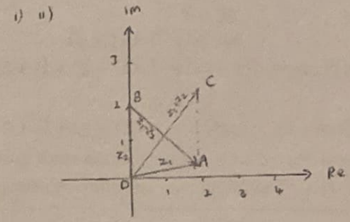
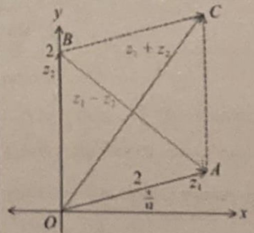
ACE Examination 2023
Year 12 Mathematics Extension 2 Yearly Examination
Worked solutions and marking guidelines

Section I

	Solution	Criteria
1	Let $u = x^2 + 1$ $\frac{du}{dx} = 2x$ and $xdx = \frac{1}{2}du$ $\int \frac{x}{\sqrt[3]{x^2+1}}dx = \frac{1}{2}\int \frac{1}{\sqrt[3]{u}}du$ $= \frac{1}{2}\int u^{-\frac{1}{3}}du$ $= \frac{1}{2} \times \frac{3}{2}u^{\frac{2}{3}} + C$ $= \frac{3}{4}\sqrt[3]{(x^2+1)^2} + C$	1 Mark: C
2	$(z-(2+3i))(z-(2-3i)) = 0$ $z^2 - 2z + 3iz - 2z - 3iz + 4 + 9 = 0$ $\therefore z^2 - 4z + 13 = 0$	1 Mark: A
3	$\underset{\sim}{u} \cdot \underset{\sim}{v} = 1 \times 4 + (-1) \times 12 + (-1) \times (-3)$ $= -5$	1 Mark: A
4	$z = 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ $\bar{z} = 3\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$ $\bar{z}^3 = 3^3\left(\cos\frac{3\pi}{6} - i\sin\frac{3\pi}{6}\right)$ $= 27\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)$	1 Mark: C
5	$\sqrt{x^2+y^2+z^2} = \sqrt{1^2+(-3)^2+5^2}$ $= \sqrt{35}$	1 Mark: B
6	$v^2 + 9x^2 = k$ $v^2 = k - 9x^2$ $\ddot{x} = \frac{d}{dx}\left(\frac{v^2}{2}\right)$ $= \frac{d}{dx}\left(\frac{k}{2} - \frac{9x^2}{2}\right)$ $= -9x = -n^2x$ Hence $n = 3$ Period $T = \frac{2\pi}{n} = \frac{2\pi}{3}$	1 Mark: D
7	The converse of a statement 'If P then Q' is 'If Q then P'. The statements can be represented as: the converse of $P \Rightarrow Q$ is $Q \Rightarrow P$ or $P \Leftarrow Q$. The converse of a true statement need not be true.	1 Mark: A
8	$v = f(x)$ $a = v\frac{dv}{dx} = f(x)f'(x)$	1 Mark: D
9	Using the conjugate root theorem $(2+i)$ and $(2-i)$ are both roots of the equation $z^3 + pz + q = 0$. $(2+i) + (2-i) + \alpha = 0$ (sum of the roots) $\alpha = -4$ $(2+i) \times (2-i) \times (-4) = -q$ (product of the roots) $(4+1) \times (-4) = -q$ $q = 20$ $(2+i)(2-i) + (2-i) \times (-4) + (2+i) \times (-4) = p$ $p = -11$ $\therefore p = -11$ and $q = 20$	1 Mark: B
10	Use the substitution $u = \cos x$ $\frac{du}{dx} = -\sin x$ $du = -\sin x dx$ When $x = 0$ then $u = 1$ and when $x = \frac{\pi}{2}$ then $u = 0$ $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sqrt{1-\cos^2 x}}dx = \int_1^0 \frac{-1}{\sqrt{1-u^2}}du$ $= [\sin^{-1}u]_0^1$ $= \frac{\pi}{2}$	1 Mark: D

Section II

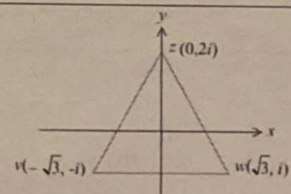
	Solution	Criteria
11(a) (i)	$i\bar{z} = i(4 - i) = 4i + 1$ $= 1 - 4i$	1 Mark: Correct answer.
11(a) (ii)	$\frac{1}{z} = \frac{1}{4-i} \times \frac{4+i}{4+i} = \frac{4+i}{16+1}$ $= \frac{4}{17} + \frac{1}{17}i$	1 Mark: Correct answer.
11(b)	$\lvert\overrightarrow{OA}\rvert = \sqrt{2^2 + 6^2 + (-3)^2}$ $= \sqrt{49} = 7$ $\widehat{OA} = \frac{\overrightarrow{OA}}{\lvert\overrightarrow{OA}\rvert}$ $= \frac{1}{7}(2\underset{\sim}{i} + 6\underset{\sim}{j} - 3\underset{\sim}{k})$	2 Marks: Correct answer. 1 Mark: Finds the magnitude of $\overrightarrow{OA}$.
11(c)	Let $u = x^2 + 4$ $\frac{du}{dx} = 2x$ or $\frac{1}{2}du = xdx$ When $x = 0$ then $u = 4$ and when $x = \sqrt{5}$ then $u = 9$ $\int_0^{\sqrt{5}} \frac{x}{\sqrt{x^2+4}}dx = \frac{1}{2}\int_4^9 \frac{1}{\sqrt{u}}du = \frac{1}{2}\int_4^9 u^{-\frac{1}{2}}du$ $= \frac{1}{2}\left[2u^{\frac{1}{2}}\right]_4^9$ $= [\sqrt{9} - \sqrt{4}]$ $= 1$ Q11 c) Alternate method. $\int_0^{\sqrt{5}} \frac{x}{\sqrt{x^2+4}}dx = \frac{1}{2}\int_0^{\sqrt{5}} 2x(x^2+4)^{-\frac{1}{2}}dx$ $= \frac{1}{2}\left[\frac{(x^2+4)^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^{\sqrt{5}}$ $= \left[(x^2+4)^{\frac{1}{2}}\right]_0^{\sqrt{5}}$ $= (\sqrt{5}^2+4)^{\frac{1}{2}} - (0+4)^{\frac{1}{2}}$ $= 9^{\frac{1}{2}} - 4^{\frac{1}{2}}$ $= 3 - 2$ $= 1$	2 marks: Correct answer. 1 mark: Sets up the integral in terms of u or as reverse chain rule.

	11c) Alternate version #2 $\int_0^{\sqrt{5}} \frac{x}{\sqrt{x^2+4}}dx$ Let $u^2 = x^2 + 4$ $2u\,du = 2x\,dx$ $dx = \frac{u}{x}du$ $= \int_2^3 \frac{x}{u} \times \frac{u}{x}du$ at $x = 0$, $u^2 = 4$, $u = 2$ $= \int_2^3 1\,du$ at $x = \sqrt{5}$, $u^2 = 5+4$, $u = 3$ $= [x]_2^3$ $= 3 - 2 = 1$	
11(d) (i)	*For question given* d) $z_1 = 2\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}$ $\frac{\pi}{12} = 15°$ $\doteqdot 1.93 + i\,0.259$ $A = \begin{pmatrix} 2\cos\frac{\pi}{12} \\ \sin\frac{\pi}{12} \end{pmatrix}$ $B = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ i) ii)  *As question was supposed to be written*  	1 Mark: Correct answer.
11(d) (ii)	See Argand diagram above.	1 Mark: Correct answer. Accept any equivalent vector.
11(d) (iii)	*For question given* iii) $\text{Arg}(z_1 + z_2)$ is angle the vector makes with the positive real axis $z_1 + z_2 = 2\cos\frac{\pi}{12} + i\left(2 + \sin\frac{\pi}{12}\right)$ $C = \begin{pmatrix} 2\cos\frac{\pi}{12} \\ 2 + \sin\frac{\pi}{12} \end{pmatrix}$ $\text{Arg}(z_1 + z_2) = \tan^{-1}\left(\frac{2 + \sin\frac{\pi}{12}}{2\cos\frac{\pi}{12}}\right)$ ← Exact form $\doteqdot 49°$	2 Marks: Correct answer or finding angle as inverse tan. 1 Mark: Shows some understanding.

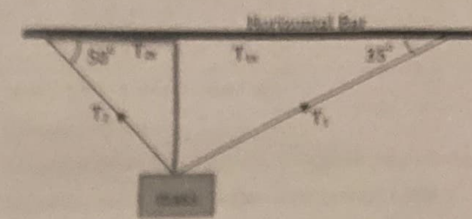
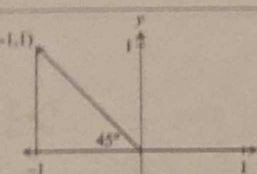
	For question as it was supposed to be written Vectors OC and AB form a parallelogram However $OA = OB = 2$. Hence $OBCA$ is a rhombus. Diagonal of a rhombus bisects the angle through which it passes $\therefore \angle AOC = \frac{1}{2} \times \left(\frac{\pi}{2} - \frac{\pi}{12}\right) = \frac{5\pi}{24}$ $\arg(z_1 + z_2) = \frac{\pi}{12} + \frac{5\pi}{24}$ $= \frac{7\pi}{24}$	
11(e)	$\int \frac{1}{\sqrt{12+4x-x^2}}dx = \int \frac{1}{\sqrt{12-(x^2-4x)}}dx$ $= \int \frac{1}{\sqrt{16-(x^2-4x+4)}}dx$ $= \int \frac{1}{\sqrt{16-(x-2)^2}}dx$ $= \sin^{-1}\left(\frac{x-2}{4}\right) + C$	2 Marks: Correct answer. 1 Mark: Completes the square.
11(f)	Step 1: To prove true for $n = 1$ LHS $= 1 + x$ RHS $= \frac{x^{1+1}-1}{x-1} = \frac{(x+1)(x-1)}{(x-1)} = 1 + x$ Result is true for $n = 1$ Step 2: Assume true for $n = k$ $S_k = \frac{x^{k+1}-1}{x-1}$ Step 3: To prove true for $n = k+1$ $S_{k+1} = \frac{x^{k+2}-1}{x-1}$ $S_k + T_{k+1} = S_{k+1}$ LHS $= \frac{x^{k+1}-1}{x-1} + x^{k+1}$ $= \frac{x^{k+1}-1}{x-1} + \frac{x^{k+1}(x-1)}{(x-1)}$ $= \frac{x^{k+1}-1+x^{k+2}-x^{k+1}}{x-1}$ $= \frac{x^{k+2}-1}{x-1}$ = RHS Step 4: True by induction	3 marks: Correct answer. 2 marks: Proves the result true for $n = 1$ and attempts to use the result of $n = k$ to prove the result for $n = k+1$ 1 mark: Proves the result true for $n =$ 1.

12(a)	$\int x\, e^{-x} dx$ $u = x \quad \frac{du}{dx} = 1$ $\frac{dv}{dx} = e^{-x} \quad v = -e^{-x}$ $\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$ $\int x\, e^{-x}dx = x \times (-e^{-x}) - \int -e^{-x} \times 1\, dx$ $= -xe^{-x} + \int e^{-x}dx$ $= -xe^{-x} - e^{-x} + C$	3 Marks: Correct answer. 2 Marks: Makes significant progress. 1 Mark: Correctly applies integration by parts.
12(b) (i)	$(1+ia)^4 = 1 + 4ia - 6a^2 - 4ia^3 + a^4$	1 Mark: Correct answer.
12(b) (ii)	$(1+ia)^4$ is real if $4a - 4a^3 = 0$ Then $4a(1-a^2) = 0$ $\therefore a = 0, \pm 1$	2 Marks: Correct answer. 1 Mark: Finds an equation for a or equivalent merit.
12(c) (i)	$\frac{5x^3-3x^2+2x-1}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$ Using partial fractions to find A, B, C and D $5x^3 - 3x^2 + 2x - 1 = Ax(x^2+1) + B(x^2+1) + (Cx+D)x^2$ $= Ax^3 + Ax + Bx^2 + B + Cx^3 + Dx^2$ $= (A+C)x^3 + (B+D)x^2 + Ax + B$ Equating the coefficients $A + C = 5$ (1) $B + D = -3$ (2) $A = 2$ and $B = -1$ Equation (1) $2 + C = 5$ or $C = 3$ Equation (2) $B + D = -3$ or $D = -2$ $\therefore A = 2, B = -1, C = 3$ and $D = -2$	2 Marks: Correct answer. 1 Mark: Makes progress in finding A, B, C or D.
12(c) (ii)	$\int \frac{5x^3-3x^2+2x-1}{x^2(x^2+1)}dx = \int \frac{2}{x} - \frac{1}{x^2} + \frac{3x-2}{x^2+1}dx$ $= \int \frac{2}{x} - \frac{1}{x^2} + \frac{3x}{x^2+1} - \frac{2}{x^2+1}dx$ $= 2\ln\|x\| + \frac{1}{x} + \frac{3}{2}\ln(x^2+1) - 2\tan^{-1}x + C$	2 Marks: Correct answer. 1 Mark: Correctly finds one of the integrals.

12(d) (i)	$\ddot{x} = -12\sin 2t$ $\dot{x} = -12\int \sin 2t\,dt$ $= -12\int 2\sin t\cos t\,dt$ $= -12\sin^2 t + C$ When $t = 0,\ \dot{x} = 6$ hence $C = 6$ $\therefore \dot{x} = -12\sin^2 t + 6$ 12d) Alternate method $\ddot{x} = -12\sin 2t$ $x = -12\int \sin 2t\ dt$ $= \frac{+12}{2}\cos 2t + c$ $= 6\cos 2t + c$ at $t = 0\ x = 0\ v = 6$ $6 = 6\cos 0 + c$ $\therefore c = 0$ $\dot{x} = 6\cos 2t$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
12(d) (ii)	$x = \int -12\sin^2 t + 6dt$ $= \int -12\left(\frac{1-\cos 2t}{2}\right) + 6dt = \int -6 + 6\cos 2t + 6dt$ $= \int 6\cos 2t\,dt$ $= 3\sin 2t + C$ When $t = 0, x = 0$ hence $C = 0$ $\therefore x = 3\sin 2t$ Then $\ddot{x} = -12\sin 2t$ $= -4 \times 3\sin 2t$ $= -4x$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Finds $x = 3\sin 2t + C$ or equivalent merit.

13(a)	$\int \frac{1}{9x^2+6x+5}dx = \int \frac{dx}{9\left[\left(x^2+\frac{2}{3}x\right)\right]+5}$ $= \int \frac{dx}{9\left[\left(x+\frac{1}{3}\right)^2-\frac{1}{9}\right]+5}$ $= \int \frac{dx}{9\left(x+\frac{1}{3}\right)^2+4}$ $= \int \frac{dx}{(3x+1)^2+2^2}$ $= \frac{1}{3}\int \frac{3dx}{(3x+1)^2+2^2}$ $= \frac{1}{6}\tan^{-1}\frac{(3x+1)}{2} + C$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Shows some understanding.
13(b) (i)	Line l_1 intersects the line l_2 then: $(11\underline{i} + 2\underline{j} + 17\underline{k}) + \lambda(-2\underline{i} + \underline{j} - 4\underline{k})$ $= (-5\underline{i} + 11\underline{j} + p\underline{k}) + \mu(-3\underline{i} + 2\underline{j} + 2\underline{k})$ $11 - 2\lambda = -5 - 3\mu$ ① $2 + \lambda = 11 + 2\mu$ ② $17 - 4\lambda = p + 2\mu$ ③ Equation ① $+ 2 \times$ ② $15 = 17 + \mu$ then $\mu = -2$ Equation ② $2 + \lambda = 11 - 4$ then $\lambda = 5$ Equation ③ $17 - 4 \times 5 = p + 2 \times -2$ $p = 17 - 20 + 4$ $= 1$ $\therefore$ The value of p is 1.	3 Marks: Correct answer. 2 Marks: Finds the correct values for λ and μ. 1 Mark: Shows some understanding.
13(b) (ii)	From 13(b)(i) $\lambda = 5$, $\mu = -2$ and $p = 1$ l_1: $(11\underline{i} + 2\underline{j} + 17\underline{k}) + 5(-2\underline{i} + \underline{j} - 4\underline{k}) = \underline{i} + 7\underline{j} + -3\underline{k}$ or l_2: $(-5\underline{i} + 11\underline{j} + \underline{k}) + -2(-3\underline{i} + 2\underline{j} + 2\underline{k}) = \underline{i} + 7\underline{j} + -3\underline{k}$ $\therefore$ Point of intersection is $\underline{i} + 7\underline{j} + -3\underline{k}$.	1 Mark: Correct answer.
13(c)	Pythagoras theorem $zw = \sqrt{3^2 + (\sqrt{3})^2}$ $= \sqrt{12} = 2\sqrt{3}$ $zv = \sqrt{3^2 + (\sqrt{3})^2}$ $= \sqrt{12} = 2\sqrt{3}$ $wv = \sqrt{3} + \sqrt{3}$ $= 2\sqrt{3}$ $\therefore$ Equilateral triangle.  	2 Marks: Correct answer. 1 Mark: Plots the points or makes some progress.

13(d) (i)	Amplitude of motion is 2 cm ($a = 2$) Period of motion is 4 seconds $T = \frac{2\pi}{n} = 4$ $n = \frac{\pi}{2}$ $v^2 = n^2(a^2 - x^2)$ $= \left(\frac{\pi}{2}\right)^2 (2^2 - x^2)$ Maximum velocity occurs when $x = 0$ $v^2 = \left(\frac{\pi}{2}\right)^2 \times 2^2$ $v = \pi \text{ cms}^{-1}$ 13 d) i) $T = 4$ $n = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$ $a = 2$ let start at 0 and phase c $x = 2\sin(\frac{\pi}{2}t)$ $\dot{x} = 2\frac{\pi}{2}\cos(\frac{\pi}{2}t) = \pi\cos(\frac{\pi}{2}t)$ max velocity at $x = 0$ $0 = 2\sin(\frac{\pi}{2}t)$ $\frac{\pi}{2}t = \frac{\pi}{2}$ $t = 1$ $\dot{x} = \pi\cos\frac{\pi}{2}$ max $v = \pi \text{ cm s}^{-1}$	2 Marks: Correct answer. 1 Mark: Finds the values of a and n or shows some understanding of the problem.
13(d) (ii)	Displacement is $x = \pm 1$ cm $v^2 = \left(\frac{\pi}{2}\right)^2 (2^2 - (\pm 1)^2)$ $= \frac{3\pi^2}{4}$ $v = \pm\frac{\sqrt{3}}{2}\pi \text{ cms}^{-1}$ at $x = 1$ $t = ?$ $t = \frac{\pi}{6} \cdot \frac{2}{\pi} = \frac{1}{3}$ $1 = 2\sin(\frac{\pi}{2}t)$ $\frac{1}{2} = \sin(\frac{\pi}{2}t)$ $\sin^{-1}(\frac{1}{2}) = \frac{\pi}{2}t$ $\frac{\pi}{6} = \frac{\pi}{2}t$ $v = \pi\cos\frac{\pi}{2}\cdot\frac{1}{3}$ $= \pi\cos\frac{\pi}{6}$ $= \pi \times \frac{\sqrt{3}}{2}$ Since motion is symmetrical about centre, the speed is $\frac{\sqrt{3}}{2}\pi$ where at/below the centre at 1cm.	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.

13(e)	 Since the object is at rest, the magnitude of the tension to the right and to the left must be equal $T_{1x} = T_{2x}$. Resolving the tension in the string into its x and y components gives $T_{1x} = T_1\cos 35°$ $T_{2x} = T_2\cos 50°$ $T_{1y} = T_1\sin 35°$ $T_{2y} = T_2\sin 50°$ Thus $T_1 = \frac{T_{1x}}{\cos 35°}$ and $T_2 = \frac{T_{2x}}{\cos 50°}$ So $\frac{T_1}{T_2} = \frac{T_{1x}}{\cos 35°} \times \frac{\cos 50°}{T_{2x}}$ $= \frac{\cos 50°}{\cos 35°}$ since $T_{1x} = T_{2x}$ $= 0.784698779$ $= 0.785$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
14(a) (i)	$r^2 = 1^2 + 1^2$ $r = \sqrt{2}$ $\tan\theta = \frac{1}{-1}$ $\theta = \frac{3\pi}{4}$  $\therefore -1 + i = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$	2 Marks: Correct answer. 1 Mark: Correctly determines the argument or the modulus.
14(a) (ii)	$(-1+i)^n = (\sqrt{2})^n\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)^n$ $= 2^{\frac{n}{2}}\left(\cos\frac{3n\pi}{4} + i\sin\frac{3n\pi}{4}\right)$	1 Mark: Correct answer.

14(b)	$\frac{1}{a}+\frac{1}{b}-\frac{4}{m}=\frac{1}{a}+\frac{1}{b}-\frac{4}{a+b}$ $=\frac{b(a+b)+a(a+b)-4ab}{ab(a+b)}$ $=\frac{(a+b)^2-4ab}{ab(a+b)}$ $=\frac{a^2-2ab+b^2}{ab(a+b)}$ $=\frac{(a-b)^2}{ab(a+b)}$ Given $a>0, b>0$ then $ab(a+b)>0$ Also $(a-b)^2>0$ for all values of a and b Hence $\frac{1}{a}+\frac{1}{b}-\frac{4}{m}=\frac{(a-b)^2}{ab(a+b)}>0$ $\therefore \frac{1}{a}+\frac{1}{b}>\frac{4}{m}$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Shows some understanding.
14(c) (i)	$m\ddot{x}=F-kv$ $m\frac{dv}{dt}=F-kv$ $\frac{dv}{dt}=\frac{F-kv}{m}$ $\frac{dt}{dv}=\frac{m}{F-kv}$ $\int_0^t dt=\int_0^v \frac{m}{F-kv}\,dv$ $t-0=\frac{m}{-k}\int_0^v \frac{1}{\frac{F}{k}-v}\,dv$ $t=-\frac{m}{k}\left[\ln\left\|\frac{F}{k}-v\right\|\right]_0^v$ $=-\frac{m}{k}\left(\ln\left\|\frac{F}{k}-v\right\|-\ln\left\|\frac{F}{k}\right\|\right)$ $=-\frac{m}{k}\left(\ln\frac{\frac{F}{k}-v}{\left(\frac{F}{k}\right)}\right)$ $=-\frac{m}{k}\ln\left(1-\frac{vk}{F}\right)$ $-\frac{kt}{m}=\ln\left(1-\frac{vk}{F}\right)$ $e^{-\frac{kt}{m}}=1-\frac{vk}{F}$ $\frac{vk}{F}=1-e^{-\frac{kt}{m}}$ $v=\frac{F}{k}\left(1-e^{-\frac{kt}{m}}\right)$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Creates equation and substitutes for acceleration

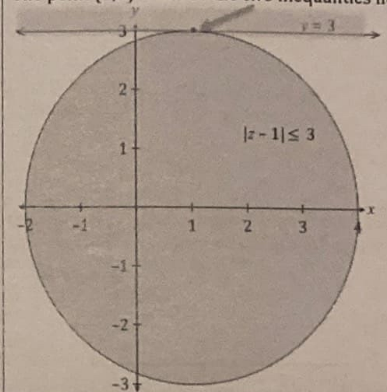
14(c) (ii)	Terminal velocity occurs at acceleration = 0, or at limit as $t\to\infty$. Thus terminal velocity is $\frac{F}{k}$. Using the equation from part i) $t=-\frac{m}{k}\ln\left(1-\frac{F}{2k}\times\frac{k}{F}\right)$ $=-\frac{m}{k}\ln\left(1-\frac{1}{2}\right)$ $=-\frac{m}{k}\ln\frac{1}{2}$ $=\frac{m}{k}\ln 2$	2 Marks: Correct answer. 1 Mark: Identifies terminal velocity or equivalent
14(c) (iii)	$v=\frac{dx}{dt}=\frac{F}{k}\left(1-e^{-\frac{kt}{m}}\right)$ $\int_0^x dx=\frac{F}{k}\int_0^t 1-e^{-\frac{kt}{m}}\,dt$ $x=\frac{F}{k}\left[t+\frac{m}{k}e^{-\frac{kt}{m}}\right]_o^t$ $=\frac{F}{k}\left(t+\frac{m}{k}e^{-\frac{kt}{m}}-0-\frac{m}{k}e^{0}\right)$ $=\frac{F}{k}\left(t+\frac{m}{k}e^{-\frac{kt}{m}}-\frac{m}{k}\right)$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
14(d)	Assume $\sqrt{4n-2}$ is rational. Then it can be written as a fraction $\frac{p}{q}$ where p and q are integers with no common factors. $\sqrt{4n-2}=\frac{p}{q}$ $4n-2=\frac{p^2}{q^2}$ $2(2n-1)q^2=p^2$ Thus p^2 is even, so p must be even. Therefore, p = 2m $2(2n-1)q^2=(2m)^2$ $=4m^2$ $(2n-1)q^2=2m^2$ Since n is a positive integer, (2n-1) is odd. Thus q^2 must be even, so q is even. This shows that p and q are both even, thus they have a common factor of 2 which contradicts the initial assumption of no common factors. Hence, $\sqrt{4n-2}$ must be irrational	2 Marks: Correct answer. 1 Mark: Shows understanding by setting up for rational proof.

15(a)	Let $z = x + iy$ Then $z^2 = (x+iy)^2 = x^2 + 2ixy - y^2$ $z^2 = \|z\|^2 - 4$ $x^2 + 2ixy - y^2 = \|x+iy\|^2 - 4$ $x^2 + 2ixy - y^2 = \left(\sqrt{x^2+y^2}\right)^2 - 4$ $x^2 + 2ixy - y^2 = x^2 + y^2 - 4$ $2ixy - y^2 = y^2 - 4$ $2y^2 - 2ixy - 4 = 0$ $2(y^2 - ixy - 2) = 0$ $(y^2 - 2) = 0$ then $y = \pm\sqrt{2}$ $ixy = 0$ then $x = 0$ Therefore $z = x + iy$ $z = \pm\sqrt{2}i$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Writes the equation using $z = x + iy$ or equivalent merit.
15(b)	Step 1: To prove true for $n = 1$ $\text{LHS} = (\cos\theta + i\sin\theta)^1 = \cos\theta + i\sin\theta$ $\text{RHS} = \cos 1\theta + i\sin 1\theta = \cos\theta + i\sin\theta$ Result is true for $n = 1$ Step 2: Assume true for $n = k$ $(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$ Step 3: To prove true for $n = k+1$ $(\cos\theta + i\sin\theta)^{k+1} = \cos(k+1)\theta + i\sin(k+1)\theta$ $\text{LHS} = (\cos\theta + i\sin\theta)^{k+1}$ $= (\cos\theta + i\sin\theta)^k \times (\cos\theta + i\sin\theta)$ $= (\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta)$ $= (\cos k\theta\cos\theta - \sin k\theta\sin\theta) + i(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$ $= \cos(k\theta + \theta) + i\sin(k\theta + \theta)$ $= \cos(k+1)\theta + i\sin(k+1)\theta$ $= \text{RHS}$ Step 4: True by induction	4 Marks: Correct answer. 3 Marks: Makes significant progress towards the solution. 2 Marks: Proves the result true for $n = 1$ and attempts to use the result of $n = k$ to prove the result for $n = k + 1$ 1 Mark: Proves the result true for $n = 1$
15(c) (i)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $= (\underset{\sim}{j} + 2\underset{\sim}{k}) - (-\underset{\sim}{i} - \underset{\sim}{j})$ $= \underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$	1 Mark: Correct answer.

15(c) (ii)	$\|\overrightarrow{AB}\| = \sqrt{x^2+y^2+z^2}$ $= \sqrt{1^2+2^2+2^2}$ $= 3$	1 Mark: Correct answer.
15(c) (iii)	$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$ $= (4\underset{\sim}{i} + \underset{\sim}{k}) - (\underset{\sim}{j} + 2\underset{\sim}{k})$ $= 4\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$ $\|\overrightarrow{BC}\| = \sqrt{x^2+y^2+z^2}$ $= \sqrt{4^2 + (-1)^2 + (-1)^2}$ $= 3\sqrt{2}$ $\cos\angle CAB = \dfrac{\overrightarrow{AB}\cdot\overrightarrow{BC}}{\|\overrightarrow{AB}\|\|\overrightarrow{BC}\|}$ $= \dfrac{1\times 4 + 2\times(-1) + 2\times(-1)}{3\times 3\sqrt{2}}$ $= \dfrac{0}{9\sqrt{2}} = 0$ $\angle CAB = 90°$ Also accept simply showing the dot product is zero with a comment indicating that means the vectors are perpendicular.	3 Marks: Correct answer. 2 Marks: Uses the dot product or the angle between two vectors. 1 Mark: Finds $\overrightarrow{BC}$ or $\|\overrightarrow{BC}\|$.

15(d)	$F = \frac{m}{x^3}(6-10x)$ $ma = \frac{m}{x^3}(6-10x)$ $v\frac{dv}{dx} = \frac{6}{x^3} - \frac{10}{x^2}$ $\int v\,dv = \int\left(\frac{6}{x^3}-\frac{10}{x^2}\right)dx$ $\frac{1}{2}v^2 = \left(\frac{6x^{-2}}{-2} - \frac{10x^{-1}}{-1}\right)+C$ $\frac{1}{2}v^2 = \left(\frac{-3}{x^2}+\frac{10}{x}\right)+C$ When $v=0$ and $x=1$ $\frac{1}{2}0^2 = \left(\frac{-3}{1^2}+\frac{10}{1}\right)+C$ $C=-7$ Hence $\frac{1}{2}v^2 = \left(\frac{-3}{x^2}+\frac{10}{x}\right)-7$ $v^2 = \left(\frac{-6}{x^2}+\frac{20}{x}\right)-14$ $= \frac{-6+20x-14x^2}{x^2}$ $v = \pm\frac{1}{x}\sqrt{2(-3+10x-7x^2)}$ Since the object was initially at rest at x = 1 and the initial force was F = m(6-10) = -4m <0. The direction of force does not change, hence the object will be moving in the negative direction so: $v = -\frac{1}{x}\sqrt{2(-3+10x-7x^2)}$	3 Marks: Correct answer. 2 Marks: Integrates and uses the initial conditions to find an expression for $\frac{1}{2}v^2$. 1 Mark: Uses $v\frac{dv}{dx}$ for acceleration.
16(a) (i)	$I_n = \int_0^1 (1-x^r)^n dx$ $= [x(1-x^r)^n]_0^1 - n\int_0^1 x(1-x^r)^{n-1}(-rx^{r-1})dx$ $= 0 - nr\int_0^1 [(1-x^r)-1](1-x^r)^{n-1}dx$ $= 0 - nr\int_0^1 (1-x^r)^n - (1-x^r)^{n-1}dx$ $I_n = nr(-I_n + I_{n-1})$ $(nr+1)I_n = nrI_{n-1}$ $I_n = \frac{nr}{nr+1}I_{n-1}$	2 Marks: Correct answer. 1 Mark: Sets up the integration by parts.

Year 12 Mathematics Extension 2

16(a) (ii)	For $r=\frac{3}{2}$ and $n=3$ $I_3 = \frac{3\times\frac{3}{2}}{3\times\frac{3}{2}+1}\times I_2$ $I_2 = \frac{2\times\frac{3}{2}}{2\times\frac{3}{2}+1}\times I_1$ $I_1 = \frac{1\times\frac{3}{2}}{1\times\frac{3}{2}+1}\times I_0$ But $I_0 = \int_0^1 (1-x^r)^0 dx = \int_0^1 1dx = 1$ $I_3 = \frac{3\times\frac{3}{2}}{3\times\frac{3}{2}+1}\times\frac{2\times\frac{3}{2}}{2\times\frac{3}{2}+1}\times\frac{1\times\frac{3}{2}}{1\times\frac{3}{2}+1}\times 1$ $= \frac{81}{220}$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
16(b) (i)	$a^{a-b} > c^{a-b}$ since $a-b>0$ (1) $b^{b-c} > c^{b-c}$ since $b-c>0$ (2) $a^{a-b}b^{b-c} > c^{a-b}c^{b-c}$ (1) x (2) $\therefore a^{a-b}b^{b-c} > c^{a-c}$	1 Mark: Correct answer.
16(c)	$\lvert z-1\rvert \le 3$ represents a region with centre (1, 0) and radius less than or equal to 3. Im(z) $\ge$ 3 represents a region above the horizontal line $y = 3$. The point (1,3) is where the two inequalities hold. 	3 Marks: Correct graphs both inequalities and indicates point of intersection. 2 Marks: Correctly graphs one inequality and attempts second. 1 Mark: Makes some progress.

16(d) (i)	$\underset{\sim}{i}$ component $6 + \lambda = 0$ $\lambda = -6$ $\underset{\sim}{j}$ component $19 - 6 \times 4 = a$ $a = -5$ $\underset{\sim}{k}$ component $-1 - 6 \times (-2) = b$ $\therefore a = -5$ and $b = 11$.	2 Marks: Correct answer. 1 Mark: Finds λ or shows some understanding.
16(d) (ii)	$\overrightarrow{OP} = (6 + \lambda)\underset{\sim}{i} + (19 + 4\lambda)\underset{\sim}{j} + (-1 - 2\lambda)\underset{\sim}{k}$ Direction vector of l_1: $\underset{\sim}{i} + 4\underset{\sim}{j} - 2\underset{\sim}{k}$ $\overrightarrow{OP}$ and l_1 are perpendicular $[(6 + \lambda)\underset{\sim}{i} + (19 + 4\lambda)\underset{\sim}{j} + (-1 - 2\lambda)\underset{\sim}{k}] \cdot (\underset{\sim}{i} + 4\underset{\sim}{j} - 2\underset{\sim}{k}) = 0$ Hence $6 + \lambda + (19 + 4\lambda)4 + (-1 - 2\lambda) - 2 = 0$ $6 + \lambda + 76 + 16\lambda + 2 + 4\lambda = 0$ $21\lambda + 84 = 0$ $\lambda = -4$ Therefore $\overrightarrow{OP} = (6 - 4)\underset{\sim}{i} + (19 + 4 \times (-4))\underset{\sim}{j} + (-1 - 2 \times (-4))\underset{\sim}{k}$ $= 2\underset{\sim}{i} + 3\underset{\sim}{j} + 7\underset{\sim}{k}$	4 Marks: Correct answer. 3 Marks: Makes significant progress towards the solution. 2 Marks: Applies the statement for perpendicular vectors. 1 Mark: Shows some understanding.