

## CS331: Introduction to Artificial Intelligence

### Written Assignment #3

Date handed out: May 11, 2018

Date due: May 21, 2018 at the start of class

Total: 28 points

This assignment is to be done individually. Please hand in a pdf on Canvas.

1. Using the full joint probability distribution below, write out what the following probability distributions look like. Notice that the **P** is in boldface to emphasize that these are distributions (ie. probability tables). This means you have to write out the probability distributions for all uninstantiated random variables eg. for (a), write out  $P(\text{Toothache}=\text{true})$  and  $P(\text{Toothache}=\text{false})$ .

<b>Toothache</b>	<b>Cavity</b>	<b>Catch</b>	<b>P(Toothache, Cavity, Catch)</b>
false	false	false	0.576
false	false	true	0.144
false	true	false	0.008
false	true	true	0.072
true	false	false	0.064
true	false	true	0.016
true	true	false	0.012
true	true	true	0.108

a)  $P(\text{Toothache})$  [2 points]

b)  $P(\text{Cavity})$  [2 points]

c)  $P(\text{Toothache} \mid \text{Cavity})$  [4 points]

2. Show that the three forms of independence below are equivalent: [3 points]

$$P(X, Y) = P(X)P(Y) \dots(1)$$

$$P(X|Y) = P(X) \dots(2)$$

$$P(Y|X) = P(Y) \dots(3)$$

3. Suppose you are given a coin that lands *heads* with probability  $x$  and *tails* with probability  $(1-x)$ .

a) Are the outcomes of successive flips of the coin independent of each other given that you know the value of  $x$ ? Justify your answer. [2 points]

b) Are the outcomes of successive flips of the coin independent of each other if you do not know the value of  $x$ ? Justify your answer. [2 points]

4. After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (ie. the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don't have the disease). The good news is that this is a rare disease, striking only 1 in 10,000 people of your age. Why is it good news that the disease is rare? What are the chances that you actually have the disease? [5 points]

5. (Adapted from Pearl 1988) Suppose you are witness to a nighttime hit-and-run accident involving a taxi in Athens. All taxis in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under the dim lighting conditions, discrimination between blue and green is 75% reliable.

a) Is it possible to calculate the most likely color for the taxi? (Hint: distinguish carefully between the proposition that the taxi *is* blue and the proposition that it *appears* blue.) [4 points]

b) What if you know that 9 out of 10 Athenian taxis are green? [4 points]