Simulation and Modeling I

Assignment 3

In queuing theory, Little's Law can be applied to any system under the requirement that the system is stable and non-preemptive. It is described as

$$N = D * X$$

and shows the relation between the number of customers in the system (N), their time spent in the system (D) and the throughput (X).

Problem 1 (2 + 2 points):

Use Little's Law and the provided formulas and your solutions for Assignment 2 Problem 4 to provide estimations for the following scenarios:

a) Consider a simple M/M/1 queue with exponentially distributed interarrival times with rate $\lambda = 0.75$ and exponentially distributed services times with rate $\mu = 1.0$.

   Please calculate the average values for:
   - utilization (U)
   - number of customers in the system (N)
   - throughput (X)
   - time spent in the system (D)

b) How do the measures D and N change for a simple M/D/1 queue with the same arrival rate and identical mean service time as the M/M/1 queue above?

Problem 2 (1 + 3 + 2 + 2 points):
A more flexible model of the M/M/1 queue

Now open the AnyLogic project file *MG1QueueNoMeasures.alp*. This model is structured according to the Client/Server architecture as presented in the lecture, but contains no provisions for collecting statistics. Extend this model so that …

a) …the simulation run stops after a specified number of customers have been served (instead of stopping after a specified stop time)!

b) …the statistical data for …

   i) ***N(t)***: number of customers in system at time ***t***
   ii) ***D(i)***: time spent in system by ***i***th customer

   …is collected in AnyLogic Statistics objects.

   Run a simulation for an M/M/1 and an M/D/1 queue with given parameters/assumptions for service and arrival rates from Problem 1, until 1 million

customers are served. Obtain the time-averaged expectation of ***N(t)*** and the expectation of ***D(i)*** from the Statistics objects. Can you confirm your calculations from Problem 1?

c) …the statistical data for the busy function ***B(t)*** (i.e., the estimate for utilization ***U***; see page 19 of material on Discrete Simulation) is collected in AnyLogic Statistics objects.

   In addition, use a counter (e.g. a plain variable) to eventually compute the throughput ***X(t)*** at time ***t*** (number of customers served per time unit). What results do you get for the same M/M/1 and M/D/1 queues from Problem 1 after 1 million customers have been served? Can you confirm your calculations from Problem 1?

d) What is your (estimate of the) coefficient of variation of the time spent in the system for the M/M/1 and the M/D/1? Make a conclusion on how this random variable might be distributed for both cases!