Page 1

$$A = \begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ 1 & -13 & 14 & -13 \end{pmatrix}$$

Set up to find the eigenvalues,

$$\det(A - \lambda I) = 0$$

That is

$$\begin{vmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ 1 & -13 & -14 & -13-\lambda \end{vmatrix} = 0$$

Determinant

The determinant of a 4x4 Matrix can be expand as:

$$\det(A-\lambda I) = (4-\lambda)\begin{vmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{vmatrix} - 8\begin{vmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{vmatrix}$$

$$+(-1)\begin{vmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{vmatrix} - (-2)\begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix}$$

Let expand along the first row

$$M_{11} = \begin{vmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{vmatrix}$$

Expand along the first row

$$= (-9-\lambda)\begin{vmatrix} 5-\lambda & -10 \\ -14 & -13-\lambda \end{vmatrix} - (-2)\begin{vmatrix} 10 & -10 \\ -13 & -13-\lambda \end{vmatrix}$$

$$+(-4)\begin{vmatrix} 10 & 5-\lambda \\ -13 & -14 \end{vmatrix}$$

Now, expand each 2x2 minor

$$\begin{vmatrix} 5-\lambda & -10 \\ -14 & -13-\lambda \end{vmatrix} = (5-\lambda)(-13-\lambda) - (-10)(-14)$$

$$= -(5-\lambda)(13+\lambda) - 140$$

$$\begin{vmatrix} 10 & -10 \\ -13 & -13-\lambda \end{vmatrix} = 10(-13-\lambda) - (-10)(-13) = -130 -$$

$$10\lambda + 130 = -10\lambda$$

$$\begin{vmatrix} 10 & 5-\lambda \\ -13 & -14 \end{vmatrix} = 10 \cdot (-14) - (-13)(5-\lambda) = -140 +$$

$$13(5-\lambda) = -140 + 65 - 13\lambda = -75$$

So

$$M_{11} = (-9-\lambda)[-(5-\lambda)(13+\lambda) - 140] + 2$$
$$[-10\lambda] - 4(-75 - 13\lambda)$$

Now expand

$$M_{11} = (-9-\lambda)[-(5-\lambda)(13+\lambda) - 140]$$
$$20\lambda + 300 + 52\lambda$$

$$P(\lambda) = a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$$

$$\lambda = \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 7,$$
$$\pm 8, \pm 9, \pm 10,$$

Evaluate $P(\lambda)$ at Integers

$$P(1) = a_4(1)^4 + a_3(1)^3 + a_3(1)^3 + a_2(1)^2$$
$$+ a_1(1)^2 + a_1(1) + a_0$$

$\lambda = 1$ is a root.

$$\lambda_4 = -21.125$$

Page 3

Set up the System for Eigenvector
plug λ_4 into $A-\lambda_4 I$:

$$A-\lambda_4 I = \begin{pmatrix} 4-(-21.125) & 8 & -1 & -2 \\ -2 & -9-(-21.125) & -2 & -4 \\ 0 & 10 & 5-(-21.125) & -10 \\ -1 & -13 & -14 & -13-(-21.125) \end{pmatrix}$$

$$25.125x_1 + 8x_2 - x_3 - 2x_4 = 0$$

$$-2x_1 + 12.125x_2 - 2x_3 - 4x_4 = 0$$
$$10x_2 + 26.125x_3 - 10x_4 = 0$$
$$-x_1 - 13x_2 - 14x_3 + 8.125x_4 = 0$$

Row3 gives

$$10x_2 + 26.125x_3 - 10x_4 = 0 \Rightarrow x_2$$
$$= \frac{10x_4 - 26.125x_3}{10}$$

Row 1:

$$25.125x_1 + 8x_2 - x_3 - 2x_4 = 0$$

$$25.125x_1 + 8\left(\frac{10x_4 - 26.125x_3}{10}\right) - x_3 - 2x_4 = 0$$

$$25.125x_1 + 8x_4 - 20.9x_3 - x_3 - 2x_4 = 0$$
$$25.125x_1 + 6x_4 - 21.9x_3 = 0$$
$$x_1 = \frac{21.9x_3 - 6x_4}{25.125}$$

Page 4

Plug x_3 into Row 2

$$-2x_1 + 12.125x_2 - 2x_3 - 4x_4 = 0$$

$$-2x_1 + 12.125\left(\frac{10x_4 - 26.125x_3}{10}\right) - 2x_3 - 4x_4$$

$$= 0$$

$$-2x_1 + 8.125x_4 - 33.726x_3 = 0$$

$$2x_1 = 8.125x_4 - 33.726x_3 = 0$$

$$2x_1 = 8.125x_4 - 33.726x_3$$

$$x_1 = \frac{8.125x_4 - 33.726x_3}{2}$$

Set equal to the previous expression for x_1:

$$\frac{21.9x_3 - 6x_4}{25.125} = \frac{8.125x_4 - 33.726x_3}{2}$$

eigenvectors

$$v_1 \begin{pmatrix} -0.036 \\ 0.958 \\ 0.025 \\ 0.563 \end{pmatrix} \qquad v_4 = \begin{pmatrix} 0.509 \\ -0.113 \\ -0.915 \\ -0.053 \end{pmatrix}$$

$$v_2 \begin{pmatrix} -0.012 \\ -0.161 \\ -0.335 \\ -0.616 \end{pmatrix}$$

$$v_3 \begin{pmatrix} -0.860 \\ 0.207 \\ -0.222 \\ 0.546 \end{pmatrix}$$

Page 5

- Importance Of Each Eigenvalue

$$\text{Improtance} = \frac{|\lambda_i|}{\sum_{j=1}^{4} |\lambda_j|} \times 100$$

$$S = |11.054| + |2.675| + |5.604| + |21.125| = 40.458$$

$$\lambda_4 : \frac{21.125}{40.458} \times 100\%$$

$$= 52.21\%$$

Eigenvalue : $\lambda_4 = -21.125$

Eigenvector : $V_4 \begin{pmatrix} 0.509 \\ -0.113 \\ -0.915 \\ -0.033 \end{pmatrix}$

Importance : 52.21%