Assignment7: Learning

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Exercise 7.3 (Weight Updates)

Our hypothesis space contains the functions $h_{\mathbf{w}}(\mathbf{x}) = F(\mathbf{w}^T\mathbf{x})$ for 2 + 1 - dimensional vectors **w, x** (using the trick $\mathbf{x}_0 = 1$ to allow for the constant term $\mathbf{w}_0$) and some fixed function F.

As the initial weights, we use $\mathbf{w}_0 = \mathbf{w}_1 = \mathbf{w}_2 = 0$. For each of the following cases, iterate the weight update rule once for each example (using the examples in the order listed). Use learning rate $\alpha = 1$.

1. Using the threshold function $F(z) = \begin{cases} 1 & z > 0, \\ 0 & otherwise. \end{cases}$

   Perceptron learning rule: $\mathbf{w}^{(i)} = \mathbf{w}^{(i-1)} + \alpha \cdot (y - h_{\mathbf{w}}(\mathbf{x}^{(i)})) \cdot \mathbf{x}^{(i)}$

   1.1. $\mathbf{w}^{(1)} = (0,0,0)^T + (2 - h_{\mathbf{w}}((0,0,0) \cdot (1,2,0)^T)) \cdot (1,2,0)^T = (2,4,0)^T$

   1.2. $\mathbf{w}^{(2)} = (2,4,0)^T + (2 - h_{\mathbf{w}}((2,4,0) \cdot (1,3,1)^T)) \cdot (1,3,1)^T = (3,7,1)^T$

2. Using the logistic function $F(z) = 1/(1 + e^{-x})$

   Logistic learning rule: $\mathbf{w}^{(i)} = \mathbf{w}^{(i-1)} + \alpha \cdot (y - h_{\mathbf{w}}(\mathbf{x}^{(i)})) \cdot h_{\mathbf{w}}(\mathbf{x}^{(i)}) \cdot (1 - h_{\mathbf{w}}(\mathbf{x}^{(i)})) \cdot \mathbf{x}^{(i)}$

   2.1. $h_{\mathbf{w}}((0,0,0) \cdot (1,2,0)^T) = 0.5$
   $\mathbf{w}^{(1)} = (0,0,0)^T + (2 - 0.5) \cdot 0.5 \cdot (1 - 0.5) \cdot (1,2,0)^T = (0.375, 0.75, 0)^T$

   2.2. $h_{\mathbf{w}}((0.375, 0.75, 0) \cdot (1,3,1)^T) \approx 0.93$
   $\mathbf{w}^{(1)} = (0.375, 0.75, 0)^T + (2 - 0.93) \cdot 0.93 \cdot (1 - 0.93) \cdot (1,3,1)^T \approx (0.44, 0.96, 0.07)^T$