

1. Question

Theory and math calculation of MAP classification

find the MAP classification with minimum

Let  $x_{\text{Sample}}$  be the data with  $x_{\text{Sample}} \in \mathbb{R}^3$

$$\Rightarrow \hat{\theta}_{\text{MAP}} = \operatorname{argmax}_{\theta} P(\theta | x_{\text{Sample}}) \quad \text{by maximum likelihood}$$

$$= \operatorname{argmax}_{\theta} \prod_{i=1}^N P(\theta | x_{\text{Sample}_i}) = \operatorname{argmax}_{\theta} \prod_{i=1}^N P(x_{\text{Sample}_i} | \theta) P(\theta)$$

$$\begin{aligned} \Rightarrow \operatorname{argmax}_{\theta} \prod_{i=1}^N \ln P(\theta | x_{\text{Sample}_i}) &= \operatorname{argmax}_{\theta} \prod_{i=1}^N \ln P(x_{\text{Sample}_i} | \theta) P(\theta) \\ &= \operatorname{argmax}_{\theta} \sum_{i=1}^N \ln P(x_{\text{Sample}_i} | \theta) P(\theta) \end{aligned}$$

Since the  $x_{\text{Sample}}$  are  $\in \mathbb{R}^3$  mixture Gaussian data,  $x = x_{\text{Sample}}$

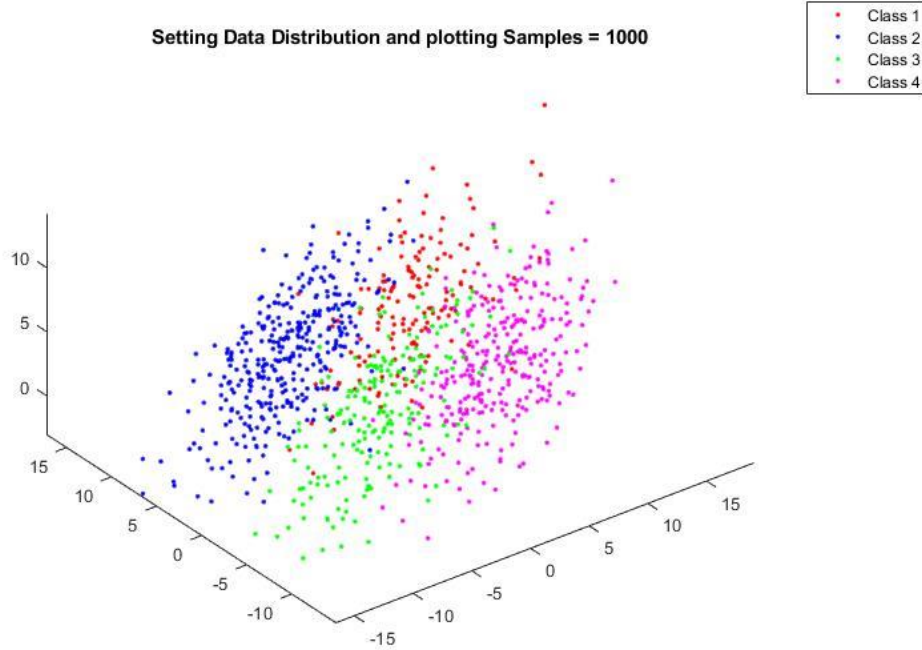
$$\Rightarrow \operatorname{argmax}_{\theta} \sum_{i=1}^N \ln \frac{1}{\sqrt{2\pi}^k \sigma^2} e^{-\frac{1}{2\sigma^2} (x_i - \mu)^T (x_i - \mu)}$$

$$= \operatorname{argmax}_{\theta} \sum_{i=1}^N \ln \frac{1}{\sqrt{2\pi}^k \sigma^2} - \frac{1}{2\sigma^2} [(x_i - \mu)^T (x_i - \mu)]$$

$$= \operatorname{argmax}_{\theta} \sum_{i=1}^N \ln \frac{1}{\sqrt{2\pi}^k} - \ln \sigma - (x_i - \mu)^T (x_i - \mu)$$

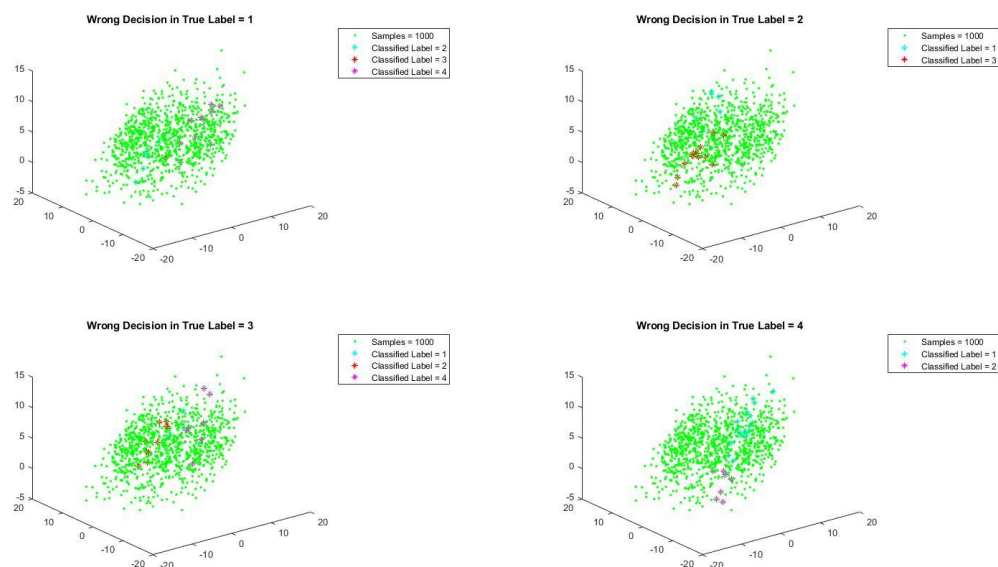
$$= \operatorname{argmax}_{\theta} \sum_{i=1}^N -\ln \sigma - (x_i - \mu)^T \sigma^{-1} (x_i - \mu)$$

$$\Rightarrow \hat{\theta}_{\text{MAP}} = \operatorname{argmin}_{\theta} \sum_{i=1}^N \ln \sigma + \frac{(x_i - \mu)^T (x_i - \mu)}{\sigma}$$



The distribution data is generated with  
 Prior = [0.15,0.35,0.2,0.3] where true sample number = [154,351,197,298]  
 $\text{Sigma} = 0.8 * \begin{bmatrix} 5 & 1 & 2 & 1 \\ 1 & 5 & 0 & 2 \\ 2 & 0 & 2 & 0 \\ 3 & 0 & 0 & 3 \end{bmatrix} / \sqrt{2}$  \* covarianceVector for each class  
 covarianceVector = [1.3<sup>2</sup> 0 0; 0 1.2<sup>2</sup> 0; 0 0 1.4<sup>2</sup>]

Apply this data distribution to the MAP classifier:



These plots show the MAP classification result  
 We get the confusion matrix = [137,6,1,10;6,333,12,0;3,10,177,7;20,0,7,271]

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Exam 2

Where represent the right classification of

Class 1 = 137

Class 2 = 333

Class 3 = 177

Class 4 = 271 in 1000 true samples

Thus, we can calculate the whole MAP classification accuracy rate =  $(137+333+177+271) / 1000 = 91.8\%$

And we get the decision error of each class is

Class 1 error = 0.11

Class 2 error = 0.05

Class 3 error = 0.10

Class 4 error = 0.09

With neural network = find the maximum likelihood  $x$ =samples  $l$ =label

$$\Rightarrow \hat{\theta}_{ML} = \underset{\theta}{\operatorname{argmax}} \prod_{i=1}^N P(x_i, l_i | \theta)$$

$$= \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^N P(x_i, l_i | \theta) = \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^N P(l_i | x_i, \theta) P(x_i | \theta)$$

$$= \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^N \ln P(l_i | x_i, \theta) P(x_i | \theta)$$

$$\Rightarrow \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^N \ln P(l_i | x_i, \theta) + \sum_{i=1}^N \ln P(x_i | \theta)$$

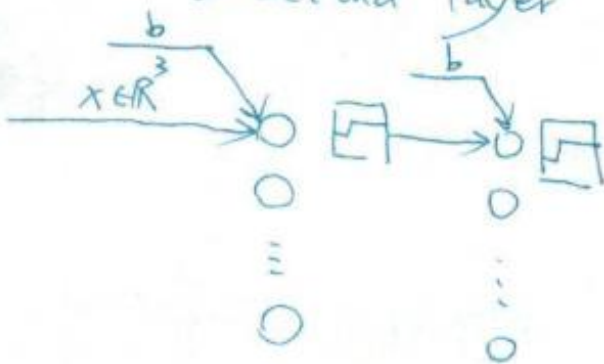
$$\Rightarrow \underset{\theta}{\operatorname{argmax}} \frac{1}{N} \sum_{i=1}^N \ln P(l_i | x_i, \theta) = \underset{\theta}{\operatorname{argmin}} - \frac{1}{N} \sum_{i=1}^N \ln P(l_i | x_i, \theta)$$

$\Rightarrow x_i$  is independent

Design a two hidden layer neural network, where two layer with weights and bias.

$\Rightarrow$  first layer with 'tanh', nonlinear.

$\Rightarrow$  second layer with 'softmax', linear.



$\Rightarrow R^3 \Rightarrow$  first layer  $\Rightarrow R^{\#node}$

$R^{\#node} \Rightarrow$  second layer  $\Rightarrow R^4$  (4 classification)

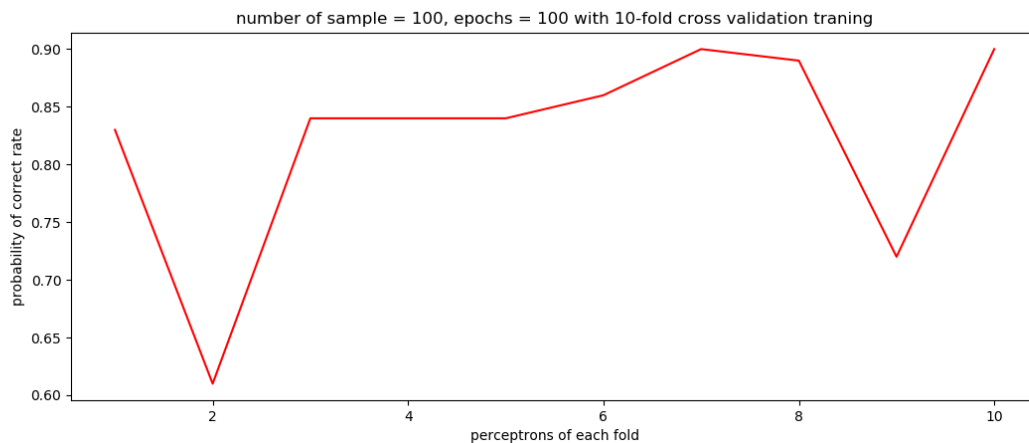
then soft  $f(x) = \ln(1 + e^x)$

where take the best model for prediction

(softmax will more prefer in higher accuracy of estimation).

a. Training 100 data for getting the model in Keras library in python

```
2019-11-26 11:56:36.626017: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1618] Found device 0 with properties:
name: GeForce RTX 2060 major: 7 minor: 5 memoryClockRate(GHz): 1.83
pciBusID: 0000:01:00:0
2019-11-26 11:56:36.628563: I tensorflow/stream_executor/platform/default/dlopen_checker_stub.cc:25] GPU libraries are statically linked, skip dlopen che
ck.
2019-11-26 11:56:36.630399: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1746] Adding visible gpu devices: 0
2019-11-26 11:56:36.631867: I tensorflow/core/platform/cpu_feature_guard.cc:142] Your CPU supports instructions that this TensorFlow binary was not compi
led to use: AVX2
2019-11-26 11:56:36.635608: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1618] Found device 0 with properties:
name: GeForce RTX 2060 major: 7 minor: 5 memoryClockRate(GHz): 1.83
pciBusID: 0000:01:00:0
2019-11-26 11:56:36.637793: I tensorflow/stream_executor/platform/default/dlopen_checker_stub.cc:25] GPU libraries are statically linked, skip dlopen che
ck.
2019-11-26 11:56:36.639810: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1746] Adding visible gpu devices: 0
2019-11-26 11:56:37.053994: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1159] Device interconnect StreamExecutor with strength 1 edge matrix:
2019-11-26 11:56:37.056166: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1165] 0
2019-11-26 11:56:37.057225: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1178] 0: N
2019-11-26 11:56:37.058849: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1304] Created TensorFlow device (/job:localhost/replica:0/task:0/device:GP
U:0 with 4606 MB memory) -> physical GPU (device: 0, name: GeForce RTX 2060, pci bus id: 0000:01:00:0, compute capability: 7.5)
2019-11-26 11:56:37.761780: I tensorflow/stream_executor/platform/default/dso_loader.cc:44] Successfully opened dynamic library cublas64_100.dll
Accuracy Rate on Train Data 100: 0.8999999761581421
[0.26291084468364717, 0.9200000166893005]
Accuracy Rate on Test Data 100: 0.9200000166893005
The confusion matrix is as below:
[[10 1 1 2]
 [ 1 32 0 0]
 [ 0 1 15 0]
 [ 1 0 1 35]]
Running Time = 186.8040363
Press any key to continue . . .
```



Training the 100 samples data with neural network with 2 layers, with 1<sup>st</sup> layer setting the 'tanh' activation function, and the 2<sup>nd</sup> layer setting the 'softplus' activation function.

After the training, the best node, bias and weight will pass to the model of Keras object.  
Let this object test the test data of 100 samples.

The result shows the confusion matrix, which allow us to find the accurate estimation  
In 100 testing sample and 100 10-fold training.  
True number of elements = [14,33,16,37]

Correct in label 1 = 10  
Correct in label 2 = 32  
Correct in label 3 = 15

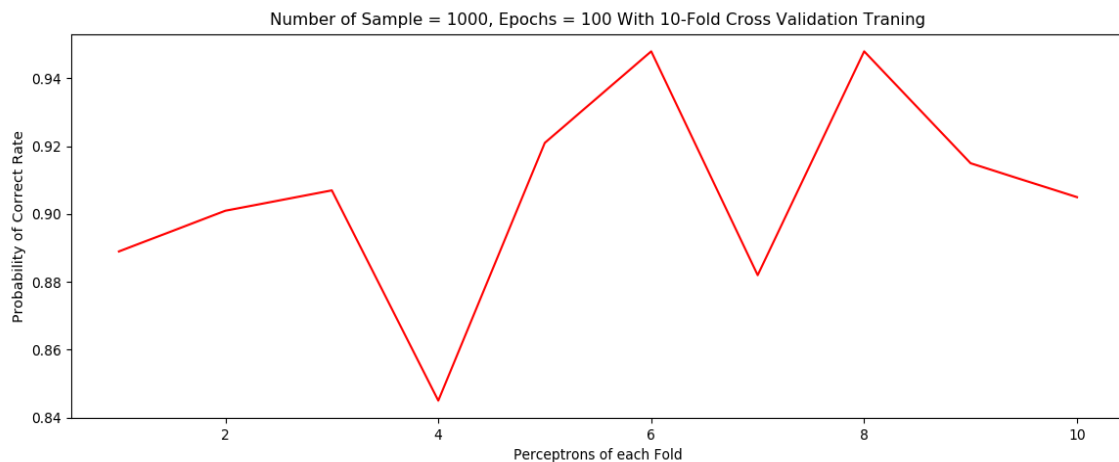


Correct in label 4 = 35    Accuracy =  $(10+32+15+35)/100 = 0.92$

In this case, we can observe that there might have bad training case beget to low probability of accuracy rate.

b. Training 1000 data for getting the model in Keras library in python

```
name: GeForce RTX 2060 major: 7 minor: 5 memoryClockRate(GHz): 1.83
pciBusID: 0000:01:00.0
2019-11-26 11:15:14.389384: I tensorflow/stream_executor/platform/default/dlopen_checker_stub.cc:25] GPU libraries are statically linked, skip dlopen check.
2019-11-26 11:15:14.391043: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1746] Adding visible gpu devices: 0
2019-11-26 11:15:14.392416: I tensorflow/core/platform/cpu_feature_guard.cc:142] Your CPU supports instructions that this TensorFlow binary was not compiled to use: AVX2
2019-11-26 11:15:14.396127: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1618] Found device 0 with properties:
name: GeForce RTX 2060 major: 7 minor: 5 memoryClockRate(GHz): 1.83
pciBusID: 0000:01:00.0
2019-11-26 11:15:14.398924: I tensorflow/stream_executor/platform/default/dlopen_checker_stub.cc:25] GPU libraries are statically linked, skip dlopen check.
2019-11-26 11:15:14.400861: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1746] Adding visible gpu devices: 0
2019-11-26 11:15:14.838404: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1159] Device interconnect StreamExecutor with strength 1 edge matrix:
2019-11-26 11:15:14.840284: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1165]      0
2019-11-26 11:15:14.841407: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1178] 0:  N
2019-11-26 11:15:14.842676: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1304] Created TensorFlow device (/job:localhost/replica:0/task:0/device:GPU:0 with 4606 MB
memory) -> physical GPU (device: 0, name: GeForce RTX 2060, pci bus id: 0000:01:00.0, compute capability: 7.5)
2019-11-26 11:15:15.533489: I tensorflow/stream_executor/platform/default/dso_loader.cc:44] Successfully opened dynamic library cublas64_100.dll
Accuracy Rate on Train Data 1000: 0.930000071525574
[0.2001308917992677, 0.9340000152587891]
Accuracy Rate on Test Data 1000: 0.9340000152587891
The confusion matrix is as below:
[[124   1   0  16]
 [  8 330  15   0]
 [  0  12 191   0]
 [ 13   0  1 289]]
Running Time = 2318.3332645
Press any key to continue . . .
```



Same as the former training in 100 samples. We take the first layer with activation function 'tanh', and the second layer with activation function 'softplus'.

After getting the best model in the training, fit the model to test samples.

The result shows the confusion matrix, which allow us to find the accurate estimation

In 1000 testing sample and 1000 10-fold training.

True number of elements = [141,353,203,303]

Correct in label 1 = 124

Correct in label 2 = 330

Correct in label 3 = 191

Correct in label 4 = 289    Accuracy =  $(124+330+191+289)/1000 = 0.934$

Same here we could observe that some bas case might be caught for training, but the accuracy rate is better than 100 sample case. Which means the probability of pick bad training module would be lower.

c. Training 10000 data with epoch 30 for getting the model in Keras library in python

```
Training Samples with Neural Network
Training for 10000 samples...
2019-11-26 12:08:52.870565: I tensorflow/stream_executor/platform/default/dso_loader.cc:44] Successfully opened dynamic library nvcuda.dll
2019-11-26 12:08:52.889310: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1618] Found device 0 with properties:
name: GeForce RTX 2060 major: 7 minor: 5 memoryClockRate(GHz): 1.83
pciBusID: 0000:01:00:0
2019-11-26 12:08:52.891498: I tensorflow/stream_executor/platform/default/dlopen_checker_stub.cc:25] GPU libraries are statically linked, skip dlopen check.
2019-11-26 12:08:52.893127: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1746] Adding visible gpu devices: 0
2019-11-26 12:08:52.894493: I tensorflow/core/platform/cpu_feature_guard.cc:142] Your CPU supports instructions that this TensorFlow binary was not compiled to use: AVX2
2019-11-26 12:08:52.898533: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1618] Found device 0 with properties:
name: GeForce RTX 2060 major: 7 minor: 5 memoryClockRate(GHz): 1.83
pciBusID: 0000:01:00:0
2019-11-26 12:08:52.901131: I tensorflow/stream_executor/platform/default/dlopen_checker_stub.cc:25] GPU libraries are statically linked, skip dlopen check.
2019-11-26 12:08:52.902839: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1746] Adding visible gpu devices: 0
2019-11-26 12:08:53.314299: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1159] Device interconnect StreamExecutor with strength 1 edge matrix:
2019-11-26 12:08:53.315878: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1165]
2019-11-26 12:08:53.316784: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1178] 0:  N
2019-11-26 12:08:53.318088: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1304] Created TensorFlow device (/job:localhost/replica:0/task:0/device:GPU:0 with 4606 MB memory)
-> physical GPU (device: 0, name: GeForce RTX 2060, pci bus id: 0000:01:00:0, compute capability: 7.5)
2019-11-26 12:08:54.053277: I tensorflow/stream_executor/platform/default/dso_loader.cc:44] Successfully opened dynamic library cublas64_100.dll
Accuracy Rate on Train Data 10000: 0.9279999732571191
[0.21498292227983476, 0.930899977684021]
Accuracy Rate on Test Data 10000: 0.930899977684021
The confusion matrix is as below:
[[1308  58  14 137]
 [  4 3346 129  0]
 [ 14 173 1789 49]
 [ 103  0 10 2866]]
Running Time = 11690.229969499998
Press any key to continue . . .
```



Same method as former two experiment

We get the confusion matrix that we could get the accurate rate and the wrong estimation number

True number of elements in 10000 data = [1517,3479,2025,2979]

Correct in label 1 = 1308

Correct in label 2 = 3346

Correct in label 3 = 1789

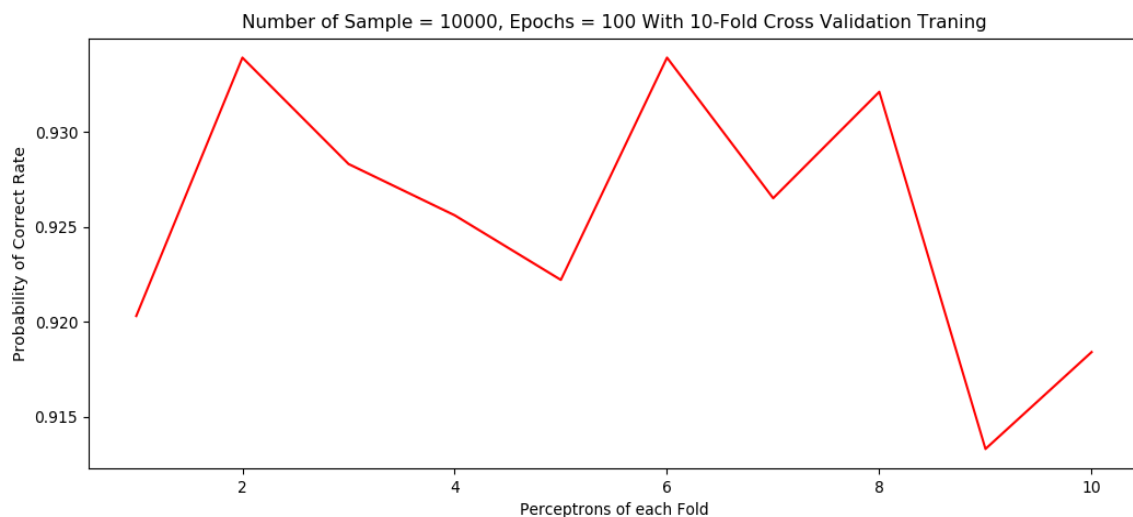
Correct in label 4 = 2866      Accuracy = 9309/10000 = 0.9309

This result does not show a better accuracy than previous two experiment since the epoch = 30 which is lower than 100

Thus, take epochs = 100 in the following experiment

d. Training 10000 data with epoch 100 for getting the model in Keras library in python

```
2019-11-26 17:19:50.339493: I tensorflow/stream_executor/platform/default/dso_loader.cc:44] Successfully opened dynamic library cudart64_100.dll
Using TensorFlow backend.
Training Samples with Neural Network
Training for 10000 samples...
2019-11-26 17:19:53.494953: I tensorflow/stream_executor/platform/default/dso_loader.cc:44] Successfully opened dynamic library nvcuda.dll
2019-11-26 17:19:53.516182: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1618] Found device 0 with properties:
name: GeForce RTX 2060 major: 7 minor: 5 memoryClockRate(GHz): 1.83
pciBusID: 0000:01:00.0
2019-11-26 17:19:53.518668: I tensorflow/stream_executor/platform/default/dlopen_checker_stub.cc:25] GPU libraries are statically linked, skip dlopen check.
2019-11-26 17:19:53.521407: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1746] Adding visible gpu devices: 0
2019-11-26 17:19:53.523411: I tensorflow/core/platform/cpu_feature_guard.cc:142] Your CPU supports instructions that this TensorFlow binary was not compiled to use: AVX2
2019-11-26 17:19:53.527470: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1618] Found device 0 with properties:
name: GeForce RTX 2060 major: 7 minor: 5 memoryClockRate(GHz): 1.83
pciBusID: 0000:01:00.0
2019-11-26 17:19:53.530077: I tensorflow/stream_executor/platform/default/dlopen_checker_stub.cc:25] GPU libraries are statically linked, skip dlopen check.
2019-11-26 17:19:53.532934: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1746] Adding visible gpu devices: 0
2019-11-26 17:19:53.963873: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1159] Device interconnect StreamExecutor with strength 1 edge matrix:
2019-11-26 17:19:53.965539: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1165] 0
2019-11-26 17:19:53.966486: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1178] 0: N
2019-11-26 17:19:53.967837: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1304] Created TensorFlow device (/job:localhost/replica:0/task:0/device:GPU:0 with 4606 MB memory) -> physical GPU (device: 0, name: GeForce RTX 2060, pci bus id: 0000:01:00.0, compute capability: 7.5)
2019-11-26 17:19:54.664314: I tensorflow/stream_executor/platform/default/dso_loader.cc:44] Successfully opened dynamic library cublas64_100.dll
Accuracy Rate on Train Data 10000: 0.9300000071525574
[0.17136851172447204, 0.9384999871253967]
Accuracy Rate on Test Data 10000: 0.9384999871253967
The confusion matrix is as below:
[[1323  26  25 143]
 [ 11 3397  71   0]
 [ 22 197 1787 19]
 [ 76   0  25 2878]]
Running Time = 41445.527304999996
Press any key to continue . . .
```



Here I tried the different if it can get a better result for accurate rate

Increase the epoch from 30 to 100.

The accurate rate: 0.9384

This accurate is a little better than the former 1000 training case

Conclusion of Question 1:

By the previous experiment, NN training is a little better than MAP classification. But it depends on the amount of training sample. As the number of training epochs increase, the accuracy supposed to be increase.



Question 2

Similar as Question 1.5 part 3.

create a neural network but with only 1 layer.

$\Rightarrow$  data =  $[X_1, X_2]^T$   $X_1$  is the input  $X_2$  is the target

$$\hat{\theta}_{NN} = \operatorname{argmax}_{\theta} \sum_{i=1}^N \ln P(x_i, d_i | \theta) \quad \text{where } d_i \approx f(x_i, \theta)$$

$$\Rightarrow \hat{\theta}_{NN} = \operatorname{argmax}_{\theta} \sum_{i=1}^N \ln P(d_i | x_i, \theta) \quad \begin{array}{l} \text{since independent model} \\ \Rightarrow \text{prior } P(x_i | \theta) \text{ ignored} \end{array}$$

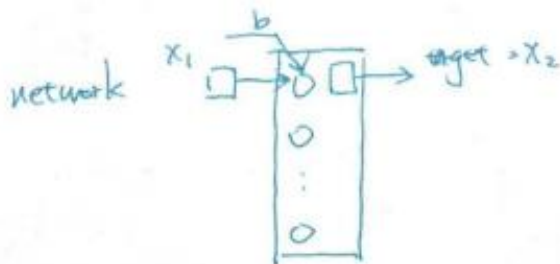
$$\Rightarrow N = \ln\left(\frac{1}{\sqrt{2\pi}\sigma^2}\right) + \frac{(x-\mu)^2}{2\sigma^2} \quad (\text{by gaussian formula.})$$

$$\Rightarrow \text{find minimum of } N \Rightarrow N = -(x-\mu)^2$$

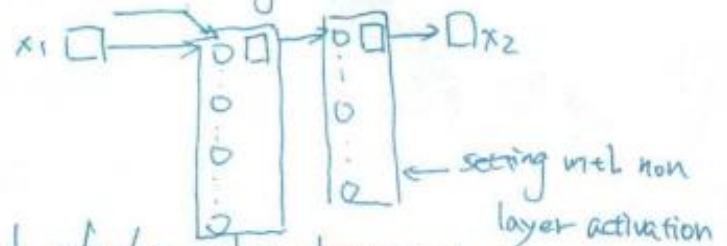
$$\Rightarrow \hat{\theta}_{NN} = \operatorname{argmin}_{\theta} - \sum_{i=1}^N (x_i - \mu)^2$$

$$\Rightarrow \hat{\theta}_{NN} = \operatorname{argmin}_{\theta} - \frac{1}{N} \sum_{i=1}^N (d_i - f(x_i, \theta))^2 \quad \text{where means MSE}$$

Train the data find MSE be converged when difference  $< 0.1$ .



$\Rightarrow$  could be design as

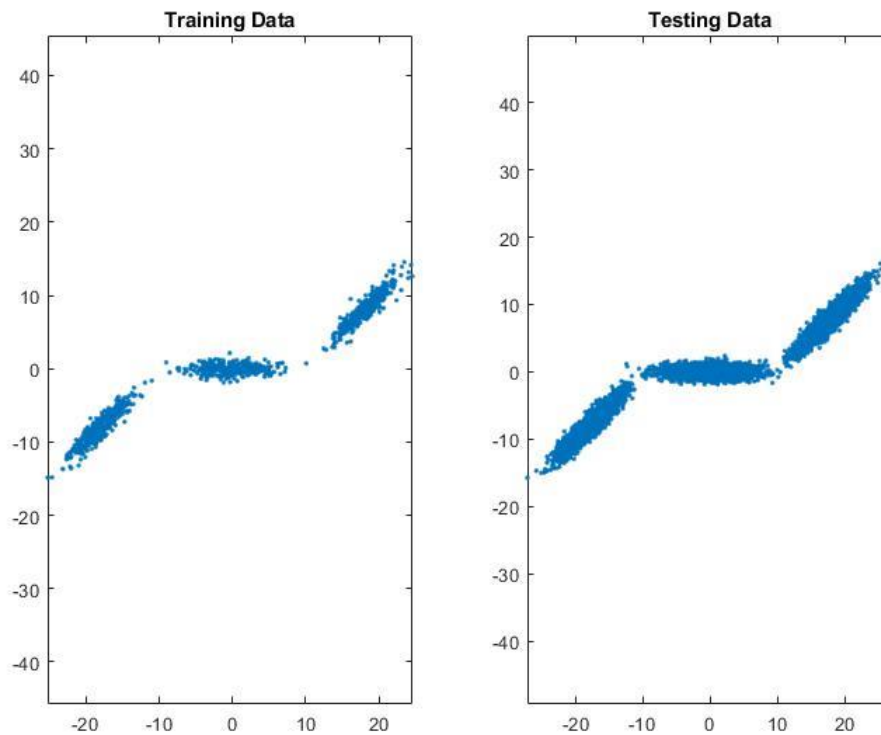


Select between logistic and softmax when determine the final model for testing 10000 data.

Theory and mathematical calculation:

Training and experiment:

The original data of training and testing data



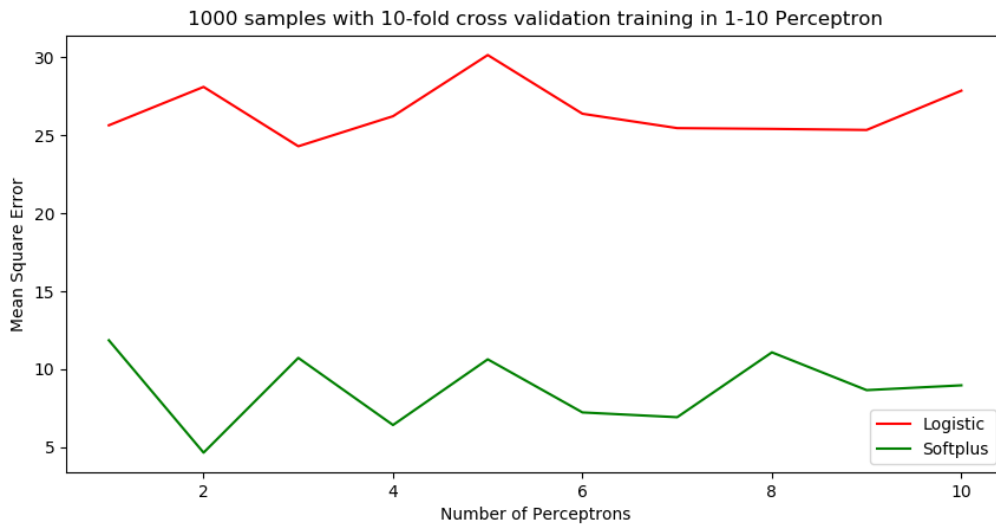
The plots show the original data of training and testing which are 1000 and 10000 samples  
Training is based on finding the mean square error of inputs  $x_1$ . When finding a newMSE, compare the newMSE to the old MSE value, if the difference is smaller than a value (define as epsilon), determine as converged.

With the method finding the best argmin value in the mathematical part show as the best model

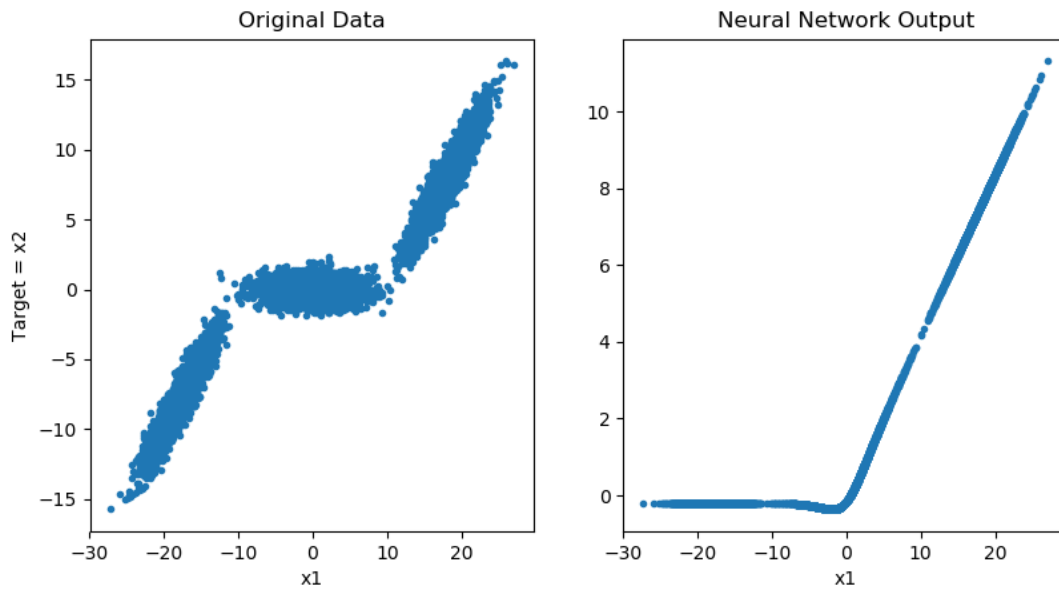
Separate the training data as 10-Fold to train and validate the model.  
Then apply this model to 10000 testing data

Tried the following epoch to observe that the final prediction is getting like the original testing data  
epochs = 1  
epochs = 30  
epochs = 50  
epochs = 100

## Exam 2

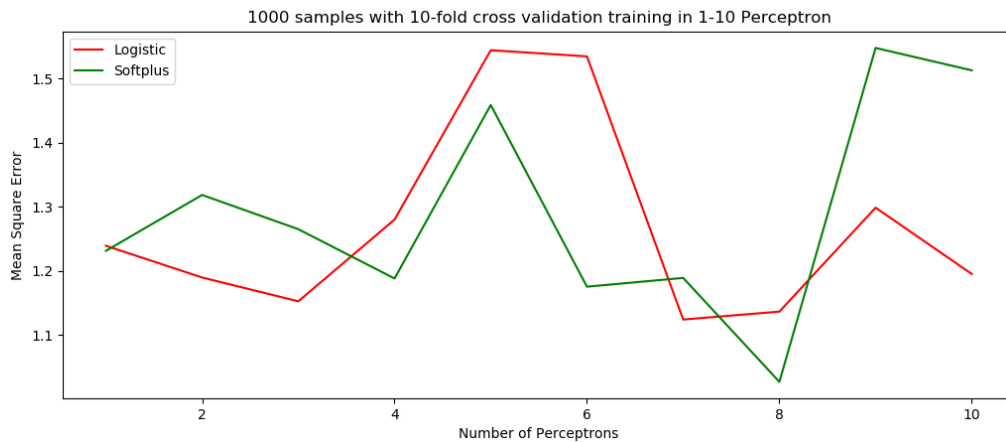


This result shows the NN training only with epochs = 1, both logistic and softplus have large MSE  
The result could be bad as below

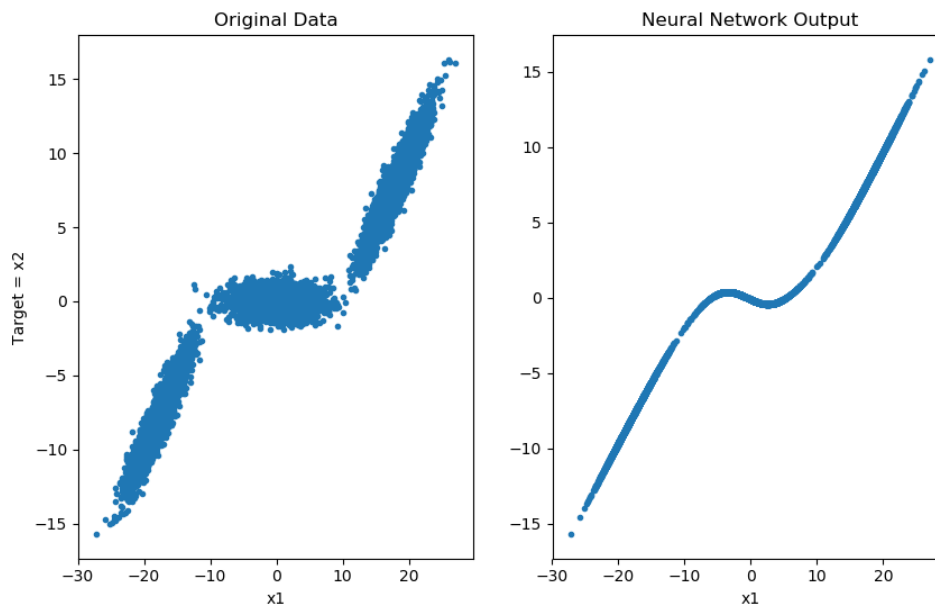


Now we increase the number of epochs to 50 see that if it is better

## Exam 2



With 50 Epochs, the MSE become smaller. Thus, we might have better estimation



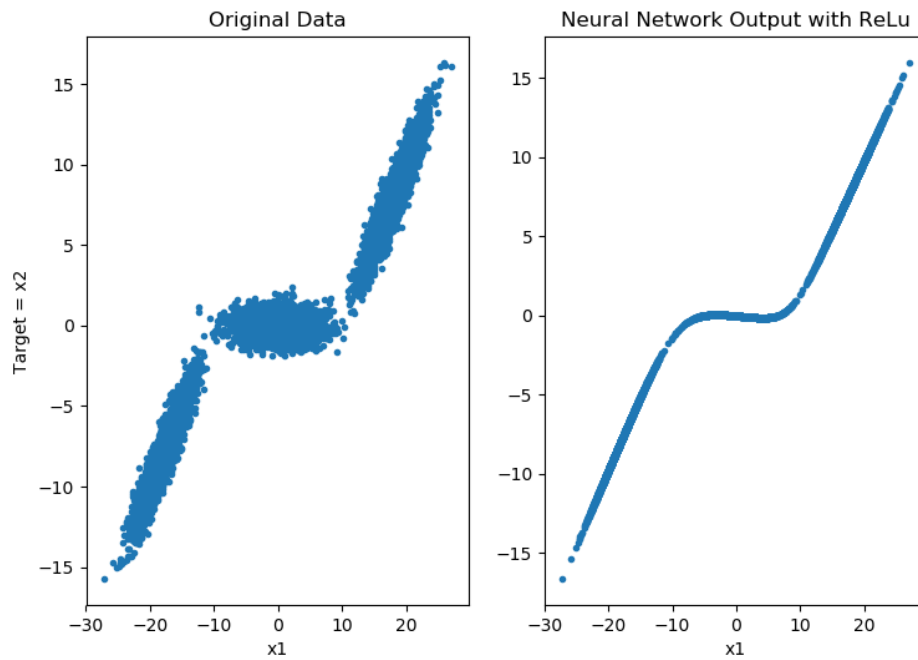
```
0.9630711078643799
Score in each train_validation determination 0.9630711078643799
MSE on Test Data: 0.6719991505146027
Running Time = 4749.9818935
Press any key to continue . . .
```

These plot shows better than the former one. But the middle data show two curves (not sharp turn) because the data distribution of Target ( $x_2$ ) is around 0 with Gaussian distribution. The MSE performance = 0.6720

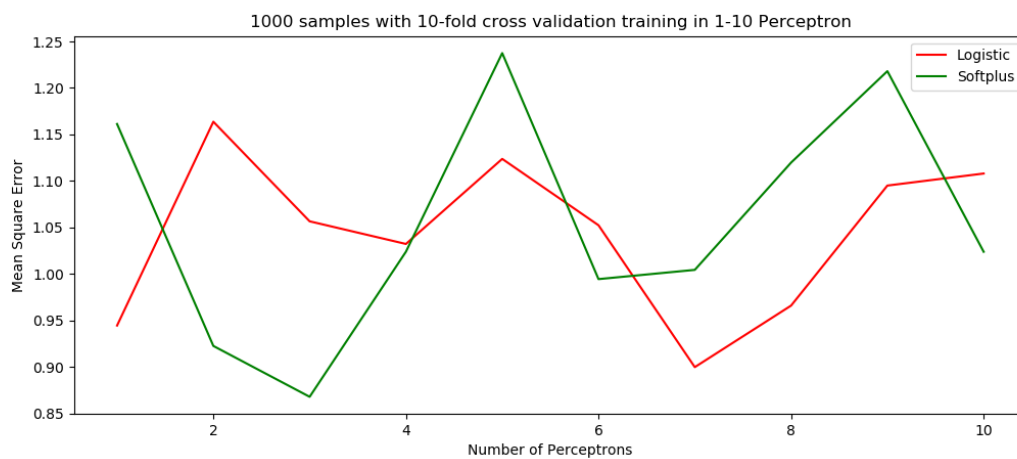
The Neural Network training is supposed to be like linear combination of two activation function. The expect result will be like SmoothReLU.

Thus, try the relu function in Keras to show the case

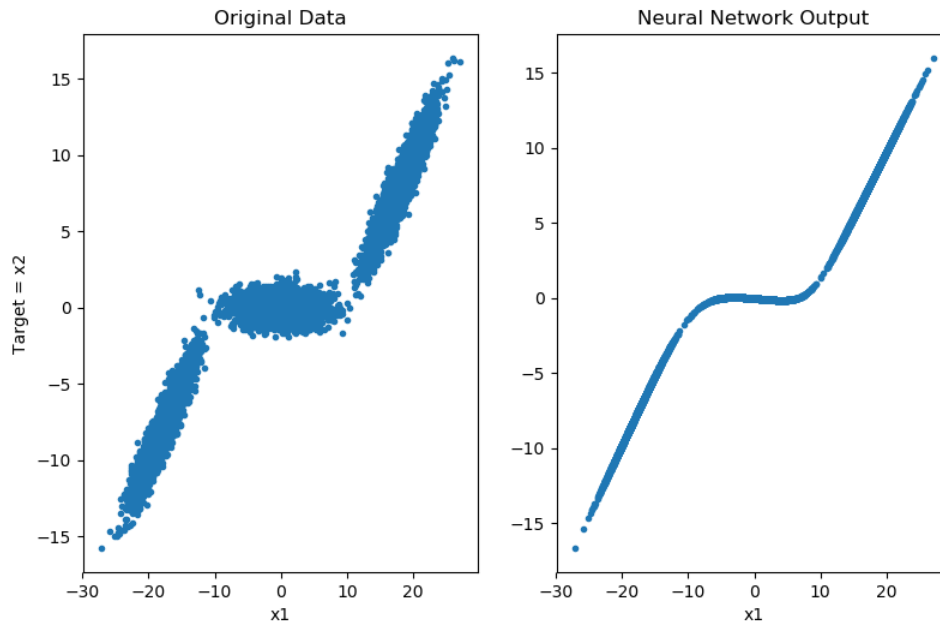




ReLu shows the expected result of Neural Network training result  
Now we try to increase the epochs = 100 to see if it is getting similar to ReLu to check the neural network is create correctly



The plot shows the logistic and softplus activation function's MSE performance in each perceptron



The 100 epochs result seems better than the 50-epochs' one. This indicate the neural network is created correctly.

Here the performance MSE = 0.6016

```
0.7053458261489868
Score in each train_validation determination 0.7053458261489868
MSE on Test Data: 0.600523956155777
10000/10000 [=====] - 0s 28us/step
10000/10000 [=====] - 0s 28us/step
10000/10000 [=====] - 0s 27us/step
10000/10000 [=====] - 0s 26us/step
10000/10000 [=====] - 0s 27us/step
10000/10000 [=====] - 0s 28us/step
10000/10000 [=====] - 0s 26us/step
10000/10000 [=====] - 0s 29us/step
10000/10000 [=====] - 0s 27us/step
10000/10000 [=====] - 0s 27us/step
10000/10000 [=====] - 0s 30us/step
10000/10000 [=====] - 0s 28us/step
10000/10000 [=====] - 0s 26us/step
10000/10000 [=====] - 0s 27us/step
10000/10000 [=====] - 0s 27us/step
10000/10000 [=====] - 0s 27us/step
MSE on Test Data in Smooth ReLu: 0.6015588176727295
Running Time = 6372.7752212000005
```

Conclusion of Question 2:

The more training epochs, the more like the desire result. Also, the Mean Square Error will become smaller. The final 100 epochs training is similar to the ReLu activation function which is desire that the neural network being.

Reference:

<https://keras.io/backend/>  
<https://github.com/keras-team/keras/tree/master/examples>  
<https://www.tensorflow.org/>  
<https://www.tensorflow.org/tutorials/keras/classification>  
[https://scikit-learn.org/stable/model\\_selection.html#model-selection](https://scikit-learn.org/stable/model_selection.html#model-selection)  
[https://matplotlib.org/3.1.1/api/\\_as\\_gen/matplotlib.pyplot.html](https://matplotlib.org/3.1.1/api/_as_gen/matplotlib.pyplot.html)  
<https://www.mathworks.com/help/thingspeak/create-and-train-a-feedforward-neural-network.html>  
<https://www.mathworks.com/help/deeplearning/ref/network.html>  
<https://www.mathworks.com/help/deeplearning/ref/mse.html>  
<https://www.mathworks.com/help/deeplearning/ug/create-and-train-custom-neural-network-architectures.html>  
<https://www.mathworks.com/help/deeplearning/ref/train.html#namevaluepairarguments>

Code Resource:

<https://github.com/MakiseYuki/EECE5644-Machine-Learning/tree/master/Exam%202>