Existence of Weak Coalitional Equilibrium: Allowing for Overlapping Coalitions

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October 11, 2022

Abstract

In this paper, I consider Ray and Vohra (1997)'s Coalitional Equilibrium, and, I show the methodological advantage of taking the notion of "an improvement for a group" to mean that there is a joint action of the group that induces a strict improvement in utility *for all* its members. This is opposed to assuming that no agent in the group is worse off while one is strictly better off. I show that, when this interpretation is taken, the sufficient conditions for existence of Ray and Vohra (1997)'s Coalitional Equilibrium can be weakened. I do so by showing that, taking the interpretation that no joint deviation induces a strict improvement for all agents, the existence of Coalitional Equilibrium is implied by the existence of a Nash equilibrium of an auxiliary game. Further to this, I show that the proof of existence can be extended to a generalisation of the concept, where groups may overlap but do not necessarily include the grand coalition. This allows for coalition configurations to be taken from a specific set of covers, while still ensuring existence. This provides the first step in answering the question of the existence of solution concepts with overlapping coalitions, while a deviation of the grand coalition is not permitted.

Keywords: Existence Result, Binding Agreements, Overlapping Coalitions.
JEL: C70, C71

1 Introduction

Coalitional Equilibrium, introduced by Ray and Vohra (1997), provides the analogous of Nash equilibrium for disjoint groups of agents that make agreements over how they play within a normal form game. This concept ensures that no group, when holding correct beliefs about the action profiles agreed upon by other groups, can deviate to a new action profile, while ensuring an improvement for all its members.[1] To pin this concept down further, if there is only a single group, containing all players, this coincides with a notion of Pareto efficiency; on the other extreme, if each individual is within a group on their own this coincides exactly with Nash equilibrium. However, the notion of what is meant by "no group can jointly deviate to a new joint action, ensuring an improvement for all agents within the group" can be interpreted in at least two ways.

Firstly, it could be that a deviation is only seen as an improvement if *all* members within the group have a *strict* improvement in their utility. This would capture a situation where any agent within the group would propose such a deviation, and all would have strict incentives to accept. This notion is captured by the concept of ruling out the possibility of a deviation that feasibly Strongly Pareto Dominates the action profile in hand. It must be feasible in the sense that a group may only change the actions of its members, not the entire action profile. Therefore, this would be a notion of equilibrium where the action profile being taken is feasibly Weakly Pareto Efficient.[2] Secondly, it could be that a deviation is seen as an improvement if *all*

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[1] As groups to do so in a binding way, the group itself need not be concerned with the prospect that some agents within the group, if they could do so privately, would like to choose a new action. This is distinct from concepts such as coalition proof Nash equilibrium of Bernheim et al. (1987) which ensures such consistency.

[2] The notion is referred to as Weakly Pareto Efficient as it is a weaker notion than Pareto Efficiency, as it could be that an outcome for which there exists another outcome that makes all players strictly better off while one receives the same utility would be Weakly Pareto Efficient while not Pareto Efficient.

members within the group are no worse off, while at least *one* member has a *strict* improvement in their utility. This would capture a situation where some agent within the group would propose such a deviation, and all agents within the group would have no reason to object. This instead captures a notion of feasible Pareto Domination, or that the action being taken is feasibly Pareto Efficient.

For the most part, the literature within cooperative game theory makes little distinction between these two notions.[3] There are a number of reasons for this. Firstly, in a generic game, it is not possible that there is a joint deviation that feasibly Pareto Dominates an action profile without Strongly Pareto Dominating it, as there is no possibility for indifference. Secondly, in games with transferable utility, the workhorse model of cooperative games, if there exists a joint deviation that feasibly Pareto Dominates the current profile, then there also exists a joint deviation that feasibly Strongly Pareto Dominates the current profile. Nonetheless, I show that making such a distinction can have important implications for the existence of Coalitional Equilibrium.

The sufficient conditions for the existence of a Coalitional Equilibrium provided by Ray and Vohra (1997) are applicable to the interpretation that there can be no group with a feasible Pareto Improvement, and therefore are also applicable to there is no group with a feasible Strong Pareto improvement. The conditions of existence, which are formally stated in section 2, are such that, if all groups have a joint action space that is convex, compact, and non-empty and the utility of *every* member of the group is quasi-concave and continuous in the joint action of the group, then a Coalitional Equilibrium exists. These conditions rely on the proof of Shafer and Sonnenschein (1975)'s existence of equilibrium in games with abstract preferences. Instead, in this work, I focus on the equilibrium notion of there is no group with a feasible Strong Pareto improvement, which I will refer to as a *weak* Coalitional Equilibrium. This allows for a weakening of the conditions for existence. Specifically, if all groups have a joint action space that is convex, compact, and non-empty and the utility of *a single* member of the group is quasi-concave and continuous in the joint action of the group, then a weak Coalitional Equilibrium exists. Not only does this provide a weakening of the sufficient conditions for the existence of a weak Coalitional Equilibrium, but the proof of this result relies directly on the existence of a Nash equilibrium (Nash, 1950) of an auxiliary game, where a single agent within the group, who has quasi-concave and continuous utility in the joint action of the group, is permitted to choose the action profile for the entire group itself. This provides a further conceptual link between Coalitional equilibrium and Nash equilibrium.

Via the use of a simple three-player game, I show that these sufficient conditions can provide existence in cases that a Coalitional equilibrium does not exist for some configurations. Further to this, I show that considering weak Coalitional equilibrium can have non-trivial effects on equilibrium binding agreements (Ray and Vohra, 1997). Equilibrium binding agreements is a refinement of (weak) Coalitional equilibrium that studies the stable groups that can form when they play a la (weak) Coalitional equilibrium upon some agents breaking away from the coalition they are member of. This is as weak Coalition equilibrium allows for a larger set of action profiles to be considered equilibrium for a given configuration. Therefore there may be a direct increase in the set of equilibrium binding agreements. However, it may also restrict the set of equilibrium binding agreements as it can include new constraints on the binding agreements larger groups could take. This is as it may be, for example, a configuration a subgroup did not consider as stable before is under weak Coalitional equilibrium. Therefore they may have to consider a different constraint when considering whether to break from their current group.

Further to this, I extend this notion to accommodate the possibility of overlapping groups. Overlapping coalitions capture a number of economically relevant applications such as international trade agreements where, for example, Russia and Vietnam have a free trade agreement and therefore have a binding agreement, Vietnam and Sudan also have a free trade agreement, while no such agreement between Russia and Sudan

[3] See Ray (1989) for an early discussion of this.

exists.[4] Despite their empirical prevalence, for the most part, the literature has not analysed the case of overlapping coalitions, while the set of all players cannot jointly deviate. Instead, there has been a focus upon the case where either groups have a partition structure, or that all possible groups are permitted. For instance, notions such as Aumann (1961)'s α- and β-core as well as Chander and Tulkens (1997)'s γ-core assume that all players can react to a deviation of a group, assuming all possible groups are permitted. In this case, it may not be possible to have Russia directly react to a deviation of Sudan. Nonetheless, Coalitional Equilibrium and weak Coalitional Equilibrium need not rely on the partition structure of groups to be well defined. However, when groups overlap, a conceptual issue arises as to what is meant by a binding agreement. Taking Coalitional Equilibria, as well as weak Coalitional Equilibria, to be such that those within the overlap may respect any agreement they make allows for the most stringent conditions, as ruling out any such deviations rules out the smaller set of deviations that they would have if they would be required to respect all agreements.[5] With this, we can take (weak) Coalitional Equilibrium to be defined as is. When doing so, the proof of the existence of weak Coalitional Equilibrium still applies in a broad set of situations. Specifically, when we can partition the set of groups into *partial covers*: within any partial cover all groups have some other group they overlap with, no group outside of the partial cover overlaps with any group within the partial cover, and for all partial covers there is at least one player who is within all groups within said partial cover. This allows the proof of existence to extend naturally. Specifically, if for all partial covers, the action space of the partial cover is convex, compact, and non-empty, and, a single member of the partial cover who is within all groups has a utility that is quasi-concave and continuous in the joint action of the partial cover, then a weak Coalitional Equilibrium exists. This provides a first step in answering the question of providing an existence result for cooperative solution concepts where coalitions may overlap, but may not allow for all possible groups.

With this, I turn to the formal definitions and results.

2 Model

Let $\Gamma = \langle N, (u_i, A_i)_{i \in N} \rangle$ be a normal form game, where N is a set of players, A_i is a set of actions for player $i \in N$, $A := \times_{j \in N} A_i$ is the set of action profiles, and $u_i : A \to \mathbb{R}$ is a utility function for player $i \in N$. For a group of players, or *coalition*, $S \subset N$ let $A_S := \times_{i \in S} A_i$. Similarly, for $a \in A$ and a coalition $S \subseteq N$, let $u_S(a) := (u_i(a))_{i \in S}$. Let π denote a partition of N, which defines the set of feasible coalitions. That is, $\pi = \{S^1, ..., S^K\}$ is such that $\bigcup_{S^k \in \pi} S^k = N$ and $S^k \cap S^l = \emptyset$ for all $S^k, S^l \in \pi$, $S^k \neq S^l$. For clarity, I will take $\gg$ to be the ordering such that $u_S(a'_S, a_{-S}) \gg u_S(a)$ if and only if $u_i(a'_S, a_{-S}) > u_i(a)$ for some $i \in S$ and $u_j(a'_S, a_{-S}) \geq u_j(a)$ for all $j \in S$. I now formally state the definition of Coalitional Equilibrium as given in Ray and Vohra (1997); Ray (2007).

Definition 1 (Coalitional Equilibrium (Ray and Vohra, 1997; Ray, 2007))**.** *An action profile $a^* \in A$ is a Coalitional Equilibrium (with respect to π) if for no coalition $S \in \pi$ there is a joint action $a'_S \in A_S$ such that $u_S(a'_S, a^*_{-S}) \gg u_S(a^*)$.*

I now restate the result of existence provided in Ray and Vohra (1997) and Ray (2007).

Proposition (Ray and Vohra (1997); Ray (2007))**.** *Suppose that for a set of feasible coalitions π, for all $S \in \pi$ $i \in S$, A_i is non-empty, compact, and convex, and u_i is continuous and quasi-concave in A_S, then a Coalitional Equilibrium (with respect to π) exists.*

[4] See WTO's Participation in RTAs for more detail.

[5] For the most part, solutions that allow for all coalitions abstract from the reasonability of threats, for instance, Aumann (1961). One example of ensuring reasonable threats that allows for *any* possible set of feasible coalitions is in Gavan (2022).

As outlined in the introduction, these rely on the utility of each agent being quasi-concave and continuous in A_S. However, when we consider a weak version of coalitional equilibrium, where only feasible weak Pareto dominance is considered, these conditions can be weakened. To clarify this distinction, I define the notion of weak Coalitional Equilibrium.

Definition 2. *An action profile $a^* \in A$ is a weak Coalitional Equilibrium (with respect to π) if for no coalition $S \in \pi$ there is a joint action $a'_S \in A_S$ such that $u_i(a'_S, a^*_{-S}) > u_i(a^*)$ for all $i \in S$.*

The weakened result of existence is as follows in proposition 1.

Proposition 1. *For a given partition π, suppose that for all i, A_i is compact and convex, and for all $S \in \pi$ there is some $j \in S$ such that u_j is continuous and quasi-concave in A_S. Then a weak Coalitional Equilibrium (with respect to π) exists.*

Notice that this is a weakening of the conditions provided in the proposition of Ray and Vohra (1997) and Ray (2007), as there need only be a single player within the coalition whose utility is continuous and quasi-concave in A_S, rather than all. To sketch the logic of this result, note that if there is a player within a coalition whose utility is continuous and quasi-concave in A_S, we may provide them with the right to choose the joint action for everyone within their coalition. Label such a player the "coalitional leader". Do so for every coalition. Now allow them to play a la Nash equilibrium. It is immediate that there can be no other joint action within the coalition that strongly Pareto dominates the joint action they have chosen, as the coalitional leader must be choosing the joint action that makes them the best off, holding the action of those outside the coalition constant. Note that this proof method in essence allows the coalitional leader to act as a principal in the sense of contract theory, see, for example, Bolton and Dewatripont (2004), as the coalitional leader need only to choose whatever is best for themselves, while respecting any exogenous constraints that may be in place due to the game, or via a further refinement of the coalitional equilibria.[6] This logic prevails despite the possibility of externalities across the coalitional leaders via the use of the game. This provides us with a further conceptual link between the two notions of binding agreements. The formal proof is given below.

Proof. Relabel the elements of the partition such that $\pi = \{S^1, S^2, ..., S^m\}$. Take $j^l \in S^l$ such that u_{j^l} is continuous and quasi-concave in A_{S^l}. Take an auxiliary game $\Gamma' = \langle(\{1, 2, ..., m\}, (A'_i, u'_i)_{i\in\{1,2,...,m\}})\rangle$ such that $A'_i = A_{S^i}$ and $u'_i(a) = u_{j^i}(a)$ for all a. Note that all conditions for existence of Nash equilibrium provided by Nash (1950) are satisfied. Therefore a Nash equilibrium of Γ' exists. Let a^* denote the Nash equilibrium of Γ' and suppose it is not a weak Coalitional Equilibrium of Γ. It must be that for some coalition $S^l \in \pi$ there is some a'_{S^l} such that $u_i(a'_{S^l}, a^*_{-S^l}) > u_i(a^*)$ for all $i \in S^l$. However, by definition $j^l \in S^l$. By definition of a Nash equilibrium of Γ' it must be that $a^*_{S^l} \in \operatorname{argmax}_{a_{S^l}\in A_{S^l}} u_{j^l}(a_{S^l}, a^*_{-S^l})$, and therefore there can be no such a'_{S^l} that improves the utility for $j^l \in S^l$, a contradiction. $\square$

Notice that the weakening of these conditions extends to ensuring existence for any set of coalitions. This is as the existence of such a coalitional leader for every possible set of feasible coalitions does not imply the conditions of Ray and Vohra (1997) and Ray (2007). This allows a game to have existence of a weak Coalitional Equilibrium for all sets of feasible coalitions, while the sufficient conditions for existence of Coalitional Equilibrium may not apply to some such sets of feasible coalitions. This is formalised in the following corollary.

[6]For instance, the Equilibrium Binding Agreements of Ray and Vohra (1997) are analogous to ensuring participation constraints on the set of coalitional equilibria.

Corollary 1. *Suppose that for all i, A_i is compact and convex, and u_i is continuous and, for all π there is some $j \in S \in \pi$ such that u_j is quasi-concave in A_S. Then a weak coalitional equilibrium exists for all π.*

This is further illustrated with the following example.

Example 1. Consider the following 3 player game. Suppose that player 1 is seeking an object that player 2 hides on behalf of player 3. To do so, player 1 and player 2 choose a position on the unit interval. Player 1 hopes to choose a position as close to the chosen position of player 2 for this element of the game. While player 3 hopes that the position player 2 has chosen is furthest away from the position player 1 has chosen. Further to this, players 2 and 3 participate in a public goods contribution, where they may choose a contribution level on the unit interval. The good is efficient for them, but player 1 dislikes this good. With this, let $A_1 = A_3 = [0,1]$ and $A_2 = [0,1]^2$, where the typical elements are given by $a_1 \in A_1$, $(a_{2,1}, a_{2,2}) \in A_2$ and $A_3 = [0,1]$. Let $u_1(a) = -(a_1 - a_{2,1})^2 - \frac{3}{4}(a_{2,2} + a_3)$, $u_2(a) = \frac{3}{4}(a_{2,2} + a_3) - a_{2,2}$ and $u_3(a) = \frac{3}{4}(a_{2,2} + a_3) - a_3 + (a_1 - a_{2,1})^2$.

Notice the following. Player 1's utility is quasi-concave in A and continuous. Player 2's utility is also quasi-concave in A and continuous. However, player 3's utility is quasi-concave in A_3, but not $A_{\{2,3\}}$, $A_{\{1,3\}}$ or A, while is continuous. With this, the sufficient conditions of Ray and Vohra (1997) for existence of a coalitional equilibrium do not apply to any coalition configuration π such that $3 \in S \in \pi$ and $S \neq \{3\}$, while the sufficient conditions provided for existence of a weak coalitional equilibrium, given in proposition 1, are satisfied for any coalition configuration. Further to this, a coalitional equilibrium does not exist for the coalition configuration $\pi = \{\{1\}, \{2,3\}\}$, despite this coalition configuration including the coalitions that have the most aligned preferences. To see this, suppose that a^* is a coalitional equilibrium for $\pi = \{\{1\}, \{2,3\}\}$. As 1 is in a singleton coalition, it must be that she best responds to a^*_{-1}, and therefore $a^*_1 = a^*_{2,1}$. Now consider a^*_S for $S = \{2,3\}$. It must be that there is no a'_S such that $u_S(a'_S, a^*_1) \gg u_S(a^*)$. However, consider the following a'_S: Let $a'_{2,1} = \text{argmax}_{a_{2,1} \in [0,1]} (a^*_1 - a_{2,1})^2$, which implies that $a'_{2,1} \neq a^*_1 = a^*_{2,1}$ and $a'_{2,2} = a^*_{2,2}$ and $a'_3 = a^*_3$. It follows that the utility of player 3 must have increased, while the utility of player 2 remains the same. Therefore it cannot be that a^* is a coalitional equilibrium for $\pi = \{\{1\}, \{2,3\}\}$, and as a^* was chosen arbitrarily, it follows no coalitional equilibrium can exist. However, a weak coalitional equilibrium can exist for any coalition configuration, as the sufficient conditions for existence of proposition 1 are satisfied. In particular, any action profile a^* such that $\max\{a^*_{2,2}, a^*_3\} = 1$ and $a^*_1 = a^*_{2,1}$ is a weak coalitional equilibrium for the coalition configuration $\pi = \{\{1\}, \{2,3\}\}$.

Further to this, note that this example gives some insight into how the change to weak coalitional equilibria can impact equilibrium binding agreements, introduced by Ray and Vohra (1997). In this solution coalitions may break, understanding in doing so other coalitions may also do so, where coalitions look at the (weak) coalitional equilibria from the resulting configuration. When an agent or group of agents decides to break, they first evaluate whether the remainder of the coalition they can leave would be stable, and then consider further breakings if not. Therefore this provides a farsighted core-like solution concept, where the utility an agent gets from a coalition configuration is taken to be that of the (weak) coalitional equilibrium selected.

Within this example, it is easy to see that all allocations are both Pareto Efficient as well as weakly Pareto Efficient. The Nash equilibrium is where $a_1 = a_{2,1}$ and $a_{2,2} = a_3 = 0$, and therefore an equilibrium binding agreement, if taken with respect to coalitional equilibria, must provide player 1 with a minimum utility of 0, as the non-existence of a coalitional equilibrium for the configuration $\pi = \{\{1\}, \{2,3\}\}$ results in player 1 only evaluating this possibility when considering breaking the grand coalition. However, when considering weak coalitional equilibrium as the baseline for equilibrium binding agreements this is no longer the case. By player 1 considering breaking from the grand coalition she understands that players 2 and 3 can reach an agreement that is stable in the sense that neither has an incentive to dissolve the coalition, for which the resulting equilibrium would leave player 3 with at most $-\frac{13}{12}$. With this, we can see that the impact of weak coalitional equilibrium on equilibrium binding agreements does not necessarily increase or reduce the set of resulting agreements. This is because it may increase the set of possibilities that are considered at each configuration, which may lead to a direct increase in the possible equilibrium binding agreements, or an indirect effect of making equilibrium binding agreements more difficult to satisfy due to the possibility of new stable configurations. Combining these effects can result in a non-trivial change, as is the case with this example. ▼

Finally, note that the logic and proof of proposition 1 allows for a more general set of permissible coalitions to be allowed. Specifically, suppose that $\pi = \{S^1, ..., S^k\}$ is a cover of N: $\bigcup_{S \in \pi} S = N$ and $\forall i \in N$ $\exists S \in \pi$ such that $i \in S$. Define weak Coalitional Equilibrium with respect to π as in definition 2. Allow for the possibility that some coalitions overlap: $S \cap S' \neq \emptyset$ for some $S, S' \in \pi$. If for any set of coalitions that overlap, there is a single player in all such coalitions, then the proof applies. This is formalised by the following. Suppose that for any $\pi' \subset \pi$ such that:

1. $\forall S \in \pi'$ $\exists S' \in \pi'$ such that $S \cap S' \neq \emptyset$ and
2. $\nexists S'' \in \pi$ such that $\exists S \in \pi'$ such that $S \cap S'' = \emptyset$.

Then there is some $i \in N$ such that $i \in S$ for all $S \in \pi'$. Then with analogous conditions to proposition 1 a weak Coalitional Equilibrium exists for π. This is by an identical logic as proposition 1, taking $\pi = \pi^1 \cup ... \cup \pi^k$, and taking an auxiliary game where $N' = \{1, ..., k\}$ and $j \in N'$ is such that $j \in N$ is such that $j \in S$ for all $S \in \pi^k$. Essentially, this allows for the notion of a *constellation* leader, rather than a coalitional leader, who may perform in the same way, choosing the action for all players within the constellation. This provides a first step in answering the question of existence of solution concepts including binding agreements when coalitions overlap.

Proposition 2. *For a set of feasible coalitions π, such that $\exists \pi^1, .., \pi^k \subseteq \pi$ such that:*

1. *$\forall S \in \pi^m$ $\exists S' \in \pi^m$ such that $S \cap S' \neq \emptyset$ and*
2. *$\nexists S'' \in \pi$ such that $\exists S \in \pi^m$ such that $S \cap S'' = \emptyset$.*
3. *and for all π^m there is some $j \in N$ such that $j \in S$ $\forall S \in \pi^m$.*

and for all i, A_i is compact and convex, and u_j, where $j \in S$ for all $S \in \pi^m$ is continuous and quasi-concave in A_C where $C = \bigcup_{S \in \pi^m} S$. Then a weak Coalitional Equilibrium (with respect to π) exists.

Proof. Take $\pi^1, ..., \pi^k$ be such that

1. $\forall S \in \pi^m$ $\exists S' \in \pi^m$ such that $S \cap S' \neq \emptyset$ and
2. $\nexists S'' \in \pi$ such that $\exists S \in \pi^m$ such that $S \cap S'' = \emptyset$.

Take $j^l \in \bigcap_{S \in \pi^l} S$ such that u_{j^l} is continuous and quasi-concave in A_{C^l} where $C^l = \bigcup_{S \in \pi^l} S$. Take an auxiliary game $\Gamma' = \langle (\{1, 2, ..., m\}, (A'_i, u'_i)_{i \in \{1,2,...,m\}}) \rangle$ such that $A'_i = A_{C^i}$ and $u'_i(a) = u_{j^i}(a)$ for all a. Note that all conditions for existence of Nash equilibrium provided by Nash (1950) are satisfied. Therefore a Nash equilibrium of Γ' exists. Let a^* denote the Nash equilibrium of Γ' and suppose it is not a weak Coalitional Equilibrium of Γ. It must be that for some coalition $S^l \in \pi$ there is some a'_{S^l} such that $u_i(a'_{S^l}, a^*_{-S^l}) > u_i(a^*)$ for all $i \in S^l$. However, by definition $j^l \in S$ for all $S \in \pi^l$. By definition of a Nash equilibrium of Γ' it must be that $a^*_{C^l} \in \text{argmax}_{a_{C^l} \in A_{C^l}} u_{j^l}(a_{C^l}, a^*_{-C^l})$, and therefore, as $A_S \subset A_{C^l}$ there can be no such a'_{C^l} that improves the utility for $j^l \in S$ and therefore no such a_S, a contradiction. □

Note that this again provides a link to the literature on contracts where there may be multiple agents that a principal contracts with the literature on binding agreements as the existence of a weak coalitional equilibria here uses this logic. With appropriate restrictions on the externalities of the game and structure of coalitions, the principal agent logic works as before.

Nonetheless, a number of interesting questions remain in understanding the structure of agreements and conditions needed when coalitions may overlap. The first is whether a more general set of coalitions can be permitted while maintaining minimal assumptions for existence of a Coalitional equilibrium. In the direction of seeing which coalitional equilibria can be viewed as more reasonable when coalitions may overlap, via refinements a la equilibrium binding agreements etc. is a question that poses both a technical and conceptual challenge. One direction is to understand the set of reasonable "breaking" of coalitions that can take place when coalitions overlap, in the sense of a farsighted core. However, as covers of sets are not endowed with a natural set ordering as partitions are, it is not clear what the resulting cover may be once a coalition breaks. However, due to the richness of economic situations, such as trade agreements, that can be represented with overlapping coalitions, there is large scope for insight within this line. I leave this exploration for future work.

Acknowledgements

I thank Antonio Penta for his ongoing supervision and support throughout my Ph.D.. I also thank Pia Ennuschat for her help and support.

The author has no competing interests to declare.

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