Assignment3: Decisions

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Exercise 3.3 (Decision Theory)

You are offered the following game: You pay x dollars to play. A fair coin is then tossed repeatedly until it comes up heads for the first time. Your payout is 2^n, where n is the number of tosses that occurred.

- Assume your utility function is exactly the monetary value. How much should you, as a rational agent, be willing to pay to play?

  $EMV = \sum_{n=1}^{\infty} P(n) \cdot U(n) = \sum_{n=1}^{\infty} \frac{1}{2^n} \cdot 2^n = \infty$. Hence, any finite fee is rational.

- Assume now, that your utility function for having k dollars is $U(k) = m \log_l k$ for some $m, l \in \mathbb{N}^+$. How does this change the result?

  $EMV = \sum_{n=1}^{\infty} \frac{m \log_l 2^n}{2^n} = 2 \cdot m \log_l 2.$

- What is wrong with the result from the first exercise? Which implicit assumption leads to the apparently nonsensical result? Can you think of a way to repair our utility function in a more realistic way than taking logarithms?

  The problem is that our utility function is unbounded. Utility function should be

  $$2^{E[n]} = 2^{\sum_{n=1}^{\infty} \frac{n}{2^n}} = 2^2 = 4.$$

  This utility function doesn't look like ordinal utility function, but this function make a sense. This is because it's just U(average number of tosses).