Test 4

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Exercise 1

Newton's method: $x_{k+1} = x_k - [\nabla^2 f(x_k)]^{-1} \nabla f(x_k)$

- $\nabla f(x_k) = 12x_k^3 + 4x_k$
- $\nabla^2 f(x_k) = 36x_k^2 + 4$

Answer: d

Exercise 2

Exact Line Search: $\underset{\alpha_k}{\operatorname{argmin}} f(x_k - \alpha \nabla f(x_k))$
Answer: c

Exercise 3

Gradient divergence: $f(x_{k+1}) > f(x_k)$.

$f(4 - \gamma \nabla f(4)) > f(4) \Rightarrow 4(4 - 12\gamma)^2 - 20(4 - 12\gamma) + 7 + 9 > 0 \Rightarrow (4 - 12\gamma - 1)(4 - 12\gamma - 4) > 0$.
Taking into account $\gamma > 0 \Rightarrow \gamma > 0.25$.
Answer: b

Exercise 4

Since MSE is a convex function, $N \approx \frac{2LR}{\epsilon}$ and time $\approx Nmn \Rightarrow$, where $L = \frac{\lambda_{\max}(A^T A)}{m}$, $R = \max \|x - x^\star\| \Rightarrow Nmn = \frac{2R\lambda_{\max}(A^T A)}{\epsilon} n$. Let's say $R_1 = R_2$, then $\text{time}_1 < \text{time}_2$.
Answer: a

Exercise 5

Since, $\underset{x}{\operatorname{argmin}} \frac{x^2}{4} = 0$ and $x_{k+1} = x_k - \frac{x_k}{2} = \frac{x_k}{2}$, $\|x_k - x^\star\| \to 0$
Answer: c