Quality and creativity

KATOOMBA HIGH SCHOOL

**Student Name** Malachi Hasson

# Katoomba High School

Examination Booklet

**Examination**

Ext 1 Maths

7

**Instructions**

- Write the question number you are answering in this box → 11
- Write the number of this booklet and the total number of booklets that you have used for this question (eg 1 of 3). → 1 of ____ (this booklet / number of booklets for this question)
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11)

a) $r = 0.6$

$\frac{dh}{dt} = 0.01$

circle $= \pi (0.6)^2$

$= 0.36\pi \ u^2$

cylinder = circle × height

$= 0.36\pi h$

$h = \int 0.01 \, dt$

$= 0.01t + C$

$\therefore V = 0.36\pi \times (0.01t) + C$

$= 0.0036\pi t + C$ units$^3$

$\frac{dV}{dt} = 0.0036\pi$ units$^3$/s

$= 0.0113$ units$^3$/s ✓ x

b)

i) $\underrightarrow{a} + \underrightarrow{b}$

$= i + 4j + 4i - 2j$

$= 5i + 2j$ ✓

ii) $3(i + 4j) - (4i - 2j)$

$= 3i + 12j - 4i + 2j$

$= -i + 14j$ x

c) xxx

d)

$n = 7$

$p = 0.4$

$q = 0.6$

i) $P(X=4) = \frac{7C_4}{2^7}$

$= \frac{35}{128}$ ×

ii) ×

iii) $npq = 7 \times 0.4 \times 0.6$

$= 1.68$ ✓

e)

$x^2 + 2x + 2$

$= x^2 + 2x + 1 + 1$

$= (x+1)^2 + 1$

$\int \frac{1}{x^2+2x+2}\,dx$

$= \int \frac{1}{(x+1)^2+1}\,dx$

$= \frac{1}{1}\tan^{-1}\left(\frac{x+1}{1}\right) + C$

$= \tan^{-1}(x+1) + C$ ✓✓

f) Let $t = \tan\frac{x}{2}$

$\frac{7\left(\frac{2t}{1+t^2}\right)}{2} + 2\left(\frac{1-t^2}{1+t^2}\right) = 4$ ✓

$\frac{14t}{1+t^2} + 4\left(\frac{1-t^2}{1+t^2}\right) = 8$

$\frac{14t}{1+t^2} + \frac{4-4t^2}{1+t^2} = 8$

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Start here ?

$$\frac{14t}{1+t^2} + \frac{4-4t^2}{1+t^2} = 8$$

$$14t + 4 - 4t^2 = 8 + 8t^2$$

$$-12t^2 + 14t - 4 = 0$$

$$-2(6t^2 - 7t + 2) = 0$$

$$t = \frac{-14 \pm \sqrt{14^2 - 4(-12)(-4)}}{-24}$$

$$= \frac{-14 \pm \sqrt{4}}{24}$$

$$= \frac{-14 \pm 2}{24}$$

$$t = -1 \text{ or } t = -\frac{2}{3}$$

$$\tan\frac{x}{2} = -1 \text{ or } \tan\frac{x}{2} = -\frac{2}{3}$$

$$\frac{x}{2} = -\frac{\pi}{4} \quad \text{or}$$

$$x = -45^\circ 00' \text{ or } x = -33^\circ 41'$$

$$x = 45^\circ 00' \text{ or } x = 33^\circ 41'$$

✓ ✗

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**Student Name** Malachi Haskin

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## Examination Booklet

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Maths Ext 1

11

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$\int \frac{x}{(5-x^2)^3} dx$ (12) (a)

$u = 5 - x^2$

$\frac{du}{dx} = -2x$

$dx = \frac{du}{-2x}$

$\int \frac{5-x^2}{(5-(5-x^2)^2)} \times \frac{du}{-2x}$

$= \int \frac{5-x^2}{(5-(25-10x^2+x^4)(-2x)}$

$= \int \frac{5-x^2}{(-2x)(5-25+10x^2-x^4)}$

$= \int \frac{5-x^2}{-2x(-20+10x^2-4x^4}$

$= \int \frac{5-x^2}{}$

$= \frac{1}{\frac{5}{5-x^2} - 5-x^2} \times \frac{du}{-2x}$

$= \frac{1}{\left(\frac{5-(5-x^2)(5-x^2)}{5-x^2}\right)} \frac{du}{-2x}$

XX

b) ${}^9C_4 \times {}^7C_2$

$= 2646$ ways ✓

c)

i) $P(-2) = 2(-2)^3 + 3(-2)^2 - 29(-2) - 60$

$= -6$

$\therefore$ remainder $= -6$ ✓

ii) $P(-3) = 2(-3)^3 + 3(-3)^2 - 29(-3) - 60$

$= 0$

$\therefore$ Factor ✓

iii)

$$x+3 \overline{)\, 2x^3 + 3x^2 - 29x - 60}$$

$2x^2 - 3x - 20$

$-(2x^3 + 6x^2)$ ↓

$-3x^2 - 29x$

$-(-3x^2 - 9x)$ ↓

$-20x - 60$

$-(-20x - 60)$

$0$ ✓✓

$P(x) = (x+3)(2x^2 - 3x - 20)$

$= (x+3)(2x^2 - 8x + 5x - 20)$

$= (x+3)(2x(x-4) + 5(x-4))$

$= (x+3)(x-4)(2x+5)$ ✓

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Start here

a) i)

$R\sin(x+\alpha) = R(\sin x\cos\alpha + \cos x\sin\alpha)$

$= R\sin x\cos\alpha + R\cos x\sin\alpha$

$= \sqrt{3}\sin x + 3\cos x$

$\therefore \sqrt{3} = R\cos\alpha$ ① & $3 = R\sin\alpha$ ②

① $^2$ + ② $^2$

$3 + 3^2 = R^2\cos^2\alpha + R^2\sin^2\alpha$

$12 = R^2(1)$

$R^2 = 12$

$R = \sqrt{12} \quad \{R > 0\}$

$\sqrt{3} = \sqrt{12}\cos\alpha$

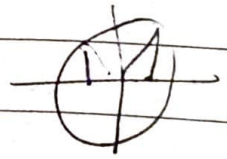
$\cos\alpha = \frac{\sqrt{3}}{\sqrt{12}}$

$\alpha = \cos^{-1}\left(\frac{\sqrt{3}}{\sqrt{12}}\right)$

$\alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2\sqrt{3}}\right)$

$= \cos^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{3}$ or $-\frac{2\pi}{3}$



$\sqrt{12}\sin\left(x + \frac{\pi}{3}\right)$ √√

ii) $\sqrt{12}\sin\left(x+\frac{\pi}{3}\right) = \sqrt{3} \quad 0 \le x \le 2\pi$

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**Student Name**

Malachi Hasker

# Katoomba High School

## Examination Booklet

**Examination**

Maths Ext 1

**Instructions**

- Write the question number you are answering in this box ⇒ 12
- Write the number of this booklet and the total number of booklets that you have used for this question (eg [1] of [3]). ⇒ [2] of [2]

  [2] = this booklet; [2] = number of booklets for this question
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$\sin\left(x+\frac{\pi}{3}\right)=\frac{\sqrt{3}}{\sqrt{12}}$ $\quad u=x+\frac{\pi}{3}$

$\sin\left(x+\frac{\pi}{3}\right)=\frac{1}{2} \quad 0\le x\le 2\pi$

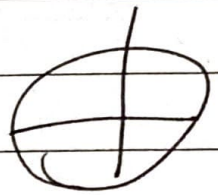
$\sin(u)=\frac{1}{2} \quad 0\le u-\frac{\pi}{3}\le 2\pi$

$\sin^{-1}\left(\frac{1}{2}\right)=x+\frac{\pi}{3} \quad 0\le x+\frac{\pi}{3}\le 2\pi$

$\frac{\pi}{6}=x+\frac{\pi}{3}$ or $\frac{5\pi}{6}=x+\frac{\pi}{3}$

$x=-\frac{\pi}{6}$ $\quad x=\frac{1}{2}$

$x=\frac{11\pi}{6}$ or $\frac{\pi}{2}$ ✓✓



e) $\int \pi r^2\,dx$

$=\pi\int_0^{\pi} y^2\,dx$

$=\pi\int_0^{\pi}\left(3\cos\frac{x}{2}\right)^2 dx$

$=\pi\times 2\times 3\int_0^{\pi}\frac{1}{2}\cos\frac{1}{2}x\,dx$

$=\pi\times 6\left[\sin\frac{1}{2}x\right]_0^{\pi}$

$=\pi\times 6\left(\sin\left(\frac{1}{2}\times\pi\right)\right)-\pi\times 6(\sin 0)$

$=6\pi$ units$^3$ ✓✗✗

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## Examination Booklet

**Examination**

Maths Ext 1

9

**Instructions**

- Write the question number you are answering in this box ⟹ 13
- Write the number of this booklet and the total number of booklets that you have used for this question (eg [1] of [3]). ⇨ [1] of [ ]
  (this booklet / number of booklets for this question)
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13)

a) $\frac{dT}{dt} = -k(T-A)$

ii) $T = A + Be^{-kt}$

$\frac{dT}{dt} = \frac{d}{dt}(A + Be^{-kt})$

$= -kBe^{-kt}$

$\frac{dT}{dt} = -k(T-A)$

$= -k((A+Be^{-kt}) - A)$

$= -k(Be^{-kt})$

$= -kBe^{-kt}$ ✓

$= \frac{dT}{dt}$

∴ satisfies ✓

ii) $60 = A + Be^{-k(0)}$

From the question $A = 12$.

$60 = A + B$

$30 = A + Be^{-k(25)}$

xxx

b) $\int_0^{\frac{1}{2}} \sqrt{\frac{x}{1-x}}\, dx$

$x = \sin^2\theta$

$\frac{dx}{d\theta} = \frac{d}{d\theta}(\sin^2\theta)$

$= \frac{d}{d\theta}\left(\frac{1}{2}(1-\cos 2\theta)\right)$

$= \frac{d}{d\theta}\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)$

$= -\frac{1}{2}(2)(-1)\sin(2\theta)$

$= \sin 2\theta$

$d\theta = \frac{dx}{\sin 2\theta}$ $\quad dx = \sin 2\theta\, d\theta$

$\frac{1}{2} = \sin^2\theta$

$\sin\theta = \sqrt{\frac{1}{2}}$

1 $\theta = \frac{\pi}{4}$

$0 = \sin^2\theta$

$\sin\theta = 0$

$\theta = 0$

$\therefore$ ~~8~~

$\therefore \int_0^{\frac{\pi}{4}} \sqrt{\frac{\sin^2\theta}{1-\sin^2\theta}} \times \sin 2\theta\, d\theta$

$= \int_0^{\pi/4} \sqrt{\frac{\sin^2\theta}{\cos^2\theta}} \times \sin 2\theta\, d\theta$

$= \int_0^{\pi/4} \frac{\sin\theta}{\cos\theta} \times \sin 2\theta\, d\theta$

$= \int_0^{\pi/4} \frac{\sin\theta}{\cos\theta} \times 2\sin\theta\cos\theta\, d\theta$

$= \int_0^{\pi/4} 2\sin^2\theta\, d\theta$

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Start here

$$\int_0^{\pi/4} 2\sin^2\theta \, d\theta$$

$$= 2\int_0^{\pi/4} (1 - \cos^2\theta) d\theta$$

$$= 2\int_0^{\pi/4} \frac{1}{2}(1 - \cos 2\theta) d\theta$$

$$= \int_0^{\pi/4} 1 \, d\theta - \int_0^{\pi/4} \cos 2\theta \, d\theta$$

$$= [\theta]_0^{\pi/4} - \frac{1}{2}\int_0^{\pi/4} 2\cos 2\theta \, d\theta$$

$$= \frac{\pi}{4} - 0 - \frac{1}{2}\left[\sin 2\theta\right]_0^{\pi/4}$$

$$= \frac{\pi}{4} - \frac{1}{2}(1 - 0)$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

$$= \frac{\pi - 2}{4}$$

√√√

c i) x

ii) xx

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Malachi Haskew

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Examination Booklet

**Examination**

Maths Ext 1

**Instructions**

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13)

d) $\frac{d}{dx}(x \sin^{-1} x)$

* $u = x \qquad v = \sin^{-1} x$

$u' = 1 \qquad v' = \frac{1}{\sqrt{1-x^2}}$

$= vu' + uv'$

$= \sin^{-1} x\,(1) + (x)\left(\frac{1}{\sqrt{1-x^2}}\right)$

$= \sin^{-1} x + \frac{x}{\sqrt{1-x^2}}$ ✓✓

e) $\sqrt{6^2 + 3^2}$

i) $= \sqrt{45}$

$= 3\sqrt{5}$ ✓

ii) $\sqrt{26}$ ✓

iii) $-21$ ✓

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**Student Name** Malachi Haskew

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Examination Booklet

**Examination**

Maths Ex 1

4

**Instructions**

- Write the question number you are answering in this box ⟹ 14
- Write the number of this booklet and the total number of booklets that you have used for this question (eg [1] of [3]). ⇨ [1] of [ ]
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14) a) i)

$\ddot{y} = -g$

$\dot{y} = \int \ddot{y}\, dt$

$= -gt + C$

~~when $t = 0$ $\dot{y} = V$~~

~~$V = -g(0) + C$~~

~~$V = C$~~

vertical velocity component:

$V \sin\theta$

$\dot{y} = -gt + C$

when $t = 0$ $V_y = V\sin\theta$

$V\sin\theta = -g(0) + C$

$C = V\sin\theta$ (oops i kind proved it unnecissarily)

$\dot{y} = -gt + V\sin\theta$

$0 = -gt + V\sin\theta$

$gt = V\sin\theta$

$t_A = \frac{V\sin\theta}{g}$ ← time to apex

$x(2t_A) = V\left(\frac{2V\sin\theta}{g}\right)\cos\theta$

$0p = \frac{2V^2\sin\theta\cos\theta}{g}$ ✓✓

ii)

$$x = V\left(\frac{2V}{\sqrt{3}g}\right)\cos\frac{\pi}{3}$$

$$= \frac{V^2}{\sqrt{3}g}$$

$$y = -\frac{1}{2}g\left(\frac{2V}{\sqrt{3}g}\right)^2 + V\left(\frac{2V}{\sqrt{3}g}\right)\sin\frac{\pi}{3}$$

$$= -\frac{1}{2}g\left(\frac{4V^2}{3g^2}\right) + \frac{2V^2}{\sqrt{3}g} \times \frac{3}{\sqrt{4}}$$

$$= -\frac{2V^2}{3g} + \frac{6V^2}{\sqrt{12}g}$$

$$= \frac{-2V^2}{3g} + \frac{3V^2}{\sqrt{3}g}$$

$$= \frac{-2V^2}{3g} + \frac{3\sqrt{3}V^2}{3g}$$

$$= \frac{3\sqrt{3}V^2 - 2V^2}{3g}$$

$$= \frac{V^2(3\sqrt{3} - 2)}{3g}$$

✓ + +

Solve simultaniously for V?

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Start here

$y = e^{mx}$

$y' = me^{mx}$

$y'' = m^2e^{mx}$

$m^2e^{mx} + me^{mx} - 6e^{mx} = 0$

~~$m(me^{mx} + e^{mx})$~~

$e^{mx}(m^2+m) - 6e^{mx} = 0$

$e^{mx}(m^2+m-6) = 0$ ← can I ln here because $\ln(0)$ = undefined

solve this.

No.

$e^{mx}(m^2+m) = 6e^{mx}$

$mx(m^2+m) = \ln(6e^{mx})$

$mx(m^2+m-6) = 0$

$mx($

$m = 0 \quad \text{or} \quad \frac{1 \pm \sqrt{1^2 - 4(-6)}}{2}$

$m = 0 \quad \text{or} \quad m = 3 \quad \text{or} \quad m = -2$ ✓x

c) $\int_{-1}^{1} \cos^{-1} x \, dx$ xx