



Student name:

Malachi Harker

2023

YEAR 12
TRIAL HSC
EXAMINATION

Mathematics Extension 1

36/70

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks:
70

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 60 marks

- Attempt all questions
- Allow about 1 hour and 45 minutes for this section

Year 12 Mathematics Extension 1 Section I - Answer Sheet

Student Name/Number Malachi Hasper

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
A B C D

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A B C D

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct
A B C D

1. A B C D
✓ 2. A B C D
correct → 3. A B C D
✓ 4. A B C D
5. A B C D
6. A B C D
7. A B C D
✓ 8. A B C D
✓ 9. A B C D
✓ 10. A B C D

Section I**10 marks****Attempt questions 1 - 10****Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for questions 1-10

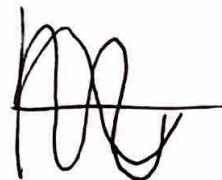
1. What is the domain of $f(x) = \sin^{-1}(3x)$?

(A) $-\frac{1}{3} \leq x \leq \frac{1}{3}$

(B) $-\frac{\pi}{6} \leq x \leq \frac{\pi}{6}$

(C) $-3 \leq x \leq 3$

(D) $-\pi \leq x \leq \pi$



2. Which of the following integrals is an expression for $\int \sin^2 4x dx$?

(A) $\int \frac{(1 + \cos 8x)}{2} dx$

(B) $\int \frac{(1 - \cos 8x)}{2} dx$

(C) $\int \frac{(1 + \sin 8x)}{2} dx$

(D) $\int \frac{(1 - \sin 8x)}{2} dx$

$$\frac{dt}{dN} = \frac{1}{0.1N - 0.2}$$

$$\int \frac{1}{0.1N - 0.2} dt$$

3. If $\frac{dN}{dt} = 0.1(N - 500)$ which of the following is the correct expression for N ?

(A) $-500 - 100e^{0.1t}$

(B) $-100 - 500e^{0.1t}$

(C) $500 + 100e^{0.1t}$

(D) $100 + 500e^{0.1t}$

$$\frac{dN}{dt} = 0.1N - 50$$

~~$$dN = (0.1N) dt - 50 dt$$~~

~~$$N = \text{_____} + 0.1$$~~

4. Which one of the following vectors is parallel to the vector $\overrightarrow{OE} = -3\mathbf{i} - 6\mathbf{j}$?

(A) $\overrightarrow{OA} = -2\mathbf{i} + 4\mathbf{j}$

(B) $\overrightarrow{OB} = -5\mathbf{i} + 10\mathbf{j}$

(C) $\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j}$

(D) $\overrightarrow{OD} = 4\mathbf{i} - 8\mathbf{j}$

guess

5. Which of the following is the exact value of $\int_{\frac{3}{\sqrt{2}}}^3 \frac{4}{\sqrt{9-x^2}} dx$?

~~(A)~~ $-\pi$

~~(B)~~ $-\frac{\pi}{4}$

(C) $\frac{\pi}{4}$

(D) π

$$4 \int \left\{ \sin^{-1} \frac{x}{4} \right\}$$

~~sin $\frac{x}{4}$~~

6. Mathematical induction is used to prove

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \text{ for all positive integers } n$$

Which of the following is the correct expression for part of the induction proof?

(A) LHS = $\frac{(k+1)(k+2)(2k+3)}{6} + k^2$

(B) LHS = $\frac{(k+1)(k+2)(2k+3)}{6} + (k+1)^2$

(C) LHS = $\frac{k(k+1)(2k+1)}{6} + k^2$

(D) LHS = $\frac{k(k+1)(2k+1)}{6} + (k+1)^2$

7. Consider the differential equation $\frac{dy}{dx} = \frac{2x}{y}$.

Which of the following equations best represents this relationship between x and y ?

(A) $y^2 = x^2 + C$

(B) $y^2 = 2x^2 + C$

(C) $y = x \ln|y| + C$

(D) $y = 2x \ln|y| + C$

$$\begin{aligned} dy &= 2x \, dx \\ \frac{y^2}{2} &= x^2 + C \\ y^2 &= x^2 + C \end{aligned}$$

~~$y^2 =$~~

8. The vectors $5\mathbf{i} + 2\mathbf{j}$ and $4\mathbf{i} - x\mathbf{j}$ are perpendicular. What is the value of x ?

(A) -10

(B) -2

(C) 3

(D) 10

$$(5\mathbf{i} + 2\mathbf{j}) \cdot (4\mathbf{i} - x\mathbf{j}) = 0$$

$$4 \times 5 + 2 \times (-x) = 0$$

$$20 - 2x = 0$$

$$2x = 20$$

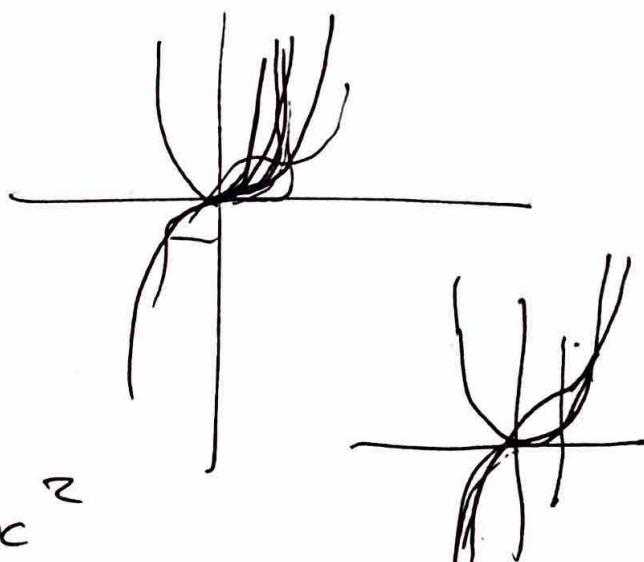
~~$x = 10$~~

9. Consider a projectile launched with an initial velocity v at an angle θ to the horizontal. Assume that $g = 9.8 \text{ ms}^{-2}$ and that air resistance is negligible. Which of the following statements is correct?
- The acceleration of the projectile remains constant during its entire flight.
 - The acceleration of the projectile decreases during its upward flight.
 - The acceleration of the projectile is greatest during its upward flight.
 - The acceleration of the projectile increases during its downward flight.

10. What is the area between the curves $y = 2x^2$ and $y = x^3$ and the x -axis from $x = 1$ to $x = 4$?

- 21 square units
- $21\frac{3}{4}$ square unit
- 22 square units
- $22\frac{2}{3}$ square units

guess



$$\int_0^1 x^3 - 2x^2$$

$$\left[\frac{1}{4}x^4 \right]_0^1 - \left[\frac{2}{3}x^3 \right]_0^1$$

$$\frac{1}{4} - \frac{2}{3}$$

$$= + \frac{5}{12}$$

Term
Ex: $\frac{3}{4}, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}$
 $\frac{38}{38}, \frac{15}{19}, \frac{38}{50}, \frac{36}{40}$

Simplify: $\frac{3}{4}, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}$
 $\frac{58}{60}, \frac{28}{32}, \frac{86}{100}, \frac{77}{100}$

Section II**60 marks****Attempt all questions****Allow about 1 hour and 45 minutes for this section**

Answer each question in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)**Marks**

- (a) The height of the liquid in a cylindrical container with radius 0.6 m is decreasing at the rate of 0.01 ms^{-1} .
Find the rate of change of the volume. Answer correct to three significant figures. 2

- (b) If $u = l + 4j$ and $v = 4l - 2j$. Calculate:
(i) $u + v$ 1
(ii) $3u - v$ 1

- (c) Use the principle of mathematical induction to prove that for all positive integers n : 3

$$1 \times 2^0 + 2 \times 2^1 + 3 \times 2^2 + \dots + n \times 2^{n-1} = 1 + (n-1) 2^n$$

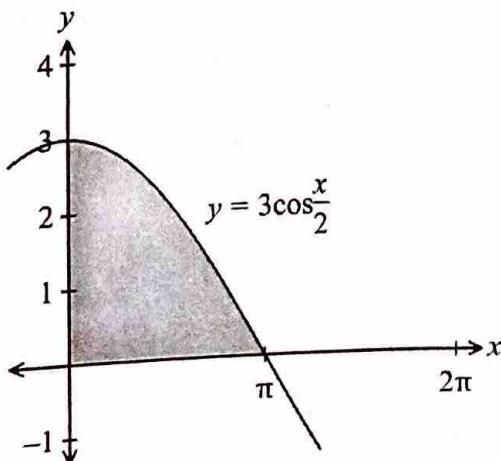
- (d) A binomial distribution is $X \sim \text{Bin}(7, 0.4)$. Find:
(i) $P(X = 4)$ 1
~~(ii)~~ (ii) $E(X)$ 1
(iii) $\text{Var}(X)$ 1

- (e) By completing the square, find $\int \frac{1}{x^2 + 2x + 2} dx$ 2

- (f) Using the t formulae solve the equation $\frac{7\sin x}{2} + 2\cos x = 4$ for $0 \leq x \leq 180^\circ$.
Answer correct to the nearest minute. 3

Question 12 (15 marks)**Marks**

- (a) Find $\int \frac{x}{(5-x^2)^3} dx$. Use the substitution $u = 5 - x^2$ or otherwise 2
- (b) A committee containing 4 men and 2 women is to be formed from a group of 9 men and 7 women. In how many different ways can the committee be formed? 1
- (c) Consider the polynomial $P(x) = 2x^3 + 3x^2 - 29x - 60$.
- Find the remainder when $P(x)$ is divided by $(x + 2)$. 1
 - Use the factor theorem to show that $(x + 3)$ is a factor of $P(x)$. 1
 - Factorise $P(x)$ completely. 3
- (d) (i) Express $\sqrt{3}\sin x + 3\cos x$ in the form $R\sin(x + \alpha)$ for $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$. 2
- (ii) Hence solve the equation $\sqrt{3}\sin x + 3\cos x = \sqrt{3}$ for $0 \leq x \leq 2\pi$. 2
- (e) The region bounded by the graph $y = 3\cos \frac{x}{2}$ and the x -axis between $x = 0$ and $x = \pi$ is rotated about the x -axis to form a solid. 3



Find the exact volume of the solid.

Question 13 (15 marks)**Marks**

- (a) Newton's law of cooling states that when an object at temperature $T^{\circ}\text{C}$ is placed in an environment at temperature $A^{\circ}\text{C}$, the rate of temperature loss is given by the equation:

$$\frac{dT}{dt} = -k(T - A)$$

where t is the time in minutes and k is a positive constant.

- (i) Show that $T = A + Be^{-kt}$ satisfies the above equation. 1

- (ii) An object whose initial temperature is 60°C is placed in a room in which the surrounding temperature is maintained at 12°C . After 25 minutes, the temperature of the object is 30°C . How long will it take for the object's temperature to reduce to 15°C ? Answer correct to the nearest minute.

~~X~~

- (b) Evaluate $\int_0^{\frac{1}{2}} \sqrt{\frac{x}{1-x}} dx$. Use the substitution $x = \sin^2 \theta$. 3

- (c) A shooter hits the target 87% of the time. In a competition he will have fifty shots at the target.

- ~~X~~ (i) What is the probability he hits 40 targets? Answer correct to four decimal places. 1
- ~~X~~ (ii) What is the probability he misses at most two times? Answer correct to four decimal places. 2

- (d) Find $\frac{d}{dx}(x \sin^{-1} x)$. 2

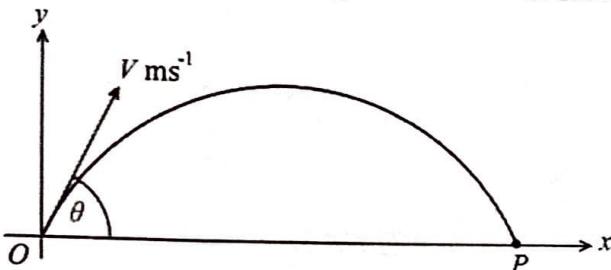
- (e) Let $a = 6\mathbf{i} - 3\mathbf{j}$ and $b = -\mathbf{i} + 5\mathbf{j}$.

- (i) Find the magnitude of a 1
- (ii) Find the magnitude of b 1
- (iii) What is the scalar product of a and b ? 1

Question 14 (15 marks)**Marks**

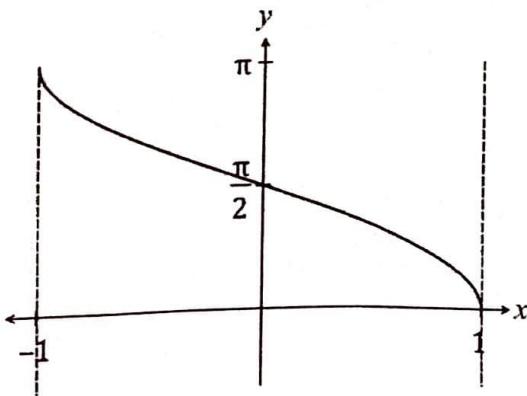
- (a) There were twenty projectiles fired from the origin O with initial velocity $V \text{ ms}^{-1}$ at an angle θ to the horizontal. The equations of motion are given by:

$$x = Vt\cos\theta \text{ and } y = -\frac{1}{2}gt^2 + Vt\sin\theta \quad (\text{Do NOT prove these equations.})$$



- (i) The projectiles reach the ground at P . What is the distance OP , in terms of V, g and θ ? 2
- (ii) A particular projectile is fired so that $\theta = \frac{\pi}{3}$. Find the angle this projectile makes with the horizontal when $t = \frac{2V}{\sqrt{3}g}$. 3
- (b) The equation $y = e^{mx}$ satisfies the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$. 2
What are the possible values of m ?

- (c) The graph of $y = \cos^{-1}x$ $\{x : -1 \leq x \leq 1\}$ is shown below.



- (i) Find the area bounded by the graph shown above, the x -axis and the line with the equation $x = -1$. 2
- (ii) Find the exact volume of the solid of revolution formed if the graph shown above is rotated about the y -axis. 3

- (d) Prove by mathematical induction $8^{2n+1} + 6^{2n-1}$ is divisible by 7, for any integer $n \geq 1$. 3

End of paper