Journal of Machine Learning Research 1 (2000) 1-48 Submitted 4/00; Published 10/00

Accelerated oblique random survival forests

Byron C. Jaeger bjaeger@wakehealth.edu
Department of Biostatistics and Data Science
Wake Forest University School of Medicine
Winston-Salem, NC 27157, USA

Sawyer Welden swelden@wakehealth.edu
Department of Biostatistics and Data Science
Wake Forest University School of Medicine
Winston-Salem, NC 27157, USA

Kristin Lenoir klenoir@wakehealth.edu
Department of Biostatistics and Data Science
Wake Forest University School of Medicine
Winston-Salem, NC 27157, USA

Jaime L Speiser jspeiser@wakehealth.edu
Department of Biostatistics and Data Science
Wake Forest University School of Medicine
Winston-Salem, NC 27157, USA

Matthew Segar Matthew.Segar@UTSouthwestern.edu
Division of Cardiology, Department of Internal Medicine,
University of Texas Southwestern Medical Center, Dallas

Nicholas M. Pajewski npajewsk@wakehealth.edu
Department of Biostatistics and Data Science
Wake Forest University School of Medicine
Winston-Salem, NC 27157, USA

Editor: TBD

Abstract

The oblique random survival forest (ORSF) is an ensemble method for supervised learning that extends the random survival forest (RSF). Trees in the ORSF are grown using linear combinations of variables to create branches in the tree, whereas in the RSF a single variable is used. ORSF ensembles often have higher prediction accuracy than RSF ensembles, but the additional computational overhead of fitting ORSF ensembles limits their scope of application. In addition, few methods have been developed for interpretation of ORSF ensembles. In this article, we introduce and evaluate methods to accelerate the ORSF (that is, reduce computational overhead) and compute the importance of individual variables in the ORSF We show that our strategy to accelerate the ORSF is up to 500 times faster than existing software for ORSFs (the `obliqueRSF` R package), and that prediction accuracy of the accelerated ORSF is equivalent or superior to that of existing ORSF methods. We estimate importance of variables for the ORSF by negating each coefficient used for the given variable in linear combinations, and then computing the reduction in out-of-bag accuracy. We show with simulation that 'negation importance' can discriminate between signal and noise variables, and it outperforms several state-of-the-art variable importance techniques in this task when there is correlation among predictors.

Keywords: Random Forests, Survival, Efficient, Variable Importance

1. Introduction

Risk prediction can reduce the burden of disease by educating patients and providers and guiding strategies to prevent and treat disease in a wide range of medical domains (Moons et al., 2012b,a). The random survival forest (RSF), a supervised learning algorithm that can engage with censored outcomes, is frequently used for risk prediction. Notable characteristics of the RSF include uniform convergence of its ensemble survival function to the true population survival function when the predictor space is discrete. In addition, software implementing the RSF is freely available, extremely efficient, and full of tools to interpret and explain the RSF. However, there remains considerable potential to improve the RSF in risk prediction tasks where training samples are not large enough to guarantee asymptotic properties or predictor spaces are non-discrete (that is, predictors are continuous).

RSFs may be axis based or oblique. The axis based RSF uses a single predictor whereas the oblique RSF uses a linear combination of predictors to create branches in trees. While axis based decision boundaries are always perpendicular to the axis of the relevant predictor, linear combinations of predictors create oblique decision boundaries that are neither parallel nor perpendicular to axes of their contributing predictors. Prior work has found the oblique RSF has higher prediction accuracy than the axis based RSF in general benchmarks (Jaeger et al., 2019) and that oblique splitting is particularly effective when predictor spaces are non-discrete. However, existing methods to implement oblique RSFs use fully trained models in each non-leaf node to identify linear combinations of predictors, exponentially increasing the number of operations required for the oblique RSF versus its axis based counterpart. In addition, standard methods to estimate the importance of individual variables in the RSF are less effective in the oblique RSF.

In this article, we introduce methods to increase computational efficiency and estimate variable importance (VI) for oblique RSFs. We evaluate computational efficiency, prediction accuracy, and the ability to discriminate between signal and noise variables using the proposed methods compared to standard and state-of-the-art methods. To accomplish these goals, we analyze 23 distinct risk prediction tasks from real data and conduct a simulation study. All methods introduced in this article for oblique RSFs are available in the `aorsf` R Package.

2. Related work

Breiman (2001) introduced the axis-based random forest (RF) and the oblique RF (oRF). Axis based RFs recursively split data using a single predictor at each non-leaf node in decision trees, whereas oRFs use multiple predictors in linear combination. The splits are called 'oblique' because linear combinations of predictors create decision boundaries that are neither parallel nor perpendicular to their axes have higher prediction accuracy in external data compared to their axis based counterparts. Several studies investigating RFs for classification and regression have noted that oblique RFs have lower generalization than their axis-based counterparts.

The axis-based RF proposed in Breiman (2001) was extended to survival outcomes by Ishwaran et al. (2008) and the oblique RF la soon after RFs for classification and regression were developed by Breiman (2001), and oblique RFs for survival were Several studies have shown that oblique recursive partitioning improves the generalization error of RFs for classification and regression, b

Several supervised learning algorithms can develop prediction functions for right-censored time-to-event outcomes, henceforth referred to as survival outcomes. Ishwaran et al. (2008) developed the RF for survival, an extension of the RF for regression and classification developed by Breiman (2001). Fast algorithms to fit the RF for survival are available in the `randomForestSRC` R package (Ishwaran and Kogalur, 2019). A similar implementation of the RF for survival can be found in the `ranger` R package (Wright and Ziegler, 2017), which is particularly suited for high dimensional data. The RF for survival can also be fit using unbiased recursive partitioning (Hothorn et al., 2006) via the `party` R package (Hothorn et al., 2010).

Zhu et al. (2015)

Zhu (2013)

Menze et al. (2011) introduced an oblique RF (oRF) for classification that utilizes fully trained models at non-leaf nodes to identify linear combinations of predictors. Tomita et al. (2020) introduced sparse projections for oRFs for classification. Katuwal et al. (2020) developed a heterogeneous oRF for classification that applies several linear classifiers at each non-leaf node. Compared to classification, fewer studies have developed oRFs for survival. Jaeger et al. (2019) introduced an oRF for survival that applies penalized regression at each non-leaf node and disseminated algorithms to fit the oRF for survival in the `obliqueRSF` R package (Jaeger, 2018).

Variable importance (VI) for oRFs VI has been estimated for oRFs using analysis of variance (ANOVA),6 but this only applies in the rare case where p-values are calculable for coefficients in linear combinations. Permutation importance is used to estimate VI in RFs,14 but is not ideal for oRFs because it does not account for the coefficients used in linear combinations. Shapley VI, a method based in game theory,15 has excellent asymptotic properties for interpretation,16,17 but is computationally infeasible and can only be approximated efficiently for standard decision trees.18

3. Novel techniques for oblique random survival forests

3.1 Partial training at non-leaf nodes

3.2 Negation variable importance

We propose a new VI method, "negation importance", that can be used for any oRF and accounts for the coefficients in linear combinations.

4. Numeric experiments

4.1 Benchmark of prediction accuracy

4.1.1 Learners

In the current study, we consider four classes of learners: random forests, boosting ensembles, regression models, and neural networks (Table 1). For random forest learners, the number of observations required in terminal nodes was fixed at 10, the number of randomly selected predictors was the nearest integer to the square root of the total number of predictors, and the number of trees in the ensemble was 500.

Learner Class	Software	Learners	Description
Random Survival Forests			
Standard	`RandomForestSRC`	`rfsrc-standard`	Axis based survival trees following Leo Breiman's original random forest algorithm, with cut-points selected to maximize a log-rank statistic.

Oblique	`obliqueRSF`
	`aorsf`
		`obliqueRSF-net`
		`aorsf-net`
		`aorsf-cph`$(i = 1)$
		`aorsf-cph`$(i \leq 15)$
		`aorsf-extratrees`
			Oblique survival trees following Leo Breiman's random forest algorithm. Linear combinations of inputs are derived using `glmnet` in `obliqueRSF-net` and `aorsf-net`, using Newton Raphson scoring for the Cox partial likelihood function in `aorsf-cph`$(i = 1)$ and `aorsf-cph`$(i \leq 15)$, and chosen randomly from a uniform distribution in `aorsf-extratrees`. Cut-points are selected to maximize a log-rank statistic.

Extremely Randomized	`ranger`	`ranger-extratrees`	Axis-based survival trees grown with randomly selected features and cut-points
Conditional Inference	`party`	`party-cif`	Axis based survival trees grown using unbiased recursive partitioning.
Boosting ensembles			
Trees	`xgboost`	`xgboost-cox`	The Cox partial likelihood function is maximized additively with decision trees. Nested cross validation (5 folds) is applied to tune the number of trees grown.
Models	`xgboost`	`xgboost-aft`	The accelerated failure time likelihood function is maximized additively with decision trees. Nested cross validation (5 folds) is applied to tune the number of trees grown.
Regression models			
Cox Net	`glmnet`	`glmnet-cox`	The Cox model is fit using an elastic net penalty. Nested cross validation (5 folds) is applied to tune penalty terms.
Neural networks			
Cox Time	`survivalmodels`	`nn-cox`	A neural network based on the proportional hazards model with time-varying effects

Table 1: Learning algorithms assessed in numeric studies

4.1.2 Evaluation of prediction accuracy

Our primary metric for evaluating the accuracy of predicted risk is the integrated and scaled Brier score (Graf et al., 1999). For observation i in the testing data, let $\widehat{S}(t \mid \boldsymbol{x}_i)$ be the predicted probability of survival up to a given prediction horizon of $t > 0$ and let $\boldsymbol{x}_i$ be the vector of predictor values. Define

$$\widehat{\mathrm{BS}}(t) = \frac{1}{N_{\text{test}}} \sum_{i=1}^{N_{\text{test}}} \{\widehat{S}(t \mid \boldsymbol{x}_i)^2 \cdot I(T_i \leq t, \delta_i = 1) \cdot \widehat{G}(T_i)^{-1} + [1 - \widehat{S}(t \mid \boldsymbol{x}_i)]^2 \cdot I(T_i > t) \cdot \widehat{G}(t)^{-1}\}$$

where $\widehat{G}(t)$ is the Kaplan-Meier estimate of the censoring distribution. As $\widehat{\mathrm{BS}}(t)$ is time dependent, integration over time provides a summary measure of performance over a range of plausible prediction horizons. The integrated $\widehat{\mathrm{BS}}(t)$ is defined as

$$\widehat{\mathcal{BS}}(t_1, t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \widehat{\mathrm{BS}}(t) dt. \tag{1}$$

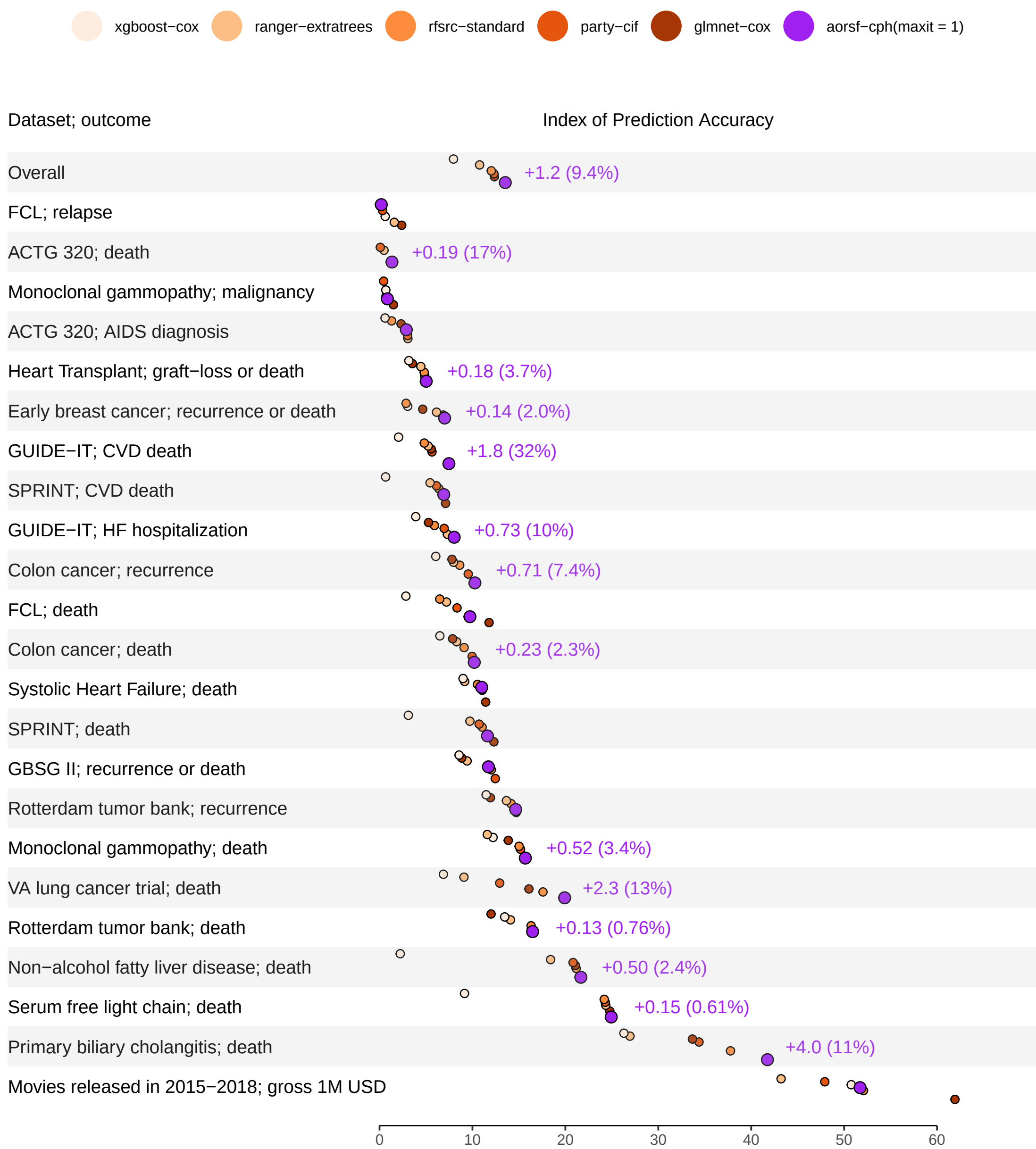
In our results, t_1 and t_2 are the 25th and 75th percentile of event times, respectively. $\widehat{\mathcal{BS}}(t_1, t_2)$, a sum of squared prediction errors, can be scaled to produce a measure of explained residual variation (that is, an R^2 statistic) by computing

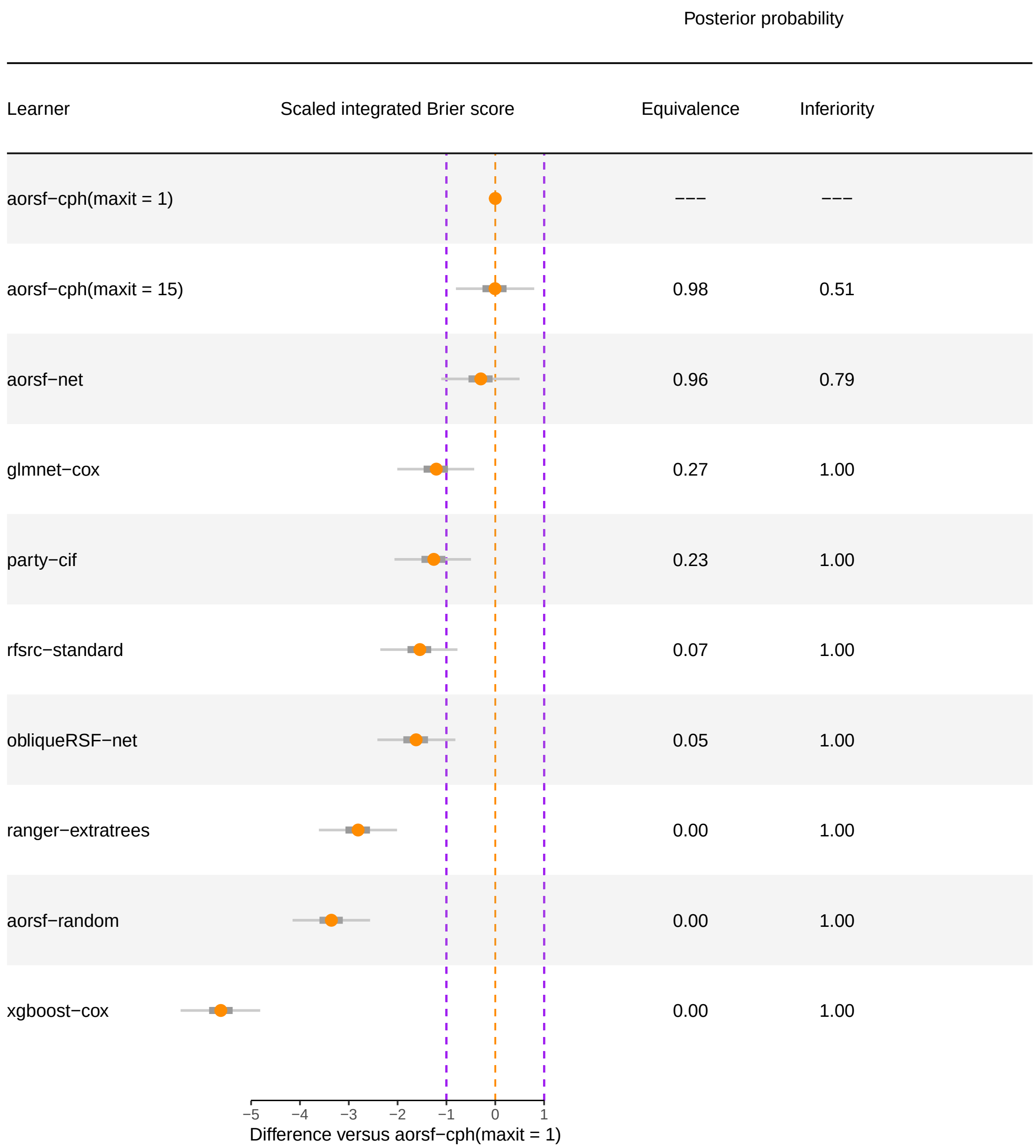
$$R^2 = 1 - \frac{\widehat{\mathcal{BS}}(t_1, t_2)}{\widehat{\mathcal{BS}}_0(t_1, t_2)} \tag{2}$$

where $\widehat{\mathcal{BS}}_0(t_1, t_2)$ is the integrated Brier score when a Kaplan-Meier estimate for survival based on the training data is used as the survival prediction function $\widehat{S}(t)$. We refer to this R^2 statistic as the index of prediction accuracy and we scale its values by 100 to avoid unnecessary leading zero's. For example, we present 25 if R^2 is 0.25 and present 10.2 if the difference between two R^2 is 0.102.

4.1.3 Statistical analysis

4.1.4 Results


xgboost−cox
ranger−extratrees
rfsrc−standard
party−cif
glmnet−cox
aorsf−cph(maxit = 1)
Dataset; outcome
Index of Prediction Accuracy
Overall
+1.2 (9.4%)
FCL; relapse
ACTG 320; death
+0.19 (17%)
Monoclonal gammopathy; malignancy
ACTG 320; AIDS diagnosis
Heart Transplant; graft−loss or death
+0.18 (3.7%)
Early breast cancer; recurrence or death
+0.14 (2.0%)
GUIDE−IT; CVD death
+1.8 (32%)
SPRINT; CVD death
GUIDE−IT; HF hospitalization
+0.73 (10%)
Colon cancer; recurrence
+0.71 (7.4%)
FCL; death
Colon cancer; death
+0.23 (2.3%)
Systolic Heart Failure; death
SPRINT; death
GBSG II; recurrence or death
Rotterdam tumor bank; recurrence
Monoclonal gammopathy; death
+0.52 (3.4%)
VA lung cancer trial; death
+2.3 (13%)
Rotterdam tumor bank; death
+0.13 (0.76%)
Non−alcohol fatty liver disease; death
+0.50 (2.4%)
Serum free light chain; death
+0.15 (0.61%)
Primary biliary cholangitis; death
+4.0 (11%)
Movies released in 2015−2018; gross 1M USD
0
10
20
30
40
50
60


Posterior probability
Learner
Scaled integrated Brier score
Equivalence
Inferiority
aorsf−cph(maxit = 1)

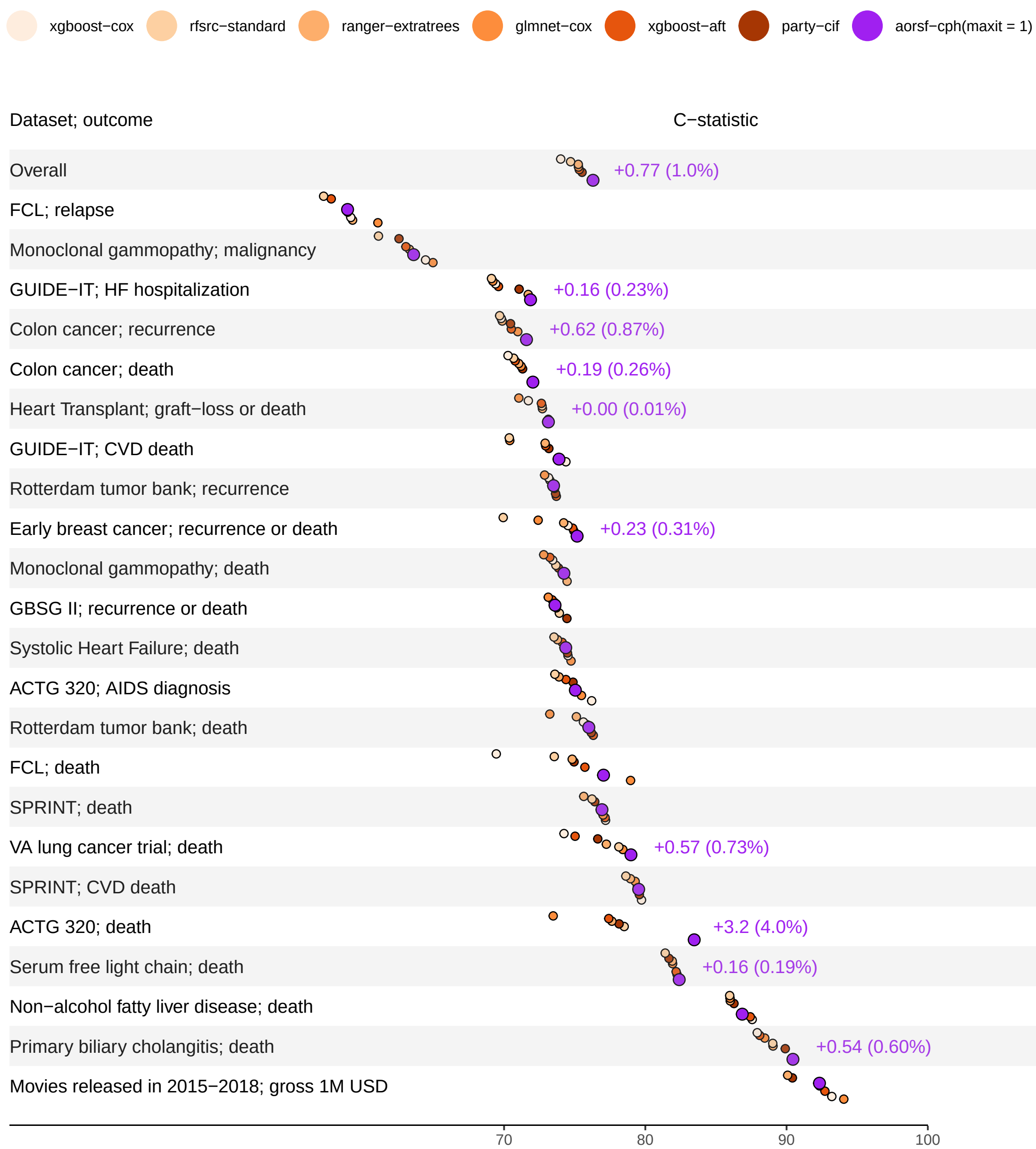
---
aorsf−cph(maxit = 15)
0.98
0.51
aorsf−net
0.96
0.79
glmnet−cox
0.27
1.00
party−cif
0.23
1.00
rfsrc−standard
0.07
1.00
obliqueRSF−net
0.05
1.00
ranger−extratrees
0.00
1.00
aorsf−random
0.00
1.00
xgboost−cox
0.00
1.00
−5
−4
−3
−2
−1
0
1
Difference versus aorsf−cph(maxit = 1)

4.2 Benchmark of variable selection

Acknowledgments

Research reported in this publication was supported by the Center for Biomedical Informatics, Wake Forest University School of Medicine

Appendix A.


xgboost−cox
rfsrc−standard
ranger−extratrees
glmnet−cox
xgboost−aft
party−cif
aorsf−cph(maxit = 1)
Dataset; outcome
C−statistic
Overall
+0.77 (1.0%)
FCL; relapse
Monoclonal gammopathy; malignancy
GUIDE−IT; HF hospitalization
+0.16 (0.23%)
Colon cancer; recurrence
+0.62 (0.87%)
Colon cancer; death
+0.19 (0.26%)
Heart Transplant; graft−loss or death
+0.00 (0.01%)
GUIDE−IT; CVD death
Rotterdam tumor bank; recurrence
Early breast cancer; recurrence or death
+0.23 (0.31%)
Monoclonal gammopathy; death
GBSG II; recurrence or death
Systolic Heart Failure; death
ACTG 320; AIDS diagnosis
Rotterdam tumor bank; death
FCL; death
SPRINT; death
VA lung cancer trial; death
+0.57 (0.73%)
SPRINT; CVD death
ACTG 320; death
+3.2 (4.0%)
Serum free light chain; death
+0.16 (0.19%)
Non−alcohol fatty liver disease; death
Primary biliary cholangitis; death
+0.54 (0.60%)
Movies released in 2015−2018; gross 1M USD
70
80
90
100

	Performance metric (SD)		Computing time, seconds			
			Fit model		Predict risk	
	Scaled Brier	C-Statistic	Median	Ratio	Median	Ratio
Overall						
aorsf-cph(maxit = 15)	0.136 (0.123)	0.763 (0.077)	2.460	2.95	0.202	0.974
aorsf-cph(maxit = 1)	0.135 (0.124)	0.763 (0.077)	0.833	1.00	0.207	1.00
aorsf-net	0.133 (0.127)	0.762 (0.077)	53.399	64.1	0.200	0.966
glmnet-cox	0.124 (0.136)	0.753 (0.081)	0.209	0.251	0.004	0.019
party-cif	0.123 (0.111)	0.755 (0.074)	1.583	1.90	4.360	21.0
rfsrc-standard	0.120 (0.128)	0.747 (0.080)	1.554	1.86	0.150	0.725
obliqueRSF-net	0.120 (0.093)	0.762 (0.076)	268.416	322.0	18.002	86.9
ranger-extratrees	0.108 (0.097)	0.753 (0.073)	0.217	0.261	0.211	1.02
aorsf-random	0.102 (0.092)	0.740 (0.072)	1.857	2.23	0.191	0.924
xgboost-cox	0.080 (0.114)	0.740 (0.095)	3.645	4.37	0.005	0.024
xgboost-aft	-3.86 (5.01)	0.753 (0.078)	12.587	15.1	0.008	0.039
ACTG 320; AIDS diagnosis, n = 1151, p = 12						
aorsf-random	0.032 (0.017)	0.749 (0.035)	1.404	2.75	0.119	1.02
obliqueRSF-net	0.031 (0.018)	0.751 (0.035)	28.321	55.4	15.117	129.1
ranger-extratrees	0.030 (0.014)	0.739 (0.035)	0.358	0.701	0.159	1.36
party-cif	0.030 (0.024)	0.749 (0.034)	1.631	3.19	4.292	36.6
aorsf-cph(maxit = 15)	0.029 (0.023)	0.755 (0.038)	1.146	2.24	0.116	0.991
aorsf-cph(maxit = 1)	0.029 (0.023)	0.750 (0.040)	0.511	1.00	0.117	1.00
aorsf-net	0.024 (0.027)	0.750 (0.036)	19.546	38.2	0.119	1.01
glmnet-cox	0.023 (0.026)	0.755 (0.038)	0.179	0.349	0.003	0.026
rfsrc-standard	0.013 (0.034)	0.736 (0.037)	0.638	1.25	0.060	0.514
xgboost-cox	0.006 (0.045)	0.762 (0.035)	3.745	7.33	0.003	0.026
xgboost-aft	-8.44 (1.61)	0.744 (0.034)	11.231	22.0	0.006	0.051
ACTG 320; death, n = 1151, p = 12						
aorsf-cph(maxit = 15)	0.013 (0.017)	0.831 (0.050)	0.683	2.35	0.076	1.03
aorsf-cph(maxit = 1)	0.013 (0.018)	0.835 (0.050)	0.291	1.00	0.074	1.00
aorsf-random	0.011 (0.013)	0.802 (0.060)	1.081	3.72	0.086	1.16
obliqueRSF-net	0.010 (0.010)	0.824 (0.048)	8.654	29.8	11.022	148.8
aorsf-net	0.008 (0.030)	0.821 (0.054)	15.008	51.6	0.085	1.14
ranger-extratrees	0.005 (0.015)	0.776 (0.055)	0.040	0.139	0.136	1.84
party-cif	0.001 (0.024)	0.781 (0.055)	1.610	5.54	4.228	57.1
xgboost-cox	-0.003 (0.003)	0.500 (0.000)	0.113	0.389	0.002	0.027
rfsrc-standard	-0.014 (0.048)	0.785 (0.058)	0.084	0.287	0.036	0.480
glmnet-cox	-0.036 (0.073)	0.735 (0.109)	0.285	0.981	0.002	0.027

(continued)

	Scaled Brier	C-Statistic	Median	Ratio	Median	Ratio
xgboost-aft	-22.6 (6.75)	0.774 (0.058)	10.436	35.9	0.005	0.068
Colon cancer; death, n = 929, p = 12						
aorsf-cph(maxit = 1)	0.102 (0.016)	0.720 (0.015)	0.818	1.00	0.208	1.00
aorsf-cph(maxit = 15)	0.101 (0.016)	0.720 (0.015)	1.917	2.34	0.205	0.984
party-cif	0.100 (0.012)	0.713 (0.013)	1.178	1.44	4.053	19.5
aorsf-random	0.098 (0.012)	0.719 (0.015)	1.640	2.00	0.191	0.918
aorsf-net	0.097 (0.016)	0.720 (0.015)	51.648	63.1	0.195	0.938
rfsrc-standard	0.091 (0.018)	0.707 (0.013)	1.462	1.79	0.149	0.716
obliqueRSF-net	0.090 (0.007)	0.720 (0.014)	241.184	294.8	44.328	213.1
ranger-extratrees	0.083 (0.008)	0.710 (0.015)	0.571	0.698	0.251	1.21
glmnet-cox	0.079 (0.016)	0.712 (0.020)	0.110	0.135	0.004	0.019
xgboost-cox	0.065 (0.014)	0.703 (0.015)	3.225	3.94	0.004	0.019
xgboost-aft	-1.09 (0.188)	0.708 (0.013)	10.975	13.4	0.006	0.029
Colon cancer; recurrence, n = 929, p = 12						
aorsf-cph(maxit = 1)	0.103 (0.020)	0.716 (0.019)	0.819	1.00	0.210	1.00
aorsf-cph(maxit = 15)	0.102 (0.019)	0.715 (0.018)	1.939	2.37	0.205	0.975
aorsf-net	0.099 (0.020)	0.716 (0.019)	50.594	61.8	0.192	0.913
party-cif	0.096 (0.016)	0.705 (0.017)	1.142	1.39	3.756	17.9
obliqueRSF-net	0.090 (0.010)	0.715 (0.018)	250.487	305.8	42.658	203.0
aorsf-random	0.089 (0.014)	0.705 (0.018)	1.653	2.02	0.190	0.905
rfsrc-standard	0.086 (0.021)	0.697 (0.017)	1.473	1.80	0.151	0.719
ranger-extratrees	0.080 (0.011)	0.699 (0.018)	0.498	0.608	0.243	1.15
glmnet-cox	0.078 (0.017)	0.710 (0.023)	0.111	0.135	0.004	0.019
xgboost-cox	0.061 (0.013)	0.698 (0.019)	2.951	3.60	0.004	0.019
xgboost-aft	-1.16 (0.240)	0.705 (0.019)	11.566	14.1	0.006	0.029
Early breast cancer; recurrence or death, n = 614, p = 1692						
obliqueRSF-net	0.076 (0.024)	0.754 (0.031)	1867.133	705.0	14.418	37.5
aorsf-cph(maxit = 15)	0.072 (0.027)	0.752 (0.028)	5.022	1.90	0.398	1.04
aorsf-cph(maxit = 1)	0.070 (0.028)	0.752 (0.029)	2.648	1.00	0.384	1.00
party-cif	0.069 (0.020)	0.749 (0.034)	8.450	3.19	4.065	10.6
ranger-extratrees	0.061 (0.022)	0.742 (0.032)	0.220	0.083	0.179	0.467
glmnet-cox	0.046 (0.031)	0.724 (0.036)	5.730	2.16	0.006	0.016
xgboost-cox	0.030 (0.028)	0.745 (0.032)	2.586	0.977	0.006	0.016
rfsrc-standard	0.029 (0.034)	0.699 (0.030)	0.606	0.229	0.617	1.61
aorsf-random	0.028 (0.015)	0.692 (0.034)	3.622	1.37	0.359	0.934
aorsf-net	0.025 (0.062)	0.750 (0.027)	451.897	170.6	0.381	0.991
xgboost-aft	-2.92 (0.594)	0.749 (0.028)	10.199	3.85	0.009	0.024

(continued)

	Scaled Brier	C-Statistic	Median	Ratio	Median	Ratio
FCL; death, n = 541, p = 7						
glmnet-cox	0.118 (0.034)	0.790 (0.034)	0.080	0.331	0.002	0.039
aorsf-cph(maxit = 15)	0.098 (0.044)	0.771 (0.034)	0.440	1.81	0.051	1.01
aorsf-cph(maxit = 1)	0.097 (0.043)	0.770 (0.034)	0.243	1.00	0.051	1.00
aorsf-net	0.095 (0.044)	0.762 (0.034)	13.416	55.3	0.052	1.01
obliqueRSF-net	0.089 (0.032)	0.764 (0.035)	97.197	400.4	5.365	105.1
party-cif	0.083 (0.042)	0.750 (0.036)	0.281	1.16	1.574	30.8
aorsf-random	0.082 (0.034)	0.756 (0.032)	0.459	1.89	0.052	1.02
ranger-extratrees	0.072 (0.019)	0.748 (0.036)	0.032	0.130	0.080	1.58
rfsrc-standard	0.065 (0.053)	0.736 (0.035)	0.111	0.458	0.040	0.784
xgboost-cox	0.028 (0.055)	0.694 (0.117)	0.325	1.34	0.002	0.039
xgboost-aft	-2.63 (0.596)	0.757 (0.034)	7.205	29.7	0.005	0.098
FCL; relapse, n = 541, p = 7						
glmnet-cox	0.024 (0.020)	0.611 (0.033)	0.086	0.236	0.002	0.027
ranger-extratrees	0.016 (0.016)	0.593 (0.024)	0.031	0.086	0.081	1.07
obliqueRSF-net	0.010 (0.018)	0.589 (0.023)	221.459	610.5	10.676	141.1
xgboost-cox	0.006 (0.016)	0.591 (0.030)	1.383	3.81	0.003	0.040
aorsf-random	0.006 (0.020)	0.585 (0.026)	0.697	1.92	0.071	0.937
party-cif	0.003 (0.020)	0.589 (0.021)	0.280	0.772	1.775	23.5
aorsf-cph(maxit = 15)	0.002 (0.023)	0.590 (0.027)	0.709	1.95	0.075	0.997
aorsf-cph(maxit = 1)	0.002 (0.023)	0.589 (0.026)	0.363	1.00	0.076	1.00
aorsf-net	0.001 (0.023)	0.587 (0.027)	20.256	55.8	0.075	0.996
rfsrc-standard	-0.033 (0.031)	0.572 (0.025)	0.917	2.53	0.087	1.16
xgboost-aft	-0.858 (0.311)	0.578 (0.031)	6.260	17.3	0.005	0.066
GBSG II; recurrence or death, n = 686, p = 10						
obliqueRSF-net	0.125 (0.015)	0.747 (0.018)	283.036	530.9	10.993	80.3
party-cif	0.125 (0.021)	0.745 (0.020)	0.465	0.873	2.516	18.4
aorsf-net	0.123 (0.024)	0.741 (0.019)	37.619	70.6	0.132	0.963
aorsf-cph(maxit = 15)	0.121 (0.026)	0.739 (0.018)	1.202	2.25	0.136	0.993
rfsrc-standard	0.120 (0.026)	0.739 (0.019)	1.528	2.87	0.113	0.825
aorsf-cph(maxit = 1)	0.117 (0.025)	0.736 (0.018)	0.533	1.00	0.137	1.00
aorsf-random	0.108 (0.022)	0.729 (0.025)	1.226	2.30	0.130	0.948
ranger-extratrees	0.094 (0.014)	0.737 (0.023)	0.061	0.113	0.141	1.03
glmnet-cox	0.088 (0.017)	0.731 (0.021)	0.091	0.170	0.003	0.022
xgboost-cox	0.086 (0.019)	0.736 (0.021)	2.596	4.87	0.003	0.022
xgboost-aft	-1.10 (0.176)	0.734 (0.021)	10.121	19.0	0.006	0.044
GUIDE-IT; CVD death, n = 894, p = 59						

(continued)

	Scaled Brier	C-Statistic	Median	Ratio	Median	Ratio
aorsf-cph(maxit = 1)	0.075 (0.019)	0.739 (0.030)	0.578	1.00	0.130	1.00
aorsf-net	0.074 (0.020)	0.738 (0.030)	27.827	48.2	0.134	1.03
aorsf-cph(maxit = 15)	0.072 (0.020)	0.738 (0.029)	1.369	2.37	0.129	0.993
obliqueRSF-net	0.061 (0.015)	0.736 (0.029)	220.667	382.1	11.349	87.2
party-cif	0.056 (0.015)	0.732 (0.030)	1.431	2.48	3.202	24.6
glmnet-cox	0.056 (0.041)	0.704 (0.097)	0.497	0.860	0.002	0.015
ranger-extratrees	0.052 (0.013)	0.729 (0.032)	0.521	0.902	0.182	1.40
rfsrc-standard	0.048 (0.026)	0.704 (0.029)	0.776	1.34	0.061	0.470
aorsf-random	0.034 (0.012)	0.694 (0.033)	1.064	1.84	0.134	1.03
xgboost-cox	0.021 (0.066)	0.744 (0.024)	3.668	6.35	0.003	0.023
xgboost-aft	-4.95 (0.885)	0.729 (0.027)	11.776	20.4	0.007	0.054
GUIDE-IT; HF hospitalization, n = 894, p = 59						
aorsf-net	0.081 (0.019)	0.719 (0.023)	54.048	60.8	0.207	1.03
aorsf-cph(maxit = 15)	0.081 (0.019)	0.719 (0.023)	2.465	2.77	0.198	0.986
aorsf-cph(maxit = 1)	0.080 (0.020)	0.719 (0.024)	0.889	1.00	0.200	1.00
ranger-extratrees	0.073 (0.012)	0.717 (0.024)	0.566	0.636	0.202	1.01
obliqueRSF-net	0.073 (0.012)	0.715 (0.025)	382.769	430.5	13.800	68.9
party-cif	0.070 (0.012)	0.711 (0.024)	1.381	1.55	3.363	16.8
rfsrc-standard	0.059 (0.019)	0.691 (0.023)	1.511	1.70	0.118	0.591
glmnet-cox	0.053 (0.023)	0.692 (0.030)	0.433	0.488	0.003	0.015
aorsf-random	0.051 (0.011)	0.682 (0.025)	2.100	2.36	0.196	0.980
xgboost-cox	0.039 (0.019)	0.694 (0.027)	2.967	3.34	0.002	0.010
xgboost-aft	-2.12 (0.324)	0.696 (0.025)	12.660	14.2	0.006	0.030
Heart Transplant; graft-loss or death, n = 3787, p = 52						
aorsf-net	0.051 (0.007)	0.730 (0.013)	126.867	33.3	1.065	1.03
aorsf-cph(maxit = 1)	0.050 (0.007)	0.731 (0.014)	3.809	1.00	1.034	1.00
aorsf-cph(maxit = 15)	0.050 (0.007)	0.730 (0.013)	10.098	2.65	1.029	0.995
party-cif	0.048 (0.006)	0.731 (0.012)	10.755	2.82	39.378	38.1
rfsrc-standard	0.048 (0.009)	0.727 (0.012)	3.702	0.972	1.115	1.08
obliqueRSF-net	0.048 (0.006)	0.729 (0.013)	365.863	96.1	161.863	156.6
ranger-extratrees	0.044 (0.006)	0.727 (0.013)	5.396	1.42	3.471	3.36
glmnet-cox	0.035 (0.005)	0.710 (0.017)	1.362	0.358	0.010	0.010
xgboost-cox	0.032 (0.008)	0.717 (0.015)	3.871	1.02	0.016	0.016
aorsf-random	0.028 (0.004)	0.691 (0.014)	6.272	1.65	1.040	1.01
xgboost-aft	-4.26 (0.534)	0.726 (0.012)	12.730	3.34	0.009	0.009
Monoclonal gammopathy; death, n = 1384, p = 8						
aorsf-cph(maxit = 15)	0.158 (0.017)	0.743 (0.011)	3.015	2.22	0.332	1.00

(continued)

	Scaled Brier	C-Statistic	Median	Ratio	Median	Ratio
aorsf-cph(maxit = 1)	0.157 (0.017)	0.742 (0.011)	1.360	1.00	0.332	1.00
obliqueRSF-net	0.155 (0.013)	0.743 (0.011)	240.551	176.9	16.910	50.9
aorsf-net	0.154 (0.017)	0.741 (0.011)	90.724	66.7	0.325	0.979
party-cif	0.152 (0.016)	0.739 (0.012)	1.585	1.17	6.240	18.8
rfsrc-standard	0.150 (0.018)	0.737 (0.011)	2.484	1.83	0.192	0.579
aorsf-random	0.147 (0.014)	0.736 (0.012)	2.999	2.21	0.316	0.951
glmnet-cox	0.139 (0.022)	0.728 (0.014)	0.121	0.089	0.004	0.012
xgboost-cox	0.122 (0.014)	0.734 (0.012)	3.480	2.56	0.005	0.015
ranger-extratrees	0.116 (0.007)	0.745 (0.012)	0.063	0.046	0.202	0.607
xgboost-aft	-0.863 (0.151)	0.732 (0.013)	11.184	8.23	0.006	0.018
Monoclonal gammopathy; malignancy, n = 1384, p = 8						
glmnet-cox	0.015 (0.011)	0.650 (0.046)	0.101	0.138	0.002	0.014
aorsf-cph(maxit = 15)	0.009 (0.014)	0.639 (0.031)	1.456	2.00	0.141	0.995
aorsf-cph(maxit = 1)	0.008 (0.014)	0.636 (0.030)	0.728	1.00	0.142	1.00
ranger-extratrees	0.007 (0.007)	0.633 (0.030)	0.058	0.080	0.162	1.14
aorsf-random	0.007 (0.014)	0.628 (0.029)	1.149	1.58	0.140	0.983
xgboost-cox	0.007 (0.014)	0.644 (0.040)	1.932	2.65	0.002	0.014
aorsf-net	0.006 (0.014)	0.634 (0.029)	23.343	32.1	0.141	0.996
obliqueRSF-net	0.005 (0.012)	0.626 (0.032)	39.064	53.6	16.273	114.6
party-cif	0.004 (0.013)	0.626 (0.031)	1.558	2.14	5.620	39.6
rfsrc-standard	-0.011 (0.018)	0.611 (0.032)	0.793	1.09	0.075	0.528
xgboost-aft	-5.45 (1.09)	0.630 (0.037)	9.725	13.4	0.006	0.042
Movies released in 2015-2018; gross 1M USD, n = 551, p = 46						
glmnet-cox	0.619 (0.029)	0.941 (0.009)	0.180	0.226	0.004	0.019
aorsf-net	0.531 (0.024)	0.929 (0.010)	52.636	66.0	0.182	1.00
aorsf-cph(maxit = 15)	0.523 (0.022)	0.927 (0.010)	2.814	3.53	0.182	1.00
rfsrc-standard	0.521 (0.023)	0.923 (0.011)	1.705	2.14	0.103	0.567
aorsf-cph(maxit = 1)	0.517 (0.024)	0.923 (0.011)	0.797	1.00	0.181	1.00
xgboost-cox	0.508 (0.030)	0.932 (0.010)	12.812	16.1	0.006	0.033
party-cif	0.479 (0.025)	0.904 (0.015)	0.393	0.493	2.302	12.7
ranger-extratrees	0.432 (0.025)	0.901 (0.016)	0.058	0.073	0.108	0.594
obliqueRSF-net	0.323 (0.021)	0.911 (0.015)	163.647	205.2	27.707	152.7
aorsf-random	0.300 (0.025)	0.850 (0.023)	1.511	1.90	0.170	0.934
xgboost-aft	-0.483 (0.078)	0.927 (0.011)	50.372	63.2	0.009	0.047
Non-alcohol fatty liver disease; death, n = 17549, p = 24						
aorsf-cph(maxit = 15)	0.217 (0.010)	0.868 (0.006)	50.582	2.70	43.776	1.15
aorsf-cph(maxit = 1)	0.217 (0.010)	0.869 (0.006)	18.704	1.00	37.967	1.00

(continued)

	Scaled Brier	C-Statistic	Median	Ratio	Median	Ratio
aorsf-net	0.214 (0.010)	0.865 (0.006)	478.494	25.6	37.231	0.981
obliqueRSF-net	0.213 (0.010)	0.868 (0.006)	1560.671	83.4	6530.150	172.0
rfsrc-standard	0.212 (0.011)	0.860 (0.007)	12.257	0.655	1.293	0.034
glmnet-cox	0.211 (0.011)	0.860 (0.006)	1.509	0.081	0.464	0.012
party-cif	0.208 (0.009)	0.863 (0.006)	72.349	3.87	716.204	18.9
ranger-extratrees	0.184 (0.007)	0.860 (0.006)	44.210	2.36	96.730	2.55
aorsf-random	0.144 (0.006)	0.838 (0.007)	25.720	1.38	37.114	0.978
xgboost-cox	0.022 (0.015)	0.876 (0.006)	9.865	0.527	1.013	0.027
xgboost-aft	-7.06 (0.796)	0.874 (0.006)	29.664	1.59	1.001	0.026
Primary biliary cholangitis; death, n = 276, p = 19						
aorsf-cph(maxit = 1)	0.418 (0.048)	0.905 (0.023)	0.191	1.00	0.045	1.00
aorsf-cph(maxit = 15)	0.408 (0.047)	0.902 (0.023)	0.443	2.32	0.047	1.05
aorsf-net	0.402 (0.044)	0.901 (0.024)	14.653	76.7	0.046	1.03
rfsrc-standard	0.378 (0.048)	0.890 (0.026)	0.101	0.527	0.038	0.854
obliqueRSF-net	0.363 (0.037)	0.903 (0.024)	109.549	573.2	1.750	39.1
aorsf-random	0.345 (0.038)	0.889 (0.022)	0.433	2.27	0.046	1.03
party-cif	0.344 (0.039)	0.899 (0.027)	0.194	1.02	0.401	8.95
glmnet-cox	0.337 (0.053)	0.885 (0.027)	0.101	0.526	0.002	0.045
ranger-extratrees	0.270 (0.034)	0.890 (0.029)	0.026	0.135	0.038	0.855
xgboost-cox	0.263 (0.095)	0.879 (0.026)	4.438	23.2	0.002	0.045
xgboost-aft	-0.985 (0.345)	0.881 (0.024)	9.390	49.1	0.007	0.146
Rotterdam tumor bank; death, n = 2982, p = 11						
aorsf-net	0.170 (0.014)	0.764 (0.011)	156.389	51.8	1.283	0.947
obliqueRSF-net	0.167 (0.012)	0.764 (0.012)	453.760	150.2	117.582	86.7
aorsf-cph(maxit = 15)	0.167 (0.014)	0.762 (0.011)	7.285	2.41	1.353	0.998
aorsf-cph(maxit = 1)	0.165 (0.015)	0.760 (0.012)	3.021	1.00	1.356	1.00
party-cif	0.164 (0.012)	0.762 (0.012)	4.837	1.60	28.933	21.3
rfsrc-standard	0.163 (0.014)	0.759 (0.011)	3.315	1.10	1.024	0.755
aorsf-random	0.157 (0.012)	0.755 (0.012)	5.742	1.90	1.275	0.941
ranger-extratrees	0.141 (0.007)	0.751 (0.011)	3.462	1.15	2.475	1.83
xgboost-cox	0.135 (0.014)	0.756 (0.012)	3.849	1.27	0.020	0.015
glmnet-cox	0.120 (0.010)	0.732 (0.011)	0.211	0.070	0.025	0.018
xgboost-aft	-1.36 (0.159)	0.763 (0.011)	15.373	5.09	0.009	0.007
Rotterdam tumor bank; recurrence, n = 2982, p = 11						
obliqueRSF-net	0.151 (0.011)	0.739 (0.010)	533.082	162.9	134.838	89.5
aorsf-net	0.150 (0.013)	0.738 (0.010)	170.007	52.0	1.437	0.954
aorsf-cph(maxit = 15)	0.148 (0.013)	0.736 (0.010)	8.035	2.46	1.523	1.01

(continued)

	Scaled Brier	C-Statistic	Median	Ratio	Median	Ratio
party-cif	0.147 (0.012)	0.736 (0.010)	5.107	1.56	31.190	20.7
aorsf-cph(maxit = 1)	0.147 (0.013)	0.735 (0.010)	3.272	1.00	1.506	1.00
aorsf-random	0.143 (0.010)	0.733 (0.009)	6.299	1.93	1.394	0.926
rfsrc-standard	0.142 (0.013)	0.733 (0.010)	3.258	0.996	0.925	0.614
ranger-extratrees	0.137 (0.007)	0.736 (0.009)	3.429	1.05	2.687	1.78
glmnet-cox	0.119 (0.008)	0.729 (0.010)	0.252	0.077	0.022	0.014
xgboost-cox	0.115 (0.008)	0.732 (0.011)	3.302	1.01	0.022	0.015
xgboost-aft	-1.06 (0.148)	0.737 (0.010)	14.130	4.32	0.010	0.007
Serum free light chain; death, n = 7874, p = 10						
aorsf-cph(maxit = 15)	0.249 (0.013)	0.824 (0.008)	19.646	2.34	17.424	0.761
aorsf-cph(maxit = 1)	0.249 (0.014)	0.824 (0.007)	8.398	1.00	22.885	1.00
aorsf-net	0.249 (0.013)	0.821 (0.008)	305.832	36.4	21.792	0.952
glmnet-cox	0.248 (0.012)	0.819 (0.007)	0.615	0.073	0.234	0.010
obliqueRSF-net	0.247 (0.012)	0.820 (0.008)	1107.351	131.9	817.888	35.7
ranger-extratrees	0.243 (0.010)	0.819 (0.008)	15.165	1.81	15.048	0.658
party-cif	0.243 (0.012)	0.817 (0.008)	19.999	2.38	155.997	6.82
rfsrc-standard	0.242 (0.013)	0.814 (0.008)	6.781	0.807	0.620	0.027
aorsf-random	0.230 (0.012)	0.815 (0.008)	14.450	1.72	20.496	0.896
xgboost-cox	0.091 (0.035)	0.822 (0.008)	6.066	0.722	0.136	0.006
xgboost-aft	-2.69 (0.251)	0.822 (0.008)	18.993	2.26	0.650	0.028
SPRINT; CVD death, n = 9361, p = 174						
glmnet-cox	0.071 (0.011)	0.793 (0.014)	13.662	1.13	0.027	0.006
aorsf-cph(maxit = 15)	0.070 (0.007)	0.795 (0.013)	36.982	3.06	4.419	0.951
aorsf-net	0.069 (0.009)	0.794 (0.013)	358.365	29.7	4.222	0.909
aorsf-cph(maxit = 1)	0.069 (0.007)	0.795 (0.013)	12.084	1.00	4.645	1.00
obliqueRSF-net	0.067 (0.006)	0.796 (0.014)	1138.135	94.2	1242.623	267.5
rfsrc-standard	0.064 (0.008)	0.786 (0.015)	5.499	0.455	1.247	0.268
party-cif	0.061 (0.005)	0.796 (0.013)	49.198	4.07	194.676	41.9
ranger-extratrees	0.054 (0.004)	0.789 (0.014)	8.810	0.729	9.762	2.10
aorsf-random	0.027 (0.002)	0.745 (0.016)	16.090	1.33	4.634	0.998
xgboost-cox	0.007 (0.019)	0.797 (0.013)	7.793	0.645	0.031	0.007
xgboost-aft	-10.7 (0.853)	0.794 (0.013)	23.758	1.97	0.017	0.004
SPRINT; death, n = 9361, p = 174						
glmnet-cox	0.123 (0.012)	0.770 (0.010)	5.748	0.327	0.072	0.005
aorsf-cph(maxit = 15)	0.117 (0.009)	0.770 (0.010)	55.731	3.17	13.261	0.895
aorsf-cph(maxit = 1)	0.116 (0.009)	0.769 (0.009)	17.581	1.00	14.818	1.00
aorsf-net	0.114 (0.010)	0.769 (0.010)	614.727	35.0	11.692	0.789

(continued)

	Scaled Brier	C-Statistic	Median	Ratio	Median	Ratio
obliqueRSF-net	0.113 (0.007)	0.767 (0.010)	3120.498	177.5	1215.032	82.0
rfsrc-standard	0.110 (0.009)	0.762 (0.010)	8.948	0.509	0.805	0.054
party-cif	0.107 (0.007)	0.764 (0.010)	51.458	2.93	222.920	15.0
ranger-extratrees	0.097 (0.005)	0.756 (0.010)	15.249	0.867	11.477	0.774
aorsf-random	0.053 (0.003)	0.719 (0.011)	23.393	1.33	15.908	1.07
xgboost-cox	0.031 (0.022)	0.772 (0.009)	9.917	0.564	0.082	0.006
xgboost-aft	-4.39 (0.355)	0.772 (0.010)	24.150	1.37	0.027	0.002
Systolic Heart Failure; death, n = 2231, p = 41						
glmnet-cox	0.114 (0.012)	0.747 (0.012)	0.272	0.113	0.010	0.013
obliqueRSF-net	0.114 (0.011)	0.747 (0.012)	388.113	161.6	56.165	70.8
aorsf-net	0.112 (0.013)	0.743 (0.012)	119.347	49.7	0.813	1.02
aorsf-cph(maxit = 15)	0.111 (0.013)	0.744 (0.012)	6.734	2.80	0.798	1.01
party-cif	0.111 (0.010)	0.745 (0.012)	4.287	1.79	18.913	23.8
aorsf-cph(maxit = 1)	0.110 (0.015)	0.744 (0.012)	2.401	1.00	0.793	1.00
rfsrc-standard	0.105 (0.012)	0.735 (0.012)	2.526	1.05	0.273	0.345
ranger-extratrees	0.092 (0.008)	0.738 (0.013)	3.196	1.33	1.399	1.76
xgboost-cox	0.090 (0.012)	0.745 (0.011)	4.108	1.71	0.010	0.013
aorsf-random	0.081 (0.006)	0.731 (0.013)	4.722	1.97	0.779	0.983
xgboost-aft	-1.97 (0.226)	0.741 (0.010)	13.112	5.46	0.008	0.010
VA lung cancer trial; death, n = 137, p = 8						
aorsf-net	0.200 (0.049)	0.792 (0.036)	10.492	91.2	0.024	1.00
aorsf-cph(maxit = 1)	0.199 (0.049)	0.790 (0.036)	0.115	1.00	0.024	1.00
aorsf-cph(maxit = 15)	0.198 (0.050)	0.789 (0.037)	0.225	1.95	0.024	1.00
rfsrc-standard	0.176 (0.046)	0.781 (0.035)	0.063	0.546	0.026	1.08
glmnet-cox	0.161 (0.044)	0.784 (0.041)	0.071	0.614	0.002	0.083
aorsf-random	0.151 (0.047)	0.772 (0.044)	0.295	2.56	0.023	0.958
party-cif	0.129 (0.039)	0.766 (0.036)	0.098	0.855	0.121	5.04
obliqueRSF-net	0.127 (0.035)	0.791 (0.034)	60.532	525.9	0.660	27.5
ranger-extratrees	0.091 (0.036)	0.772 (0.038)	0.020	0.175	0.026	1.09
xgboost-cox	0.069 (0.066)	0.742 (0.055)	1.400	12.2	0.002	0.083
xgboost-aft	-0.010 (0.131)	0.750 (0.044)	5.934	51.5	0.005	0.208

References

Leo Breiman. Random forests. *Machine Learning*, 45(1):5–32, 2001.

Erika Graf, Claudia Schmoor, Willi Sauerbrei, and Martin Schumacher. Assessment and comparison of prognostic classification schemes for survival data. *Statistics in Medicine*, 18(17-18):2529–2545, 1999. URL `https://doi.org/10.1002/(SICI)1097-0258(19990915/30)18:17/18%3C2529::AID-SIM274%3E3.0.CO;2-5`.

Torsten Hothorn, Kurt Hornik, and Achim Zeileis. Unbiased recursive partitioning: A conditional inference framework. *Journal of Computational and Graphical Statistics*, 15 (3):651–674, 2006.

Torsten Hothorn, Kurt Hornik, Carolin Strobl, and Achim Zeileis. Party: a laboratory for recursive partytioning, 2010.

H. Ishwaran and U.B. Kogalur. *Random Forests for Survival, Regression, and Classification (RF-SRC)*, 2019. URL `https://cran.r-project.org/package=randomForestSRC`. R package version 2.8.0, available at `https://cran.r-project.org/package=randomForestSRC`.

Hemant Ishwaran, Udaya B Kogalur, Eugene H Blackstone, and Michael S Lauer. Random survival forests. *The Annals of Applied Statistics*, pages 841–860, 2008.

Byron Jaeger. *obliqueRSF: Oblique Random Forests for Right-Censored Time-to-Event Data*, 2018. URL `https://CRAN.R-project.org/package=obliqueRSF`. R package version 0.1.0, available at `https://CRAN.R-project.org/package=obliqueRSF`.

Byron C Jaeger, D Leann Long, Dustin M Long, Mario Sims, Jeff M Szychowski, Yuan-I Min, Leslie A Mcclure, George Howard, and Noah Simon. Oblique random survival forests. *The Annals of Applied Statistics*, 13(3):1847–1883, 2019.

Rakesh Katuwal, Ponnuthurai Nagaratnam Suganthan, and Le Zhang. Heterogeneous oblique random forest. *Pattern Recognition*, 99:107078, 2020.

Bjoern H Menze, B Michael Kelm, Daniel N Splitthoff, Ullrich Koethe, and Fred A Hamprecht. On oblique random forests. In *Joint European Conference on Machine Learning and Knowledge Discovery in Databases*, pages 453–469. Springer, 2011.

Karel GM Moons, Andre Pascal Kengne, Diederick E Grobbee, Patrick Royston, Yvonne Vergouwe, Douglas G Altman, and Mark Woodward. Risk prediction models: Ii. external validation, model updating, and impact assessment. *Heart*, 98(9):691–698, 2012a.

Karel GM Moons, Andre Pascal Kengne, Mark Woodward, Patrick Royston, Yvonne Vergouwe, Douglas G Altman, and Diederick E Grobbee. Risk prediction models: I. development, internal validation, and assessing the incremental value of a new (bio) marker. *Heart*, 98(9):683–690, 2012b.

Tyler M Tomita, James Browne, Cencheng Shen, Jaewon Chung, Jesse L Patsolic, Benjamin Falk, Carey E Priebe, Jason Yim, Randal Burns, Mauro Maggioni, et al. Sparse projection oblique randomer forests. *Journal of machine learning research*, 21(104), 2020.

Marvin N. Wright and Andreas Ziegler. ranger: A fast implementation of random forests for high dimensional data in c++ and r. *Journal of Statistical Software*, 77(1):1–17, 2017. doi: 10.18637/jss.v077.i01. URL `https://www.jstatsoft.org/index.php/jss/article/view/v077i01`.

Ruoqing Zhu. *Tree-based Methods for Survival Analysis and High-dimensional Data*. PhD thesis, The University of North Carolina at Chapel Hill, 2013.

Ruoqing Zhu, Donglin Zeng, and Michael R Kosorok. Reinforcement learning trees. *Journal of the American Statistical Association*, 110(512):1770–1784, 2015.