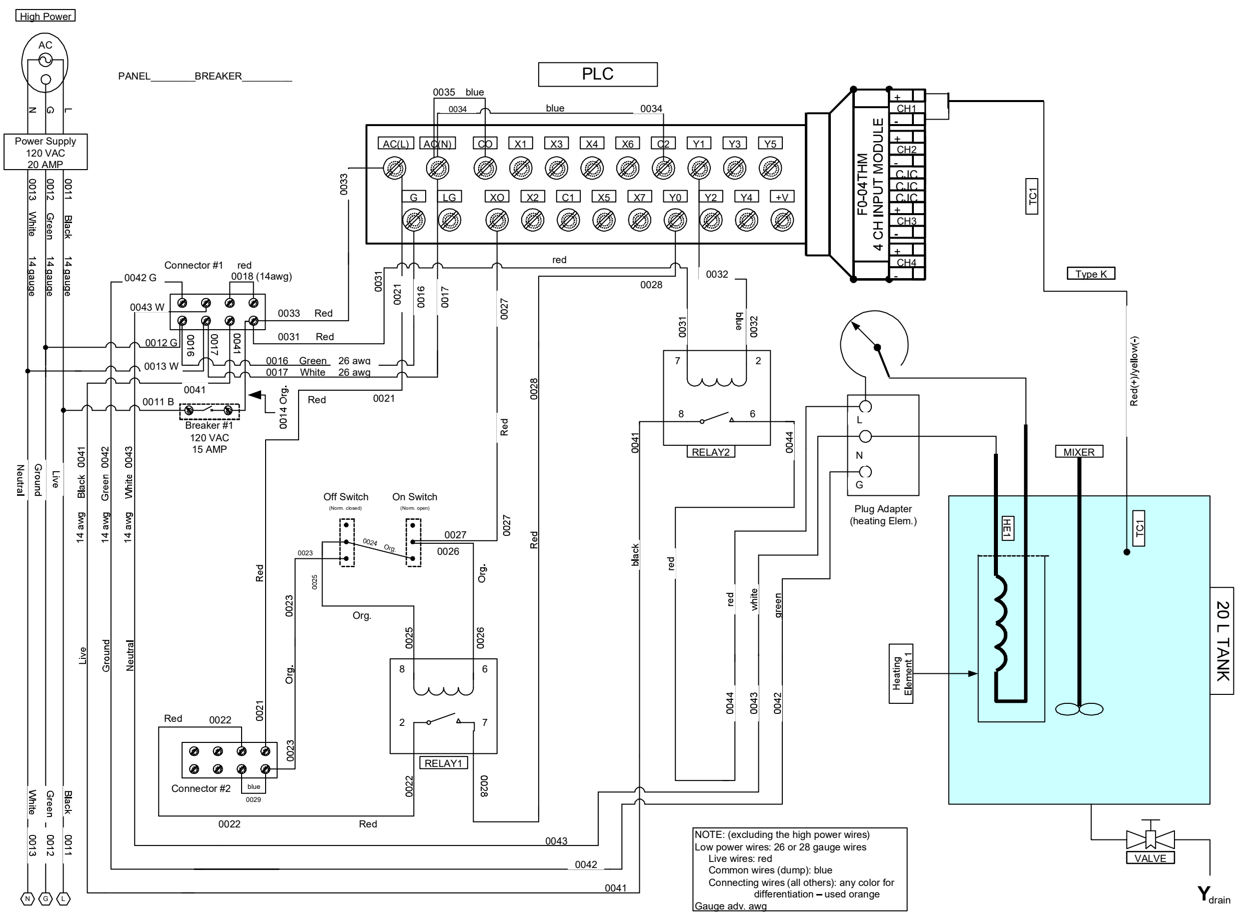


Figure 7.2-4 Wiring diagram

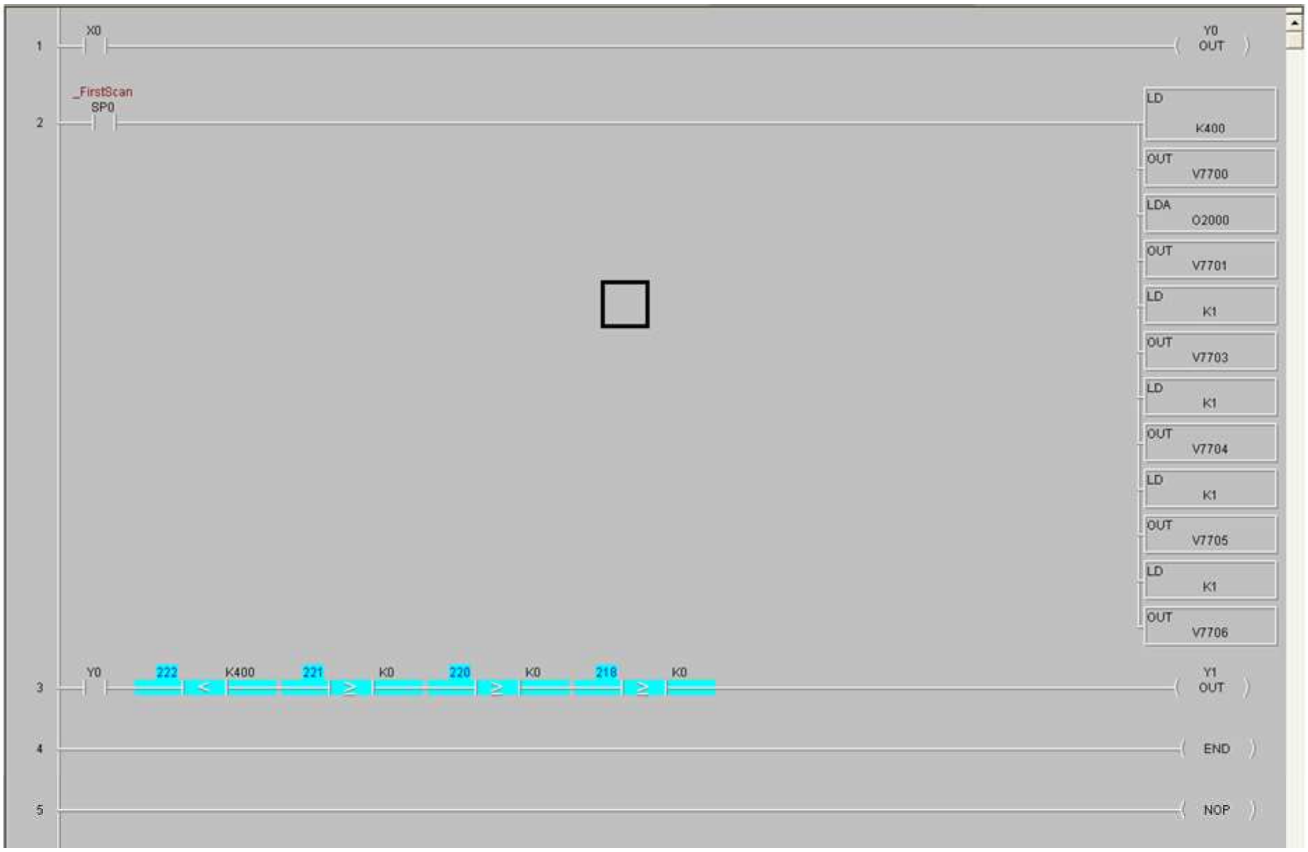


Figure 7.2-5 Ladder program for the water tank system.

Figure 7.2-5 shows the ladder program to turn on the electrical heater when the water temperature is below the set point. The following description of the program was prepared by Ashley Cheney.

"The graphical language used for this system is known as ladder logic. Ladder logic is the most commonly used language for the programming of PLCs (Programmable Logic Controllers). Specifically, ladder logic, also referred to as a ladder diagram, is similar to a schematic for a set of relay circuits. The name is based on the observation that when using this language for programming the layout resembles ladders, with two vertical "rails" and a series of horizontal "rungs," or blocks, between them. In reference to the print out of the ladder logic interface page within this section of the packet, one will notice that it follows the bases of ladders. The ladder logic is programmed using the computer software, DirectSoft32, and is designed to turn on/off the heater in real time when the temperature reading of the water in the tank is below/above the set point. In other words the program controls the setup of the thermocouple module, F0-04THM. Each block or address that is tagged onto the ladder has a significant meaning to the program of the control system. The remaining document outlines each address that is associated and implicated into the program of the system.

- When the start push button, located on the system, is pressed the XO input is energized. In reference to the interface page, the XO input is addressed on line (1). Running down line (1) also has a connection to the output YO. This means that once the input XO is energized the output YO will also become energized. The YO output is connected to Relay 1 (in reference to the wiring diagram), so when YO is energized, Relay 1 is also energized, which it's purpose is to keep the system on without having to continuously hold down the push start button (further explanation is under the *electrical operations* of this manual).

- To base this program to follow the philosophy of the ladder logic, line (2) will carry out the following steps for the programming of the control process. Line (2) starts with an initialization address noted as "_FirstScan SPO". This address is promoted for initializing the program to process. Each block that is attached to line (2) carries out a significant implication that is executed throughout the performance of the system. Line (2) is only ran through once during each execution of the program. The following sub points layout the significance of each block on line (2):
  - **LD K400**:
    This address loads (LD) a constant (K) that specifies the number of input channels to scan and the data format. The setup of the F0-04THM thermocouple module has 4 input channels; the 400 address specifies that there are 4 channels available in the system.

  - **OUT V7700**
    This address outputs (OUT) a special V-memory location assigned to option slot 1 (address described above) that specifies the data format and the number of channels to scan. In this case, the address 7700 inquires that all 4 channels will be scanned and will follow the data parameters designated in this program. (Regardless of how many channels are actually being used within the system; the program will account for all 4).

  - **LDA O2000**
    This address loads (LDA-load address) an octal value for the first V-memory location that will be used to store the incoming data. For instance, O2000 entered, under this slot, using the LDA instruction would designate the following addresses: Ch1 – V2000/2001, Ch2 – V2002/2003, Ch3 – V2004/2005, Ch4 – V2006/2007. In other words, this address specifies that all the 4 channels will be placed in a V-memory location within the program, whether or not all for 4 channels are activated.

  - **OUT V7701**
    The octal address (O2000) is stored here. Special V-memory location V7701 is assigned to the option slot and acts as a pointer, which means the CPU will use the octal value in this location to determine exactly where to store the incoming data. These specific V-memory slots are registered within the program to process the input data from each active channel of the thermal module to follow a specific basis of parameters.

  - **LD K1**
    This address loads (LD) a constant that signifies the input type. K1 selects K type thermocouple with CJC (cold junction compensation) enabled.
    - The CJC is calibrated to operate in a still air environment. If the module is used in an application that has forced convection cooling, an error of 2-3 °C may be introduced. CJC is used to compensate for this error. The particular PLC model used for this control system (DL05) the CJC is automatically implemented and enabled.

  - **OUT V7703**
    This address outputs a special V-memory location assigned to the option slot (the address described above) that specifies the thermocouple input type or voltage range selection. CJC is disabled when voltage is selected; not applied to this system. One will

notice that LD K1 is an address for 4 of the blocks in the ladder of line 2. Each LD input address requires a V-memory location to output the specific parameter. Each V-memory location signifies a different parameter. So LD K1 placed in the output OUT V7703 (type of thermocouple) signifies different parameter than the LD K1 address placed in the output OUT V7704 (scale units), which will be clarified. The input type (K1) just happens to be the same input selection value in all input parameters, which makes sense because this system is only using a type K thermocouple; this can be altered with the necessary modifications within the ladder code.

- **LD K1**
  This address loads a constant that specifies the Units Code (temperature scale and data format). K1 selects °C and magnitude + sign bit data format.

- **OUT V7704**
  This address loads a special V-memory location assigned to the option slot (the address described above) that specifies the temperature scale and data format selections.

- **LD K1**
  This address loads a constant that enables/disables the thermocouple burnout detection function. K1 selects burnout function disabled.
    - A Burnout is referred as a sensor failure indication. Thermocouples can fail by a break in the junction or extension wire opening up. This burnout function detection alerts the operator or process computer that a sensor problem exists and where/what to look for.

- **OUT V7705**
  This address loads a special V-memory location assigned to the option slot that specifies the thermocouple burnout detection enable/disable.

- **LD K1**
  Loads a constant that specifies the thermocouple burnout data value at burnout. K1 specifies a down scale (lower than normal output) value of 0000h to be written to the channel input register when a thermocouple burnout occurs.

- **OUT V7706**
  This address loads a special V-memory location assigned to the option slot (the address described above) that specifies the thermocouple up scale/down scale burnout value. The value is written to the channel input register when a thermocouple burnout occurs. For this system, we have addressed the burnout detection to be disabled it doesn't matter, in this case, whether to specify an upscale or downscale burnout value.

- Line (3) first starts with the output Y0. X0 energizes Y0 (specified in the description of line (1) mentioned previously) than output Y1 will ALSO be energized. Notice, running down line (3) Y0 also has a connection to the output Y1. Also, one will notice that there is a data argument between the outputs Y0 and Y1. This argument signifies that if the temperature value of channel 1 thermocouple (V2000 – refer to the address LDA 02000 described above) is less than 25 °C than Y1 will be energized. In reference to the wiring diagram, Y1 is an output for the heater. So if Y1 is energized then heater will turn on and the temperature will obviously increase. If the temperature value of channel 1 thermocouple becomes greater than 25 °C then the output Y1 will **not** be energized and the heater will turn off.

- Line (4) and (5) mark the end of the ladder program

****Note****

All this information was pulled from the document under this section of the turn over package, titled, "Ch15 F0-04THM 4-Channel Thermocouple Input." Further information is mentioned within that document, which describes and lists all various input selections and different data formats of all the different parameters that can be applied to the thermocouple module. This document can also be retrieved from this link:

http://www.automationdirect.com/static/manuals/d0optionsm/ch15.pdf

Appendix A

Previous Exams

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____
LAST NAME, FIRST

Quiz #1

Note: Your answers must be correct to 3 significant figures and have the appropriate units.

I. $$\frac{0.8}{s\left(s^2+2.8s+1\right)} = \frac{\_\_}{\_\_} + \frac{B}{s+r_1} + \frac{C}{s+r_2}$$

In this equation, $r_1 > r_2$

(1) $A =$ ____________

(2) $B =$ ____________

II. Given $Y(s) = \dfrac{k_1}{(s+k_1)(s+k_2)}$. Determine

(3) $y(t) =$

(4) $y(t \to \infty) =$

____

III. (5) Two consecutive, first order reactions take place in a perfectly mixed, isothermal continuous reactor (CSTR).

$$\mathbf{A} \xrightarrow{k_1} \mathbf{B} \xrightarrow{k_2} \mathbf{C}$$

Volumetric flow rates (F) and density are constant. The reactor operates at steady state. The inlet stream to the reactor contains only A. The tank volume that maximized the concentration of component B in the product stream is given by

A) $\dfrac{F}{k_1 + k_2}$ **B)** $\dfrac{F}{k_1 k_2}$ **C)** $\dfrac{F}{(k_1 k_2)^{0.5}}$ **D)** $\dfrac{F}{(k_1 k_2)^{2}}$ **E)** None of the above

IV. A tank containing 3.8 m^3 of 20% (by volume) NaOH solution is to be purged by adding pure water at a rate of 4.5 m^3/h. If the solution leaves the tank at a rate of 4.5 m^3/h, determine the time necessary to purge 80% of the NaOH by mass from the tank. Assume perfect mixing. Specific gravity of pure NaOH is 1.22.

(6) t(h) = ________

V. (7) Find the Laplace transform of $e^{-3t}\cos 2t$

____

VI. (8) The inverse of $\hat{F}(s) = \dfrac{s+1}{s^2 - 6s + 13}$ is

A) $0.5e^{3t}\cos 2t - 2e^{3t}\sin 2t$

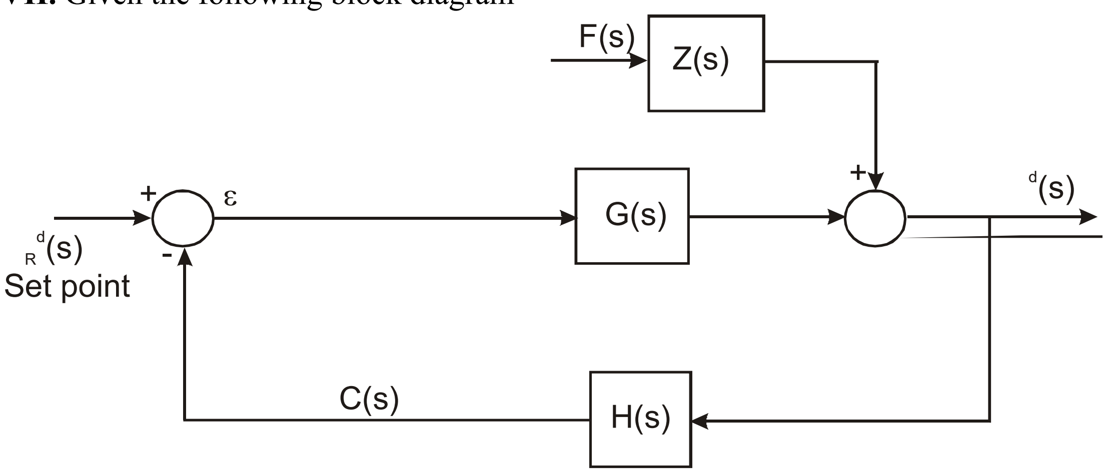
B) $e^{3t}\cos 2t + 2e^{3t}\sin 2t$

C) $0.5e^{3t}\cos 2t + e^{3t}\sin 2t$

D) $0.5e^{3t}\cos 2t + 2e^{3t}\sin 2t$

E) None of the above

VII. Given the following block diagram



9) $\dfrac{T^{d}(s)}{T_{R}^{d}(s)} =$

10) $\dfrac{T^{d}(s)}{F(s)} =$

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Quiz #2

Note: Your answers must be correct to 3 significant figures and have the appropriate units.

I. The temperature of a CSTR is controlled by an electronic (4 to 20 mA) feedback control system containing (1) a 50 to 300°F temperature transmitter, (2) a PI controller with integral time set at 4 minutes and proportional band at 25, and (3) a control valve with linear trim, air-to-open action, and C_v = 20 gpm/psi$^{0.5}$ through which cooling water flows. The pressure drop across the valve is a constant 20 psi.

1) If the steady state controller output, *CO*, is 10 mA, how much cooling water is going through the valve?

____

2) If a sudden disturbance increases reactor temperature by 10°F, what will be the immediate change on the controller output?

ΔCO = __________

II. (3) Find the Laplace transform of $e^{-3t}\sin 2t$

____

III. (4) A thermometer having a time constant of 1 min is initially at 50°C. It is immersed in a bath maintained at 100°C at $t = 0$. Determine the temperature reading at $t = 1.2$ min.

____

IV. **(5)** A thermometer having a time constant of 0.2 min is placed in a temperature bath, and after the thermometer comes to equilibrium with the bath, the temperature of the bath is increased linearly with time at a rate of 1°C/min. The difference between the indicated temperature and the bath temperature can be obtained from the following equation

A) $Y(s) = \frac{1}{s}\frac{0.2}{(s+1)}$

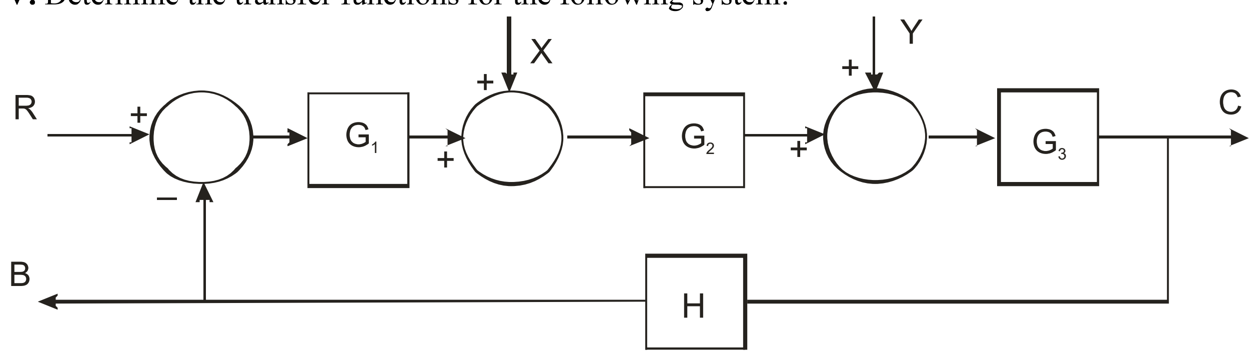
B) $Y(s) = \frac{1}{s}\frac{1}{(0.2s+1)}$

C) $Y(s) = \frac{1}{s^2}\frac{0.2}{(s+1)}$

D) $Y(s) = \frac{1}{s^2}\frac{1}{(0.2s+1)}$

E) None of the above

V. Determine the transfer functions for the following system:



(6) — =

(7) — =

VI. (8) There are 3460 pounds of water in the jacket of a reactor that are initially at 145°F. At time equal zero, 70°F cooling water is added to the jacket at a constant rate of 400 pounds per minute. The holdup of water in the jacket is constant since the jacket is completely filled with water and excess water is removed from the system on pressure control as cold water is added. Water in the jacket is perfectly mixed. How many minutes does it take the jacket to reach 99°F if no heat is transferred into the jacket?

____

9) You enter a cold room in a house and adjust a simple thermostat to heat the room to a more comfortable level. A simple thermostat is an on-off switch which is in the "on" position if the room temperature is below the desired setting and is in the "off" position otherwise. **If you want the room temperature to increase quickly, should you set the thermostat setting to the desired temperature or set it much higher than the desired temperature?**

____

A) All the way up
B) Set to desired temperature
C) Either setting will heat the room at the same rate
D) Can't determine from the information given

10) Your answer to Question 9 is correct because __________

A) A higher setting will produce hotter air in the furnace which will heat the house faster
B) Heat transfer is proportional to temperature difference so a higher setting will heat the house faster
C) The furnace heats at the same rate as long as the desired room temperature hasn't been reached yet
D) A higher setting will move air through the furnace at a faster rate which will heat the house faster
E) Can't determine unless heating rate of the furnace is known
F) The furnace is designed to most efficiently heat a home if the thermostat is set to the desired temperature

Answers to Quizzes

Quiz #1

I. (1) $A = 0.8$ **(2)** $B = 0.1715$

II. (3) $y(t) = \dfrac{k_1}{k_2 - k_1}[\exp(-k_1 t) - \exp(-k_2 t)]$ **(4)** $y(t \to \infty) = 0$

III. (5) C) $\dfrac{F}{(k_1 k_2)^{0.5}}$

IV. (6) t **= 1.36 h**

V. (7) $\mathscr{L}\{e^{-3t}\cos 2t\} = \dfrac{s+3}{(s+3)^2+4}$

VI. (8) $f(t) = e^{3t}\cos 2t + 2e^{3t}\sin 2t$

VII. 9) $\dfrac{T^{\mathrm{d}}(s)}{T_{\mathrm{R}}{}^{\mathrm{d}}(s)} = \dfrac{G(s)}{1+H(s)G(s)}$ **10)** $\dfrac{T^{\mathrm{d}}(s)}{F(s)} = \dfrac{Z(s)}{1+H(s)G(s)}$

Quiz #2

I. 1) F **= 33.54 gpm 2)** ΔCO **= 2.56 mA**

II. (3) $\mathscr{L}\{e^{-3t}\sin 2t\} = \dfrac{2}{(s+3)^2+4}$

III. (4) $y(1.2)$ **= 84.94°C**

IV. (5) D

V. (6) $\dfrac{\quad}{\quad} = \dfrac{G_1 G_2 G_3}{1 + HG_1 G_2 G_3}$ **(7)** $\dfrac{\quad}{\quad} = \dfrac{HG_3}{1 + HG_1 G_2 G_3}$

VI. (8) 8.22 min

VII. 9) C 10) C

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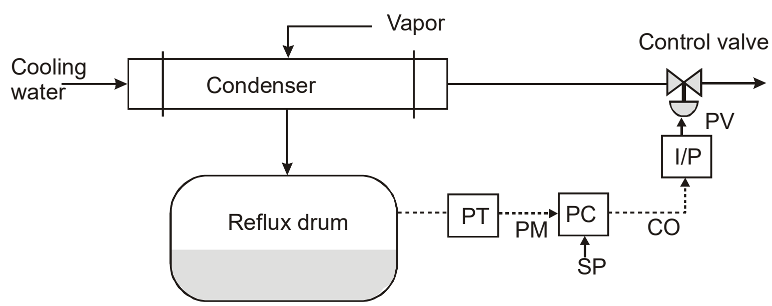
Quiz #3

Note: Your answers must be correct to 3 significant figures and have the appropriate units.

I.[1] The overhead vapor from a depropanizer distillation column is totally condensed in a water-cooled condenser at 120°F and 230 psig. The vapor is 98 mol % propane and 2 mol % isobutene. The vapor design flow rate is 30,000 lb/h and average latent heat of vaporization is 128 Btu/lb. Cooling water inlet and outlet temperatures are 75 and 100°F, respectively. The condenser heat transfer area is 1000 ft^2. The cooling water pressure drop through the condenser at design rate is 50 psi. A linear-trim control valve (air-to-closed) is installed in the cooling water line. The pressure drop over the valve is 25 psi at design with the valve half open. The process pressure is measured by an electronic (4-20 mA) pressure transmitter whose range is 150-350 psig. An analog electronic proportional controller with a gain of 4 is used to control process pressure by manipulating cooling water flow. The electronic signal from the controller (CO) is converted into a pneumatic signal in the I/P transducer.



1) Calculate the cooling water flow rate (gpm) at design conditions. Water density is 62.3 lb/ft^3 and 1 ft^3 = 7.48 gal

____

307 gpm

2) If the cooling water flow rate is 300 gpm at design conditions, calculate the size coefficient (C_v) of the control valve.

C_v = __________

C_v = 120 gpm/psi$^{0.5}$

(3) Calculate the value of the signal PM at design condition ____________

PM = 10.4 mA

(4) Calculate the value of the signal PV at design conditions ____________

CO = **9 psig**

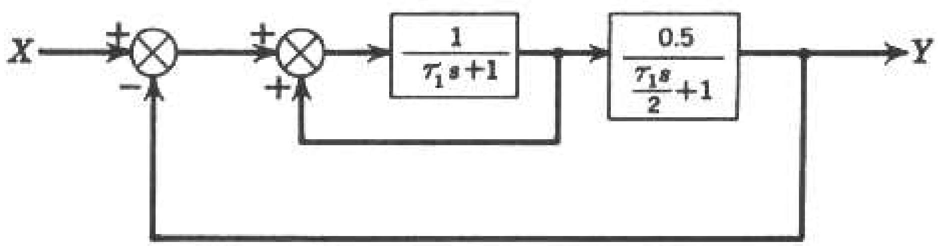
(5) Suppose the process pressure jumps 20 psi, determine value for ΔCO ____________

ΔCO = **– 6.4 mA**

II. (6) There are 3460 pounds of water in the jacket of a reactor that are initially at 145°F. At time equal zero, 70°F cooling water is added to the jacket at a constant rate of 416 pounds per minute. The holdup of water in the jacket is constant since the jacket is completely filled with water and excess water is removed from the system on pressure control as cold water is added. Water in the jacket is perfectly mixed. How many minutes does it take the jacket to reach 99°F if a constant 362,000 Btu/h of heat is transferred into the jacket from the reactor, starting at time equal to zero when the jacket is at 145°F? $C_{p.water}$ = 1.0 Btu/lb· °F

11.88 min

III. (7) The transfer functions for this system is: __________



a) $Y(s)/X(s) = \dfrac{0.5}{(\tau_1 s+1)^2}$

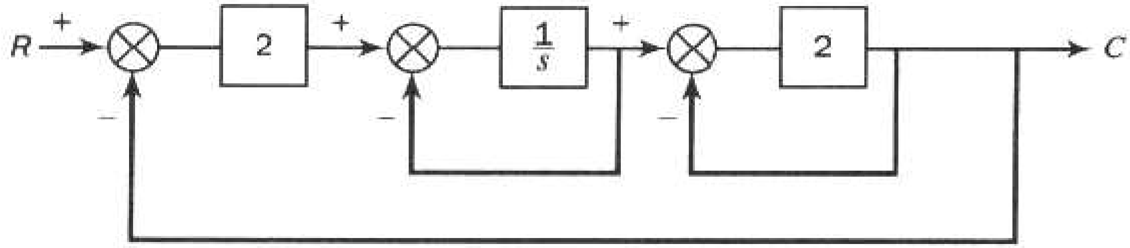
b) $Y(s)/X(s) = \dfrac{0.5}{\tau_1 s(\tau_1 s+1)}$

c) $Y(s)/X(s) = \dfrac{1}{(\tau_1 s+1)^2}$ **Ans**

d) $Y(s)/X(s) = \dfrac{1}{\tau_1 s(\tau_1 s+1)}$

e) None of the above

IV. (8) The transfer functions for this system at s = 1 is: ________



$C(s)/R(s)$ = **0.4**

V. (9) Find the inverse of $\hat{F}(s) = \dfrac{s+6}{s^2 - 6s + 18}$

$f(t) = e^{3t}\cos 3t + 3e^{3t}\sin 3t$

VI. (10)^2 A pneumatic PI temperature controller has an output pressure of 10 psig when the set point and process temperature coincide. The set point is suddenly increased by 15°F (i.e. a step change in error is introduced), and the following data are obtained:

Time, s	0-	0+	20	60	90
psig	10	8	7	5	3.5

Determine the actual gain (psig/°F)

K_c = **– 0.1333 psig/°F**

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Quiz #4

Note: Your answers must be correct to 3 significant figures and have the appropriate units.

I.[1] A circulating chilled-water system is used to cool an oil stream from 90 to 70°F in a tube-in-shell heat exchanger shown in Figure 1 (next page). The temperature of the chilled water entering the process heat exchanger is maintained constant at 50°F by pumping the chilled water through a cooler located upstream of the process heat exchanger.

The design chilled-water for normal conditions is 1000 gpm, with chilled water leaving the process heat exchanger at 60°F. Chilled-water pressure drop through the process heat exchanger is 15 psi at 1000 gpm. Chilled-water pressure drop through the refrigerated cooler is 15 psi at 1000 gpm.

The temperature transmitter on the process oil stream leaving the heat exchanger has a range of 40-200°F. The range of the orifice-differential pressure flow transmitter on the chilled water is 0-2000 gpm. All instrumentation is electronic (4 to 20 mA). Assume the chilled-water pump is centrifugal with a flat pump curve (total pressure drop across the system is constant).

1) Assuming linear trim determine C_v for the chilled-water control valve that is 40 percent open at the 1000 gpm design rate and has a maximum flow of 2000 gpm. **158.1 gpm/psi$^{0.5}$**

2) If C_v = 200 gpm/psi$^{0.5}$, the total pressure drop through the system is ΔP_T = **186.25 psi**

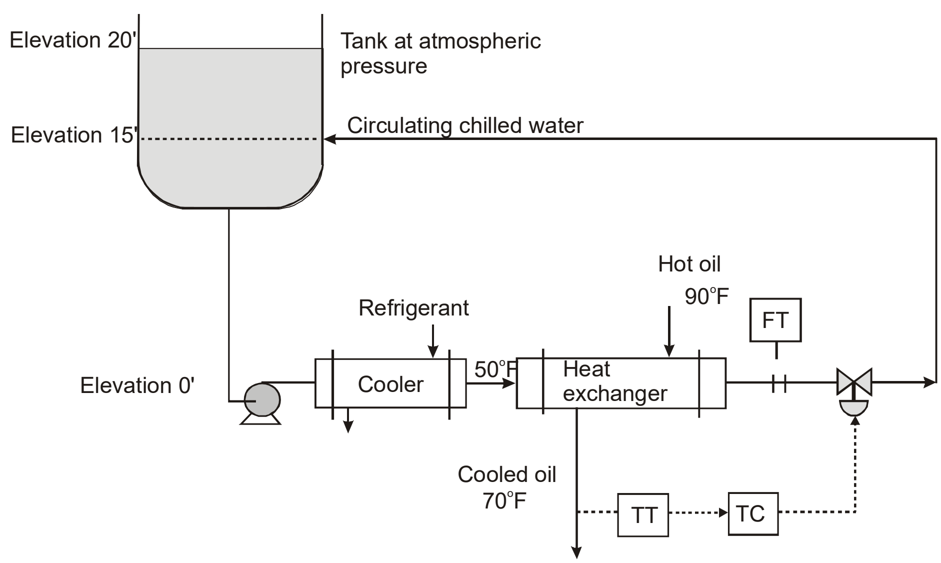


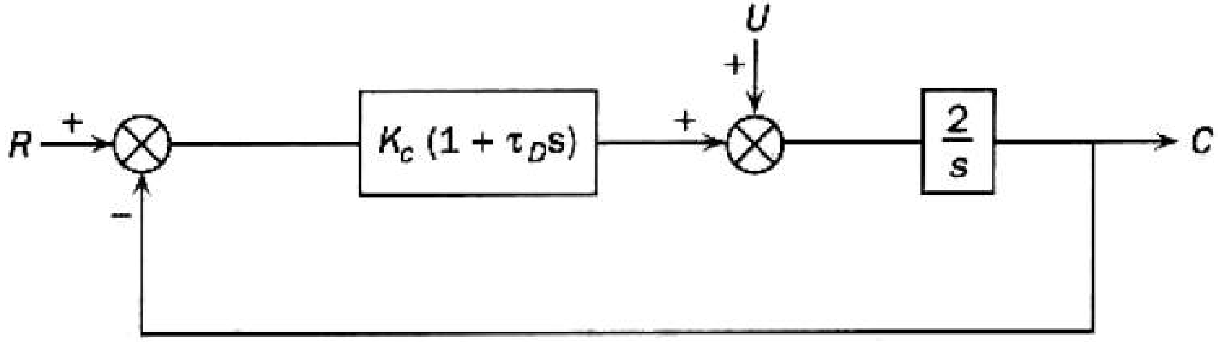
Figure 1 A circulating chilled-water system

3) The value of the signal from the temperature transmitter is PM_T = **7.0 mA**

4) The value of the signal from the flow transmitter is PM_F = **8 mA**

5) If C_v = 200 gpm/psi$^{0.5}$ and the total pressure drop through the system is 186.25 psi, determine the fraction open of the chilled-water control valve when the chilled-water flow rate is reduced to 500 gpm. $f_{(x)}$ = **0.1870**

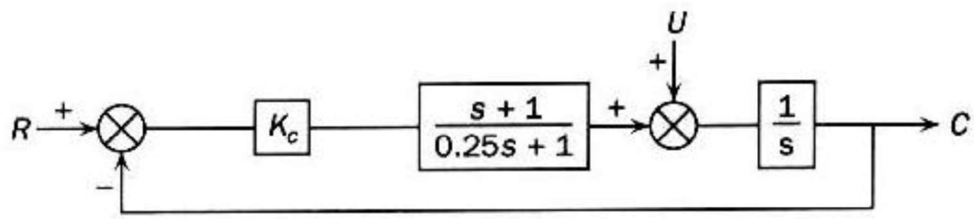
II. Consider the following control system with $U(s) = 1/s$, $K_c = 2$ and $\tau_D = 1$.



6) Determine $C(t = 1.25)$: $C(t = 1.25) = 0.3161$

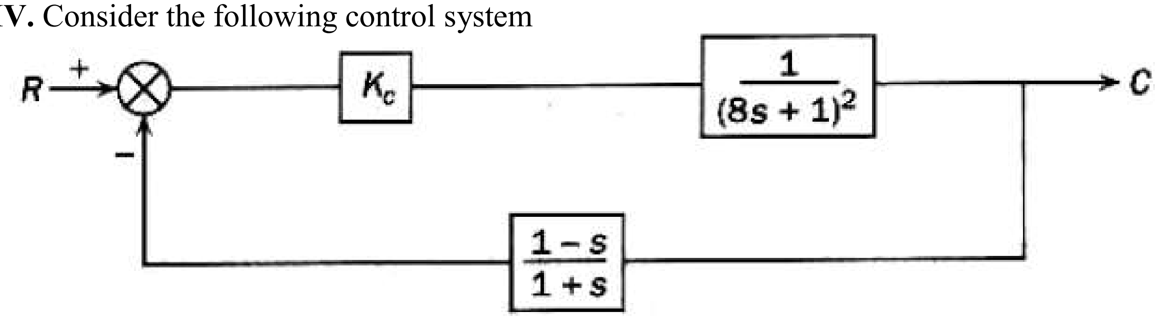
7) The offset is $C(\infty) - R(\infty)$ = **0.5**

III. Consider the following control system with $U(s) = 4/s$, $K_c = 2$



(8) The offset is $C(\infty) - R(\infty)$ = **2**

IV. Consider the following control system



(9) The ultimate gain is K_{cu} = **9**

(10) The ultimate frequency is ω_u = **0.3536 rad/s**

Appendix B

Previous Exams

CHE 426 (Fall 2011) ____________________

Quiz #1

Note: Your answers must be correct to 3 significant figures and have the appropriate units.

I. $\dfrac{5}{s\left(s^2+4s+3\right)} = \dfrac{A}{s} + \dfrac{B}{s+r_1} + \dfrac{C}{s+r_2}$

In this equation, $r_1 < r_2$

(1) $\boldsymbol{B = -2.5}$

(2) $\boldsymbol{C = 0.83333}$

II. (3) Given $f(t) = 3 - 4(t-1)U(t-1) + 4(t-3)U(t-3)$, determine

a) $f(2) = -1$ ________

b) $f(5) = -5$ ________

III. (4) A tank containing 5 m^3 of 20% (by volume) NaOH solution is to be purged by adding pure water at a rate of 4 m^3/h. If the solution leaves the tank at a rate of 4 m^3/h, determine the time necessary to purge 90% of the NaOH by mass from the tank. Assume perfect mixing. Specific gravity of pure NaOH is 1.22. **2.88 h**

IV. (5) Find the Laplace transform of $e^{-2t}\cos 3t$ $\mathscr{L}\{e^{-2t}\cos 3t\} = \dfrac{s+2}{(s+2)^2+9}$

V. (6) Find the inverse of $\hat{F}(s) = \dfrac{s+3}{s^2-6s+18}$ $f(t) = e^{3t}\cos 3t + 2e^{3t}\sin 3t$

VI. Figure 6 shows the schematic of a process for treating residential sewage. In this simplified process, sewage (without bacteria) at a rate of 6000 gal/min is pumped into a well-mixed aeration tank where the concentration of bacteria $C_{B,\text{aration}}$ is maintained at 0.25 lb/gal. The treated sewage is then pumped to a settling tank where the bacterial is separated and recycled back to the aeration tank. The treated sewage leaving the settling tank has no bacteria in it while the recycle sewage contains a bacterial concentration of 1.0 lb/gal. Both the aeration and the settling tanks have the same volume of 5×10^6 gallons. You can assume the liquid (sewage) density remains constant throughout the process and neglect the mass loss due to the generation of CO_2 leaving the aeration tank.

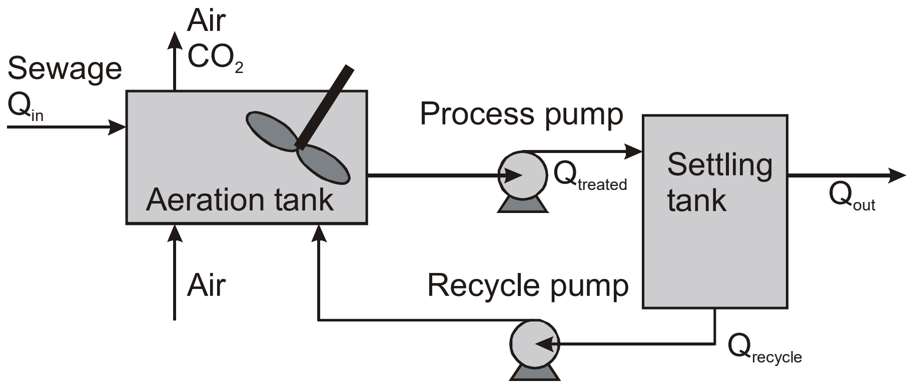


Figure 6 A process for treating residential sewage.

If 6000 gal/min of sewage enters and leaves the treatment facility, determine the two volumetric flow rates Q_{treated} and Q_{recycle}.

7) Q_{treated} = _____________ **8)** Q_{recycle} = _____________

Q_{treated} = **8000 gal/min**

Q_{recycle} = **2000 gal/min**

VII. (9) Obtain the Laplace transform of the equation $c(t) = U(t-4)[1 - e^{-(t-4)/4}]$

$$C(s) = \frac{e^{-4s}}{s(4s+1)}$$

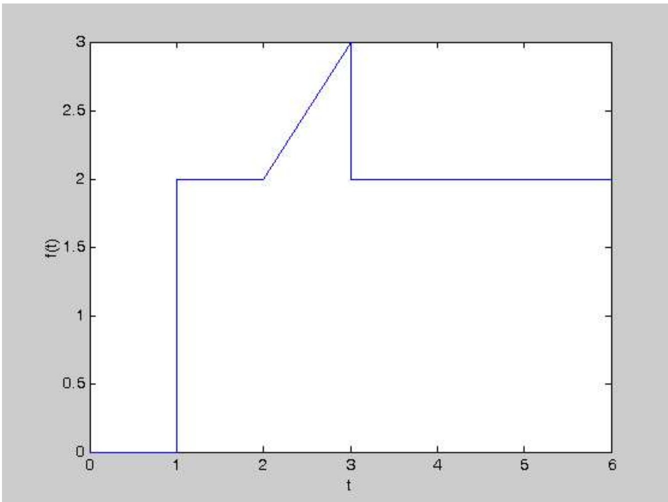
VIII. 10) Given $F(s) = \dfrac{1}{s(s+1)^2}$ find $\lim_{t\to\infty} f(t)$.

$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\,F(s) = \mathbf{1}$

Quiz #2

Note: Your answers must be correct to 3 significant figures and have the appropriate units.

I. Express the function given the graph in the t-domain



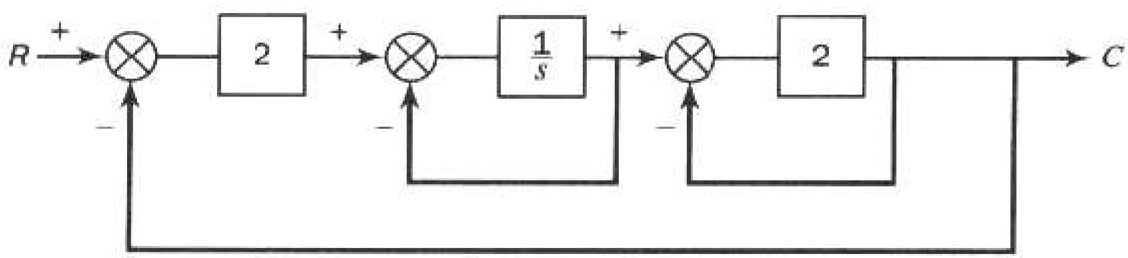
1) $f(t) = 2u(t - 1) + (t - 2)u(t - 2) - (t - 3)u(t - 3) - u(t - 3)$

II.) 2) Given $f(s) = (1 - 2e^{-s} + e^{-2s})/s^2$

$$f(t) = t - 2(t - 1)u(t - 1) + (t - 2)u(t - 2)$$

III. (3) Find the inverse of $\hat{F}(s) = \dfrac{s+2}{s^2 - 8s + 20}$ $\qquad f(t) = e^{4t}\cos 2t + 3e^{4t}\sin 2t$

III. (4) The transfer functions for this system at $s = 0.5$ is: ________



$C(s)/R(s) =$ **0.47059**

IV. An engineer has designed a system in which a positive-displacement pump is used to pump water from an atmosphere tank into a pressurized tank operating at 150 psig. A control valve is installed between the pump discharge and the pressurized tank.

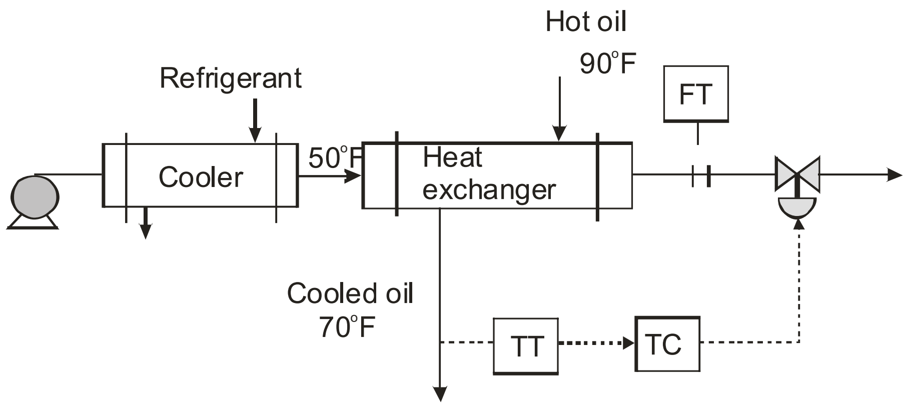
With the pump running at a constant speed and stroke length, 350 gpm of water is pumped when the control valve is wide open and the pump discharge pressure is 200 psig.

If the control valve is pinched back to 60 percent open, determine

(5) the water flow rate **350 gpm**

(6) If the pump reduces speed so that the water flow rate is 250 gpm at 60 percent valve open, determine the pump discharge pressure. **220.9 psig**

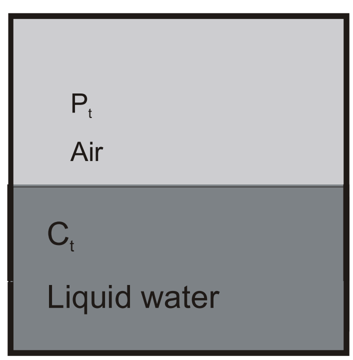
V. Consider the following control system with all instrumentation in electronic (4 to 20 mA)



7) If the temperature transmitter has a range of 50-100°F, the value from the temperature transmitter is: **10.4 mA**

8) If the range of the orifice-differential pressure flow transmitter on the water line is 0-2000 gpm, the value from the flow transmitter is for a water flow rate of 900 gpm is**: 7.24 mA**

VI. A 2 L glass jar is half filled with water and half filled with air at a temperature of 293°K. After 92 mg (=10^{-3} mol, mw = 92) of liquid toluene is added, the jar is sealed. At 293°K, the Henry's law constant for toluene is $K_{H,g}$ = 0.15 [M·atm^{-1}]. Note: M= mol/L, gas constant = 82.05×10^{-3} L·atm/mol·K



Determine:

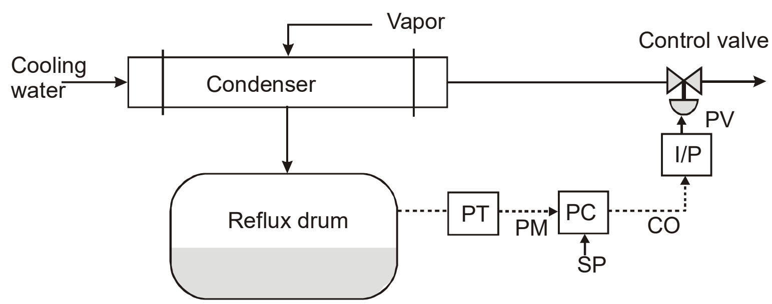
(9) the equilibrium partial pressure (atm) of toluene in the gas phase: **5.23×10^{-3} atm**

(10) the equilibrium concentration (M) of toluene in the water and: **7.86×10^{-4} M**

Quiz #3

Note: Your answers must be correct to 3 significant figures and have the appropriate units.

I.[1] The overhead vapor from a depropanizer distillation column is totally condensed in a water-cooled condenser at 120°F and 230 psig. The vapor is 98 mol % propane and 2 mol % isobutene. The vapor design flow rate is 40,000 lb/h and average latent heat of vaporization is 128 Btu/lb. Cooling water inlet and outlet temperatures are 75 and 100°F, respectively. The condenser heat transfer area is 1000 ft^2. The cooling water pressure drop through the condenser at design rate is 50 psi. A linear-trim control valve (air-to-closed, when CO = 20 mA, PV = 15 psig) is installed in the cooling water line. The pressure drop over the valve is 25 psi at design with the valve half open. The process pressure is measured by an electronic (4-20 mA) pressure transmitter whose range is 150-400 psig. An analog electronic proportional controller with a gain of 2 is used to control process pressure by manipulating cooling water flow. The electronic signal from the controller (CO) is converted into a pneumatic signal in the I/P transducer.



1) Calculate the cooling water flow rate (gpm) at design conditions. Water density is 62.3 lb/ft^3 and 1 ft^3 = 7.48 gal. **409 gpm**

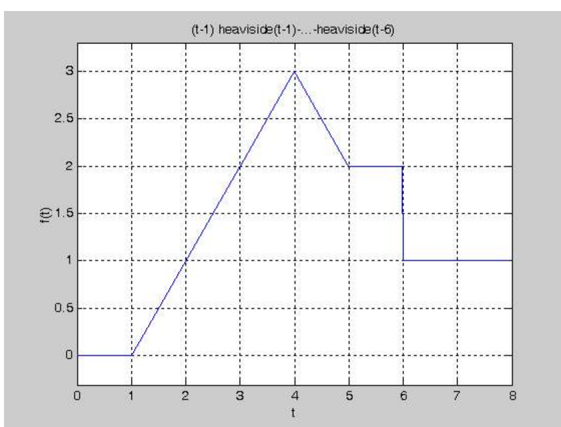
2) If the cooling water flow rate is 250 gpm at design conditions, calculate the size coefficient (C_v) of the control valve. C_v = 100 gpm/psi$^{0.5}$

(3) Calculate the value of the signal PM at design condition . **9.12 mA**

(4) Calculate the value of the signal PV at design conditions**. 9 psig**

(5) Suppose the process pressure jumps 20 psi, determine value for CO**. 9.44 mA**

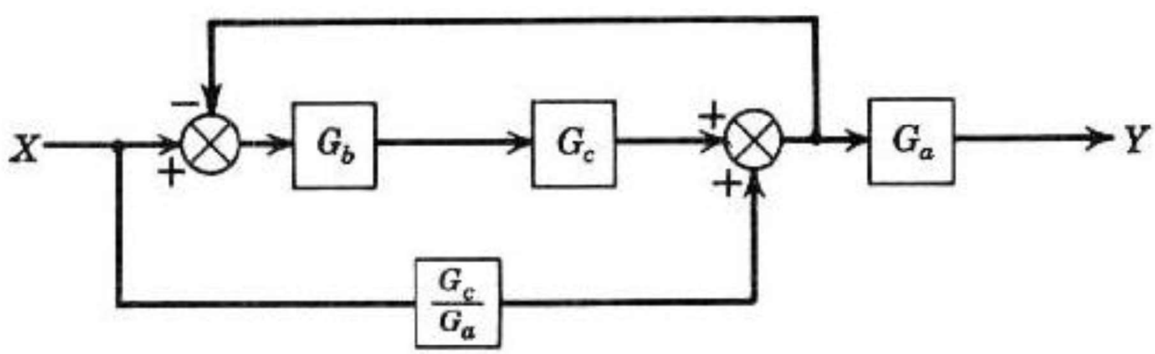
II. (6) Express the function given the graph in the *t*-domain



$f(t) = (t - 1)*u(t - 1) - 2*(t - 4)*u(t - 4) + (t - 5)*u(t - 5) - u(t - 6)$

III. (7) A thermometer having first-order dynamics with a time constant of 1 min is at 100°F. The thermometer is suddenly placed in a bath at 110°F at $t = 0$ and left there for 0.167 min, after which it is immediately returned to a bath at 100°F. Calculate the thermometer reading at $t = 0.5$ min. **101.1°F**

IV. (8) Determine the transfer functions for this system:



$$\frac{Y}{X} = \frac{G_c(1+G_aG_b)}{1+G_bG_c}$$

V. The temperature of a CSTR is controlled by an electronic (4 to 20 mA) feedback control system containing (1) a 50 to 300°F temperature transmitter, (2) a PI controller with integral time set at 4 minutes and proportional band at 25, and (3) a control valve with linear trim, air-to-open action, and $C_v = 20$ gpm/psi$^{0.5}$ through which cooling water flows. The pressure drop across the valve is a constant 20 psi.

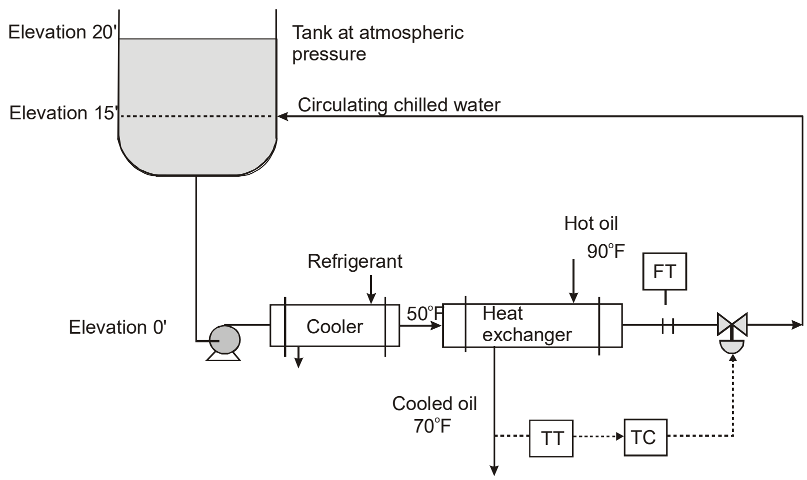
9) If the steady state controller output, *CO*, is 10 mA, how much cooling water is going through the valve? **33.54 gpm**

10) If a sudden disturbance increases reactor temperature by 10°F, what will be the immediate change on the controller output? **2.56 mA**

Quiz #4

Note: Your answers must be correct to 3 significant figures and have the appropriate units.

I.[1] A circulating chilled-water system is used to cool an oil stream from 90 to 70°F in a tube-in-shell heat exchanger shown. The temperature of the chilled water entering the process heat exchanger is maintained constant at 50°F by pumping the chilled water through a cooler located upstream of the process heat exchanger. The design chilled-water for normal conditions is 600 gpm, with chilled water leaving the process heat exchanger at 65°F. Chilled-water pressure drop through the process heat exchanger is 15 psi at 800 gpm. Chilled-water pressure drop through the refrigerated cooler is 15 psi at 800 gpm. The temperature transmitter on the process oil stream leaving the heat exchanger has a range of 40-180°F. The range of the orifice-differential pressure flow transmitter on the chilled water is 0-1500 gpm. All instrumentation is electronic (4 to 20 mA). Assume the chilled-water pump is centrifugal with a flat pump curve (total pressure drop across the system is constant). The control valve has a linear trim with C_v = 128.81 gpm/psi$^{0.5}$. The valve is 40 percent open at the 800 gpm design rate and has a maximum flow of 1500 gpm.



1) The total pressure drop through the system is **271 psi**

2) The pressure drop through the cooler and the heat exchanger at maximum flow rate is **105.47 psi**

3) The value of the signal from the temperature transmitter is **7.43 mA**

4) The value of the signal from the flow transmitter is **8.55 mA**

5) If C_v = 200 gpm/psi$^{0.5}$ and the total pressure drop through the system is 200 psi, determine the fraction open of the chilled-water control valve when the chilled-water flow rate is reduced to 600 gpm. $f_{(x)}$ = 0.**22169**

II. (6) Find the inverse of $\hat{F}(s) = \dfrac{s-12}{s^2-4s+29}$ $\qquad f(t) = e^{2t}\cos 5t - 2e^{2t}\sin 5t$

III. A tank is heated by steam condensing inside a coil. A PID controller is used to control the temperature in the tank by manipulating the steam valve position.

Process. The feed has a density ρ of 68.0 lb/ft^3 and a heat capacity c_p of 1.0 Btu/lb·°F. The volume V of liquid in the reactor is maintained constant at 200 ft^3. The coil consists of 300 ft of 4-in. schedule 40 steel pipe with outside diameter of 5 in. The overall heat transfer coefficient U, based on the outside are of the coil, has been estimated as 2.0 Btu/min·ft^2·°F. It can be assumed that its latent heat of condensation of saturated steam λ is constant at 950 Btu/lb. It can also be assumed that the inlet temperature T_i is constant.

Design Conditions. The feed flow F at design condition is 20 ft^3/min, and its temperature T_i is 100°F. The contents of the tank must be maintained at a temperature T of 150°F.

(7) Determine the steam temperature at the design condition. **236.58°F**

(8) If the steam temperature at the design condition is 250°F, determine the steam flow rate in lb/min**. 82.67 lb/min**

IV. (9) A thermometer having a time constant of 1 min is initially at 50°C. It is immersed in a bath maintained at 150°C at $t = 0$. Determine the temperature reading at $t = 1.5$ min. **127.7°C**

V. (**10)** A thermometer having a time constant of 0.5 min is placed in a temperature bath, and after the thermometer comes to equilibrium with the bath, the temperature of the bath is increased linearly with time at a rate of 10°C/min. Determine the difference between the indicated temperature and the bath temperature after 20 s. **– 2.4329°F**

Quiz #5

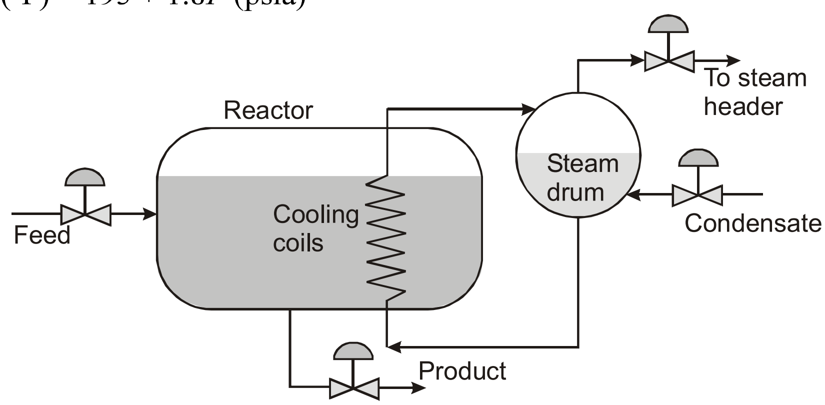
Note: Your answers must be correct to 3 significant figures and have the appropriate units.

I.[1] The formula for the flow of saturated steam through a control valve is

$$\dot{m}_{steam}\ (\text{lb/h}) = 2.1 C_V f_{(x)} \sqrt{P_1^2 - P_2^2}$$

In this equation, P_1 is upstream pressure (psia) and P_2 is downstream pressure (psia). The temperature of the steam-cooled reactor shown below is 285°F. The heat that must be transferred from the reactor into the steam generation system is 25×10^6 Btu/h. The overall heat transfer coefficient for the cooling coils is 300 Btu/h·ft²·°F. The steam discharges into a 25-psia steam header. The enthalpy difference between saturated steam and liquid condensate is 1000 Btu/lb. The vapor pressure of water can be approximated by

$$T(^\circ\text{F}) = 195 + 1.8P\ (\text{psia})$$



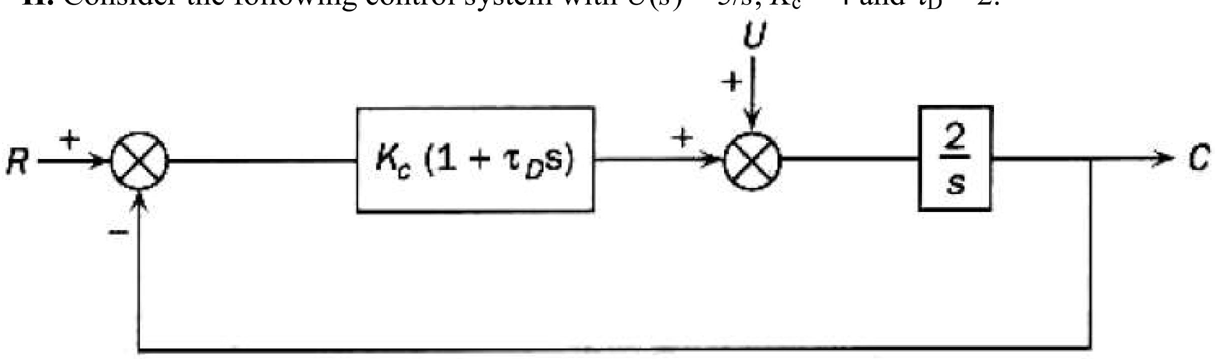
For questions (1) and (2) the drum pressure is 40 psia.

1) Calculate the area of the cooling coils. **4630 ft²**

2) Calculate the C_v for the steam valve, assuming that the valve is half open at the design conditions: $f_{(x)} = 0.5$. $C_v =$ **762 lb/h·psia**

3) If $C_v = 700$ lb/h·psia, and the temperature of the drum at the maximum heat removal (Q_{max}) of the system is 257.1°F, determine Q_{max} in Btu/h. **34.95×10⁶ Btu/h**

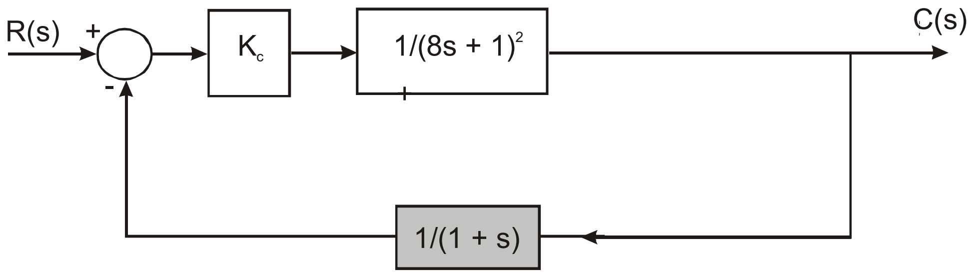
II. Consider the following control system with $U(s) = 5/s$, $K_c = 4$ and $\tau_D = 2$.



4) Determine $C(t = 1.25)$. $C(t = 1.25) = 0.5559$

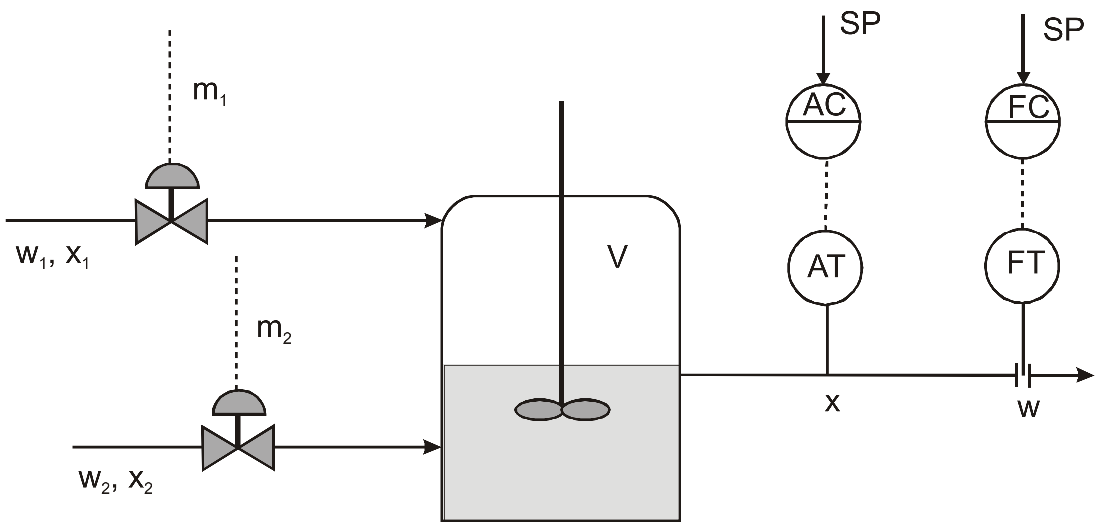
5) The offset is**:** $C(\infty) - R(\infty) =$ **1.25**

III. Consider the following control system



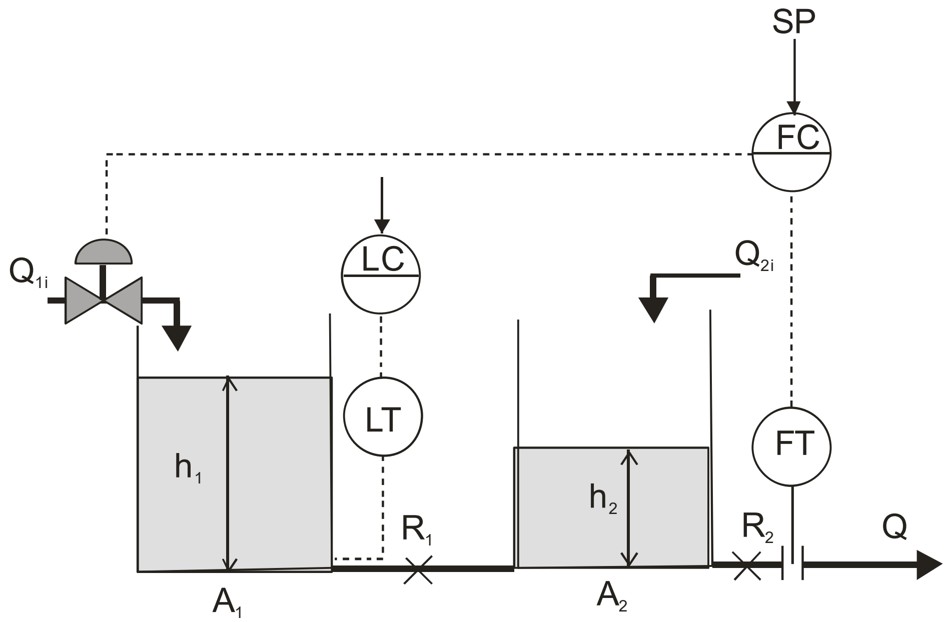
(6) The ultimate gain is K_{cu} = **20.2500**

IV. Consider a tank in which a solution (w_1, x_1) containing 10 weight % salt is blended with a concentrated solution (w_2, x_2) containing 42 weight % salt, to produce 100 lb/h of a solution (w, x) containing 20 weight % salt at design conditions.



7) Determine the open loop steady state gain on the mass percent of salt in the product stream for an increase of 2.0 lb/h in w_2. **0.2155 % salt/(lb/h)**

8) Determine the closed loop steady state gain on the mass percent of salt in the product stream for an increase of 2.0 lb/h in w_2 (and w_1 is reduced by 2.0 lb/h). **0.32 % salt/(lb/h)**



The two tanks system shown above is initially at steady state with q_{1i} = 8 cfm and q_{2i} = 4 cfm. The following data apply to the tanks: A_1 = 1 ft^2, A_2 = 1.25 ft^2, R_1 = 2 ft/cfm, and R_2 = 0.8 ft/cfm.

9) Determine open loop steady state gain for h_1 when the flow q_{2i} is increased from 4 to 6 cfm

K_{h1} = **0.8 ft/cfm**

10) Determine closed loop steady state gain for h_1 when the flow q_{2i} is increased from 4 to 6 cfm

$K_{h1,closed}$ = **– 2.0 ft/cfm**

Appendix C

Previous Exams

CHE 426 (Fall 2012)

____
LAST NAME, FIRST

Quiz #1

Note: Your answers must be correct to 3 significant figures and have the appropriate units.

I. Find the Laplace transform of (sin 3*t*)(cos 3*t*):

$$\mathscr{L}\{0.5 \sin 6t\} = \frac{3}{s^2 + 36}$$

II. The liquid-phase reaction 2A + 3B → C + D is carried out in a semi-batch reactor. The reactor initially contains 6 mol of B at a concentration of 0.020 mol/liter. A at a concentration of 0.040 mol/liter is fed to the reactor at a rate of 4 liter/min. The rate of disappearance of A is given by $r_A = -kC_AC_B$. The feed rate to the reactor is discontinued when the reactor contains 500 liters of fluid. At this time the molar extent of reaction, ζ, is 1.2 moles (Note: $N_i = N_{io} + \nu_i \zeta$). Questions 2-4 consider the conditions of the reactor at this time.

2. C_B (mol/liter) = (6 – 3×1.2)/500 = **0.0048 mol/liter** ________________

3. $\int_0^t r_A V dt$ (mol) = – 1.2×2 = **– 2.4 mol** ____________

Note: V is the volume of the fluid and the integration is to the time t when V = 500 liters

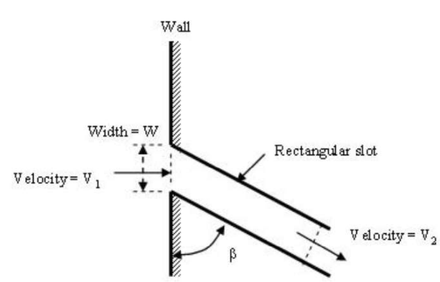
4. Fractional conversion of B = 1 – 2.4/6 = **0.6** ______________

III. (5) Find the Laplace transform of $e^{-3t}\sin 5t$: $\mathscr{L}\{\sin 5t\} = \int_0^\infty e^{-st}\sin 5t dt = \frac{5}{s^2 + 5^2}$

IV. (6) Find the inverse of $\widehat{}(s) = \frac{s}{s^2 - 6s + 18}$: $\boldsymbol{f(t) = e^{3t}\cos 3t + e^{3t}\sin 3t}$

V. (7) There are 3460 pounds of water in the jacket of a reactor that are initially at 145°F. At time equal zero, 70°F cooling water is added to the jacket at a constant rate of 400 pounds per minute. The holdup of water in the jacket is constant since the jacket is completely filled with water and excess water is removed from the system on pressure control as cold water is added. Water in the jacket is perfectly mixed. How many minutes does it take the jacket to reach 90°F if no heat is transferred into the jacket? **11.43 min**

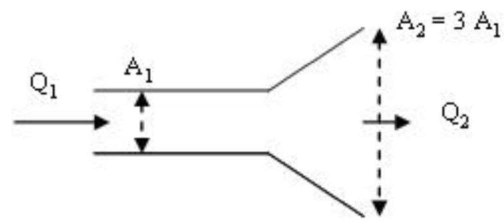
VI. Water flows into a rectangular slot that is at an angle β with respect to the wall as shown below.



The correct relation between the velocities V_1 and V_2 is

A) $V_2 = V_1/\sin(\beta)$ **(Ans.)**

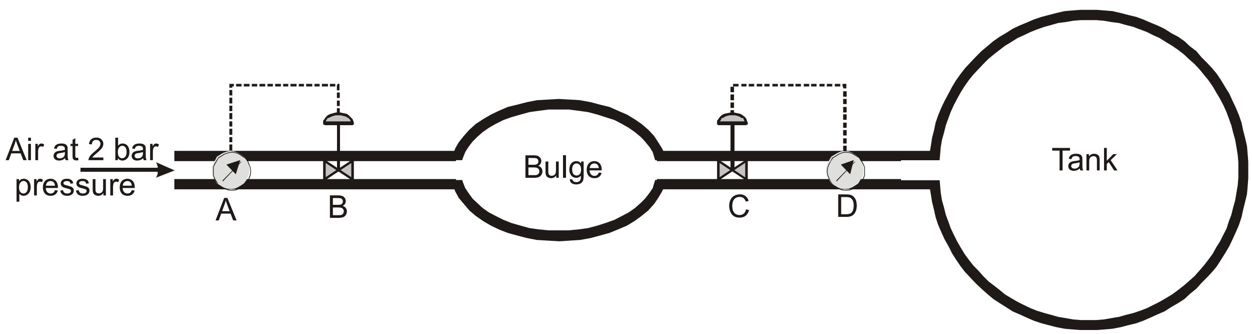
VII. (9) Air flows out of a ventilation duct as shown. The flow can be assumed to be incompressible and steady. The relation between the volume flow rate Q_1 at section 1 and Q_2 at section 2 is



C) $Q_2 = Q_1$ **(Ans.)**

VII. (10) A well-insulated pipe of 2.54 cm inside diameter carries air at 2 bar pressure and 366.5°K. It is connected to a 0.0283 m^3 insulated bulge, as shown . The air in the bulge is initially at one bar pressure and 311°K. **A** and **D** are flow meters which accurately measure the air mass flow rate. Valves **B** and **C** control the air flow into and out of the bulge. Connected to the bulge is a 0.283 m^3 rigid, adiabatic tank which is initially evacuated to a very low pressure.

At the start of the operation, valve **B** is opened to allow 4.54 g/s of air flow into the bulge; simultaneously, valve **C** is operated to transfer exactly 4.54 g/s from the bulge into the tank. These flows are maintained constant as measured by the flow meters. Air may be assumed to be an ideal gas with a specific heat ratio $\gamma = C_p/C_v = 1.4$, $C_p = 29.3$ J/mol·°K, and molecular weight = 29. Gas constant $R = 8.314$ m^3·Pa/mol·°K.



Determine $\frac{dT}{dt}$ when the temperature in the bulge is 340°K. **5.8 K/s**

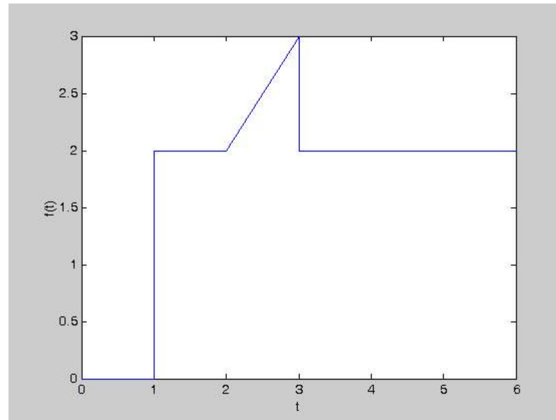
CHE 426 (Fall 2012)

LAST NAME, FIRST

Quiz #2

Note: Your answers must be correct to 3 significant figures and have the appropriate units.

I. The function given the graph in the *t*-domain is



A) $f(t) = 2u(t - 1) + (t - 2)u(t - 2) - (t - 4)u(t - 3)$

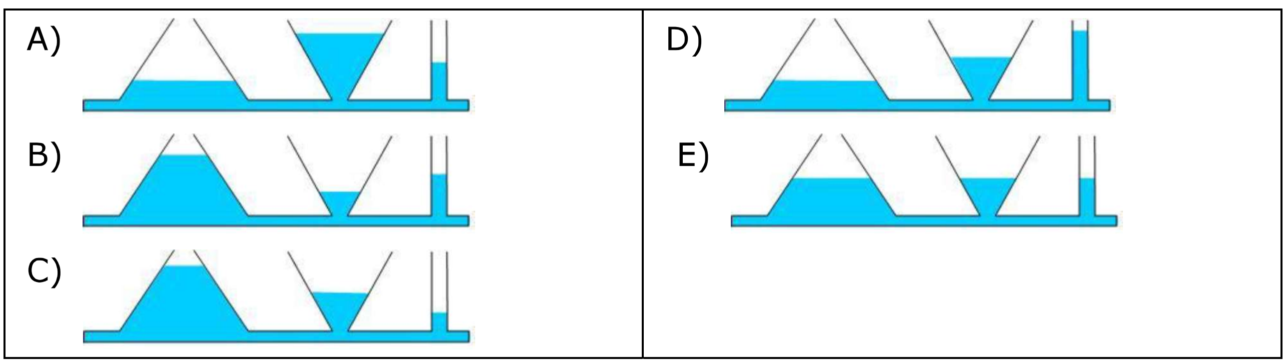
B) $f(t) = 2u(t - 1) + (t - 2)u(t - 2) - (t - 2)u(t - 3)$ **A**

C) $f(t) = 2u(t - 1) - (t - 2)u(t - 2) + (t - 2)u(t - 3)$

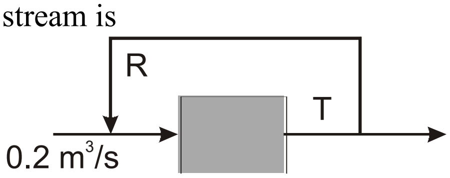
D) $f(t) = 2u(t - 1) + (t - 2)u(t - 2) + (t - 2)u(t - 3)$

E) None of the above

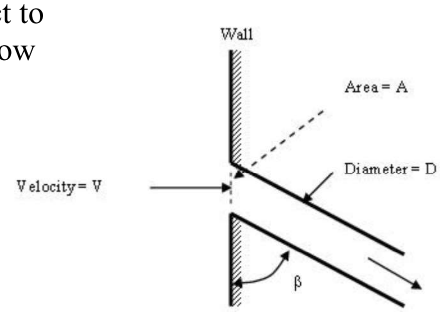
II.) Three containers connected at the base are filled with a liquid. The top of each container is open to the atmosphere and surface tension is negligible. The container shapes are all different. The figure that shows the correct fluid levels in the containers at equilibrium conditions is _E_



III. The fresh feed to the reactor is 0.2 m^3/s, the stream leaving the reactor is split into a product stream and a recycle stream that is combined with the fresh feed to the CSTR. For a steady operation and 20 % of the stream leaving the reactor is recycled, the flow rate of the recycle stream is **0.05 m^3/s**

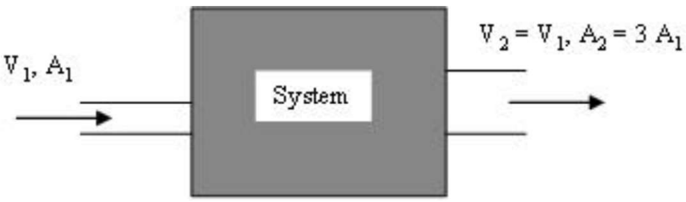


IV. Air flows into a round pipe that is at an angle β with respect to the wall as shown below. The correct expression for the mass flow rate of the air through the pipe is



C) $\dot{n} = \rho A V$ **(A)**

V. A fluid flows steadily through the system shown below. The inlet and outlet velocities are equal and the outlet area is three times the area of the inlet. The outlet density ρ_2 is



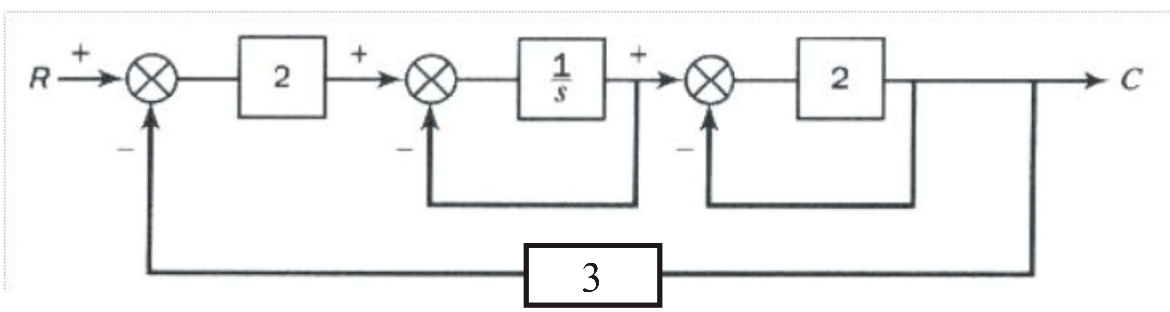
A) $\rho_2 = 3\rho_1$ **B)** $\rho_2 = \rho_1/\sqrt{3}$

C) $\rho_2 = \rho_1/3$ **(A)**

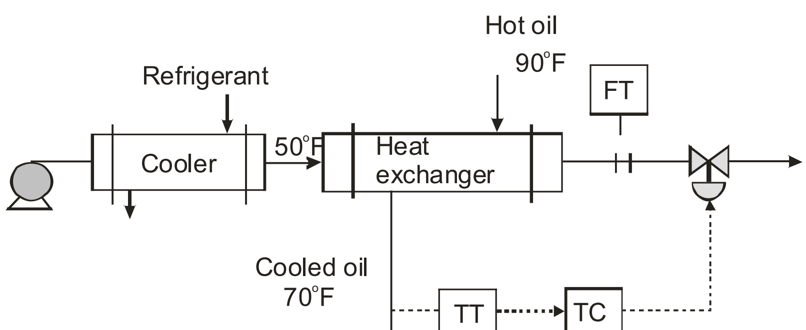
D) $\rho_2 = \rho_1$ **E)** $\rho_2 = \sqrt{3}\,\rho_1$

VI. Find the inverse of $\widehat{\ }(s) = \dfrac{s-12}{s^2-4s+29}$: $\boldsymbol{f(t) = e^{2t}\cos 5t - 2e^{2t}\sin 5t}$

VII. The transfer functions for this system at s = 0.5 is: **0.24242**



VIII. Consider the following control system with all instrumentation in electronic (4 to 20 mA)

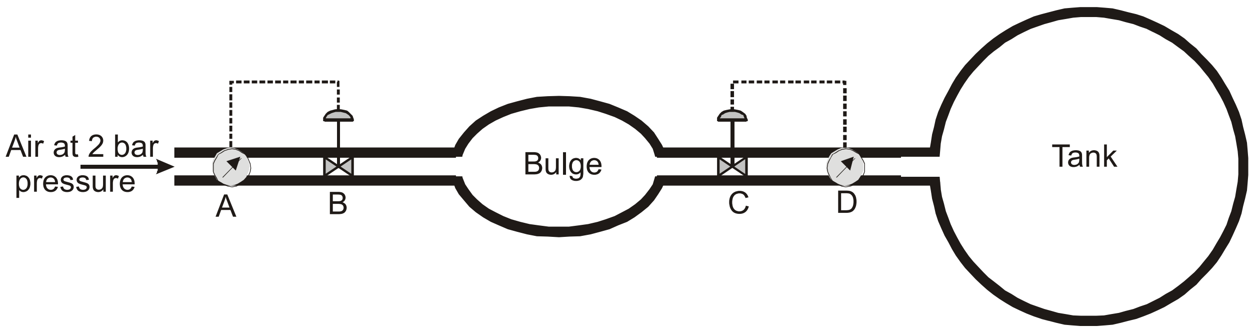


8) If the temperature transmitter has a range of 60-200°F, the value from the temperature transmitter is: **5.14 mA**

9) If the range of the orifice-differential pressure flow transmitter on the water line is 0-1500 gpm, the value from the flow transmitter is for a water flow rate of 800 gpm is**: 8.55 mA**

IX. (10) A well-insulated pipe of 2.54 cm inside diameter carries air at 2 bar pressure and 366.5^{o}K. It is connected to a 0.0283 m^3 insulated bulge, as shown . The air in the bulge is initially at one bar pressure and 311^{o}K. **A** and **D** are flow meters which accurately measure the air mass flow rate. Valves **B** and **C** control the air flow into and out of the bulge. Connected to the bulge is a 0.283 m^3 rigid, adiabatic tank which is initially evacuated to a very low pressure.

At the start of the operation, valve **B** is opened to allow 4.54 g/s of air flow into the bulge; simultaneously, valve **C** is operated to transfer exactly 4.54 g/s from the bulge into the tank. These flows are maintained constant as measured by the flow meters. Air may be assumed to be an ideal gas with a specific heat ratio $\gamma = C_p/C_v = 1.4$, $C_p = 29.3$ J/mol·^{o}K, and molecular weight = 29. Gas constant $R = 8.314$ m^3·Pa/mol·^{o}K.



Determine $\dfrac{dT}{dt}$ when the temperature in the bulge is 311^{o}K. **11.11 K/s**

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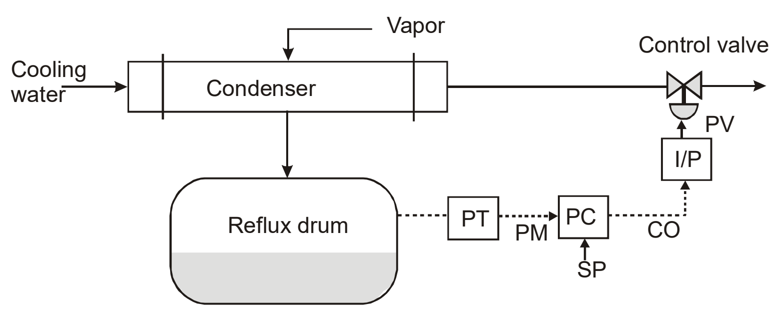
Quiz #3

Note: Your answers must be correct to 3 significant figures and have the appropriate units.

I. A sewage disposal plant has a big concrete holding tank of 100,000 gal capacity. It is three-fourth full of liquid to start with and contains 60,000 lb of organic material in suspension. Water runs into the holding tank at the rate of 20,000 gal/hr and the solution leaves at the rate of 15,000 gal/hr. How much organic material is in the tank at the end of 3 hr? **34,722 lb**

____

II.[1] The overhead vapor from a depropanizer distillation column is totally condensed in a water-cooled condenser at 120^{o}F and 230 psig. Cooling water inlet and outlet temperatures are 75 and 100^{o}F, respectively. The condenser heat transfer area is 1000 ft^2 and the overall heat transfer coefficient is 90 Btu/hr·^{o}F·ft^2. The cooling water pressure drop through the condenser at design rate is 50 psi. A linear-trim control valve (air-to-closed, when CO = 20 mA, PV = 15 psig) is installed in the cooling water line. The pressure drop over the valve is 25 psi at design with the valve half open. The process pressure is measured by an electronic (4-20 mA) pressure transmitter whose range is 150-300 psig. An analog electronic proportional controller with a gain of 3 is used to control process pressure by manipulating cooling water flow. The electronic signal from the controller (CO) is converted into a pneumatic signal in the I/P transducer. At the condenser pressure of 230 psig, CO is 12 mA.



2) Calculate the cooling water flow rate (gpm) at design conditions. Water density is 62.3 lb/ft^3 and 1 ft^3 = 7.48 gal. C_p of water is 1 Btu/lb·^{o}F. **222.1 gpm**

3) If the cooling water flow rate is 200 gpm at design conditions, calculate the size coefficient (C_v) of the control valve. $\boldsymbol{C_v}$ **= 80 gpm/psi$^{0.5}$**

(4) Calculate the value of the signal PM at design condition __**12.53 mA** __

(5) Suppose the process pressure jumps 20 psi, determine value for CO ___**5.6 mA**

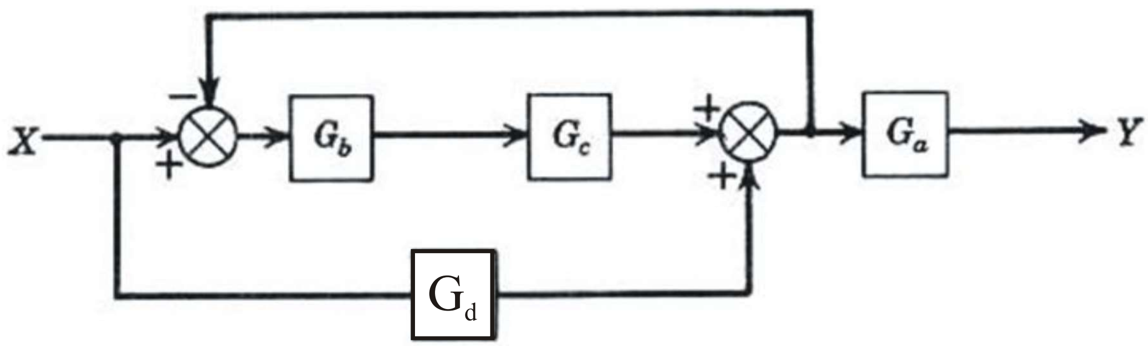
III. **(6)** Solve the following equation for $y(t)$

$$-2\int_0^t y(\tau)d\tau = \frac{dy}{dt} + y \qquad y(0) = 1$$

$$y(t) = e^{-t/2}\cos\left(\sqrt{1.75}t\right) - \frac{0.5}{\sqrt{1.75}}\, e^{-t/2}\sin\left(\sqrt{1.75}t\right)$$

IV. (7) A thermometer having first-order dynamics with a time constant of 1 min is at 100°F. The thermometer is suddenly placed in a bath at 120°F at $t = 0$ and left there for 1 min, after which it is immediately returned to a bath at 100°F. Calculate the thermometer reading at $t = 1.5$ min.
107.1°F

V. (8) Determine the transfer functions for this system:



d) $\dfrac{Y}{X} = \dfrac{G_a\left(G_bG_c + G_d\right)}{1 + G_bG_c}$ **(A)**

VI. (9) For fully-developed viscous flow in a horizontal pipe, which of the following is true:

a) pressure forces are balanced by shear forces (A) **b)** pressure forces are balanced by body forces.

c) shear forces result in fluid deceleration. **d)** pressure forces result in fluid acceleration

e) None of the above

VII. (10) For a very rough pipe wall the friction factor is constant at high Reynolds numbers. For a pipe of diameter D_1 the pressure drop over the length is Δp_1. If he diameter of the pipe is then reduced to one-half the original value what is the relation of the new pressure drop Δp_2 to the original pressure drop Δp_1 at the same mass flow rate?

a) $\Delta p_2 = 2\Delta p_1$

b) $\Delta p_2 = 16\Delta p_1$

c) $\Delta p_2 = 4\Delta p_1$

d) (A) $\Delta p_2 = 32\Delta p_1$

e) $\Delta p_2 = 8\Delta p_1$

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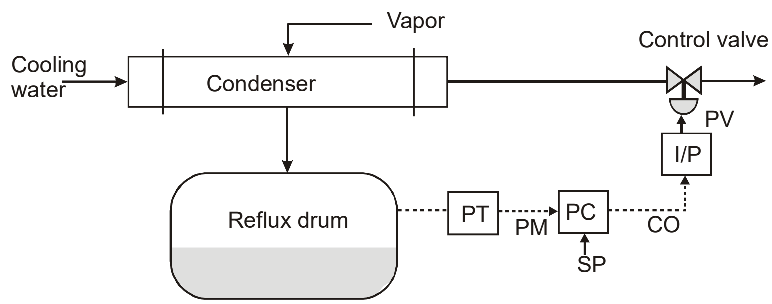
Quiz #4

Note: Your answers must be correct to 3 significant figures and have the appropriate units.

I.[1] The overhead vapor from a depropanizer distillation column is totally condensed in a water-cooled condenser at 120°F and 230 psig. Cooling water inlet and outlet temperatures are 75 and 100°F, respectively. The condenser heat transfer area is 1200 ft^2 and the overall heat transfer coefficient is 100 Btu/hr·°F·ft^2. The cooling water pressure drop through the condenser at design rate is 50 psi. A linear-trim control valve (air-to-closed, when CO = 20 mA, PV = 15 psig) is installed in the cooling water line. The pressure drop over the valve is 25 psi at design with the valve half open. The process pressure is measured by an electronic (4-20 mA) pressure transmitter whose range is 150-300 psig. An analog electronic proportional controller with a gain of 3 is used to control process pressure by manipulating cooling water flow. The electronic signal from the controller (CO) is converted into a pneumatic signal in the I/P transducer. At the condenser pressure of 230 psig, CO is 12 mA.



1) Calculate the cooling water flow rate (gpm) at design conditions. Water density is 62.3 lb/ft^3 and 1 ft^3 = 7.48 gal. C_p of water is 1 Btu/lb·°F. **295.6gpm**

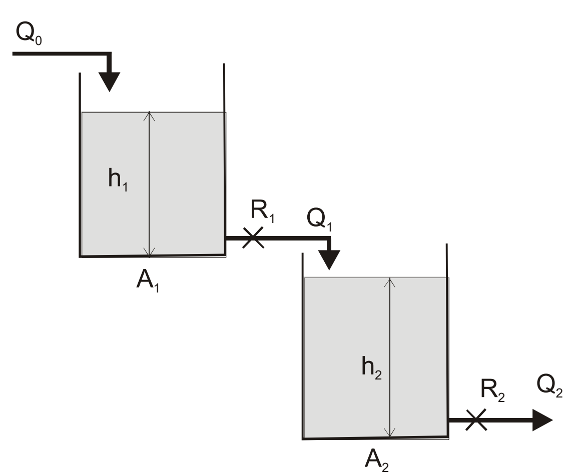
2) Suppose the process pressure jumps 12 psi, determine value for CO **_8.16 mA**

II. **(3)** Solve the following equation for $y(t)$

$$-5\int_0^t y(\tau)d\tau = 2\frac{dy}{dt} + y \qquad y(0) = 1$$

$y(t) = e^{-0.25t}\cos(1.5612t) - 0.16013e^{-0.25t}\sin(1.5612t)$

III. (4) The two-tank liquid-level system shown on the right is operating at steady state when a step change is made in the flow rate to tank 1. The transient response is critically damped where $\tau_1 = \tau_2$. If the time constant of each tank is 0.80 min, how long does it take for the change in level of the first tank to reach 90 percent of the total change? **1.84 min**



The critical damped response is given by:

$$Q_2(t) = \frac{\overset{d}{\overline{\ 2\ }}}{\overline{\ 2\ }} = 1 - \left(1+\frac{t}{\tau}\right)\exp\left(-\frac{t}{\tau}\right)$$

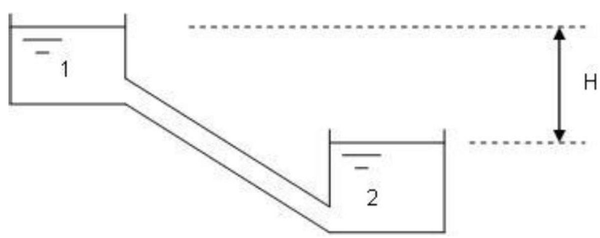
IV. (5) The steady state energy equation with head loss (h_L) and shaft work (h_s) is written as

$$\boxed{\frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} = \frac{p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_s - h_L}$$

If the left hand side of the energy equation were larger than the right hand side:

A) **(A)** the flow would not be possible.
B) the flow would come to a new equilibrium.
C) the flow would speed up.
D) the flow would stop.
E) the flow would slow down.

V. (6) A pipe connects two reservoirs at different elevations as shown in the figure below and water flows from reservoir 1 to 2. The flow is turbulent and the friction factor can be taken to be constant.



If the elevation difference H is doubled ($H_B = 2H_A$), the relation between the mass flow rate for case B and case A is:

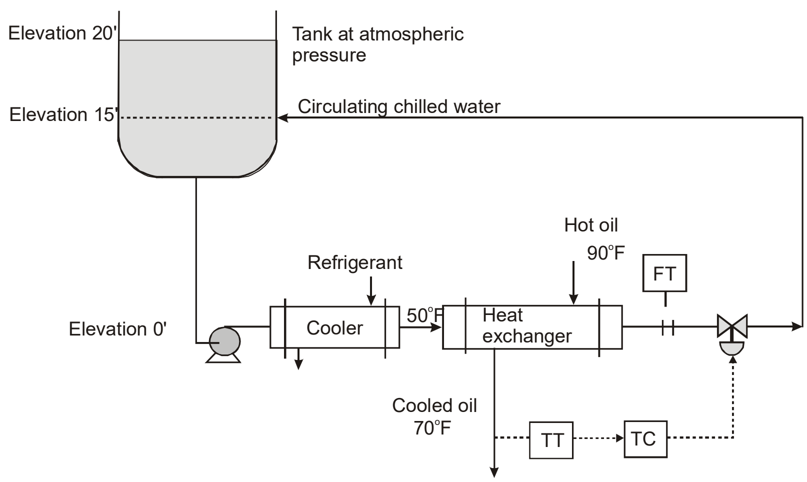
A) $\dot{m}_B = \dot{m}_A / 2^{0.5}$ **B)** $\dot{m}_B = \dot{m}_A / 2$ **C)** $\dot{m}_B = 4\,\dot{m}_A$

D) $\dot{m}_B = 2\,\dot{m}_A$ **E)** $\dot{m}_B = 2^{0.5}\,\dot{m}_A$ **(A)**

VI. A) The addition of integral action will make the system more stable.
B) Proportional controller only can make the system unstable.

A. A and B are true **B.** Only A is true **C.** Only B is true **(A)** **D.** A and B are false

VII.[1] A circulating chilled-water system is used to cool an oil stream from 90 to 70°F in a tube-in-shell heat exchanger shown. The temperature of the chilled water entering the process heat exchanger is maintained constant at 50°F by pumping the chilled water through a cooler located upstream of the process heat exchanger. The design chilled-water for normal conditions is 900 gpm, with chilled water leaving the process heat exchanger at 65°F. Chilled-water pressure drop through the process heat exchanger is 15 psi at 900 gpm. Chilled-water pressure drop through the refrigerated cooler is 15 psi at 900 gpm. The temperature transmitter on the process oil stream leaving the heat exchanger has a range of 40-180°F. The range of the orifice-differential pressure flow transmitter on the chilled water is 0-1500 gpm. All instrumentation is electronic (4 to 20 mA). Assume the chilled-water pump is centrifugal with a flat pump curve (total pressure drop across the system is constant). The control valve has a linear trim with C_v = 200 gpm/psi$^{0.5}$. The valve is 40 percent open at the 900 gpm design rate and has a maximum flow of 1800 gpm.



8) The total pressure drop through the system is **156.6 psi**

9) The value (mA) of the signal from the flow transmitter is **9.76 mA**

VIII. (10) An ester in aqueous solution is to be saponified in a CSTR. The reaction is irreversible, first order with reaction rate constant k = 0.1 min^{-1}. We need to process 100 moles/hr of 4 molar feed (4 moles/L) to 95% conversion. Determine the CSTR volume required for this process. **79.2 L**

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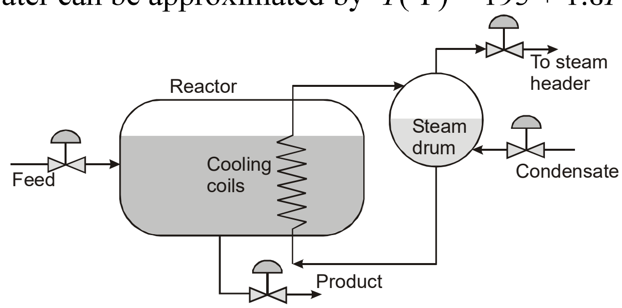
Quiz #5

Note: Your answers must be correct to 3 significant figures and have the appropriate units.

I.[1] The formula for the flow of saturated steam through a control valve is

$$\dot{m}_{steam}\,(\text{lb/h}) = 2.1 C_V f_{(x)} \sqrt{P_1^2 - P_2^2}$$

In this equation, P_1 is upstream pressure (psia) and P_2 is downstream pressure (psia). The temperature of the steam-cooled reactor shown below is 300°F. The heat that must be transferred from the reactor into the steam generation system is 30×10^6 Btu/h. The overall heat transfer coefficient for the cooling coils is 400 Btu/h·ft^2·°F. The steam discharges into a 35-psia steam header. The enthalpy difference between saturated steam and liquid condensate is 1000 Btu/lb. The vapor pressure of water can be approximated by $T(°\text{F}) = 195 + 1.8P$ (psia)

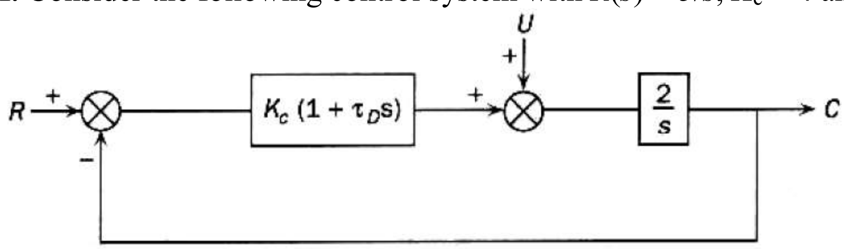


For questions (1) and (2) the drum pressure is 50 psia.

1) Calculate the area of the cooling coils: **_5,000 ft^2_**

2) Calculate the C_v for the steam valve, assuming that the valve is half open at the design conditions: $f_{(x)} = 0.5$. **C_v = 800 lb/h·psia**

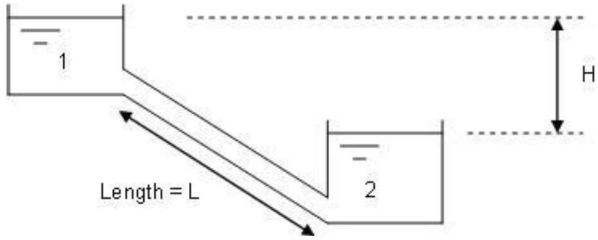
II. Consider the following control system with $R(s) = 5/s$, $K_c = 4$ and $\tau_D = 2$.



3) Determine $C(t = 1.25)$: **4.8367** ___________

4) The offset is $C(\infty) - R(\infty)$ = **5 – 5 = 0**

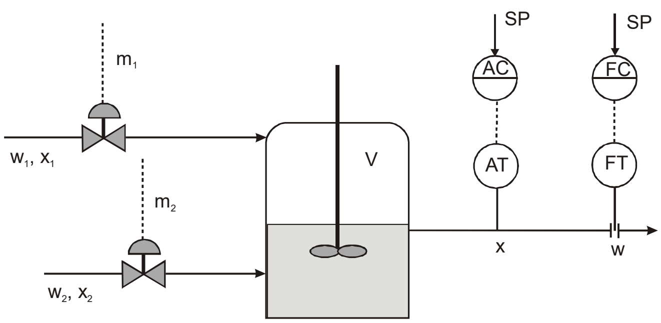
III. (5) A pipe connects two reservoirs at different elevations as shown in the figure below and water flows from reservoir 1 to 2. The flow is turbulent and the friction factor can be taken to be constant.



If the elevation difference remains the same and the pipe length L is doubled ($L_B = 2L_A$), the relation between the mass flow rate for case B and case A is:

A) $\dot{m}_B = \dot{m}_A / 2^{0.5}$ **(A)**

IV. Consider a tank in which a solution (w_1, x_1) containing 10 weight % salt is blended with a concentrated solution (w_2, x_2) containing 42 weight % salt, to produce 100 lb/h of a solution (w, x) containing 20 weight % salt at design conditions.



6) Determine the open loop steady state gain on the mass percent of salt in the product stream for a decrease of 2.0 lb/h in w_2. $\boldsymbol{K_{x2}}$ **= 0.2245 % salt/(lb/h)**

7) Determine the closed loop steady state gain on the mass percent of salt in the product stream for a decrease of 2.0 lb/h in w_2 (and w_1 is increased by 2.0 lb/h). **0.32 % salt/(lb/h)**

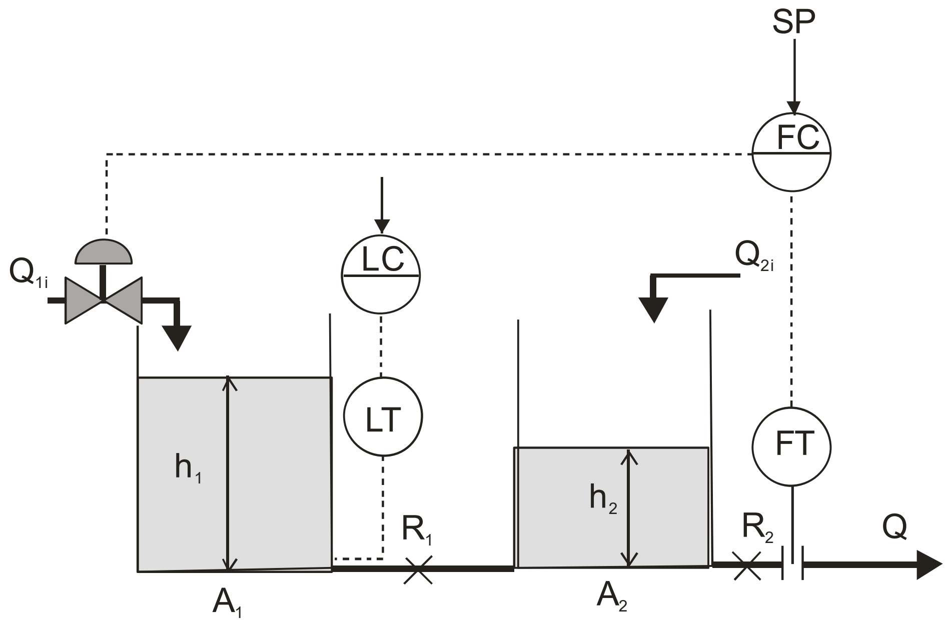
V. (8) The total head of a steady flow, H, is a measure of:

A) **(A)** the maximum height the fluid could reach with no additional work input.

B) the maximum pressure the fluid could reach with no additional work input.

C) the maximum velocity the fluid could reach with no additional work input.

D) All of the above.



VI. The two tanks system shown above is initially at steady state with q_{1i} = 8 cfm and q_{2i} = 6 cfm. The following data apply to the tanks: A_1 = 1 ft^2, A_2 = 1.25 ft^2, R_1 = 2 ft/cfm, and R_2 = 0.8 ft/cfm.

9) Determine open loop steady state gain for h_1 when the flow q_{2i} is decreased from 6 to 4 cfm

K_{h1} = **0.8 ft/cfm**

10) Determine closed loop steady state gain for h_1 when the flow q_{2i} is decreased from 6 to 4 cfm

– 2.0 ft/cfm