

1. จงแสดงวิธีทำและหาดีเทอร์มิแนนต์ของ

$$A = \begin{bmatrix} 2 & 0 & 1 & 3 & -2 \\ -2 & 1 & 3 & 2 & -1 \\ 1 & 0 & -1 & 2 & 3 \\ 3 & 1 & 2 & 4 & -3 \\ 1 & -1 & 3 & 2 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 1 & 3 & -2 \\ 1 & -1 & 2 & 3 \\ 3 & 2 & 4 & -3 \\ 1 & 3 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 3 \\ 2 & 4 & -3 \end{bmatrix} = (-6) + (16) + (6) - (-6) - (12) - (9) = 7$$

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ 3 & 4 & -3 \end{bmatrix} = (-12) + (24) + (-6) - (-12) - (24) - (-9) = 4$$

$$\begin{bmatrix} 2 & 1 & -2 \\ 1 & -1 & 3 \\ 3 & 2 & -3 \end{bmatrix} = (6) + (9) + (-4) - (6) - (12) - (-3) = -4$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 3 & 2 & -1 \\ -1 & 2 & 3 \end{bmatrix} = (6) + (3) + (-12) - (4) - (-2) - (27) = -32$$

$$\begin{bmatrix} 2 & 3 & -2 \\ -2 & 2 & -1 \\ 1 & 2 & 3 \end{bmatrix} = (12) + (-3) + (6) - (-4) - (-4) - (-18) = 43$$

$$\begin{bmatrix} 2 & 1 & -2 \\ -2 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix} = (18) + (-1) + (-4) - (-6) - (2) - (-6) = 23$$

$$\begin{vmatrix} 1 & 3 & -2 \\ 3 & 2 & -1 \\ 2 & 4 & -3 \end{vmatrix} = (-6) + (-6) + (-24) - (-8) - (-4) - (-29) = 3$$

$$\begin{vmatrix} 2 & 3 & -2 \\ -2 & 2 & -1 \\ 3 & 4 & -3 \end{vmatrix} = (-12) + (-9) + (16) - (-12) - (-8) - (16) = -3$$

$$\begin{vmatrix} 2 & 7 & -2 \\ -2 & 3 & -1 \\ 3 & 2 & -3 \end{vmatrix} = (-18) + (-3) + (8) - (-18) - (-4) - (6) = 3$$

$$\begin{vmatrix} 2 & 7 & 3 \\ -2 & 3 & 2 \\ 3 & 2 & 4 \end{vmatrix} = (24) + (6) + (-12) - (28) - (8) - (-8) = -9$$

$$|A_{21}| = (-7)(1) + (3)(4) + (2)(4) + 0 = 13$$

$$|A_{42}| = (32)(1) + (43)(3) + (-21)(2) + 0 = 115$$

$$|A_{52}| = (1)(3) + (-1)(3) + (3)(2) + (9)(3) = 33$$

$$|A| = 0 + 15 + 0 + 115 + 33 = 161$$

## 2. จงแสดงวิธีทำและหาค่า $|A^{-1}|$

นั่นคือ เริ่มต้นด้วยการหา  $A^{-1}$  จากนั้นให้ทำการหาดีเทอร์มิแนนต์ของเมทริกซ์นั้น

$$A = \begin{bmatrix} 1 & 0 & -1 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 0 & -2 & -1 \\ 1 & -3 & 1 & 2 \end{bmatrix}$$

$$A_{1,1} = \begin{vmatrix} 0 & 3 & 2 \\ 0 & -2 & -1 \\ -3 & 1 & 2 \end{vmatrix} = 0 + (9) + (0) - (12) - (0) - (0) = -3$$

$$A_{2,1} = \begin{vmatrix} 0 & -1 & 3 \\ 0 & -2 & -1 \\ -3 & 1 & 2 \end{vmatrix} = 0 + (-3) + 0 - (12) - 0 - 0 = -21$$

$$A_{3,1} = \begin{vmatrix} 0 & -1 & 3 \\ 0 & 3 & 2 \\ -3 & 1 & 2 \end{vmatrix} = 0 + (6) + 0 - (-27) - 0 - 0 = 33$$

$$A_{4,1} = \begin{vmatrix} 0 & -1 & 3 \\ 0 & 3 & 2 \\ 0 & -2 & -1 \end{vmatrix} = 0$$

$$A_{1,3} = \begin{vmatrix} 1 & 0 & 2 \\ 2 & 0 & -1 \\ 1 & -3 & 2 \end{vmatrix} = 0 + 0 + (-12) - 0 - (3) - 0 = -15$$

$$A_{2,3} = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 0 & -1 \\ 1 & -3 & 2 \end{vmatrix} = 0 + 0 + (-18) - 0 - (3) - 0 = -21$$

$$A_{3,3} = \begin{vmatrix} 1 & 0 & 3 \\ 1 & 0 & 2 \\ 1 & -3 & 2 \end{vmatrix} = 0 + (-6) + 0 - 0 - (-6) - 0 = 0$$

$$A_{1,4} = \begin{vmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & -3 & 1 \end{vmatrix} = 0 + 0 + (-18) - 0 - (6) - 0 = -24$$

$$A_{2,4} = \begin{vmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & -3 & 1 \end{vmatrix} = 0 + 0 + (6) - 0 - (6) - 0 = 0$$

$$A_{3,4} = \begin{vmatrix} 1 & 0 & -1 \\ 1 & 0 & 3 \\ 1 & -3 & 1 \end{vmatrix} = 0 + 0 + (3) - 0 - (-3) - 0 = 12$$

$$\text{adj}(A) = \begin{bmatrix} 1 & 0 & -1 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 0 & -2 & -1 \\ 1 & -3 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -3 & 10 & -15 & 24 \\ 21 & 14 & 21 & 0 \\ 33 & 2 & -3 & -12 \\ 0 & -24 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 24 & 33 & 0 \\ 70 & 14 & 2 & -24 \\ -15 & 21 & -3 & 0 \\ 24 & 0 & -12 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \text{adj}(A) = \frac{1}{84} \begin{bmatrix} -3 & 24 & 33 & 0 \\ 70 & 14 & 2 & -24 \\ -15 & 21 & -3 & 0 \\ 24 & 0 & -12 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{28} & \frac{1}{4} & \frac{11}{24} & 0 \\ \frac{5}{6} & \frac{1}{6} & \frac{1}{42} & -\frac{1}{3} \\ -\frac{5}{24} & \frac{1}{4} & -\frac{1}{24} & 0 \\ \frac{2}{7} & 0 & -\frac{1}{7} & 0 \end{bmatrix}$$

$$A_{24}^{-1} = \begin{bmatrix} -\frac{1}{28} & \frac{1}{4} & \frac{11}{24} & 0 \\ \frac{5}{6} & \frac{1}{6} & \frac{1}{42} & -\frac{1}{3} \\ -\frac{5}{24} & \frac{1}{4} & -\frac{1}{24} & 0 \\ \frac{2}{7} & 0 & -\frac{1}{7} & 0 \end{bmatrix}$$

$$A_{24}^{-1} = \begin{bmatrix} -\frac{1}{28} & \frac{1}{4} & \frac{11}{24} & 0 \\ \frac{5}{6} & \frac{1}{6} & \frac{1}{42} & -\frac{1}{3} \\ -\frac{5}{24} & \frac{1}{4} & -\frac{1}{24} & 0 \\ \frac{2}{7} & 0 & -\frac{1}{7} & 0 \end{bmatrix} \cdot \frac{-1}{24}$$

$$C_{24} = -\frac{1}{24}$$

$$|A^{-1}| = 0 + \left(-\frac{1}{28}\right)\left(-\frac{1}{3}\right)$$

$$= \frac{1}{84} \neq$$