Written Assignment #2

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1.

a) From the definition of entailment, $\alpha \models \beta$ if and only if, in every model in which α is true, β is also true. But because *True* is true in every model (by definition), it must be in every model where *False* (which happens to be none of them). Thus, $False \models True$ is true.

b) This is false because in any model, the left-hand is true but the right-hand is not, which contradicts the definition of entailment.

c) This is true because the only model in which $(A \wedge B)$ is true is the one where A and B are both true and in that model $(A \Leftrightarrow B)$ is also true.

d) This is false because in the model A and B are both false, the left-hand side is true but the right-hand side is false.

2.

a) This is neither valid nor unsatisfiable because assigning false to Smoke makes the statement true while assigning true to Smoke and false to Fire makes this statement false.

Smoke	Fire	Smoke $\Rightarrow$ Fire
T	T	T
T	F	F
F	T	T
F	F	T

b)

Smoke	Heat	Fire	Smoke ⇒ Fire	Smoke ∧ Heat	(Smoke ∧ Heat) ⇒ Fire	(Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire)
T	T	T	T	T	T	T
T	T	F	F	T	F	T
T	F	T	T	F	T	T
T	F	F	F	F	T	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

Thus, because (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire) is true in every model, it is valid.

3.

a) $A \Leftrightarrow (B \vee E)$

$(A \Rightarrow (B \vee E)) \wedge ((B \vee E) \Rightarrow A)$ biconditional elimination

$(\neg A \vee (B \vee E)) \wedge (\neg(B \vee E) \vee A)$ implication elimination

$(\neg A \vee (B \vee E)) \wedge ((\neg B \wedge \neg E) \vee A)$ De Morgan's Law

$(\neg A \vee (B \vee E)) \wedge ((A \vee \neg B) \wedge (A \vee \neg E))$ distributivity of $\vee$ over $\wedge$

$(\neg A \vee (B \vee E)) \wedge (A \vee \neg B) \wedge (A \vee \neg E)$ associativity of $\wedge$

$\boxed{(\neg A \vee B \vee E) \wedge (A \vee \neg B) \wedge (A \vee \neg E)}$ associativity of $\vee$

$E \Rightarrow D$

$\boxed{\neg E \vee D}$ implication elimination

$C \wedge F \Rightarrow \neg B$

$\neg(C \wedge F) \vee \neg B$ implication elimination

$(\neg C \vee \neg F) \vee \neg B$ De Morgan's Law

$\boxed{\neg C \vee \neg F \vee \neg B}$ associativity of $\vee$

$E \Rightarrow B$

$\boxed{\neg E \vee B}$ implication elimination

$B \Rightarrow F$

$\boxed{\neg B \vee F}$ implication elimination

$B \Rightarrow C$

$\boxed{\neg B \vee C}$ implication elimination

Thus, $(\neg A \vee B \vee E) \wedge (A \vee \neg B) \wedge (A \vee \neg E) \wedge (\neg E \vee D) \wedge (\neg C \vee \neg F \vee \neg B) \wedge (\neg E \vee B) \wedge (\neg B \vee F)$
$\wedge (\neg B \vee C)$ is the CNF.

b) To prove a sentence α by resolution, we first compute the CNF of $KB \wedge \neg\alpha$, which in this case is:

$\neg\alpha = \neg(\neg A \wedge \neg B)$

$A \vee B$ De Morgan's and double negation elimination

R1: $\neg A \vee B \vee E$

R2: $A \vee \neg B$

R3: $A \vee \neg E$

R4: $\neg E \vee D$

R5: $\neg C \vee \neg F \vee \neg B$

R6: $\neg E \vee B$

R7: $\neg B \vee F$

R8: $\neg B \vee C$

R9: $A \vee B$

R10: R1 + R2 : E

R11: R10 + R6 : B

R12: R11 + R7 : F

R13: R11 + R8 : C

R14: R13 + R5 : $\neg F \vee \neg B$

R15: R14 + R12 : $\neg B$

R16: R15 + R11 : empty clause

Thus, $\neg A \wedge \neg B$ is true for this knowledge base.