Solutions Excercise 4

Maksimov, Dmitrii
dmitrii.maksimov@fau.de
ko65beyp

Ilia, Dudnik
ilia.dudnik@fau.de
ex69ahum

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Exercise 1

(a) Write down a procedure solving linear programs of the form max $\{c^T x | Ax \leq b\}$ using the Fourier-Motzkin-Elimination.

1. reduce n-variable problem to (n-1) - variable problem using Fourier-Motzkin-Elimination
2. repeat first punkt while problem is not 1-variable
3. check whether $\{x \in \mathbb{R}^1 \ : \ Ax \leq b\}$ is not empty. If non empty then go next else finish
4. traceback steps using a solution to the 1-variable to find solutions to another one

(b) Use your developed procedure to solve

$$\begin{aligned} \max \quad & 2x_1 + x_2 - x_3 \\ \text{s.t.} \quad & 3x_1 + x_2 - 2x_3 \leq 0 \\ & x_1 - 2x_2 - 4x_3 \leq -14 \\ & x_1 - 2x_2 - 4x_3 \leq -8 \\ & -x_1 + x_2 + 4x_3 \leq 14 \\ & -2x_1 - 5x_2 + x_3 \leq -6 \end{aligned}$$

According to the algorithms, let's start with eliminating x_3

$$\begin{cases} x_3 \geq \frac{3x_1+x_2}{2} \\ x_3 \geq \frac{x_1-2x_2+14}{4} \\ x_3 \geq \frac{-x_1+3x_2+8}{2} \\ x_3 \leq \frac{14+x_1-x_2}{4} \\ x_3 \leq 6 + 2x_1 + 5x_2 \end{cases}$$

Now, pair up the lower bounds and the upper ones

$$\begin{cases} \frac{3x_1+x_2}{2} \leq \frac{14+x_1-x_2}{4} \\ \frac{3x_1+x_2}{2} \leq -6+2x_1+5x_2 \\ \frac{x_1-2x_2+14}{4} \leq \frac{14+x_1-x_2}{4} \\ \frac{x_1-2x_2+14}{4} \leq -6+2x_1+5x_2 \\ \frac{-x_1+3x_2+8}{2} \leq \frac{14+x_1-x_2}{4} \\ \frac{-x_1+3x_2+8}{2} \leq -6+2x_1+5x_2 \end{cases}$$

Just repeate step 1 and step 2 for other variables

$$\begin{cases} x_1 \leq \frac{14-3x_2}{5} \\ x_1 \geq -9x_2+12 \\ x_2 \geq 0 \\ x_1 \geq \frac{-22x_2+38}{7} \\ x_1 \geq 7x_2+2 \\ x_1 \geq \frac{-7x_2+20}{5} \end{cases} \implies \begin{cases} \frac{-9x_2+12}{2} \leq \frac{14-3x_2}{5} \\ \frac{-22x_2+38}{7} \leq \frac{14-3x_2}{5} \\ 7x_2+2 \leq \frac{14-3x_2}{5} \\ \frac{-7x_2+20}{5} \leq \frac{14-3x_2}{5} \\ x_2 \geq 0 \end{cases} \implies \begin{cases} x_2 \geq \frac{46}{42} \\ x_2 \geq \frac{92}{89} \\ x_2 \leq \frac{4}{38} \\ x_2 \geq \frac{6}{4} \\ x_2 \geq 0 \end{cases}$$

Since $\{x \in \mathbb{R}^1 : Ax \leq b\}$ is empty, the problem is infeasible.

Exercise 2

(a) Let C be the convex hull of the four points $P_1 = (0,0)$, $P_2 = (2,1)$, $P_3 = (-1,3.5)$ and $P_4 = (-1,1)$.

i. Describe C through linear inequalities

$$C = \begin{cases} \frac{5}{6}x_1 + x_2 \leq \frac{8}{3} \\ -\frac{x_1}{2} + x_2 \geq 0 \\ x_1 + x_2 \geq 0 \\ x_1 \geq -1 \end{cases}$$

ii. Give the point $Q = (1,1)$ as convex combination of $P_1, \dots, P_4$. Convex combination for Q

$$Q = \sum_{i=1}^{4} \lambda_i P_i, \sum_{i=1}^{4} \lambda_i = 1.$$

Since, It's obvious that Q lies on a line $P_4P_2 \longrightarrow \lambda_1 = \lambda_3 = 0$. Hence

$$Q = \lambda P_4 + (1-\lambda)P_2 \Longrightarrow \lambda_4 = \frac{1}{3}, \lambda_2 = \frac{2}{3}$$

The rest of the exercises seemed very difficult and incomprehensible.