Introduction

The Turkey data consisted of 13 observations on individual birds, each of which was fed with one of three different feed types until slaughter. The response variable for this analysis is weight (lb) at slaughter for each bird. A complicating factor is that the ages of the birds at slaughter were not uniform and therefore, age was included as a potential predictor along with feed type. The data are show in Table 1.

Table 1: Turkey data.

Age (weeks)	Weight (pounds)	Feed Type
28	13.3	a
20	8.9	a
32	15.1	a
22	10.4	a
29	13.1	b
27	12.4	b
28	13.2	b
26	11.8	b
21	11.5	c
27	14.2	c
29	15.4	c
23	13.1	c
25	13.8	c

The central question for this analysis is to consider any evidence of a differential effect of feed type on weight gain. Table 2 shows the mean weight gain by fed type with upper and lower 95% confidence limits based on the student t distribution. The picture given in Table 2 is mixed. The sample mean estimates suggest that mean weight gain increased from feed type a to b to c. However, the group-wise confidence intervals (displayed in Figure 1) suggest that likely range of true population mean weigh gain cannot be assumed statistically different (i.e. they overlap).

Table 2: Mean Weight gain & 95% CI by feed type.

Feed Type	Mean	Lower 95% CL	Upper 95% CL
a	11.93	7.48	16.37
b	12.62	11.58	13.67
c	13.60	11.81	15.39

A non-trivial complication of these data is that the turkeys were not all slaughtered at the same age nor were the ages at slaughter equally represented in each feed type group. Therefore, it seems sensible to correct for the different ages of the birds when attempting to compare the performance of the three feed types. A scatter plot of age versus weight gain in displayed in Figure 2. It is evident from the plot that a straight line relationship may exist between age and weight for the three feed type and this will form the basis for the statistical analysis.

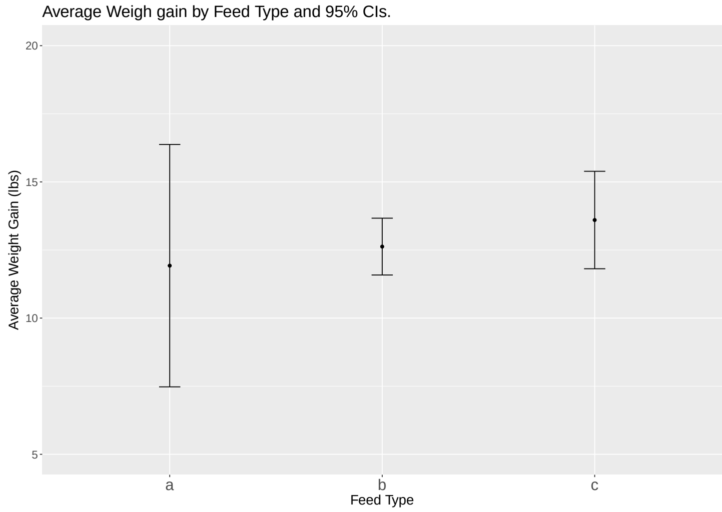

Figure 1. Plot of Average Weight Gain by Feed Type.

Model I

A linear regression model was fitted to these data with age as a continuous predictor and feed as a categorical predictor. To allow maximum flexibility in the initial model, an interaction of age and feed type was included.

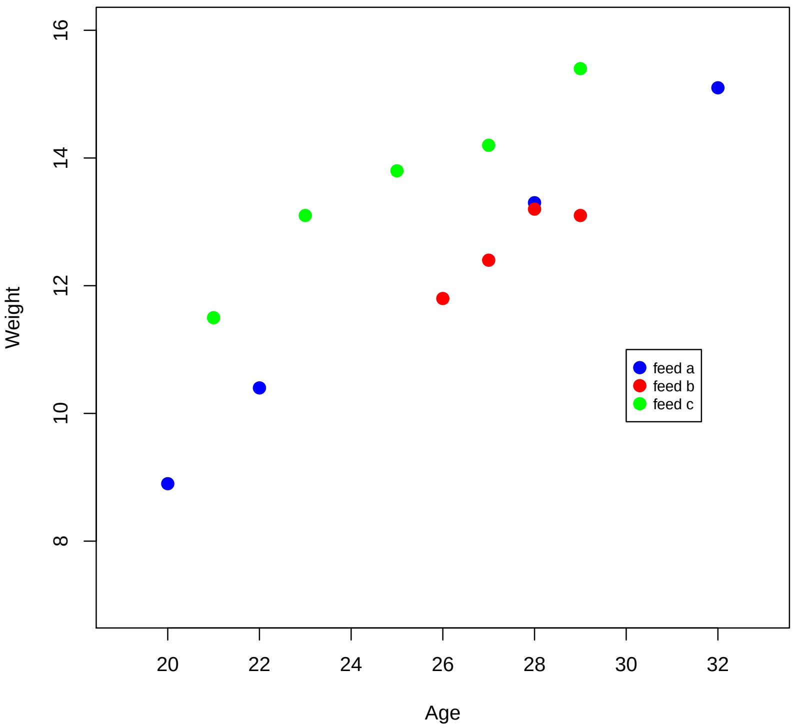

Figure 2. Plot of Age versus Weight - Turkey data.

This model allows for different straight lines (differing in both intercept and slope) to be fitted to each of the three feed types; a model that would be compatible with the different feed types resulting in different rates of weight gain over time. The model was parameterised to allow the null hypothesis of 'common slopes' to be tested.

The model was:

$$y_i = \beta_0 + \beta_1(age_i) + \beta_2(\delta_{ib}) + \beta_3(\delta_{ic}) + \beta_4(\delta_{ib} \times age_i) + \beta_5(\delta_{ic} \times age_i) + \epsilon_i$$

where y_i is the weight for individual i, $\delta_{ib} = 1$ *iff* individual i is getting feed type b and zero otherwise (and similarly for δ_{ic}). This leads to the following regression line equations for each feed type:
type a $= \beta_0 + \beta_1 \times (age)$;
type b $= (\beta_0 + \beta_2) + (\beta_1 + \beta_4) \times (age)$;
type c $= (\beta_0 + \beta_3) + (\beta_1 + \beta_5) \times (age)$.

A test of the hull hypothesis $H_0 : \beta_4 = \beta_5 = 0$ can be interpreted as a test of the 'common slopes assumption', i.e. the the slope for the three feed types is the same but that their intercepts might differ. This hypothesis was tested using the standard formulation for a general linear hypothesis (see R code). The F-test statistic was found to be 0.52 on 2 and 7 degreed of freedom, yielding a p-value of 0.616. Therefore, we failed to reject the null hypothesis and assume that a common slopes model is adequate for the data.

Model II

The following common slope model was then fitted to the data:

$$y_i = \beta_0 + \beta_1(age_i) + \beta_2(\delta_{ib}) + \beta_3(\delta_{ic}) + \epsilon_i$$

The leads to the following regression line equations for each feed type:

type a $= \beta_0 + \beta_1 \times (age)$;
type b $= (\beta_0 + \beta_2) + \beta_1 \times (age)$;
type c $= (\beta_0 + \beta_3) + \beta_1 \times (age)$.

This model allows for a common slope for the relationship between age and weight for each feed type, but different intercepts. The effect of age was statistically significant with a F-test statistics of 357.5 on 1 and 9 degrees of freedom, yielding a p-value < 0.0001. The slope parameter for age was 0.49, suggesting that there was an average weekly increase in weight of 0.49 lb for turkeys on all feed types (95% CI 0.43–0.55). The effect of feed type was also statistically significant, with an F-test statistic of 68.8 on 2 and 9 degrees of freedom yielding a p-value < 0.0001. A quadratic effect of age was also included to test for any evidence of a non straight line relationship between weight gain and age. The quadratic parameter was found to be statistically non-significant (p-value=0.32) and was removed from the model.

Comparison among feed types

A central interest of this analysis was the comparison of effect of the different feed types on weight gain. This analysis gives evidence to suggest that feed type c gives the highest average weight among the turkeys, with a estimated average weight for type c 1.92 lb in excess of type a (95% CI 1.46 to 2.37 lb); and 2.19 lb in excess of type b (95% CI 1.71 to 2.67 lb).

The average weights given by feed types a and b were not found to be statistically different (p-value = 0.24). The point estimate for this difference was an average weight gain for type a 0.27 lb in excess of b, but with a 95% CI between -0.22 (i.e. lower than b) to 0.77 lb (i.e in excess of b).

In conclusion we determine that feed type c yields the highest weights on average, and there is no evidence to suggest that feeds types a and b give different average weights. In addition, although there is strong evidence that weight increases with the age of the bird, there is little evidence in these data to suggest that the rate of weight gain is different between the three feed types used.

```
setwd("DATA")
## read in the data
turkey = read.table("turkey.txt",header=T,sep='')
attach(turkey)
## get plot of feed type means and error bars
#install.packages('ggplot2')
library(ggplot2)
pdf('error_bars.pdf')
group_means=by(weight,feed,t.test)
group_means=matrix(c(unlist(group_means[['a']][5:4]),unlist(group_means[['b']][5:4]),unlist(group_means[['c']][5:4])),nrow=3,ncol=3,byrow=T)
group_means=data.frame(cbind(group_means,c('a','b','c')))
colnames(group_means)=c('mean','lcl','ucl','Feed.Type')
ggplot(group_means, aes(x=Feed.Type,y=group_means[,1])) +
  geom_errorbar(aes(ymin=lcl, ymax=ucl), width=.1) +
  geom_line() + geom_point() +expand_limits(y=c(5,20))+ylab("Average Weight Gain (lbs)")+xlab('Feed Type')+labs(title="Average Weigh gain by Feed Type and 95% CIs.")+theme(text = element_text(size=16),axis.text.x=element_text(size=18))
dev.off()
## get plot of data and optionally output to pdf graphics file
#pdf(file="turkey.pdf")
plot(age[feed=='a'],weight[feed=='a'],pch=20,col='blue',xlab='Age',ylab='Weight',xlim=c(19,33),ylim=c(7,16),cex=2)
points(age[feed=='b'],weight[feed=='b'],pch=20,col='red',cex=2)
points(age[feed=='c'],weight[feed=='c'],pch=20,col='green',cex=2)
legend(30,11,legend=c('feed a','feed b','feed c'),cex=.75,pch=rep(20,3),col=c('blue','red','green'),pt.cex=2)
#dev.off
## fit full interaction model
source("anovatab.R")
fit1=lm(weight~age+factor(feed)+age:factor(feed))
anovatab(fit1)
## test fo rinclusion of the inteaction
drop1(fit1,test='F')
## retreat to common slopes model
fit2=update(fit1,.~.-age:factor(feed))
anovatab(fit2)
drop1(fit2,test='F')
summary(fit2)
confint(fit2)

## check for quadratic effect in age
fit2a=update(fit2,.~.+I(age^2))
drop1(fit2a,test='F')

library(multcomp)
# compariing feed types b and c
L1=matrix(c(0, 0, 1, -1),nrow=1)
glh1=glht(fit2,linfct=L1)
summary(glh1,test=Ftest())
confint(glh1)

# compariing feed types a and b
L2=matrix(c(0, 0, 1, 0),nrow=1)
glh2=glht(fit2,linfct=L2)
summary(glh2,test=Ftest())
confint(glh2)

# compariing feed types a and c
L3=matrix(c(0, 0, 0, 1),nrow=1)
glh3=glht(fit2,linfct=L3)
summary(glh3,test=Ftest())
confint(glh3)
```