Solutions 01 BayesianClassifier

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Exercise 1

A computer science student is annoyed that the two thirds of the e-mails he/she receives is spam. Therefore, he/she decides to write a classifier that should decide whether an incoming e-mail is spam (class y=1) or ham (class y=0). For classification the Bayes classifier is used. The student notices that in spam and ham mails, certain words occur with different probability. Therefore, the student bases the classification on the words $\boldsymbol{x}$ = {*Viagra, bet, student, sports, cinema*}. By inspecting all his/her previous mails, the student estimates that in the ham mails, the probabilities are 0% for Viagra, 10% for bet, 40% for student, 30% for sports, and 10% for cinema. In the spam mails, the words Viagra occurs in 50%, bet in 30%, student in 5%, and sports and cinema in 2% of the mails.

The student does not count how often each word occurs, but only whether it is present in the mail. For simplicity, he/she assumes that the words occur independently.

(a) Has student consider all the postulates of pattern recognition?

For this example 4 postulates shoul be considered:

1 Sample

There are two classes for student's emails and classes have their own features. Their e-mails are tha data $\triangleq \Omega$ and there are data for each 2 classes.

$$\omega = \{^1f(x), ^2f(x)\} \in \Omega$$

2 Features

Since the words $\in \mathbf{x}$ occur with different probability, features can be used to distinguish one class from another.

3 Compactness

As our features are vectors we can measure distance between them. And considering the fact that some words are likely to occur in specific class distance should be small between members of one class and large for different ones.

4 Similarity

A distance measure shoud be chosen to check whether patterns be similar.

(b) Write down the priors for an e-mail being spam or ham.

$p(y = 1) = \frac{2}{3}, p(y = 0) = \frac{1}{3}$

(c) Write down the class-conditional probabilities $p(\mathbf{x}|y = 0)$ and $p(\mathbf{x}|y = 1)$ for an arbitrary feature vector $\boldsymbol{x}$.

Let $\mathbf{x} \in \mathbb{R}^n$, $y \in \{0, 1\}$ and $x_i \sim Bernoulli(p)$, then

$$p(\mathbf{x}|y = j) = \prod_{i=1}^{n} p(x_i|y = j) = \prod_{i=1}^{n} p(x_i = 1|y = j)^{x_i} \cdot (1 - p(x_i = 1|y = j)^{1-x_i}, \text{for } j \in \{0, 1\}$$

(d) Write down the Bayesian decision rule for the spam classification problem. Classify the following e-mail using the decision rule:

Hi, As we talked about yesterday, I want to make a bet with you about the upcoming soccer match. I clearly know more about sports than you. I bet 5$ against Nürnberg.

Decision rule

$$f(\mathbf{x}_{email}) = \begin{cases} 1, & p(y = 1|\mathbf{x}_{email}) \geq p(y = 0|\mathbf{x}_{email}), \\ 0, & \text{otherwise.} \end{cases}$$

$\{\text{e-mail}\} \cap \boldsymbol{x} = \{bet, sports\} \triangleq \mathbf{x}_{email}$

$$p(\mathbf{x}_{email}|y = 0) = p(bet|y = 0) \cdot p(sports|y = 0) = 0.1 \cdot 0.3 = 0.03$$

$$p(\mathbf{x}_{email}|y = 1) = p(bet|y = 1) \cdot p(sports|y = 1) = 0.3 \cdot 0.02 = 0.006$$

$$p(y = 0|\mathbf{x}_{email}) = p(y = 0) \cdot p(\mathbf{x}_{email}|y = 0)) = \frac{1}{3} \cdot 0.03 = 0.01$$

$$p(y = 1|\mathbf{x}_{email}) = p(y = 1) \cdot p(\mathbf{x}_{email}|y = 1)) = \frac{2}{3} \cdot 0.006 = 0.004$$

$$f(\mathbf{x}_{email}) = 0$$