Quality and creativity

KATOOMBA HIGH SCHOOL

Student Name

Malachi Hasken

Katoomba High School

Examination Booklet

Examination

Maths ext 2

6

Instructions

- Write the question number you are answering in this box ⟹ 11
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Start here

a) $\overline{i(4-i)}$

i) $= \overline{4i - i^2}$

$= \overline{1 + 4i}$

$= 1 - 4i$ ✓

ii) $\frac{1}{4-i} \times \frac{4+i}{4+i}$

$= \frac{4+i}{4^2 - i^2}$

$= \frac{4+i}{16+1}$

$= \frac{4}{17} + \frac{1}{17} i$ ✓

b) $|\overrightarrow{OA}| = 7$

$x = \frac{1}{7}(2i + 6j - 3k)$

$= \frac{2}{7} i + \frac{6}{7} j - \frac{3}{7} k$ ✓✓

c) $\int_0^5 x(x^2+4)^{-1}$

$= \frac{1}{2} \int_0^5 2x(x^2+4)^{-1}$

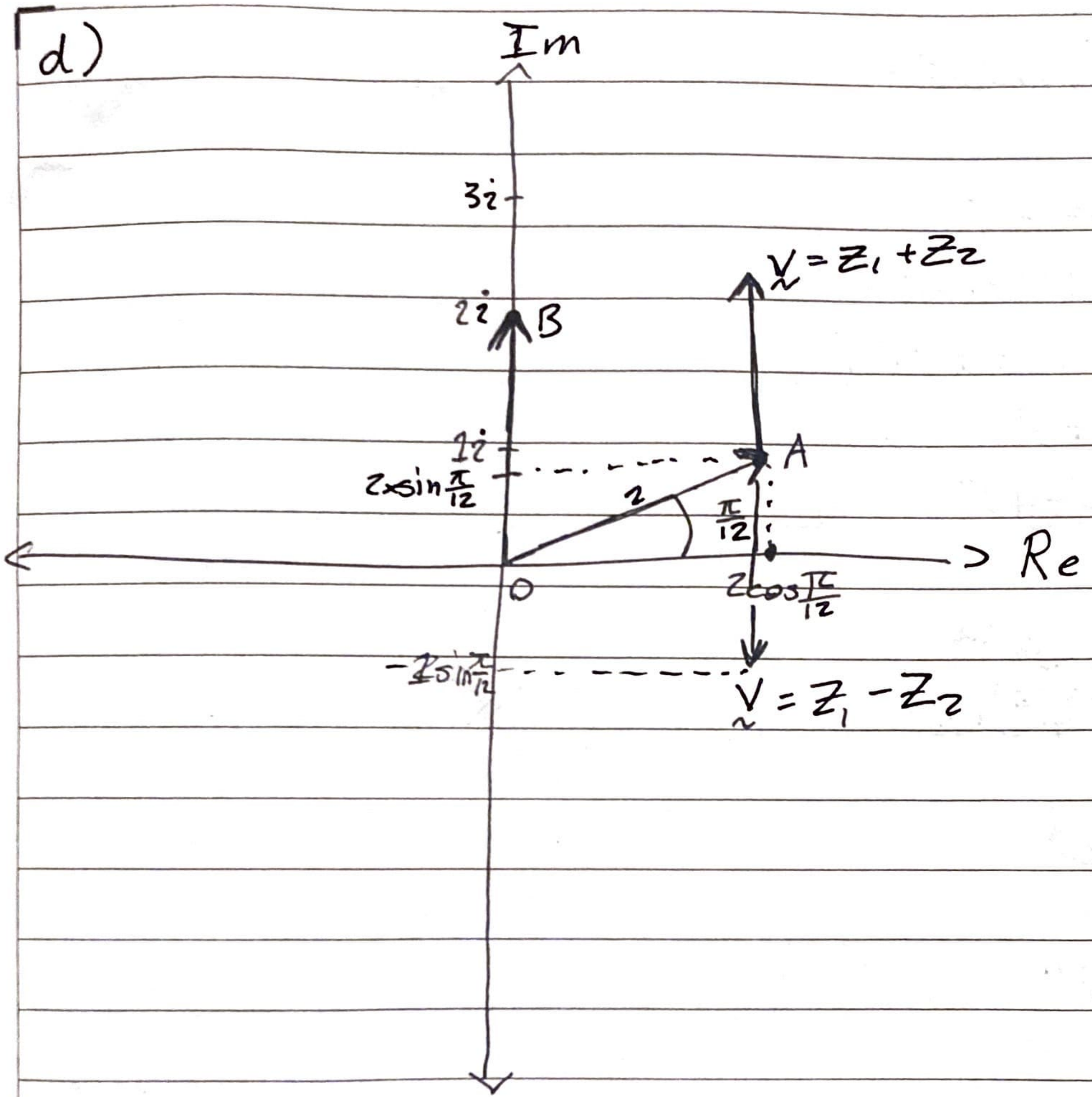
$= \frac{1}{2}\left[\frac{1}{2}\right.$ ✗✗

d)

i) ✓

ii) ×

iii) ××



$z_1 = 2(\cos\frac{\pi}{12} + i\sin(\frac{\pi}{12}))$

$\sin\frac{\pi}{12} = \frac{\cancel{\sqrt{2}}}{2}$

$z_1 = 2\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}$

Additional writing space on back page.

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Instructions

13

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12)

a) $\int x e^{-x} dx$

$u = x \qquad v' = e^{-x}$

$u' = 1 \qquad v = -e^{-x}$

$= x(-e^{-x}) - \int -e^{-x} dx$

$= -xe^{-x} + - (e^{-x}) + C$

$= -xe^{-x} - e^{-x} \quad + C$ ✓✓✓

$= \cancel{-e^{-x}(-x-1) + C}$

This line is incorrect and may cost a mark in the HSC.

b)

i) $(1+ia)^4$

$= 1^4 + \binom{4}{1}(1)^3(ia) + \binom{4}{2}(1)^2(ia)^2 + \binom{4}{3}(1)^1(ia)^3 + \binom{4}{4}(ia)^4$

$= 1 + 4ia + 6i^2a^2 + 4i^3a^3 + i^4a^4$

$= 1 + i4a - 6a^2 - 4a^3i + a^4$

$= 1 + 4ai - 6a^2 - 4a^3i + a^4$ ✓

ii) $a=0,$ ✗✗

c)

$B=-1$ $\quad C=3$

$A=2$ $\quad D=-2$

✓✓

i) $\frac{Ax^2+Bx}{x^3} + \frac{Cx+D}{x^2+1}$

$\frac{(x^2+1)(Ax^2+Bx)+x^3((Cx+D)}{x}$

$\frac{Ax}{x^2}+\frac{B}{x^2} = \frac{Ax+B}{x^2}+\frac{Cx+D}{x^2+1}$

$= Ax^3+Bx^2+Ax+B+Cx^3+Dx^2$

$A+C=5$

$B+D=-3$

$A=2$

$B=-1$

d) $\ddot{x}=12\sin 2t$

i) ~~when t=0~~

~~$\frac{1}{2}v^2 = \int 12\sin 2t\,dx$~~

$\int \ddot{x}\,dt = \int 12\sin 2t\,dt$

$v = 6\int 2\sin 2t\,dt$

$v = 6(-\cos(2t))$

$\dot{x} = -6\cos 2t + C$

when $t=0$ $\quad \dot{x}=6$

$6=-6\cos 2(0)+C$

$=-6+C$

$C=0$

$\therefore v=-6\cos 2t$

✓✓ t.e.

ii) $\int v\,dt = \int -6\cos 2t\,dt$

$=-3\int 2\cos 2t\,dt$

$x=-3\sin 2t + C$

✓ cfpa

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Start here

when $t=0$ $\quad x=0$

$-3\sin 2t + c = x$

$0 = -3\sin 2(0) + c$

$0 = 0 + c$

$c = 0$

$\therefore x = -3\sin 2t \quad \& \quad \ddot{x} = -n^2(x-c)$

$\therefore n=2 \quad \alpha=0 \quad c=0 \quad a=1$

$\ddot{x} = -2^2(x-0)$

$= -4x$

✓✓

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Start here

c) ii)

$$\int \frac{2}{x} + -\frac{1}{x^2} + \frac{3x-2}{x^2+1}\,dx$$

$$= 2\ln x + x^{-1} + \int \frac{3x-2}{x^2+1}\,dx \quad + C$$

$$= 2\ln x + \frac{1}{x} + \int \frac{3x}{x^2+1} \cancel{+} - \frac{2}{x^2+1}\,dx$$

$$= 2\ln x + \frac{1}{x} + 3\frac{1}{2}\left(\ln|x^2+1|\right) - \int \frac{2}{x^2+1}\,dx$$

$$= 2\ln x + \frac{1}{x} + \frac{3}{2}\ln|x^2+1| - 2\int \frac{1}{1+x^2}\,dx$$

$$= 2\ln|x| + \frac{1}{x} + \frac{3}{2}\ln|x^2+1| - 2\tan^{-1}(x) + C$$

✓✓

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Maths Ex 2

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13)

a) $9x^2+6x+5$

$= 9(x^2+\frac{2}{3}x+\frac{5}{9})$

$= 9(x^2+\frac{2}{3}x+(\frac{1}{3})^2-(\frac{1}{3})^2+\frac{5}{9})$

$= 9((x+\frac{1}{3})^2-\frac{1}{9}+\frac{5}{9})$

$= 9(x+\frac{1}{3})^2+4$

$\therefore \int \frac{1}{9x^2+6x+5}\,dx$

$= \int \frac{1}{9(x+\frac{1}{3})^2+4}\,dx$

$= \int \frac{1}{3^2(x+\frac{1}{3})^2+2^2}\,dx$

$= \frac{1}{2}\tan^{-1}(\frac{3x+1}{2})+C$ ✓✓✗

b) i) $11i+2j+17k+\lambda(-2i+j-4k)=-5i+11j+pk+\mu(-3i+2j+2k)$

$11i+5i+2j-11j+17k-pk-2\lambda i+\lambda j-4\lambda k -3\mu i+2\mu j+2\mu k=0$

$16i-9j+(17-p)k$ ✓✗✗

ii) ✗

c) ~~$|z-w|=|2i-\sqrt{3}|$~~

~~$=\sqrt{7}$~~

~~$|z-w|=|2i-(-$~~

$|z-w|=|2i-(\sqrt{3}-i)|$

$=|2i+i-\sqrt{3}|$

$=|3i-\sqrt{3}|$

$=\sqrt{12}$

$|z - v| = |2i - (-\sqrt{3} - i)|$

$= |2i + \sqrt{3} + i|$

$= |2i + i + \sqrt{3}|$

$= |3i + \sqrt{3}|$

$= \sqrt{12}$

$|w - v| = |\sqrt{3} - i - (-\sqrt{3} - i)|$

$= |\sqrt{3} - i + \sqrt{3} + i|$

$= |2\sqrt{3}|$

$= |\sqrt{4 \times 3}|$

$= \sqrt{12}$

$\therefore$ all the side lengths are equal ✓✓

$\therefore$ equilateral triangle

d) $a = 2$ $n = \pi/2$

i) $x = 2\cos(\frac{\pi}{2}t)$

$\dot{x} = 2 \times \frac{\pi}{2} \times (-1) \sin \frac{\pi}{2} t$

$= -\pi \sin \frac{\pi}{2} t$

$\therefore$ max velocity $= \pi$ ✓✓

ii) ~~$\cos x = \frac{1}{2}$~~

~~$x = \frac{\pi}{3}$~~

~~$\therefore \dot{x}(\frac{\pi}{3}) = -\pi \sin(\frac{\pi}{2}(\frac{\pi}{3}))$~~

$2\cos(\frac{\pi}{2}t) = 2$

$\cos(\frac{\pi}{2}t) = 1$ ✗✗

$\frac{\pi}{2}t = \cos^{-1}(1)$

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Start here

~~mass~~ c)

Let mass = 1

$\cancel{T_{1y}} = \sin 35° = \frac{T_{1y}}{T_1}$ Use the x-direction.

$T_{1y} = T_1 \sin 35°$

$T_{1x} = T_1 \cos 35°$

$T_{2y} = T_2 \sin 50°$

$T_{2x} = T_2 \cos 50°$

$T_{1y} = T_{2y}$

$T_1 \sin 35° = T_2 \sin 50°$

$\frac{T_1}{T_2} = \frac{\sin 50°}{\sin 35°}$

$= 1.34 \text{ N}$ ✓ ✗

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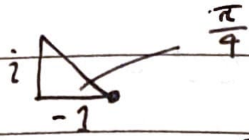
$|-1+i| = \sqrt{2}$ 14 a i



$\therefore \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

$-1+i = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ ✓✓

ii) $(-1+i)^n = (\sqrt{2})^n\left(\cos\frac{3\pi n}{4} + i\sin\frac{3\pi n}{4}\right)$ ✓

b) $\frac{b+a}{ab} > \frac{4}{m}$

$\cancel{b+a > \frac{4ab}{m}}$

$\cancel{m = \frac{4ab}{b+a}}$

$a = m - b$

$\frac{b+m-b}{(m-b)b} > \frac{4}{m}$

working

$\frac{m}{mb-b^2} > \frac{4}{m}$

$m^2 > 4mb - 4b^2$

$m^2 - 4mb > -4b^2$

$m^2 - 4mb + 4b^2$

$(m-2b)^2$

b) Proof:

$(m-2b)^2 > 0$

$m^2 - 4mb + 4b^2 > 0$

$m^2 > 4mb - 4b^2$

$m(m) > 4(mb-b^2)$

$m > \frac{4(mb-b^2)}{m}$

$\frac{m}{4(mb-b^2)} > \frac{4}{m}$

$\frac{m}{b(m-b)} > \frac{4}{m}$

from the question $a+b=m$

$\therefore$ ~~ba~~ $a = m-b$ *

$\frac{m}{ba} > \frac{4}{m}$ *

$\frac{(m-b)+b}{ba} > \frac{4}{m}$

$\frac{a+b}{ba} > \frac{4}{m}$ *

$\frac{1}{a} + \frac{1}{b} > \frac{4}{m}$ ✓✓✓

Unusual method but logic is sound.

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Start here

14)

d) Statement: If n is a positive integer then $\sqrt{4n-2}$ is irrational

if $n=1$

$\sqrt{4(1)-2} = \sqrt{4-2}$

$= \sqrt{2}$

if $n=2$

$\sqrt{4(2)-2} = \sqrt{8-2}$

$= \sqrt{6}$

contrapositive: If $\sqrt{4n-2}$ is rational then n is a ~~a~~ not a positive integer

× ×

Not proof by contradiction.

c) i) ×××

ii) ××

iii) ××

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15)

a) $z^2 = |z|^2 - 4$

Let $z = a+bi$

$(a+bi)^2 = |(a+bi)|^2 - 4$

$a^2 - b^2 + 2abi = (\sqrt{a^2+b^2})^2 - 4$

$a^2 - b^2 + 2abi = a^2 + b^2 - 4$

$-2b^2 + 2abi - 4 = 0$

$A = -2, \quad B = 2a, \quad C = -4$

$$x = \frac{-2a \pm \sqrt{4a^2 - 4(-2)(-4)}}{2(-2)}$$

$$= \frac{-2a \pm \sqrt{4a^2 - 32}}{-4}$$

$$= \frac{-2a \pm 2\sqrt{a^2-8}}{-4}$$

$$= \frac{a \pm \sqrt{a^2-8}}{2}$$

a) $z^2 = a^2 + b^2 - 4$

$z = \sqrt{a^2 \cancel{4b} + b^2 - 4}$

15)

b) RTP: $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

$n \geq 1$

Test:

Let $n = 1$

$(\cos\theta + i\sin\theta)^1 = \cos\theta(1) + i\sin\theta(1)$

$\cos\theta + i\sin\theta = \cos\theta + i\sin\theta$

$\therefore$ True for $n = 1$

Always layout as

LHS = . . .

= RHS

✓

Assume true for $n = k$ where k is a positive integer

$(\cos\theta + i\sin\theta)^k = \cos(k\theta) + i\sin(k\theta)$ ✱

Prove for $n = k+1$

RTP: $(\cos\theta + i\sin\theta)^{k+1} = \cos((k+1)\theta) + i\sin((k+1)\theta)$

~~LHS~~

$LHS = (\cos\theta + i\sin\theta)^{k+1}$

Not valid Mathematical Induction since it did not use ✱.

$= \cos((k+1)\theta) + i\sin((k+1)\theta)$ by De Moeie's Theorem ✗

$= RHS$

$\therefore$ True for $n = k+1$ ✗

$\therefore$ statement is true by the principle of mathematic induction. ✓

(I was running out of time)

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Start here

does pythag only work in ~~right~~ trig ~~or~~ will it sometimes work in others?

• Pythag. always needs right angles.

c) $\overrightarrow{OA} = -i - j$

$\overrightarrow{OB} = j + 2k$

$\overrightarrow{OC} = 4i + k$

i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= j + 2k - (-i - j)$

$= j + 2k + i + j$

$= 2j + 2k + i$

$= i + 2j + 2k$ ✓

ii) $|\overrightarrow{AB}| = 3$ ✓

iii) ~~$\overrightarrow{OB}$~~ $\overrightarrow{BC} = 4i + k - (j + 2k)$

$= 4i + k - j - 2k$

$= 4i - j - k$ ✓

$|\overrightarrow{BC}| = \sqrt{18}$

$= 3\sqrt{2}$

$\overrightarrow{AB} \cdot \overrightarrow{BC} = (3\sqrt{2})(3)\cos\theta$

$= 4 - 2 - 2$

$= 0$

$0 = (3\sqrt{2})(3)\cos\theta$

$\cos\theta = 0$

$\theta = 90°$ ✓✓

$\therefore$ Right angled triangle

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15)

d)

$2x$

x^2

$F = ma$

$F = \frac{m}{x^3}(6 - 10x)$

$\cancel{F}\ ma = \frac{m}{x^3}(6 - 10x)$

$a = \frac{1}{x^3}(6 - 10x)$ ✓

show method

$\int a\,dx = \int \frac{1}{x^3}(6 - 10x)\,dx$

$\frac{1}{2}v^2 = \int \frac{6}{x^3} - \frac{10x}{x^3}\,dx$

$= \int \frac{6}{x^3} - \frac{10}{x^2}\,dx$

$= \int 6x^{-3} - 10x^{-2}\,dx$

$\cancel{= -2\int 3x}$

$\cancel{= 6 \times \frac{-1}{4} \int -4x^{-3}\,dx - 10 \times \frac{-1}{3} \int -3x^{-2}\,dx}$

$\cancel{= -\frac{6}{4}(x^{-4}) + \frac{10}{3}(x^{-}}$

$= -3\int -2x^{-3}\,dx + 10\int -x^{-2}\,dx$

$= -3x^{-2} + 10x^{-1}$

$\frac{1}{2}v^2 = -\frac{3}{x^2} + \frac{10}{x} + C$

$v^2 = -\frac{6}{x^2} + \frac{20}{x} + C$

~~when t = 0~~ when $x = 1$ $v = 0$

$0 = -\frac{6}{1} + \frac{20}{1} + C$

$= -6 + 20 + C$

$C = -14$

$\therefore\ v = \sqrt{-\frac{6}{x^2} + \frac{20}{x} + 14}$ ✓ ✗

Direction is negative.

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Instructions

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Start here

$I_n = \int_0^1 (1-x^r)^n \, dx \qquad r>0$

$= \int_0^1$

16)

a) i) × × ×

ii) $n=1$

$I_1 = \int_0^1 (1-x^{3/2})^1 \, dx$

$= \int_0^1 1 - x^{3/2} \, dx$

$= \left[x - \frac{2x^{5/2}}{5} \right]_0^1$

$= 1 - \frac{2}{5} - (0-0)$

$= \frac{3}{5}$

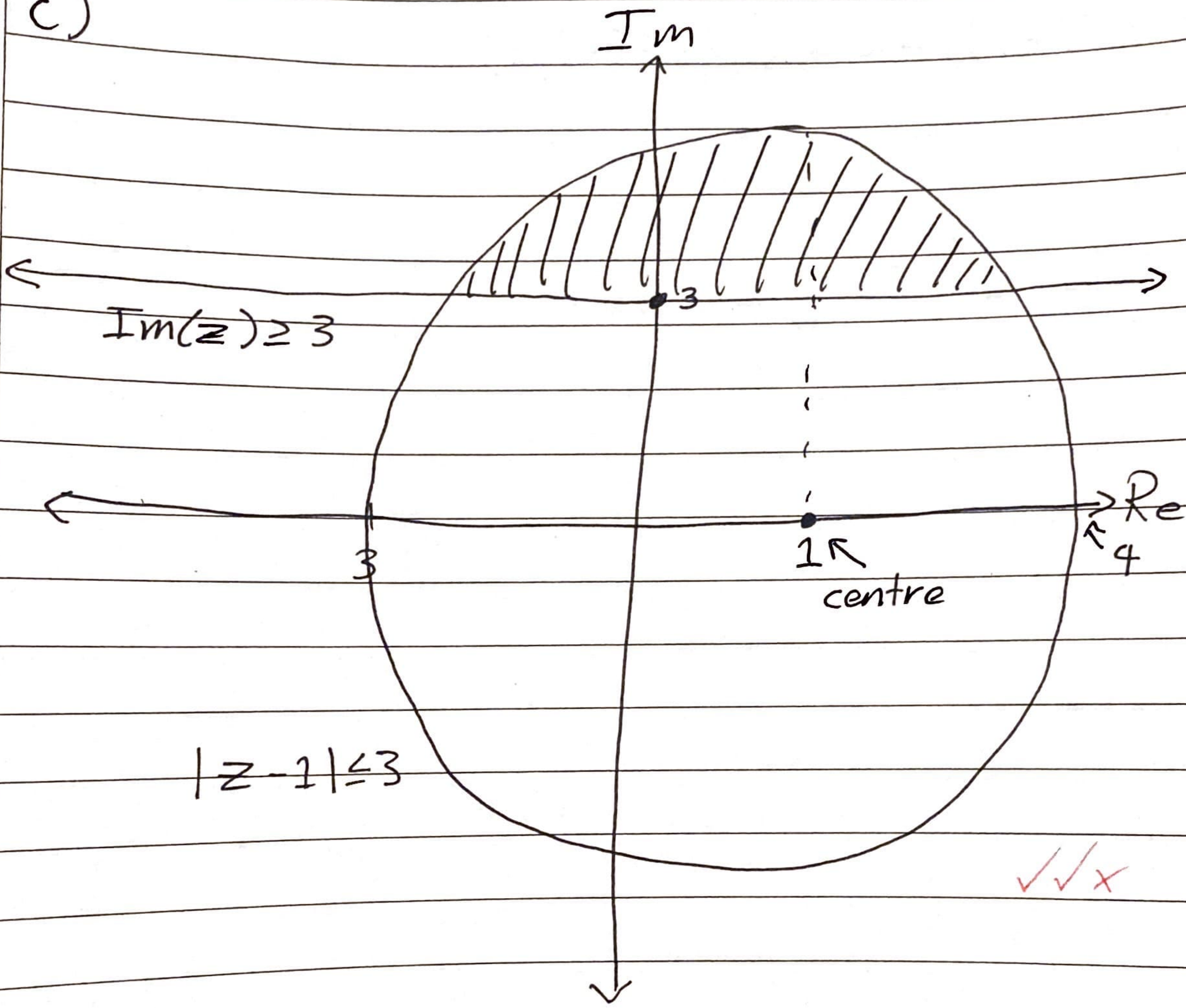
$I_2 = \frac{2(\frac{3}{2})}{2(\frac{3}{2})+1} \times \frac{3}{5}$

$= \frac{9}{20}$

$I_3 = \frac{81}{220}$ ✓✓

b) ×

c)



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d)

$r = 6i + 19j - k + \lambda(i + 4j - 2k)$

i) x x

ii) x x x x