Simulation and Modeling I

Assignment 1

Problem 1 (1.5 + 1.5 points):

Please **define events and apply rules of the probability measure for both a) and b)** to obtain your numerical results!

a) At a multi-port switch S, packets may arrive via four different input ports P1, …, P4 and are switched to different output ports. The table below shows which percentage/share of the arriving packets each input port carries and the switching error probability for each input port.

input port	share of arriving packets	switching error probability
P1	55%	0.077
P2	5%	0.0001
P3	17%	0.15
P4	23%	0.002

What is the probability that an arbitrary arriving packet is switched correctly?

b) Now consider two such multi-port switches S1 and S2 connected in such a way that all packets of input port P1 on the first switch S1 are switched so that they enter switch S2 on its input port P1 (we don't care about other traffic!). We are interested in the probability that packets along this single path are received correctly after switch S2. (We assume *perfect links*, i.e., packets are not lost *between* switches.)

What is this probability

- **if both switches are independent?**
- **if a transmission on S2 is always successful on the condition that the transmission on S1 was successful?**

Problem 2 (2 points):

Are two disjoint events A and B with $P(A) > 0$ and $P(B) > 0$ stochastically independent of one another? Give **both a formal and an informal** answer! („Informal" means to give the answer in your own words as opposed to formulas.)

Problem 3 (1 + 1 + 1 points):

Given the following experiment:
You have a box with 2 blue and 2 red balls inside. You blindly pick a ball, note its color and put it aside (not back into the box!). After that, you pick another ball from the box. Each pick gives you either a "Red" (R) or "Blue" (B) ball.

a) **Set up the probability space (S,Φ,P)** !
b) **Define random variable X: S → R** such that **X** denotes the number of occurrences of the event „Red" with respect to the experiment above!
c) **Compute the full distribution function F(x)** for random variable **X**!

Problem 4 (1.5 + 1.5 points):

A frequency-nonselective or flat fading radio channel may be accurately characterized by the Rayleigh model of fading. In this model, the received signal-to-noise ratio (SNR) per bit is described by an exponential distribution with density $f_{SNR}(x) = ce^{-cx}$, where parameter x is nonnegative ($x \geq 0$) and c is some positive constant ($c > 0$). So, SNR is a random variable and $f_{SNR}(x)$ is indeed a density function for any value of c, since

- $f_{SNR}(x) \geq 0$ for all $x \geq 0$ and $c > 0$ and

- $\int_0^\infty f_{SNR}(x)dx = 1$ for any $c > 0$.

a) **Determine the value of c** knowing that the mean value of SNR is 35 dB!

b) The probability of outage is an important performance criterion for digital communication over fading channels and is defined as the probability that the modem performs more poorly than a specified threshold. We require an SNR of at least 20 dB. Therefore, the probability of outage here is the probability that the SNR per bit is less than 20 dB. **What is the value of the probability of outage?**

Hint: Recall that y dB (per decade) may simply be replaced by $10^{0.1y}$ in your computations.