Assignment9: Learning

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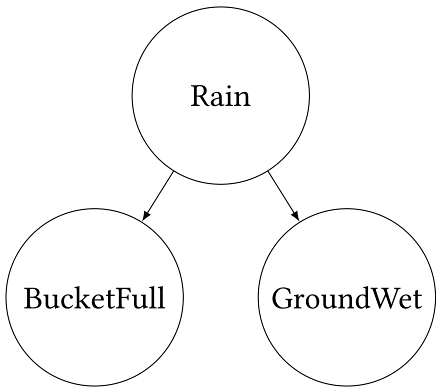
Exercise 9.1 (Statistical Learning)

We use two observations to determine if it has rained on our property: whether the ground is wet, and whether a bucket we left outside is full.

1. Model this situation as a naive Bayesian network with a boolean class and two boolean attributes.



2. Explain why this network requires 5 parameters (2n + 1 where n = 2 is the number of attributes). Choose 5 names for the parameters and use them to give the conditional probability table of the network.

   For each node that has a parent conditional probabilities are required depending on the value of the parent node. Also, distributions of parents nodes are also required. Since there are 2 children nodes and 1 parent node and dimension of each node is 2, this network requires 5 conditional probabilities(i.e. parameters).

   Let $\theta_R = P(R = yes), \theta_{BF|R} = P(BF = yes|R = yes), \theta_{BF|\neg R} = P(BF = yes|R = no)$, $\theta_{GW|R} = P(GW = yes|R = yes), \theta_{GW|\neg R} = P(GW = yes|R = no)$, where GW - GroundWet, BF - BucketFull.

3. State the formula for the likelihood of this list of 50 observations in terms of the 5 parameters.

$P(R|\theta_R, \theta_{BF|R}, \theta_{BF|\neg R}, \theta_{GW|R}, \theta_{GW|\neg R}) = \theta_R^{\#R}(1-\theta_R)^{\#\neg R} \cdot \theta_{BF|R}^{\#BF|R}(1-\theta_{BF|R})^{\#\neg BF|R} \cdot \theta_{BF|\neg R}^{\#BF|\neg R}(1-\theta_{BF|\neg R})^{\#\neg BF|\neg R} \cdot \theta_{GW|R}^{\#GW|R}(1-\theta_{GW|R})^{\#\neg GW|R} \cdot \theta_{GW|\neg R}^{\#GW|\neg R}(1-\theta_{GW|\neg R})^{\#\neg GW|\neg R}$

4. Give the Maximum Likelihood approximations for the 5 parameters given these 50 observations.

$\theta_R = \frac{\#R}{\#R + \#\neg R} = \frac{25}{50} = \frac{1}{2}, \theta_{BF|R} = \frac{\#BF|R}{\#BF|R + \#\neg BF|R} = \frac{16}{25},$
$\theta_{BF|\neg R} = \frac{\#BF|\neg R}{\#BF|\neg R + \#\neg BF|\neg R} = \frac{5}{25} = \frac{1}{5}, \theta_{GW|R} = \frac{\#GW|R}{\#GW|R + \#\neg GW|R} = \frac{15}{25} = \frac{3}{5},$
$\theta_{GW|\neg R} = \frac{\#GW|\neg R}{\#GW|\neg R + \#\neg GW|\neg R} = \frac{11}{25}$