Test 2

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Exercise 1

a) $Q = \{x \in \mathbb{R}^n | \sum_{i=1}^n x_i^2 = 1 \text{ and } x_i \geq 0 \ \forall i = 1, \dots, n\}$
Let $x = (1,0)^T, y = (0,1)^T \in Q$ and $z = (0.5, 0.5)^T \in \{\alpha x + (1-\alpha)y\},$ where $\alpha = 0.5$, then $z \notin Q$. This is because $\sum_{i=1}^2 z_i \neq 1$ - non-convex

b) $Q = \mathbb{R}^n$ - convex

c) $Q = \{x \in \mathbb{R}^n | \sum_{i=1}^n x_i < 0 \ \forall i = 1, \dots, n\}$
Let $x, y \in Q$ and $z = \alpha x + (1-\alpha)y,$ then $z \in Q$.
This is because $\sum_{i=1}^n z_i = \alpha \sum_{i=1}^n x_i + (1-\alpha) \sum_{i=1}^n y_i < 0$ - convex

d) $Q = \{0\}$ - a single point is a convex set

Answer: a

Exercise 2

Answer: c

Exercise 3

$f(x)$ - L-smooth and μ-strongly convex

a) if $f(x)$ twice differentiable, then $\lambda_{min}(\nabla^2 f(x)) \geq \mu$ and $\lambda_{max}(\nabla^2 f(x)) \leq L$
From lectures: $0 \preceq \nabla^2 f(x) \preceq L I_d$ and $\nabla^2 f(x) \succeq \mu I_d$ - True

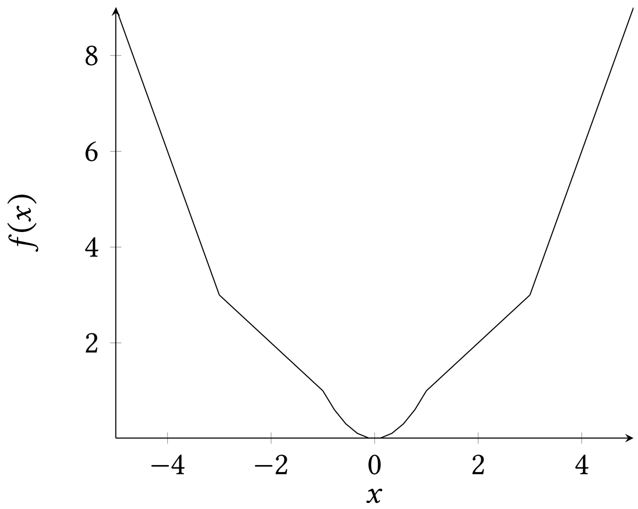
b) It can be that $L = \frac{\mu}{2}$ - **the answer is False, but I don't understand, could you explain?**

c) $\forall x, y \ f(y) \leq f(x) - \langle \nabla f(x), y - x \rangle + \frac{L}{2} \|y - x\|_2^2$
and $f(y) \geq f(x) - \langle \nabla f(x), y - x \rangle + \frac{\mu}{2} \|y - x\|_2^2$ - False ($+\langle \nabla f(x), y - x \rangle$)

d) $\forall x, y \ \langle \nabla f(y) - \nabla f(x), y - x \rangle \geq \frac{1}{\mu} \|\nabla f(x) - \nabla f(y)\|_2^2$
and $\langle \nabla f(y) - \nabla f(x), y - x \rangle \geq L \|x - y\|_2^2$. The first inequality should be $\frac{1}{L}$ rather than $\frac{1}{\mu}$ and the second one $\leq$ rather than $\geq$ - False.

Answer: a

Exercise 4



The function is convex, but not strictly convex since there are linear parts which are non-strictly convex.

Answer: b

Exercise 5

The function is L-smooth with $L = 3 \Rightarrow \max \nabla f(x) = 3$. **However, I don't understand what "for L < 3 the function is not L-smooth" means. Could you explain it?**

Answer: d