PBR cheat sheet

Maksim Jaroslavcevas

April 10, 2024

1 Rendering equation

1.1 General form

$$L_o(\mathbf{x}, \omega_o, \lambda, t) = L_e(\mathbf{x}, \omega_o, \lambda, t) + L_r(\mathbf{x}, \omega_o, \lambda, t)$$

where

- L_o is the total spectral radiance of wavelength λ directed outward along direction ω_o at time t, from a particular position x
- $\mathbf{x}$ is the world position
- ω_o is direction of the outgoing light
- λ is a light wavelength
- t is time
- $L_e(\mathbf{x}, \omega_o, \lambda, t)$ is emitted spectral radiance
- $L_r(\mathbf{x}, \omega_o, \lambda, t)$ is reflected spectral radiance

Since we will use this equation in computer graphics scope we can, eliminate irrelevant variables, such as λ and t. Result equation:

$$L_o(\mathbf{x}, \omega_o) = L_e(x, \omega_o) + L_r(\mathbf{x}, \omega_o) \tag{1}$$

As a result we get very neat function. This functions gives a color of a point at position $\mathbf{x}$ from view vector perspective, as we see final color is a sum of a emitted light and reflected light.

First and second functions work with only single light channel, but actually if we calculate each of RGB channel separately and compose into a single three-dimensional vector, result will be the same as just replacing evrething with three-dimensional vector. Next we will consider light as a three-dimensional vector, where each dimension corresponds to red, green blue channels respectively.

Now lets look at reflection function

1.2 Reflection function

$$L_r(x, \omega_o, \lambda, t) = \int_\Omega f_r(x, \omega_i, \omega_o, \lambda, t) L_i(x, \omega_i, \lambda, t)(\omega_i * N) dw_i$$

where

- **N** is a surface normal
- ω_i light vector
- $\cdots$ all other variables are repeated as in General form equation
- $L_i(x, \omega_i, \lambda, t)$ incoming light

Lets simplify this function, eliminating λ, t variables

$$L_r(x, \omega_o) = \int_\Omega f_r(x, \omega_i, \omega_o) L_i(x, \omega_i)(\omega_i * N) dw_i$$

As wee see reflected light is just an $\int$ of all incoming light from an Ω sphere. Since in computer graphics we usually have a finite number of lights, we can replace $\int$ with a Σ.

$$L_r(x, \omega_o) = \sum_{n=1}^{N} f_r(x, \omega_n, \omega_o) L_i(x, \omega_n)(\omega_n * N) \tag{2}$$

Now we get reflection is a sum of products between incoming lights, f_r functions and $(\omega_n * N)$. At this point our final equation should look something like this

$$L_o(\mathbf{x}, \omega_o) = L_e(x, \omega_o) + \sum_{n=1}^{G} f_r(x, \omega_n, \omega_o) L_i(x, \omega_n)(\omega_n * N)$$

- **G** total number of lights

Now we need to clarify f_r function, but before lets replace some variables, since we will use them quite a lot further, and not all names are quite intuitive.

$$L_o(x, V) = L_e(x, V) + \sum_{n=1}^{G} f_r(x, L_n, V) L_i(x, L_n)(L_n * N) \tag{3}$$

where

- **x** is a world position of a point
- **V** view vector
- L_n - n^{th} light vector
- N surface normal
- **G** total number of lights
- L_i - incoming light function