Assignment4: Information Value and Markov Basics

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Exercise 4.3 (Moody HMM)

Consider the Markov process from Ex. 4.2 about the roommate Moody (which in particular gives the concrete probabilities needed below). We have already modeled it as an HMM with state variables M_d and evidence variables L_d.

Because the transition model is first-order and stationary, we can collect the conditional probabilities for the state transitions into a matrix $T_{ij} = P(M_d = x_j|M_{d-1})$, where x_i, x_j are two states (i.e., two possible values of the state variable). We use S for the number of states, and T is an $S \times S$ matrix.

Because the sensor model is stationary and has the sensor Markov property, we can collect the conditional probabilities for the observations into a matrix $O_{ij} = P(L_d = y_i|M_d = x_j)$ is a state and the y_i are the possible observations. If there are N possible observations, this is an $N \times S$ matrix. For a fixed observation e the diagonal $S \times S$ matrices O_e from the lecture notes are obtained from the rows of this matrix.

1. Clarify the modeling as an HMM. Concretely:

   (a) What is S? Give the transition matrix T.

   S is the possible observations of M_d, $S \in \{h, s\}$. $T =$

M_{d-1} \ M_d	h	s
h	0.85	0.15
s	0.3	0.7

   (b) What is N? Give the sensor matrix O.

   N is the possible observations of L_d, $N \in \{j, m\}$. $O =$

L_d \ M_d	h	s
j	0.7	0.4
m	0.3	0.6

2. Now consider a fixed sequence $L_1 = e_1, L_2 = e_2$ of observations that we have made for two days. Concretely, you heard Moody play metal on day $d = 1$ and jazz on day $d = 2$.

   (a) Give the diagonal sensor matrices O_1 and O_2 corresponding to the observation at $d = 1$ and $d = 2$.

   $$O_1 = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.6 \end{pmatrix}, O_2 = \begin{pmatrix} 0.7 & 0 \\ 0 & 0.4 \end{pmatrix}$$

   (b) You are not sure what kind of mood your flatmate was in on day $d = 0$, but it was either good or bad with equal probability. The HMM algorithm for filtering and smoothing uses compact matrix/vector equation to compute f and b. Use those equation to determine the probability distribution of Moody's mood on day $d = 1$.

   - filtering

     $f_{1:t+1} = \alpha O_{t+1} T^T f_{1:t} \Rightarrow f_{1:1} = \alpha O_1 T^T f_0$, where $f_0 = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$.
     Hence, $f_{1:1} = \alpha \cdot \langle 0.1725, 0.255 \rangle = \langle 0.404, 0.596 \rangle \Rightarrow$
     $P(M_1 = h) = 0.404, P(M_1 = s) = 0.596$

   - smoothing

     $P(X_k | e_{1:t}) = \alpha \cdot f_{1:k} \cdot b_{k+1:t} \Rightarrow P(X_1 | e_{1:2}) = \alpha \cdot f_{1:1} \cdot b_{2:2}$,
     where $b_{2:2} = T O_2 b_{3:2}, b_{3:2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.
     Hence, $P(X_1 | e_{1:2}) = \alpha \cdot \langle 0.404, 0.596 \rangle \cdot \langle 0.655, 0.49 \rangle = \langle 0.475, 0.525 \rangle \Rightarrow$
     $P(M_1 = h) = 0.475, P(M_1 = s) = 0.525$