PCA matri

$$A = \begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

$$\det(A - \lambda I) = 0.$$

$$A - \lambda I = \begin{vmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{vmatrix}$$

$$\det(A - \lambda I) = (4-\lambda)\det(M_{11}) - (8)\det(M_{12}) + (-1)\det(M_{13}) + (2)\det$$

$$\det(M_{11}) = \begin{vmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{vmatrix}$$

$$= (-9-\lambda)[(5-\lambda)(-13-\lambda) - (-10)(-14)] - (-2)[(10)(-13-\lambda) - (-10)$$
$$+ (-4)[(10)(-14) - (5-\lambda)(-13)]$$

$$= (-9-\lambda)[-65 - 5\lambda + 13\lambda + \lambda^2 - 140] + 2[-130 - 10\lambda - 130] -$$
$$+ 65 - 13\lambda]$$

$$= (-9-\lambda)[\lambda^2 + 8\lambda - 205] + 2[-260 - 10\lambda] - 4[-75 - 13\lambda]$$

$$= -9\lambda^2 - 72\lambda + 1845 - \lambda^3 - 8\lambda^2 + 205\lambda - 520 - 20\lambda + 300 + 52\lambda$$

$$= -\lambda^3 - 17\lambda^2 + 165\lambda + 1625.$$

$$\det(M_{12}) = \begin{vmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{vmatrix} \quad \begin{vmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{vmatrix}$$

$$= (-2)[(5-\lambda)(-13-\lambda) - (-10)(-14)] - (-2)[0(-13-\lambda) - (-10)(-1)] +$$
$$(-4)[0(-14) - (5-\lambda)(-1)].$$

$$= (-2)[(5-\lambda)(-13-\lambda) - 140] + 2[0 - 10] - 4[0 + (5-\lambda)]$$

$$= (-2)[-65 - 5\lambda + 13\lambda + \lambda^2 - 140] + 2[-10] - 4[5-\lambda]$$

$$= (-2)[\lambda^2 + 8\lambda - 205] - 20 - 20 - 4\lambda$$

$$= -2\lambda^2 - 16\lambda + 410 - 40 - 4\lambda$$

$$= -2\lambda^2 - 12\lambda + 390$$

$$\det(M_{13}) = \begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix}$$

$$\begin{vmatrix} & -2 \\ & -4 \\ \lambda & -10 \\ 14 & -13-\lambda \end{vmatrix}$$

$M_{11}) - (8) \det(M_{12}) + (-1) \det(M_{13}) + (2) \det(M$

$)-[-10)[-14)] - (-2)[(10)(-13-\lambda) - (-10)[-13]]$
$-(5-\lambda)[+13)]$

$13\lambda + \lambda^2 - 140] + 2[-130 - 10\lambda - 130] - 4[-140$

$05] + 2[-260 - 10\lambda] - 4[-75 - 13\lambda]$

$-\lambda^3 - 8\lambda^2 + 205\lambda - 520 - 20\lambda + 300 + 52\lambda$

$$\det(M_{13}) = (-2)[10(-13-\lambda) - (-10)(-13)] - (-9-\lambda)[0(-13-\lambda) - (-10)(-1)] + (-4)[0(-13) - 10(-1)]$$

$$= (-2)[-130 - 10\lambda - 130] + (9+\lambda)[0 - 10] - 4[0 + 10]$$

$$= (-2)[-260 - 10\lambda] + (9+\lambda)[-10] - 40$$

$$= 520 + 20\lambda - 90 - 10\lambda - 40$$

$$= 10\lambda + 390$$

$$\det(M_{14}) = \begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix}$$

$$= (-2)[10(5-\lambda) - (-10)(-13)] - (-9-\lambda)[0(5-\lambda) - (-10)(-1)] + (-2)[0(-13) - 10(-1)]$$

$$= (-2)[50 - 10\lambda - 130] + (9+\lambda)[0 - 10] - 2[0 + 10]$$

$$= (-2)[-80 - 10\lambda] + (9+\lambda)[-10] - 20$$

$$= 160 + 20\lambda - 90 - 10\lambda - 20$$

$$= 10\lambda + 50$$

$$\det(A - \lambda I) = (4-\lambda)(-\lambda^3 - 17\lambda^2 + 165\lambda + 1625) - 8(-2\lambda^2 - 12\lambda + 370) + (-1)(10\lambda + 390) - (-2)(10\lambda + 50)$$

$$= \cancel{(4-\lambda}$$

$$= -4\lambda^3 - 68\lambda^2 + 660\lambda + 6500 + \lambda^4 + 17\lambda^3 - 165\lambda^2 - 1625\lambda + 16\lambda^2 + 96\lambda - 2960 - 10\lambda - 390 + 20\lambda + 100$$

$$= \cancel{-4\lambda^3} - 4\lambda^3 - 68\lambda^2 + 660\lambda + 6500 + \lambda^4 + 17\lambda^3 - 165\lambda^2 - 1625\lambda + 16\lambda^2 + 96\lambda - 2960 - 10\lambda - 390 + 20\lambda + 100$$

$$= \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500$$

~~So $p(x) = x$~~

So $p(\lambda) = \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500$

By Analyzing the polynomial's behavior.

- for $\lambda = 0$. $p(\lambda) = 3500 > 0$.
- for $\lambda = -8$ $p(-8) = (-8)^4 + 13(-8)^3 - 219(-8)^2 - 835(-8) + 3500$
  $= 4096 + 13(-512) - 219 \cdot 64 + 835 \cdot 8 + 3500$
  $= 14276 - 20672 = -6396 < 0$

for ⊛ $\lambda = -6 \iff p(-6) = (-6)^4 + 13(-6)^3 - 219(-6^2) - 835(-6) + 3500$

$= 1296 - 2808 - 7884 + 5010 + 3500$

$= -886 < 0$

The nearest root is betwen $(-6, -5)$.

⊛ $\lambda = 5 \quad p(5) = (5)^4 + 13(5)^3 - 219(5^2) - 835(5) + 3500$

$= 625 + 13(55) - 219(25) - 4175 + 3500$

$= 625 + 715 - 5475 - 4175 + 3500$

$= -5,433 < 0$

⊛ for $\lambda = -5.603$

$p(-5.603) = (-5.603)^4 + 13(-5.603)^3 - 219(-5.603)^2 - 835(-5.603) + 3500$

$= 985.97 - 2291.97 - 6869.97 + 4685.51 + 3500$

$= 0$

⊛ for $\lambda = 2.606$

$p(2.606) = (2.606)^4 + 13(2.603)^3 - 219(2.606)^2 - 835(2.606) + 3500$

$= 46.13 + 230.13 - 1487.13 - 2176.01 + 3500$

$= 0$

⊛ for $\lambda = 11.058$

$p(11.058) = (11.058)^4 + (13)(11.058)^3 - 219(11.058)^2 - 835(11.058)^2 + 3500$

$= 14934.97 + 17589.97 - 26745.97 - 9233.43 + 3500$

$= 0$

⊛ for $\lambda = -21.061$

$p(-21.061) = (-21.061)^4 + (13)(-21.061)^3 - 219(-21.061)^2 - (835)(-21.061) + 3500$

$= 196683.97 - 120853.97 - 97089.97 + 17585.94 + 3500$

$= 0$

$\Rightarrow$ eigen values, $\lambda_1 = -5.603$, $\lambda_2 = 2.606$, $\lambda_3 = 11.058$, $\lambda_4 = -21.061$

By replacing the eigen values in the matrix

for $\lambda = -5.603$

$$\begin{bmatrix} 9.603 & 8 & -1 & -2 \\ -2 & -3.393 & -2 & -4 \\ 0 & 10 & 10.603 & -10 \\ -1 & -13 & -14 & -7.397 \end{bmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

By solving this we get $= (0.5, 1, 0.5, 1)$.

⊛ for $\lambda_2 = 2.606$

$$\begin{pmatrix} 1.394 & 8 & -1 & -2 \\ -2 & -11.606 & -2 & -4 \\ 0 & 10 & 2.394 & -10 \\ -1 & -13 & -14 & -15.606 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

By solving that we get $\lambda_2 = [-2, 1, -1, 1]$

⊛ for $\lambda_3 = 11.058$

$$\begin{pmatrix} -7.058 & 8 & -1 & -2 \\ -2 & -20.058 & -2 & -4 \\ 0 & 10 & -6.058 & -10 \\ -1 & -13 & -14 & -24.058 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

By solving that $\lambda_3 = [-1, 1, 0, 1]$

⊛ for $\lambda_4 = -21.061$

$$\begin{pmatrix} 25.061 & 8 & -1 & -2 \\ -2 & -20.058 & -2 & -4 \\ 0 & 10 & -6.058 & -10 \\ -1 & -13 & -14 & \cancel{-24.058}\ 8.061 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 25.061 & 8 & -1 & -2 \\ -2 & -20.058 & -2 & -4 \\ 0 & 10 & +26.061 & -10 \\ -1 & -13 & -14 & 8.061 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

By solving that we get $\lambda_4 = [1, 0, 0, 1]$