Simulation and Modeling I, Assignment 3

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Exercise 1

Use Little's Law and the provided formulas and your solutions for Assignment 2 Problem 4 to provide estimations for the following scenarios:

(a) Consider a simple M/M/1 queue with exponentially distributed interarrival times with rate $\lambda = 0.75$ and exponentially distributed services times with rate $\mu = 1.0$.

Calculate the average values for:

- utilization (U) $U = \frac{\lambda}{\mu} = 0.75$
- number of custtomers in the system (N) $N = \frac{U}{1-U} = 3$
- throughput(X)

  Since $U < 1$: $X = m \cdot U \cdot \mu$, where m - the number of servers in a system = 1 $\implies X = 0.75$
- time spent in the system (D), using Little's Law $D = \frac{N}{X} = 4$

(b) How do the measures D and N change for a simple M/D/1 queue with the same arrival rate and identical mean service time as the M/M/1 queue above?

- Let S - the service time, then $D = W + E[S] = \frac{U(c_s^2+1)}{2\mu(1-U)} + \mu$.
  Since, $c_s = 0$: $D = \frac{U}{2\mu(1-U)} + \mu = 2.5$.
- $N = D \cdot X = 1.875$

Exercise 2

Extend the model so that

(a) the simulation run stops after a specified number of customers have been served(instead of stopping after a specified stop time)!

(b) the statistical data for

(i) $N(t)$: number of customers in system at time t

NumCustomInSystem
20,000,005 samples [0...43]. Mean=3.002

(a) for an M/M/1

NumCustomInSystem
20,000,000 samples [0...26]. Mean=1.872

(b) for an M/D/1

Figure 1: Expectation of N(t)

(ii) $D(i)$: time spent in system by $i - th$ customer

CustimerTimeInSystem
10,000,000 samples [5.136E-7...49.539]. Mean=4.002

(a) for an M/M/1

CustimerTimeInSystem
10,000,000 samples [1...25.879]. Mean=2.497

(b) for an M/D/1

Figure 2: Expectation of D(i)

As we can see calculatios are approximately equal.

(c) the statistical data for

(i) $B(t)$ utilization

Utilization
20,000,000 samples [0...1]. Mean=0.75

(a) for an M/M/1

Utilization
20,000,000 samples [0...1]. Mean=0.75

(b) for an M/D/1

Figure 3: Expectation of B(t)

(ii) $X(t)$: number of customers served per time unit

Throughput
10,000 samples [0.675...0.843]. Mean=0.75

(a) for an M/M/1

Throughput
10,000 samples [0.666...0.854]. Mean=0.75

(b) for an M/D/1

Figure 4: Expectation of X(t)

As we can see calculatios are equal.

(d) What is your (estimate of the) coefficient of variation of the time spent in the system for the M/M/1 and the M/D/1? Make a conclusion on how this random variable might be distributed for both cases!

Coefficient of variation: $C_X = \frac{\sigma_X}{E[X]}$

- M/M/1

Mean	3.999
Min	1.565E-7
Max	49.308
Deviation	3.991

Figure 5: Statistics of M/M/1

  Hence, $C_X \approx 1$. It's obvious, since this is exponential distribution.

- M/D/1

Mean	2.497
Min	1
Max	25.879
Deviation	1.797

Figure 6: Statistics of M/D/1

  Hence, $C_X \approx 0.72$. Taking into account that we are in Erlangen let it be the Erlang distribution(joke). Exponential distrubition is a special case of the Erlang distribution with $k = 1$. Coefficient of variation of Erlang distribution: $C = \frac{1}{\sqrt{k}}$. Hence, it seems to be the Erlang distribution witk k = 2.