
ACCELERATED AND INTERPRETABLE OBLIQUE RANDOM SURVIVAL FORESTS

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ABSTRACT

The oblique random survival forest (RSF) is an ensemble supervised learning method for right-censored outcomes. Trees in the oblique RSF are grown using linear combinations of predictors, whereas in the standard RSF, a single predictor is used. Oblique RSF ensembles have high prediction accuracy, but assessing many linear combinations of predictors induces high computational overhead. In addition, few methods have been developed for estimation of variable importance (VI) with oblique RSFs. We introduce a method to increase computational efficiency of the oblique RSF and a method to estimate VI with the oblique RSF. Our computational approach uses Newton-Raphson scoring in each non-leaf node. We estimate VI by negating each coefficient used for a given predictor in linear combinations, and then computing the reduction in out-of-bag accuracy. In benchmarking experiments, we find our implementation of the oblique RSF is hundreds of times faster, with equivalent prediction accuracy, compared to existing software for oblique RSFs. We find in simulation studies that ‘negation VI’ discriminates between relevant and irrelevant predictors more accurately than permutation VI, Shapley VI, and a technique to measure VI using analysis of variance. All oblique RSF methods in the current study are available in the `aorsf` R package.

Keywords Oblique · Survival · Random Forests · Fast · Variable Importance

1 Introduction

Risk prediction may reduce the burden of disease by guiding strategies for prevention and treatment in a wide range of domains [Moons et al., 2012a,b]. The random survival forest (RSF; Ishwaran et al. [2008], Hothorn et al. [2006]) is a supervised learning algorithm that has been used frequently for risk prediction [Wang and Li, 2017]. Similar to random forests (RFs) for classification and regression [Breiman, 2001], The RSF is a large set of de-correlated and randomized decision trees, with each tree contributing to the ensemble’s prediction function. Notable characteristics of the RSF include uniform convergence of its ensemble survival prediction function to the true survival function, first shown by Ishwaran and Kogalur [2010] and later by Cui et al. [2017] under more general conditions. However, Cui et al. [2017] noted that the RSF is at a disadvantage when predictors are correlated and some are not relevant to the censored outcome, which is a strong possibility when large clinical and ‘omic’ databases are leveraged for risk prediction.

A potential approach to improve the RSF when predictors are correlated and some are not relevant to the censored outcome is to use oblique trees instead of axis based trees. Axis based trees split data using a single predictor, creating decision boundaries that are perpendicular or parallel to axes of the predictor space [see Breiman et al., 2017, Chapter 2]. Oblique trees split data using a linear combination of predictors, creating decision boundaries that are neither parallel nor perpendicular to axes of their contributing predictors [see Breiman et al., 2017, Chapter 5]. Menze et al. [2011] examined prediction accuracy of RFs in the presence of correlated predictors and found that oblique RFs had substantially higher prediction accuracy compared to axis-based RFs. Similarly, Jaeger et al. [2019] found that growing RSFs with oblique rather than axis-based survival trees reduced the RSF’s concordance error, with improvements ranging from 2.5% to 24.9% depending on the data analyzed.

Despite the potential for higher accuracy, oblique trees have at least two notable drawbacks compared to axis-based trees. First, finding a locally optimal oblique decision rule may require exponentially more computation than an axis-based rule. If p predictors are potentially used to split n observations, up to $\mathcal{O}(n^p)$ oblique splits can be assessed versus $\mathcal{O}(n \cdot p)$ axis-based splits [Heath et al., 1993, Murthy et al., 1994]. Second, estimating variable importance (VI) using permutation (a standard method for RFs) may be less effective in ensembles of oblique trees, as permuting the values of one predictor may not destabilize decisions that are based on linear combinations of predictors. Although VI is one of the most widely used strategies to interpret random forests [Ishwaran and Lu, 2019], few studies have investigated VI for oblique random forests [see Menze et al., 2011, Section 5], and fewer have investigated VI specifically for the oblique RSF.

The aim of this paper is to introduce methodology that improves the computational efficiency and interpretation of oblique RSFs. Section 2 reviews prior studies that have developed methods related to those introduced in the current study. In Section 3, we reduce the computational cost of oblique RSFs (*i.e.*, accelerate them) with a scalable algorithm to identify linear combinations of coefficients. In Section 4, we improve the interpretability of oblique RSFs with ‘negation VI’, a method to estimate VI that flips the sign of coefficients in linear combinations of predictors instead of permuting predictor values. We evaluate these methods with general benchmarking experiments and simulation studies in Section 5. In Section 6, we summarize results from the current study and present ideas connecting the current work to existing frameworks and methods for RSFs that future studies may engage with.

2 Related work

Sections 2.1 and 2.2 briefly summarize prior studies that have developed methods related to the oblique RSF and VI, respectively.

2.1 Axis-based and oblique random forests

After Breiman [2001] introduced the axis-based and oblique RF, numerous methods were developed to grow oblique RFs for classification or regression tasks [Menze et al., 2011, Zhang and Suganthan, 2014, Rainforth and Wood, 2015, Zhu et al., 2015, Poona et al., 2016, Qiu et al., 2017, Tomita et al., 2020, Katuwal et al., 2020]. However, oblique splitting approaches for classification or regression may not generalize to censored outcomes [*e.g.*, see Zhu, 2013, Section 4.5.1], and most research involving the RSF has focused on forests with axis-based trees [Wang and Li, 2017].

Building on prior research for bagging survival trees [Hothorn et al., 2004], Hothorn et al. [2006] developed an axis-based RSF in their framework for unbiased recursive partitioning, more commonly referred to as the conditional inference forest (CIF). Zhou et al. [2016] developed a rotation forest based on the CIF and Wang and Zhou [2017] developed a method for extending the predictor space of the CIF. Ishwaran et al. [2008] developed an axis-based RSF with strict adherence to the rules for growing trees proposed in Breiman [2001]. Jaeger et al. [2019] developed the

oblique RSF following the bootstrapping approach described in Breiman’s original RF and incorporating early stopping rules from the CIF.

Fast algorithms to fit axis-based RSFs are available in the randomForestSRC R package [Ishwaran and Kogalur, 2019] and the ranger [Wright and Ziegler, 2017] R package. randomForestSRC provides a unified interface to grow RFs in a wide range of analyses, and ranger is designed to grow RFs efficiently using high dimensional data. Fast algorithms to fit the CIF are provided by the party R package [Hothorn et al., 2010], which provides a computational toolbox for recursive partitioning using conditional inference trees. Jaeger et al. [2019] developed the obliqueRSF package and found it was approximately 30 times slower than party and nearly 200 times slower than randomForestSRC. Few studies have developed software with fast algorithms for oblique RSFs that have comparable speed compared to algorithms for axis-based RSFs.

2.2 Variable importance

Several techniques to estimate VI have been developed since Breiman [2001] introduced permutation VI, which is defined for each predictor as the difference in a RF’s estimated prediction error before versus after the predictor’s values are randomly permuted. Strobl et al. [2007] identified bias in permutation VI driven by variable selection bias and effects induced by bootstrap sampling, and proposed an unbiased permutation VI measure based on unbiased recursive partitioning [Hothorn et al., 2006]. Menze et al. [2011] introduced an approach to estimate VI for oblique RFs that computes an analysis of variance (ANOVA) table in non-leaf nodes to obtain p-values for each predictor contributing to the node. The ANOVA VI¹ is then defined for each predictor as the number of times a p-value associated with the predictor is ≤ 0.01 while growing a forest. Lundberg and Lee [2017] introduced a method to estimate VI using SHapley Additive exPlanation (SHAP) values, which estimate the contribution of a predictor to a model’s prediction for a given observation. SHAP VI is computed for each predictor by taking the mean absolute value of SHAP values for that predictor across all observations in a given set. With the exception of Menze et al. [2011], few studies have evaluated estimation of VI using oblique RFs, and fewer have examined VI specifically for the oblique RSF.

3 The accelerated oblique random survival forest

This section describes our approach to reduce computational overhead of the oblique RSF. Consider the usual framework for right-censored time-to-event outcomes with training data

$$\mathcal{D}_{\text{train}} = \{(T_i, \delta_i, \mathbf{x}_i)\}_{i=1}^{N_{\text{train}}}.$$

Here, T_i is the event time if $\delta_i = 1$ or the censoring time if $\delta_i = 0$, and \mathbf{x}_i is a vector of predictors values. Assuming there are no ties, let $t_1 < \dots < t_m$ denote the m unique event times in $\mathcal{D}_{\text{train}}$.

To accelerate the oblique RSF, we propose to identify linear combinations of predictor variables in non-leaf nodes by applying Newton Raphson scoring to the partial likelihood function of the Cox regression model:

$$L(\boldsymbol{\beta}) = \prod_{i=1}^m \frac{e^{\mathbf{x}_{j(i)}^T \boldsymbol{\beta}}}{\sum_{j \in R_i} e^{\mathbf{x}_j^T \boldsymbol{\beta}}}, \quad (1)$$

where R_i is the set of indices, j , with $T_j \geq t_i$ (i.e., those still at risk at time t_i), and $j(i)$ is the index of the observation for which an event occurred at time t_i . Newton Raphson scoring is an exceptionally fast estimation procedure, and the survival package [Therneau, 2022a] includes documentation that outlines how to efficiently program it [Therneau, 2022b]. Briefly, a vector of estimated regression coefficients, $\hat{\boldsymbol{\beta}}$, is updated in each step of the procedure based on its first derivative, $U(\hat{\boldsymbol{\beta}})$, and second derivative, $H(\hat{\boldsymbol{\beta}})$:

$$\hat{\boldsymbol{\beta}}^{k+1} = \hat{\boldsymbol{\beta}}^k + U(\hat{\boldsymbol{\beta}} = \hat{\boldsymbol{\beta}}^k) H^{-1}(\hat{\boldsymbol{\beta}} = \hat{\boldsymbol{\beta}}^k)$$

For statistical inference, it is recommended to continue updating $\hat{\boldsymbol{\beta}}$ by completing additional iterations of Newton Raphson scoring until a convergence threshold is met. However, since an estimate of $\hat{\boldsymbol{\beta}}$ is created by the first iteration of Newton Raphson scoring, only one iteration of Newton Raphson scoring is needed to identify a valid linear combination of predictors. Moreover, computing U and H requires computation and exponentiation of the vector $\mathbf{x}\hat{\boldsymbol{\beta}}$, but these steps can be skipped on the first iteration of Newton Raphson scoring if an initial value of $\hat{\boldsymbol{\beta}} = \mathbf{0}$ is chosen, allowing for

¹Menze et al. [2011] name their method ‘oblique RF VI’, but we use the name ‘ANOVA VI’ in this article to avoid confusing Menze’s approach with other approaches to estimate VI for oblique RFs.

a reduction in computing operations and removing the need to scale predictor values prior to initiating the Newton Raphson algorithm.² In Section 5.1.6, we formally test whether growing oblique survival trees using one iteration of Newton Raphson scoring provides equivalent prediction accuracy compared to trees where iterations are completed until a convergence threshold is met.

Algorithm 1 presents our approach to fitting an oblique survival tree in the accelerated oblique RSF using default values from the `aorsf` R package. Several steps are taken to reduce computational overhead. First, memory is conserved by conducting bootstrap resampling via random integer-valued weights, rather than using a bootstrapped copy of the original data. Memory conservation also takes place in terminal nodes, where we restrict estimation of the survival and cumulative hazard function to event times that occur among observations in the node. Second, early stopping is applied to the tree-growing procedure if a statistical criterion is not met. In our case, the criterion is based on the magnitude of a log-rank test statistic corresponding to splitting the data at a current node. Third, instead of greedy recursive partitioning, we use ‘good enough’ partitioning. More specifically, instead of computing a log-rank test statistic for several different linear combinations of variables and proceeding with the highest scoring option, we identify an optimal cut-point for one linear combination of variables and assess whether using this combination will create a split that passes the criterion for splitting a node. If it does not pass the criterion, then another linear combination will be tested, with the maximum number of attempts set by the parameter `n_retry`. Often a ‘good-enough’ split can be found in just one attempt when the training set is large, which gives the accelerated oblique RSF a computational advantage in larger training sets compared to greedy partitioning.

4 Negation variable importance

This Section introduces negation VI, which is similar to permutation VI in that it measures how much a model’s prediction error increases when a variable’s role in the model is de-stabilized. Specifically, negation VI measures the increase in an oblique RF’s prediction error after flipping the sign of all coefficients linked to a variable (*i.e.*, negating them). As the magnitude of a coefficient increases, so does the probability that negating it will change the oblique RF’s predictions. For the current study, we use Harrell’s concordance (C)-statistic [Harrell et al., 1982] to measure change in prediction error when computing negation VI.

Negation VI has several helpful characteristics. First, negation VI generalizes to any oblique RF (*i.e.*, not just RSFs) using any valid error function, making it both general and flexible.³ Second, since the coefficients in each non-leaf node of an oblique tree are adjusted for the accompanying predictors, negation VI may provide better estimation of VI in the presence of correlated variables compared to standard VI techniques. Third, unlike permutation, negation is non-random and hence reproducible without setting a random seed. Additionally, since negation VI does not permute variables, the analyst need not worry about impossible combinations of predictors that may occur when one predictor is randomly permuted, such as having a negative status for type 2 diabetes and having Hemoglobin A1c level $\geq 6.5\%$ (a value indicative of type 2 diabetes) as a result of randomly permuting the values of Hemoglobin A1c.

5 Numeric experiments

Sections 5.1 and 5.2 present numerical experiments examining the accelerated oblique RSF and negation VI, respectively. The code used to run these experiments is available online at <https://github.com/bcjaeger/aorsf-bench>. All analyses were conducted using R version 4.1.3 and coordinated by the `targets` R package [Landau, 2021]. To standardize comparisons of computational efficiency, all learners and VI techniques used up to 4 processing units.

5.1 Benchmark of prediction accuracy

The aim of this numeric experiment is to evaluate the prediction accuracy of the accelerated oblique RSF compared to its predecessor (the oblique RSF from the `obliqueRSF` R package) and to several other machine learning algorithms. Inferences drawn from this experiment include equivalence and inferiority tests based on Bayesian linear mixed models.

5.1.1 Learners

We consider four classes of learners: RSFs (both axis-based and oblique), boosting ensembles, regression models, and neural networks. Specific learners from each class are summarized in Table 1. To facilitate fair comparisons, tuning

²Predictors are scaled prior to initiating the Newton Raphson algorithm to avoid exponentiation of large numbers. However, if only one iteration is completed with an initial value of 0 for $\hat{\beta}$, then $\exp(\mathbf{x}\hat{\beta}) = 1$.

³The `aorsf` package enables customized functions to be applied in lieu of the default C-statistic (see `?aorsf::orsf_vi_negate`)

Algorithm 1 Accelerated oblique random survival tree using default parameters.

Require: Training data $\mathcal{D}_{\text{train}} = \{(T_i, \delta_i, \mathbf{x}_i)\}_{i=1}^{N_{\text{train}}}$, $\text{mtry} = \sqrt{\text{ncol}(\mathbf{x}_{\text{train}})}$, $\text{n_split} = 5$, $\text{n_retry} = 3$, and $\text{split_min_stat} = 3.841459$

- 1: $\mathcal{T} \leftarrow \emptyset$
- 2: $w \leftarrow \text{sample}(\text{from} = \{0, \dots, 10\}, \text{size} = \text{nrow}(\mathbf{x}_{\text{train}}), \text{replace} = \text{T})$
- 3: $\mathcal{D}_{\text{in-bag}} \leftarrow \text{subset}(\mathcal{D}_{\text{train}}, \text{rows} = \text{which}(w > 0))$
- 4: $w \leftarrow \text{subset}(w, \text{which}(w > 0))$
- 5: $\text{node_assignments} \leftarrow \text{rep}(1, \text{times} = \text{nrow}(\mathbf{x}_{\text{in-bag}}))$
- 6: $\text{nodes_to_split} \leftarrow \{1\}$
- 7: **while** $\text{nodes_to_split} \neq \emptyset$ **do**
- 8: **for** $\text{node} \in \text{nodes_to_split}$ **do**
- 9: $\text{n_try} \leftarrow 1$
- 10: $\text{node_rows} \leftarrow \text{which}(\text{node_assignments} \equiv \text{node})$
- 11: $\text{node_cols} \leftarrow \text{sample}(\text{from} = \{1, \dots, \text{ncol}(\mathbf{x})\}, \text{size} = \text{mtry}, \text{replace} = \text{F})$
- 12: $\mathcal{D}_{\text{node}} \leftarrow \text{subset}(\mathcal{D}_{\text{in-bag}}, \text{rows} = \text{node_rows}, \text{columns} = \text{node_cols})$
- 13: $\beta \leftarrow \text{newt_raph}(\mathcal{D}_{\text{node}}, \text{weights} = \text{subset}(w, \text{node_rows}), \text{max_iter} = 1)$
- 14: $\eta \leftarrow \mathbf{x}_{\text{node}} \times \beta$
- 15: $\mathcal{C} \leftarrow \text{sample}(\text{from} = \text{unique}(\eta), \text{size} = \text{n_split}, \text{replace} = \text{F})$
- 16: $c \leftarrow \text{argmax}_{c^* \in \mathcal{C}} \{\log_rank_stat(\eta, c^*)\}$
- 17: **if** $\log_rank_stat(\eta, c) \geq \text{split_min_stat}$ **then**
- 18: $\mathcal{T} \leftarrow \text{add_node}(\mathcal{T}, \text{name} = \text{node}, \text{beta} = \beta, \text{cutpoint} = c)$
- 19: ▷ Right node logic omitted for brevity (identical to left node logic)
- 20: $\text{node_left_name} \leftarrow \max(\text{node_assignments}) + 1$
- 21: $\text{node_left_rows} \leftarrow \text{subset}(\text{node_rows}, \text{which}(\eta \leq c))$
- 22: $\text{subset}(\text{node_assignments}, \text{node_left_rows}) \leftarrow \text{node_left_name}$
- 23: **if** $\text{is_splittable}(\text{subset}(\text{node_assignments}, \text{node_left_rows}))$ **then**
- 24: $\text{nodes_to_split} \leftarrow \text{nodes_to_split} \cup \text{node_left_name}$
- 25: **else**
- 26: $\mathcal{T} \leftarrow \text{add_leaf}(\mathcal{T}, \text{data} = \text{subset}(\mathcal{D}_{\text{node}}, \text{rows} = \text{node_left_rows}))$
- 27: **end if**
- 28: **else if** $\text{n_try} \leq \text{n_retry}$ **then**
- 29: $\text{n_try} \leftarrow \text{n_try} + 1$
- 30: **go to** 11
- 31: **else**
- 32: $\mathcal{T} \leftarrow \text{add_leaf}(\mathcal{T}, \text{data} = \mathcal{D}_{\text{node}})$
- 33: **end if**
- 34: $\text{nodes_to_split} \leftarrow \text{nodes_to_split} \setminus \{\text{node}\}$
- 35: **end for**
- 36: **end while**
- 37: **return** \mathcal{T}

parameters were harmonized within each class. For example, for RSF learners, we set the minimum node size (a parameter shared by all RSF learners) as 10. Additionally, for RSF learners, the number of randomly selected predictors was the square root of the total number of predictors rounded to the nearest integer, and the number of trees in the ensemble was 500 (a common default value for the number of trees). For boosting, regression, and neural network learners, nested cross-validation was applied to tune relevant model parameters. Specifically, tuning for boosting models included identifying the number of steps to complete. For regression models, tuning was used to identify the magnitude of penalization. For neural networks, the number and density of layers was tuned.

Learner Class	Software	Learners	Description
<i>Random Survival Forests</i>			
Axis based	RandomForestSRC ranger party rotsf rsfse	rsf-standard rsf-extratrees cif-standard cif-rotate cif-spacextend	rsf-standard grows survival trees following Leo Breiman’s original random forest algorithm with variables and cut-points selected to maximize a log-rank statistic. rsf-extratrees grows survival trees with randomly selected predictors and cut-points. cif-standard uses the framework of conditional inference to grow survival trees. cif-rotate extends cif-standard by applying principal component analysis to random subsets of data prior to growing each survival tree. cif-spacextend derives new predictors for each tree in the ensemble, separately.
Oblique	obliqueRSF aorsf	obliqueRSF-net aorsf-net aorsf-fast aorsf-cph aorsf-extratrees	Oblique survival trees following Leo Breiman’s random forest algorithm. Linear combinations of inputs are derived using glmnet in obliqueRSF-net and aorsf-net, using Newton Raphson scoring for the Cox partial likelihood function in aorsf-fast (1 iteration of scoring) and aorsf-cph (up to 20 iterations), and chosen randomly from a uniform distribution in aorsf-extratrees. Cut-points are selected from 5 randomly selected candidates to maximize a log-rank statistic.
<i>Boosting ensembles</i>			
Trees	xgboost	xgboost-cox xgboost-aft	xgboost-cox maximizes the Cox partial likelihood function, whereas xgboost-aft maximizes the accelerated failure time likelihood function. Nested cross validation (5 folds) is applied to tune the number of trees grown, the minimum number of observations in a leaf node was 10, the maximum depth of trees was 6, and \sqrt{p} variables were considered randomly for each tree split, where p is the total number of predictors.
<i>Regression models</i>			
Cox Net	glmnet	glmnet-cox	The Cox proportional hazards model is fit using an elastic net penalty. Nested cross validation (5 folds) is applied to tune penalty terms.
<i>Neural networks</i>			
Cox Time	survivalmodels	nn-cox	A neural network based on the proportional hazards model with time-varying effects. Nested cross-validation was applied to select the number of layers (from 1 to 8), the number of nodes in each layer (from $\sqrt{p}/2$ to \sqrt{p}), and the number of epochs to complete (up to 500). A drop-out rate of 10% was applied during training.

Table 1: Learning algorithms assessed in numeric studies. **aorsf-fast** is the accelerated oblique random survival forest (see Algorithm 1), and each of the additional learners are compared to **aorsf-fast** in numeric studies.

5.1.2 Evaluation of prediction accuracy

Our primary metric for evaluating the accuracy of predicted risk is the integrated and scaled Brier score [Graf et al., 1999], a proper scoring rule that combines discrimination and calibration in one value and improves interpretability by adjusting for a benchmark model [Kattan and Gerds, 2018]. Consider a testing data set:

$$\mathcal{D}_{\text{test}} = \{(T_i, \delta_i, x_i)\}_{i=1}^{N_{\text{test}}}.$$

Let $\hat{S}(t | x_i)$ be the predicted probability of survival up to a given prediction time of $t > 0$. For observation i in $\mathcal{D}_{\text{test}}$, let $\hat{S}(t | x_i)$ be the predicted probability of survival up to a given prediction time of $t > 0$. Define

$$\begin{aligned} \widehat{\text{BS}}(t) = & \frac{1}{N_{\text{test}}} \sum_{i=1}^{N_{\text{test}}} \{ \hat{S}(t | x_i)^2 \cdot I(T_i \leq t, \delta_i = 1) \cdot \hat{G}(T_i)^{-1} \\ & + [1 - \hat{S}(t | x_i)]^2 \cdot I(T_i > t) \cdot \hat{G}(t)^{-1} \} \end{aligned}$$

where $\hat{G}(t)$ is the Kaplan-Meier estimate of the censoring distribution. As $\widehat{\text{BS}}(t)$ is time dependent, integration over time provides a summary measure of performance over a range of plausible prediction times. The integrated $\widehat{\text{BS}}(t)$ is defined as

$$\widehat{\text{BS}}(t_1, t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \widehat{\text{BS}}(t) dt. \quad (2)$$

In our results, t_1 and t_2 are the 25th and 75th percentile of event times, respectively. $\widehat{\text{BS}}(t_1, t_2)$, a sum of squared prediction errors, can be scaled to produce a measure of explained residual variation (*i.e.*, an R^2 statistic) by computing

$$R^2 = 1 - \frac{\widehat{\text{BS}}(t_1, t_2)}{\widehat{\text{BS}}_0(t_1, t_2)} \quad (3)$$

where $\widehat{\text{BS}}_0(t_1, t_2)$ is the integrated Brier score when a Kaplan-Meier estimate for survival based on the training data is used as the survival prediction function $\hat{S}(t)$. We refer to this R^2 statistic as the index of prediction accuracy (IPA) [Kattan and Gerds, 2018].

Our secondary metric for evaluating predicted risk is the time-dependent concordance (C)-statistic. We compute the first time-dependent C-statistic proposed by Blanche et al. [2013, Equation 3], which is interpreted as the probability that a risk prediction model will assign higher risk to a case (*i.e.*, an observation with $T \leq t$ and $\delta = 1$) versus a non-case (*i.e.*, an observation with $T > t$). Similar to the IPA, observations with $T \leq t$ and $\delta = 0$ only contribute to inverse probability of censoring weights for the time-dependent C-statistic.

Both the IPA and time-dependent C-statistic generally take values between 0 and 1. To avoid presenting an excessive amount of leading zeroes in our tables, figures, and text, we scale both the IPA and time-dependent C-statistic by 100. For example, we present a value of 25 if the IPA is 0.25, 87 if the time-dependent C-statistic is 0.87, and present 10.2 if the difference between two IPA values is 0.102

5.1.3 Data sets

We used a collection of 21 data sets containing a total of 35 risk prediction tasks (tasks per data set ranged from one to four). Participant-level data from the GUIDE-IT and SPRINT clinical trials and the ARIC, MESA, and JHS community cohort studies was obtained from the National Institute of Health Biologic Specimen and Data Repository Coordinating Center (BioLINCC). Designs and protocols for these studies have been made available [ARIC Investigators, 1989, Bild et al., 2002, Felker et al., 2017, SPRINT Research Group, 2015, Taylor Jr et al., 2005]. All other datasets were publicly available and obtained through R packages (see Appendix A.1). Across all prediction tasks, the number of observations ranged from 137 to 17,549 (median: 1,384), the number of predictors ranged from 7 to 1,692 (median: 41), and the percentage of censored observations ranged from 5.26 to 97.7 (median: 78.1) (Table A.1).

5.1.4 Monte-Carlo cross validation

For each risk prediction task, we completed 25 runs of Monte-Carlo cross validation. In each run, we used a random sample containing 50% of the available data for training and the remaining 50% for testing each of the learners described in Section 5.1.1. Then, for each learner, we computed the IPA and time-dependent C-statistic. If any learner failed to obtain predictions on any particular split of data⁴, the results for that split were omitted from downstream analyses for all learners.

5.1.5 Statistical analysis

After collecting data from 25 replications of Monte-Carlo cross validation for the 14 learners in all 35 risk prediction tasks, we analyzed the resulting 12,250 observations of IPA and, separately, time-dependent C-statistic, using a Bayesian linear mixed model. Our approach follows the ideas described by Benavoli et al. [2017] and Kuhn and Wickham [2020], who developed guidelines on making statistical comparisons between learners using Bayesian models. Specifically, we fit two models:

$$\text{IPA} = \hat{\gamma}_0 + \hat{\gamma} \cdot \text{learner} + (1 \mid \text{data/run})$$

and

$$\text{C-stat} = \hat{\gamma}_0 + \hat{\gamma} \cdot \text{learner} + (1 \mid \text{data/run}).$$

Random intercepts for specific splits of data (*i.e.*, run in the model formula) were nested within datasets. The intercept, $\hat{\gamma}_0$, was the expected value of the outcome using `aorsf-fast`, making the coefficients in $\hat{\gamma}$ the expected differences between `aorsf-fast` and other learners. Default priors from `rstanarm` were applied for model fitting [Goodrich et al., 2022].

Hypothesis testing For both the IPA and time-dependent C-statistic, we conducted equivalence and inferiority tests based on a 1 point region of practical equivalence. More specifically, we concluded that two learners had practically equivalent IPA or time-dependent C-statistic if there was a 95% or higher posterior probability that the absolute difference in the relevant metric was less than 1. We concluded that one learner was weakly superior when there was $\geq 95\%$ posterior probability that the absolute difference in the relevant metric was non-zero, and concluded superiority when when there was $\geq 95\%$ posterior probability that the absolute difference in the relevant metric was 1 or more.

5.1.6 Results

A full summary of all results presented in this Section is provided in Table A.2. In total, 871 out of 875 Monte-Carlo cross validation runs were completed. On run 13, 18, 24 and 25 for the ACTG 320 data, the `nn-cox` learner encountered an error during its fitting procedure.

Index of prediction accuracy Compared to learners that were not oblique RSFs, `aorsf-fast` had the highest IPA in 20 out of 35 risk prediction tasks, with an overall mean IPA of 12.6 (Figure 1). Compared to the learner with the second highest mean IPA (`cif-standard`), `aorsf-fast`'s mean was 1.35 points higher, a relative increase of 12.0%. The posterior probability of `aorsf-fast` and `aorsf-cph` having practically equivalent expected IPA was 0.99, and the posterior probability of `aorsf-fast` having a superior IPA to other learners ranged from 0.86 (versus `cif-standard`) to >0.999 (versus several other learners; see Figure 2)

⁴For example, when the prediction task was to predict risk of death in the ACTG 320 clinical trial (26 events total), some splits did not leave enough events in the training data to fit complex learners such as neural networks

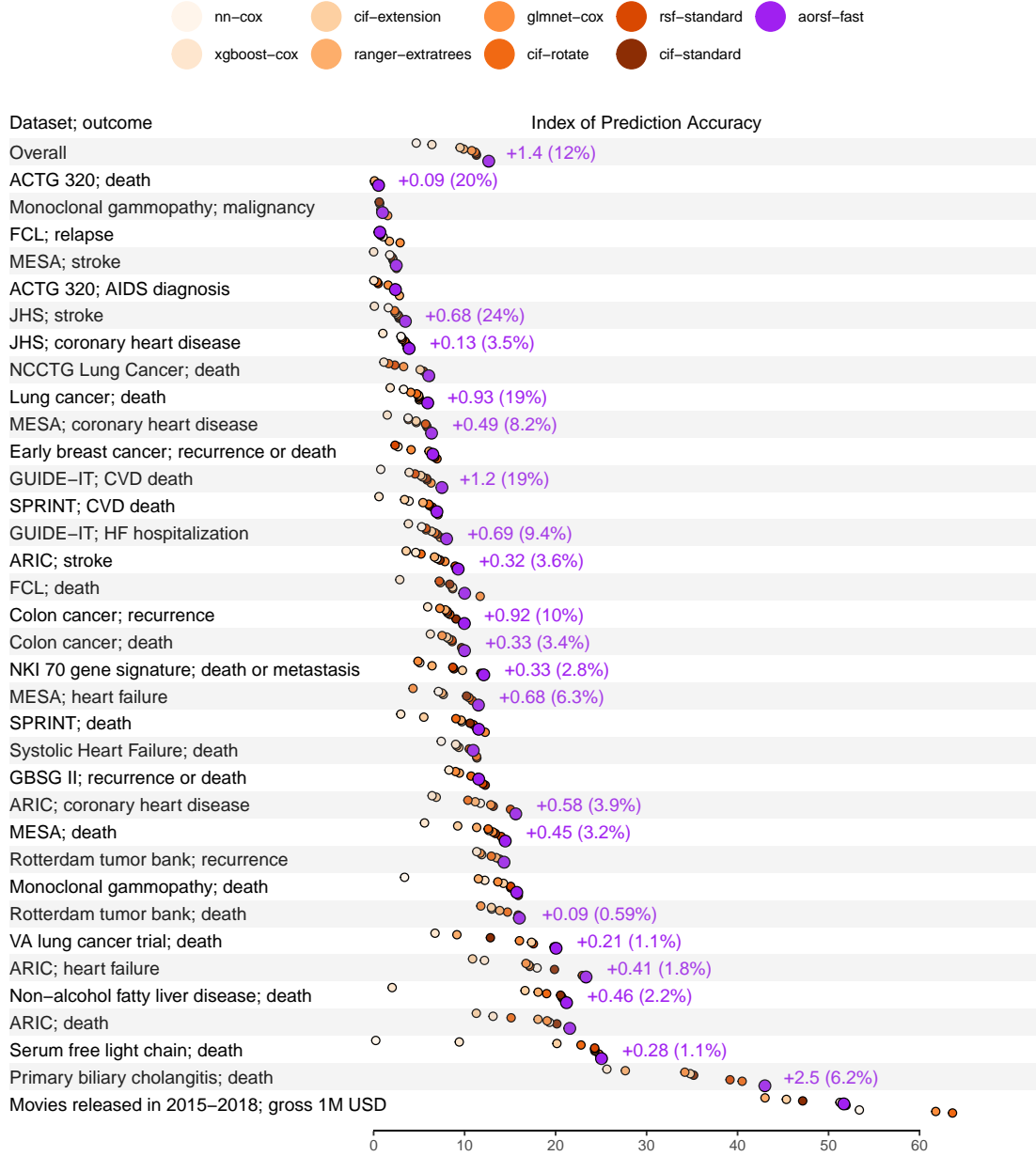


Figure 1: Index of prediction accuracy for the accelerated oblique random survival forest and other learning algorithms across multiple risk prediction tasks. Text appears in tasks where the accelerated oblique random survival forest obtained the highest index of prediction accuracy, showing the absolute and percent improvement over the second best learner. As predicted survival probabilities are not a standard output from xgboost-aft, it is not included in this figure. Also, since this figure is intended to compare aorsf-fast to learners that are not oblique random survival forests, aorsf-cph, aorsf-net, aorsf-random, and obliqueRSF-net are not included.

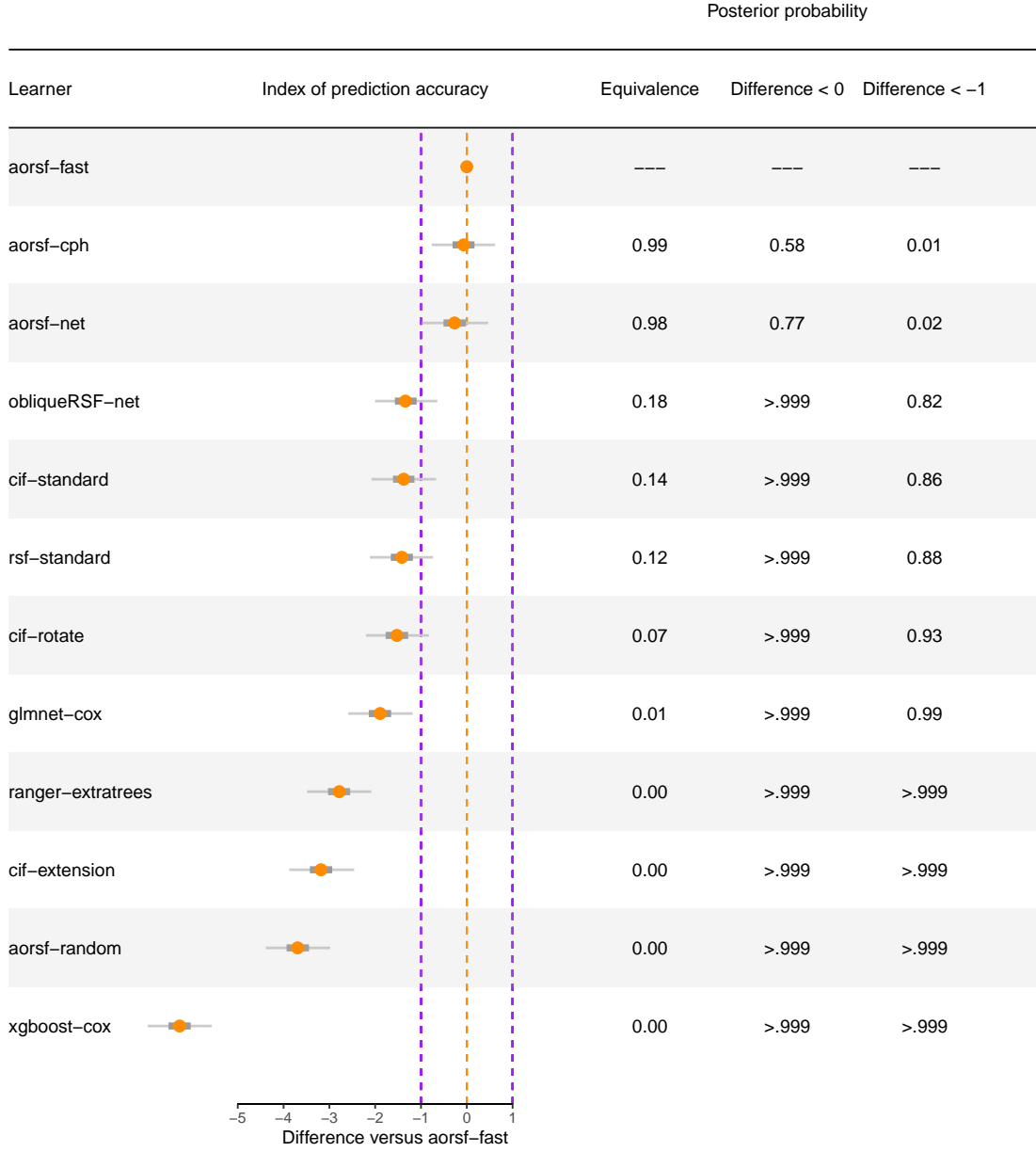


Figure 2: Expected differences in index of prediction accuracy between the accelerated oblique random survival forest and other learning algorithms. A region of practical equivalence is shown by purple dotted lines, and a boundary of non-zero difference is shown by an orange dotted line at the origin.

Time-dependent concordance statistic Compared to learners that were not oblique RSFs, `aorsf-fast` had the highest time-dependent C-statistic in 10 out of 35 risk prediction tasks, with an overall mean of 77.2 (Figure 3). Compared to the learner with the second highest mean C-statistic (`cif-standard`), `aorsf-fast`’s mean was 0.685 points higher, a relative increase of 0.896%. The posterior probability of `aorsf-fast` and `aorsf-cph` having practically equivalent expected time-dependent C-statistics was > 0.999 , and the posterior probability of `aorsf-fast` having a superior time-dependent C-statistic versus other learners ranged from 0.18 (versus `cif-standard`) to > 0.999 (versus several other learners; see Figure 4)

5.2 Benchmark of variable importance

The aim of this experiment is to evaluate negation VI and similar VI methods based on how well they can discriminate between relevant and irrelevant variables, where relevance is defined by having a relationship with the simulated outcome. We consider methods that are intrinsic to the oblique RF (*e.g.*, ANOVA VI), those that are intrinsic to the RF (*e.g.*, permutation VI), and those that are model-agnostic (*e.g.*, SHAP VI). VI methods with unavailable or still developing software were not included.⁵

5.2.1 Variable importance techniques

We compute permutation VI for axis based RSFs using the `randomForestSRC` package. We compute ANOVA VI, negation VI, and permutation VI for oblique RSFs using the `aorsf` package. For ANOVA VI, we applied a p-value threshold of 0.01, following the threshold recommended by Menze et al. [2011]. We compute SHAP VI for boosted tree models using the `xgboost` package [Chen et al., 2022], which incorporates the tree SHAP approach proposed by Lundberg et al. [2018].

5.2.2 Variable types

We considered five classes of predictor variables, with each class characterized by its variables’ relationship to a right-censored outcome on the log-hazard scale. Specifically,

- *irrelevant* variables had no relationship with the outcome.
- *main effect* variables had a linear relationship to the outcome on the log-hazard scale.
- *non-linear effect* variables had a non-linear relationship to the outcome. A normally distributed variable x was generated with a linear relationship to the outcome on the log-hazard scale, then $\tilde{x} = \sin(a \cdot \pi \cdot x)$ was retained for modeling. The constant a varied uniformly from 0.125 to 0.25.
- *combination effect* variables were formed by linear combinations of three other variables. While their combination was linearly related to the outcome on the log-hazard scale, each of the three variables contributing to the combination had no relation to the outcome.
- *interaction effect* variables were related to the outcome by multiplicative interaction with one other variable, which could have been a main effect, non-linear effect, or combination effect variable.

5.2.3 Simulated data

We initiated each set of simulated data with a random draw of size n from a p -dimensional multivariate normal distribution, yielding n observations of p predictors. Each of p predictor variables had a mean of zero, standard deviation of 1, and correlation with other predictor variables drawn at random between a lower and upper boundary. A time-to-event outcome with roughly 45% of observations censored was generated using

⁵Although the `party` package implements the approach to VI developed by Strobl et al. [2007], the developers of the `party` package note that the implementation of this approach for survival outcomes is “extremely slow and experimental” as of version 1.3.10. Therefore, it is not incorporated in the current simulation study.

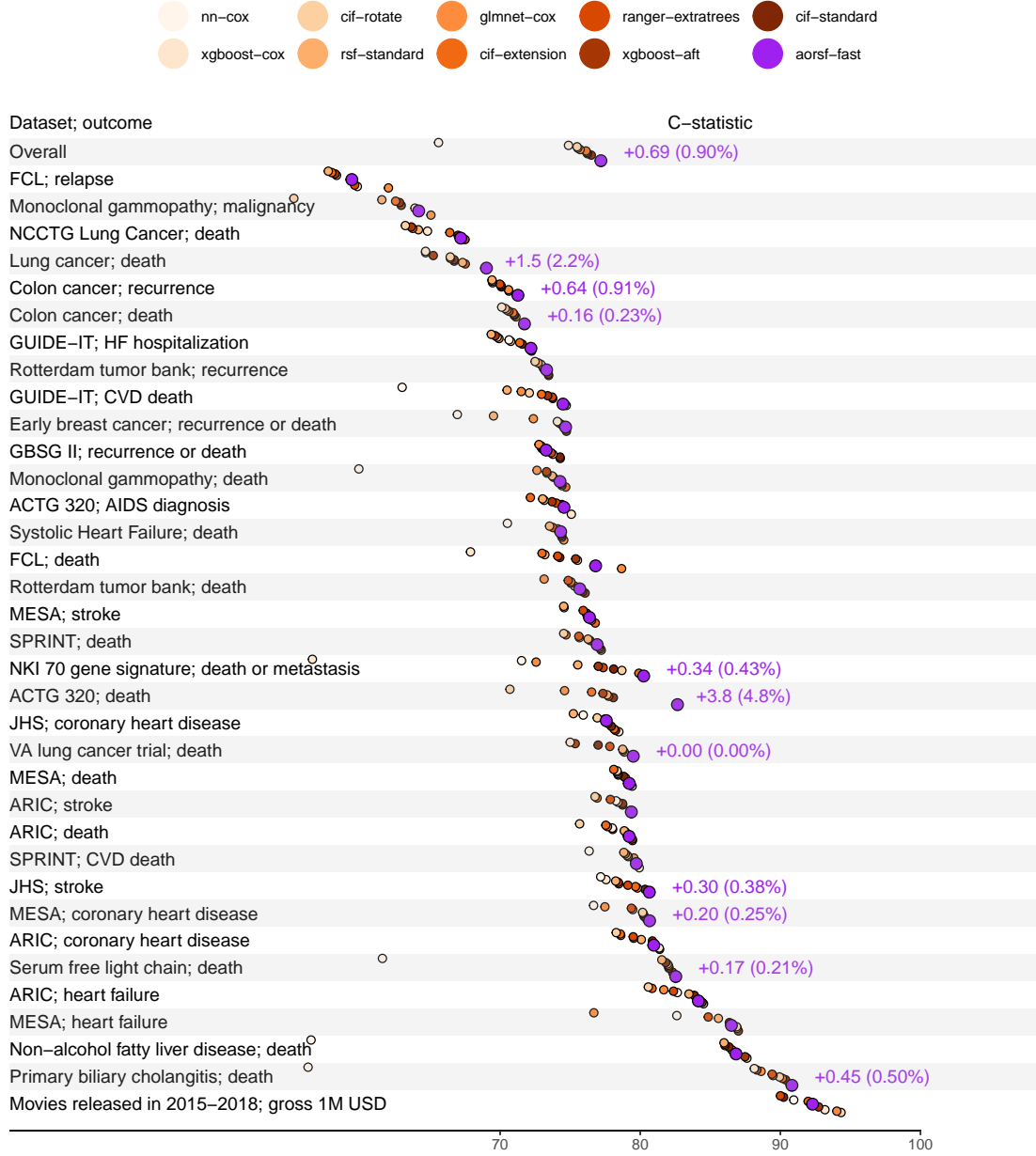


Figure 3: Time-dependent concordance statistic for the accelerated oblique random survival forest and other learning algorithms across multiple risk prediction tasks. Text appears in tasks where the accelerated oblique random survival forest obtained the highest concordance, showing the absolute and percent improvement over the second best learner. Since this figure is intended to compare aorsf-fast to learners that are not oblique random survival forests, aorsf-cph, aorsf-net, aorsf-random, and obliqueRSF-net are not included.

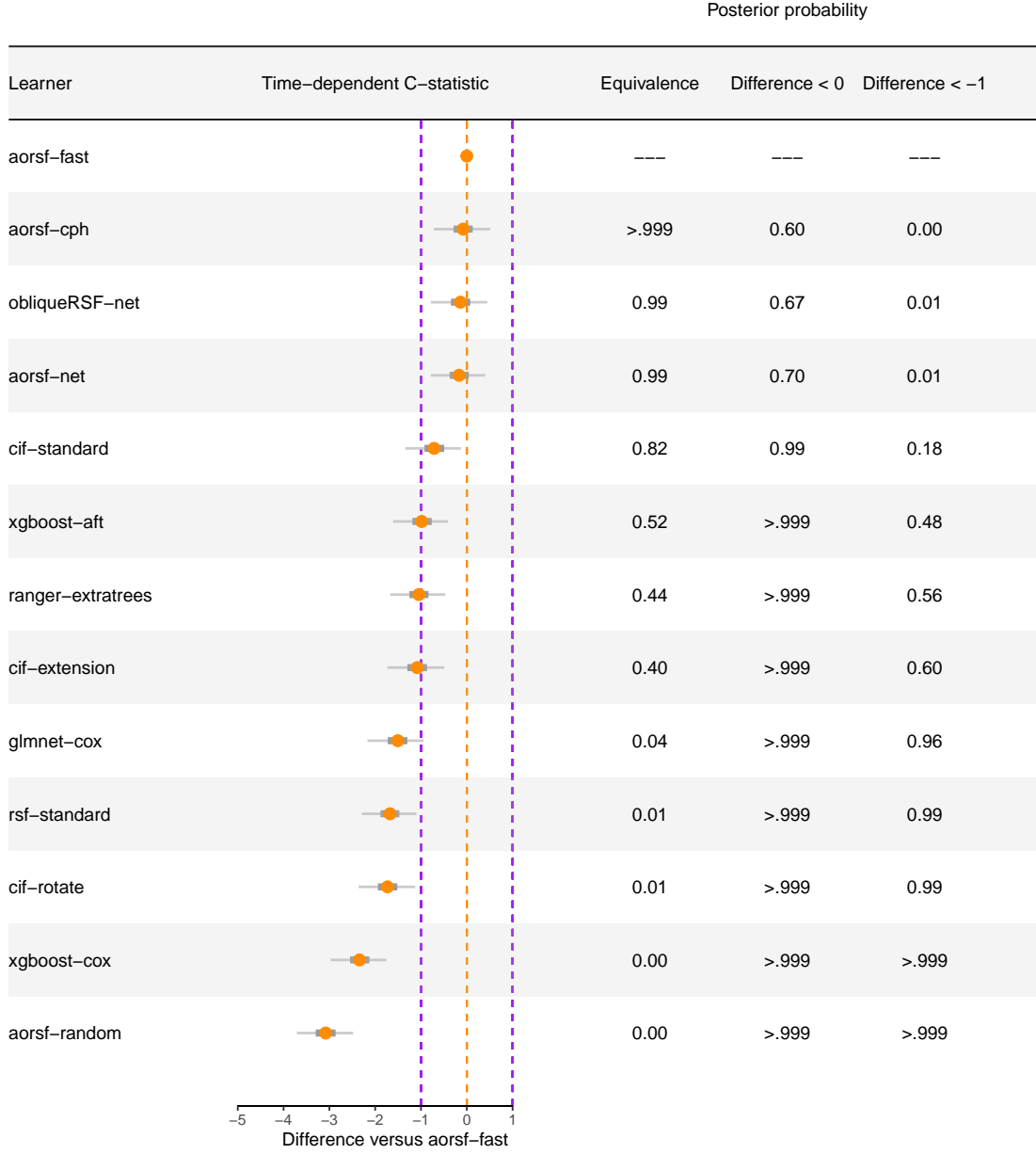


Figure 4: Expected differences in time-dependent concordance statistic between the accelerated oblique random survival forest and other learning algorithms. A region of practical equivalence is shown by purple dotted lines, and a boundary of non-zero difference is shown by an orange dotted line at the origin.

the `simSurv` package [Brilleman, 2018, Brilleman et al., 2020]. The full predictor matrix (*i.e.*, including interactions, non-linear mappings, and combinations) was used to generate the outcome. Interactions, non-linear mappings, and combinations were dropped from the predictor matrix after the outcome was generated so that VI techniques could be evaluated based on their ability to detect these effects.

5.2.4 Parameter specifications

Parameters that varied in the current simulation study included the number of observations (500, 1000, and 2500) and the absolute value of the maximum correlation between predictors (0.3, 0.15, and 0). Parameters that remain fixed throughout the study included the number of predictors in each class (15) and the effect size of each predictor (one standard deviation increase associated with a 64% increase in relative risk). Using this design for simulated data, the Heller explained relative risk (95% confidence interval) of our covariates was 88.4 (88.1, 88.7) [Heller, 2012] with 2,500 observations.

5.2.5 Evaluation of variable importance

We compared VI techniques based on their discrimination (*i.e.*, C-statistic) between relevant and irrelevant variables. Specifically, we generated a binary outcome for each predictor variable based on its relevance (*i.e.*, the binary outcome is 1 if the variable is relevant, 0 otherwise). Treating VI as if it were a ‘prediction’ for these binary outcomes yields a C-statistic which may be interpreted as the probability that the VI technique will rank a relevant variable higher than an irrelevant variable [Harrell et al., 1982].

5.2.6 Results

The three techniques that used ‘aorsf’ to estimate VI were ranked first (`aorsf-negate`; $C = 75.9$), second (`aorsf-anova`; $C = 73.9$), and third (`aorsf-permute`; $C = 73.2$) in overall mean C-statistic across all of the simulation scenarios, with `aorsf-negate` obtaining the highest C-statistic in 26 out of 36 VI tasks (Figure 5). Among the four relevant variable classes, `aorsf-negate` had the highest mean C-statistic for main effects, combination effects, and non-linear effects, with the greatest advantage of using `aorsf-negate` occurring among non-linear and combination variables. Full results from the experiment are provided in Table A.3. Computationally, ANOVA VI was faster than negation and permutation VI, with a median time of 2.88 seconds versus 20.4 and 21.8 seconds, respectively.

5.3 Benchmark of computational efficiency

The aim of this numeric experiment is to evaluate the computational efficiency of the accelerated oblique RSF compared to its predecessor (the oblique RSF from the `obliqueRSF` R package) and to several other machine learning algorithms.

5.3.1 Evaluation of computational efficiency

For each learner discussed in Section 5.1.1 and for each of the 35 risk prediction tasks analyzed in Section 5.1, we tracked the amount of time required to fit a prediction model (including time used to tune parameters) and compute predicted risk.

We performed additional benchmarks on the time required to fit 500 trees using `aorsf`, `randomForestSRC`, and `ranger`. The learners that represented these R packages were `aorsf-fast`, `rsf-standard`, and `rsf-extratrees`, respectively. To allow for controlled comparisons of computational efficiency with varying dimensions of training data, we used the same process to simulate data as described in Section 5.2.3, varying the number of observations from 100 to 10000 and the number of predictors from 10 to 1000. The minimum node size of trees in this experiment was dynamically set as the nearest integer to the number of observations in the training set divided by 10.

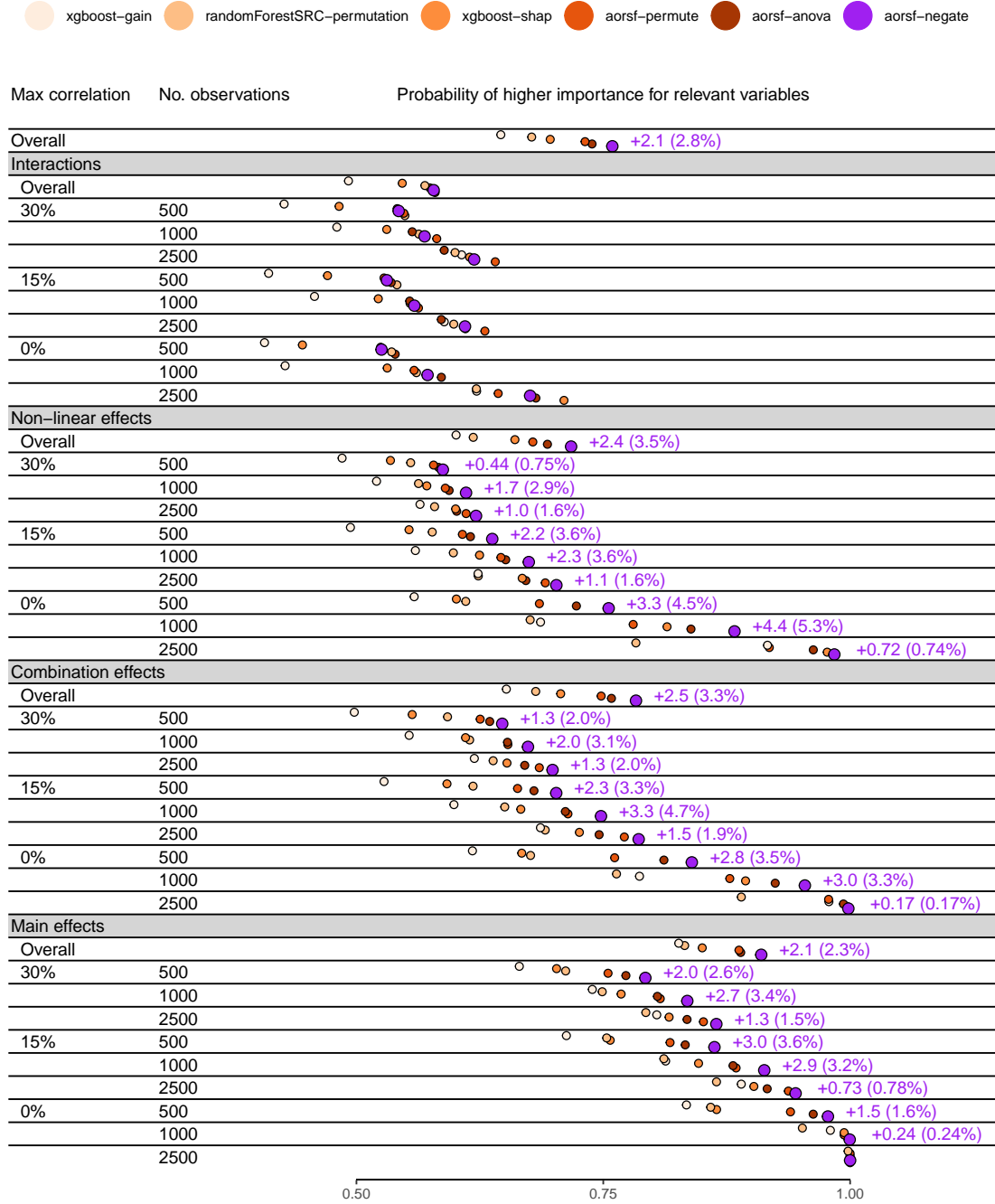


Figure 5: Concordance statistic for assigning higher importance to relevant versus irrelevant variables. Text appears in rows where negation importance obtained the highest concordance, showing absolute and percent improvement over the second best technique.

5.3.2 Results

In the analysis of 35 risk prediction tasks, `aorsf-fast` was the second fastest learner overall, with a median time to develop a risk prediction model and compute predictions about 385 milliseconds longer than `glmnet-cox` (Figure 6). Comparing median computing times, `aorsf-fast` was 379.8 times faster than its predecessor, `obliqueRSF-net`. In addition, `aorsf-fast` was 16.1, 1.55, and 2.66 faster than axis based forests grown using the `party`, `ranger`, and `randomForestSRC` packages, respectively.

In the analysis of time to fit 500 trees using simulated data, the `ranger` package exhibited the fastest computation times overall (Figure 7). `aorsf` was the second fastest when the number of predictors was 10 or 100, and `randomForestSRC` had similar computation time versus `aorsf` when 1000 predictors were present.

6 Discussion

In this paper, we have developed two contributions to the oblique RSF: (1) the accelerated oblique RSF (*i.e.*, `aorsf-fast`) and (2) negation VI. Our technique to accelerate the oblique RSF reduces the number of operations required to find linear combinations of inputs using a single iteration of Newton Raphson scoring, while our VI technique directly engages with coefficients in linear combinations of inputs to measure importance of individual variables. In numeric experiments, we found that that `aorsf-fast` is approximately 379.8 times faster than its predecessor, `obliqueRSF-net`, with a practically equivalent C-statistic. We also found that negation VI, a technique to estimate VI using the oblique RSF, detected non-linear, combination, and main effects more effectively than three standard methods to estimate VI: permutation, ANOVA, and SHAP VI. Overall, we found that estimating VI using negation instead of ANOVA increased the C-statistic for ranking a relevant variable higher than an irrelevant variable by 2.05, a relative increase of 2.78%.

6.1 Implications of our results

Accurate risk prediction models have the potential to improve healthcare by directing timely interventions to patients who are most likely to benefit. However, prediction models that cannot scale adequately to large databases or cannot be interpreted and explained will struggle to gain acceptance in clinical practice [Moss et al., 2022]. The current study advances the oblique RSF, an accurate risk prediction model, towards being accurate, scalable, and interpretable. The improved computational efficiency of the accelerated oblique RSF increases the feasibility of applying oblique RSFs in a wide range of prediction tasks. Faster model evaluation and re-fitting also improve diagnosis and resolution of model-based issues (*e.g.*, model calibration deteriorates over time). The introduction of negation VI also advances interpretability. VI is intrinsically linked to model fairness, as it can be used to identify when protected characteristics such as race, religion, and sexuality are inadvertently used (either directly or through correlates of these characteristics) by a prediction model. Since negation VI engages with the coefficients used in linear combinations of variables, a major component of oblique RSFs, it may be more capable of diagnosing unfairness in oblique RSFs compared to permutation importance and model-agnostic VI techniques.

6.2 Limitations and next steps

While the current study advances the oblique RSF towards being scalable and interpretable, there remain several limitations that can be targeted in future studies. The accelerated oblique RSF does not account for competing risks, and biased estimation of incidence may occur when competing risks are ignored. Thus, allowing the oblique RSF to account for competing risks is a high priority for future studies. In addition, the current study only considered data without missing values, only evaluated oblique RSFs that applied the log-rank statistic for node splitting, and only considered negation VI estimates based on Harrell’s C-statistic. Few studies have developed strategies to deal with missing data while growing oblique survival trees. Prior studies have found that log-rank tests can be mis-informative when survival curves cross [Li et al., 2015], and

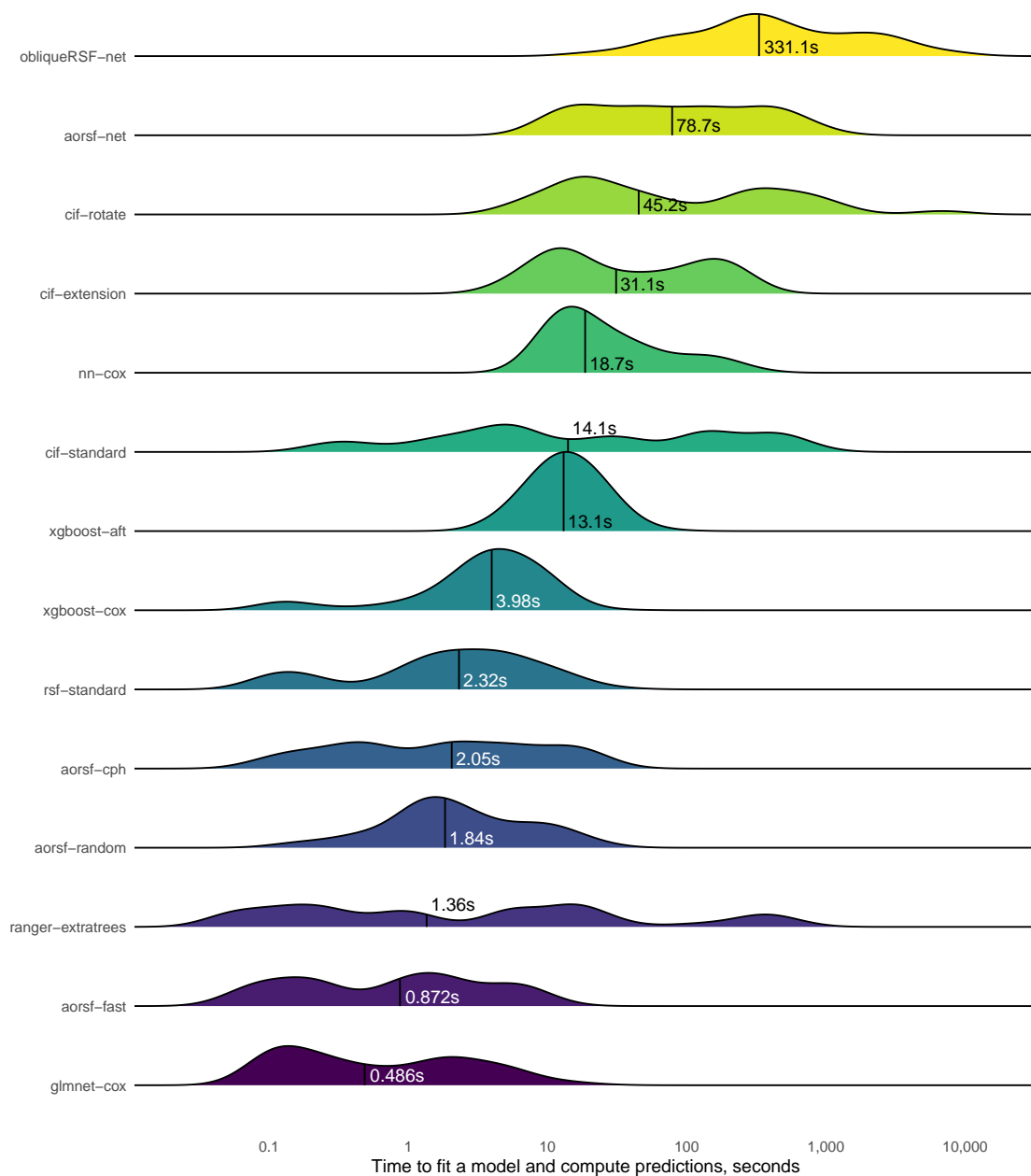


Figure 6: Distribution of time taken to fit a prediction model and compute predicted risk. The median time, in seconds, is printed and annotated for each learner by a vertical line.

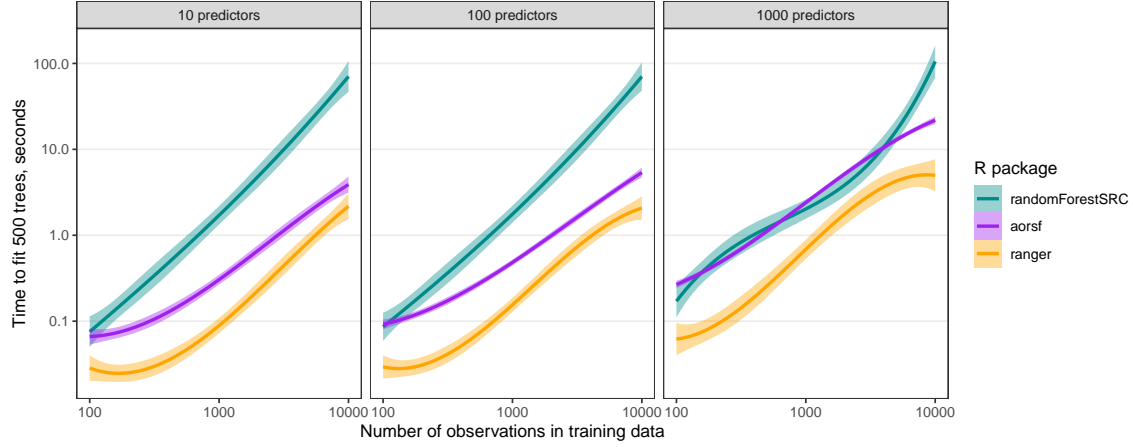


Figure 7: The expected time, in seconds, to fit an ensemble of 500 axis-based survival trees using the ranger or randomForestSRC package versus 500 oblique survival trees using the aorsf package. The ranger package is the most efficient overall, and aorsf appears to be relatively efficient in larger samples, particularly when 10 or 100 predictors are present in the training data. All three packages appear to scale linearly in computation time with the number of observations in the training data.

that Harrell’s C-statistic is dependent on the censoring distribution of the outcome [Uno et al., 2011]. Thus, a second item is to expand the range of options available to users of the aorsf package, enabling them to apply strategies for imputation of missing values and use a broad range of statistical criteria while growing oblique survival trees. Last, Cui et al. [2017] found that estimating an inverse-probability weighted hazard function at each non-leaf node of a survival tree allows the RSF to converge asymptotically to the true survival function when some variables contribute both to the risk of the event and the risk of censoring, a scenario that is very likely in the analysis of medical data. The accelerated oblique RSF could incorporate this splitting technique by using Newton Raphson scoring to fit a model for the censoring distribution after which a weighted model could be fit to the failure distribution. This final item has the highest priority, as Cui et al. [2017] showed it is a requisite condition for consistency of axis-based survival trees in fairly general settings.

Oblique RSFs have exceptional prediction accuracy and this study has shown how they can be fit with computational efficiency that rivals their axis-based counterparts. We have also introduced a general and flexible method to estimate VI with oblique RFs, and demonstrated its effectiveness specifically for the oblique RSF. Code used for the current study is available at <https://github.com/bcjaeger/aorsf-bench>, and the aorsf package is available at <https://github.com/bcjaeger/aorsf>.

Acknowledgements

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Appendix

Data sources

1. The “VA lung cancer trial” data [Kalbfleisch and Prentice, 2011] were obtained from the randomForestSRC R package [Ishwaran and Kogalur, 2019].
2. The “Colon cancer” data [Moertel et al., 1995] were obtained from the survival R package [Therneau, 2022a].
3. The “Primary biliary cholangitis” data [Therneau and Grambsch, 2000] were obtained from the aorsf R package [Jaeger, 2022].
4. The “Movies released in 2015-2018” data were obtained from the censored R package [Hvitfeldt and Frick].
5. The “GBSG II” data [Schumacher, 1994] were obtained from the TH.data R package [Hothorn, 2022].
6. The “Systolic Heart Failure” data [Hsieh et al., 2011] were obtained from the randomForestSRC R package [Ishwaran and Kogalur, 2019].
7. The “Serum free light chain” data [Dispenzieri et al., 2012, Kyle et al., 2006] were obtained from the survival R package [Therneau, 2022a].
8. The “Non-alcohol fatty liver disease” data [Allen et al., 2018] were obtained from the survival R package [Therneau, 2022a].
9. The “Rotterdam tumor bank” data [Royston and Altman, 2013] were obtained from the survival R package [Therneau, 2022a].
10. The “ACTG 320” data [Hosmer and Lemeshow, 2002] were obtained from the mlr3proba R package [Sonabend et al., 2021].
11. The “GUIDE-IT” data [Felker et al., 2017] were obtained from BioLINCC.
12. The “Early breast cancer” data [Desmedt et al., 2011, Hatzis et al., 2011, Ternès et al., 2017] were obtained from the biospear R package [Ternes et al., 2018].
13. The “SPRINT” data [SPRINT Research Group, 2015] were obtained from BioLINCC.
14. The “NKI 70 gene signature” data [Van De Vijver et al., 2002] were obtained from the OpenML R package [Casalicchio et al., 2017].
15. The “Lung cancer” data [Director’s Challenge Consortium for the Molecular Classification of Lung Adenocarcinoma, 2008] were obtained from the OpenML R package [Casalicchio et al., 2017].
16. The “NCCTG Lung Cancer” data [Loprinzi et al., 1994] were obtained from the survival R package [Therneau, 2022a].
17. The “FCL” data [Pintilie, 2006] were obtained from the randomForestSRC R package [Ishwaran and Kogalur, 2019].
18. The “Monoclonal gammopathy” data [Kyle et al., 2002] were obtained from the survival R package [Therneau, 2022a].
19. The “MESA” data [Bild et al., 2002] were obtained from BioLINCC.
20. The “ARIC” data [ARIC Investigators, 1989] were obtained from BioLINCC.
21. The “JHS” data [Taylor Jr et al., 2005] were obtained from BioLINCC.

A.1: Data sets used for numeric experiments

Label	N observations	N predictors	Outcome	N Events	% Censored
VA lung cancer trial	137	8	Death	128	6.57
Colon cancer	929	12	Recurrence	468	49.6
			Death	452	51.3
Primary biliary cholangitis	276	19	Death	111	59.8
Movies released in 2015-2018	551	46	Gross 1M USD	522	5.26
GBSG II	686	10	Recurrence Or Death	299	56.4
Systolic Heart Failure	2,231	41	Death	726	67.5
Serum free light chain	7,874	10	Death	2,169	72.5
Non-alcohol fatty liver disease	17,549	24	Death	1,364	92.2
			Recurrence	1,518	49.1
Rotterdam tumor bank	2,982	11	Death	1,272	57.3
			AIDS Diagnosis	96	91.7
ACTG 320	1,151	12	Death	26	97.7
			Cardiovascular Death	110	87.7
GUIDE-IT	894	59	Hf Hospitalization	288	67.8
			Recurrence Or Death	134	78.2
Early breast cancer	614	1,692	Cardiovascular Death	521	94.4
			Death	1,644	82.4
SPRINT	9,361	174	Death Or Metastasis	48	66.7
NKI 70 gene signature	144	77	Death	236	46.6
Lung cancer	442	24	Death	165	27.6
NCCTG Lung Cancer	228	9	Death	76	86.0
FCL	541	7	Relapse	272	49.7
			Death	963	30.4

Monoclonal gammopathy	1,384	8	Malignancy	115	91.7
MESA	6,783	48	Heart Failure	339	95.0
			Coronary Heart Disease	439	93.5
			Stroke	292	95.7
			Death	1,297	80.9
			Heart Failure	2,981	78.1
ARIC	13,623	41	Coronary Heart Disease	2,282	83.2
			Stroke	1,323	90.3
			Death	6,662	51.1
			Stroke	152	95.8
JHS	3,501	80	Coronary Heart Disease	190	94.6

A.2: Index of prediction accuracy, time-dependent concordance statistic, and computational time required to fit and compute predictions for several learning algorithms across 35 risk prediction tasks.

	Performance metric (SD)		Computation time, seconds	
	Scaled Brier	C-Statistic	Model fitting	Risk prediction
Overall				
aorsf-fast	0.126 (0.110)	0.772 (0.071)	0.620	0.145
aorsf-cph	0.126 (0.109)	0.771 (0.070)	1.912	0.148
aorsf-net	0.124 (0.112)	0.771 (0.071)	78.512	0.143
obliqueRSF-net	0.113 (0.085)	0.771 (0.071)	300.393	18.803
cif-standard	0.113 (0.098)	0.765 (0.071)	4.293	6.044
rsf-standard	0.113 (0.114)	0.755 (0.075)	2.099	0.199
cif-rotate	0.112 (0.124)	0.755 (0.081)	37.695	8.348
glmnet-cox	0.108 (0.119)	0.757 (0.077)	0.473	0.003
ranger-extratrees	0.099 (0.085)	0.762 (0.067)	0.878	1.156
cif-extension	0.095 (0.092)	0.761 (0.072)	22.321	6.949
aorsf-random	0.090 (0.081)	0.741 (0.063)	1.790	0.141
xgboost-cox	0.064 (0.100)	0.749 (0.094)	3.973	0.004
nn-cox	0.047 (0.108)	0.656 (0.138)	15.352	2.060
xgboost-aft	—	0.762 (0.076)	13.060	0.007
ACTG 320; AIDS diagnosis, $n = 1151$, $p = 12$				
aorsf-random	0.028 (0.022)	0.748 (0.038)	0.382	0.034
ranger-extratrees	0.028 (0.017)	0.740 (0.036)	0.064	0.146
obliqueRSF-net	0.027 (0.022)	0.746 (0.038)	25.980	15.236
aorsf-cph	0.025 (0.029)	0.751 (0.042)	0.424	0.033
cif-standard	0.024 (0.031)	0.744 (0.040)	1.771	4.657
aorsf-fast	0.024 (0.029)	0.746 (0.043)	0.144	0.032
cif-extension	0.023 (0.015)	0.722 (0.038)	9.848	4.083
aorsf-net	0.019 (0.034)	0.745 (0.042)	17.078	0.034
glmnet-cox	0.016 (0.030)	0.746 (0.037)	0.179	0.003
rsf-standard	0.005 (0.041)	0.730 (0.042)	0.192	0.061
cif-rotate	0.004 (0.040)	0.731 (0.038)	16.095	3.888
xgboost-cox	0.000 (0.044)	0.751 (0.033)	3.704	0.003
nn-cox	-0.001 (0.008)	0.549 (0.125)	11.075	0.668
xgboost-aft	—	0.737 (0.035)	10.015	0.006
ACTG 320; death, $n = 1151$, $p = 12$				
obliqueRSF-net	0.007 (0.011)	0.823 (0.052)	8.869	11.004
aorsf-cph	0.007 (0.018)	0.821 (0.060)	0.340	0.018
aorsf-fast	0.005 (0.019)	0.827 (0.056)	0.086	0.018
aorsf-random	0.005 (0.014)	0.788 (0.074)	0.221	0.022
cif-extension	0.001 (0.020)	0.765 (0.066)	8.436	3.493
ranger-extratrees	0.001 (0.019)	0.777 (0.069)	0.044	0.138
xgboost-cox	-0.004 (0.004)	0.500 (0.000)	0.119	0.002
nn-cox	-0.004 (0.004)	0.508 (0.119)	10.867	0.518
cif-standard	-0.005 (0.025)	0.781 (0.062)	1.726	4.648
aorsf-net	-0.005 (0.032)	0.806 (0.067)	13.309	0.022
rsf-standard	-0.031 (0.051)	0.776 (0.073)	0.090	0.035
cif-rotate	-0.037 (0.049)	0.707 (0.090)	13.920	3.444

A.2: Index of prediction accuracy, time-dependent concordance statistic, and computational time required to fit and compute predictions for several learning algorithms across 35 risk prediction tasks. (*continued*)

	Scaled Brier	C-Statistic	Model fitting	Risk prediction
glmnet-cox	-0.065 (0.095)	0.746 (0.098)	0.281	0.002
xgboost-aft	—	0.773 (0.071)	9.190	0.005
<i>ARIC; coronary heart disease, n = 13623, p = 41</i>				
aorsf-fast	0.156 (0.007)	0.810 (0.007)	5.048	1.287
aorsf-cph	0.153 (0.006)	0.809 (0.007)	14.564	1.300
aorsf-net	0.152 (0.006)	0.809 (0.007)	480.098	1.437
rsf-standard	0.150 (0.007)	0.801 (0.007)	9.407	0.991
obliqueRSF-net	0.143 (0.005)	0.811 (0.008)	2830.398	356.908
cif-standard	0.132 (0.005)	0.809 (0.007)	71.835	371.102
glmnet-cox	0.129 (0.011)	0.795 (0.008)	1.473	0.011
nn-cox	0.117 (0.029)	0.786 (0.046)	51.356	84.715
ranger-extratrees	0.112 (0.005)	0.795 (0.009)	293.864	61.837
cif-rotate	0.104 (0.004)	0.783 (0.009)	569.289	67.436
aorsf-random	0.097 (0.005)	0.772 (0.008)	10.996	1.259
cif-extension	0.069 (0.002)	0.786 (0.009)	169.206	56.091
xgboost-cox	0.064 (0.017)	0.814 (0.006)	8.544	0.015
xgboost-aft	—	0.814 (0.006)	23.781	0.013
<i>ARIC; death, n = 13623, p = 41</i>				
aorsf-net	0.217 (0.006)	0.792 (0.004)	902.210	2.343
rsf-standard	0.216 (0.006)	0.789 (0.004)	14.727	1.274
aorsf-fast	0.216 (0.007)	0.792 (0.004)	7.268	2.936
aorsf-cph	0.215 (0.006)	0.792 (0.004)	22.420	3.070
obliqueRSF-net	0.207 (0.005)	0.791 (0.004)	7329.229	331.164
cif-standard	0.201 (0.004)	0.790 (0.004)	72.253	384.744
nn-cox	0.193 (0.011)	0.780 (0.005)	103.296	92.992
glmnet-cox	0.191 (0.015)	0.777 (0.007)	2.256	0.011
ranger-extratrees	0.181 (0.004)	0.780 (0.005)	332.296	70.372
cif-rotate	0.151 (0.007)	0.757 (0.006)	589.958	66.749
xgboost-cox	0.131 (0.012)	0.794 (0.004)	11.220	0.015
aorsf-random	0.130 (0.005)	0.733 (0.005)	19.416	2.051
cif-extension	0.113 (0.002)	0.775 (0.005)	180.483	54.092
xgboost-aft	—	0.794 (0.004)	28.692	0.014
<i>ARIC; heart failure, n = 13623, p = 41</i>				
aorsf-fast	0.233 (0.006)	0.841 (0.005)	5.731	1.497
rsf-standard	0.229 (0.006)	0.835 (0.005)	11.238	1.079
aorsf-cph	0.229 (0.006)	0.841 (0.005)	16.768	1.549
aorsf-net	0.228 (0.006)	0.841 (0.005)	597.572	1.659
obliqueRSF-net	0.212 (0.005)	0.841 (0.005)	3737.617	320.616
cif-standard	0.199 (0.005)	0.839 (0.005)	71.854	391.419
nn-cox	0.180 (0.017)	0.826 (0.007)	56.056	96.125
cif-rotate	0.172 (0.006)	0.806 (0.007)	589.494	69.448
ranger-extratrees	0.170 (0.004)	0.824 (0.005)	387.176	77.513
glmnet-cox	0.167 (0.044)	0.817 (0.018)	2.141	0.011
aorsf-random	0.139 (0.005)	0.789 (0.006)	12.952	1.501
xgboost-cox	0.122 (0.017)	0.845 (0.005)	10.926	0.015

A.2: Index of prediction accuracy, time-dependent concordance statistic, and computational time required to fit and compute predictions for several learning algorithms across 35 risk prediction tasks. (*continued*)

	Scaled Brier	C-Statistic	Model fitting	Risk prediction
cif-extension	0.109 (0.003)	0.808 (0.006)	176.907	57.216
xgboost-aft	—	0.844 (0.005)	24.549	0.013
<i>ARIC; stroke, n = 13623, p = 41</i>				
aorsf-fast	0.093 (0.004)	0.794 (0.007)	4.549	1.078
aorsf-net	0.090 (0.004)	0.792 (0.007)	349.547	1.189
rsf-standard	0.090 (0.006)	0.784 (0.006)	8.991	0.984
aorsf-cph	0.090 (0.004)	0.792 (0.007)	12.609	1.077
obliqueRSF-net	0.082 (0.003)	0.791 (0.007)	1869.359	387.014
glmnet-cox	0.078 (0.004)	0.787 (0.007)	1.673	0.011
cif-standard	0.073 (0.003)	0.787 (0.007)	73.989	364.863
nn-cox	0.070 (0.012)	0.783 (0.010)	32.099	82.720
ranger-extratrees	0.067 (0.003)	0.779 (0.008)	251.130	50.542
aorsf-random	0.059 (0.004)	0.750 (0.008)	9.179	1.091
cif-rotate	0.052 (0.003)	0.768 (0.009)	586.962	68.410
xgboost-cox	0.046 (0.014)	0.794 (0.006)	7.316	0.015
cif-extension	0.036 (0.002)	0.769 (0.009)	169.866	53.493
xgboost-aft	—	0.793 (0.006)	20.363	0.013
<i>Colon cancer; death, n = 929, p = 12</i>				
aorsf-fast	0.100 (0.014)	0.717 (0.012)	0.237	0.049
aorsf-cph	0.099 (0.014)	0.717 (0.011)	0.636	0.049
cif-standard	0.097 (0.013)	0.710 (0.012)	0.745	3.685
aorsf-net	0.096 (0.014)	0.717 (0.012)	47.226	0.048
aorsf-random	0.096 (0.010)	0.716 (0.011)	1.717	0.044
obliqueRSF-net	0.089 (0.006)	0.717 (0.012)	252.190	16.726
cif-rotate	0.086 (0.017)	0.705 (0.014)	13.242	3.448
rsf-standard	0.086 (0.019)	0.704 (0.011)	1.901	0.150
ranger-extratrees	0.083 (0.007)	0.710 (0.012)	0.560	0.248
cif-extension	0.080 (0.006)	0.709 (0.011)	8.397	3.363
glmnet-cox	0.075 (0.016)	0.711 (0.019)	0.128	0.003
xgboost-cox	0.062 (0.013)	0.701 (0.013)	3.217	0.003
nn-cox	-0.003 (0.003)	0.508 (0.033)	12.769	1.106
xgboost-aft	—	0.706 (0.013)	10.290	0.006
<i>Colon cancer; recurrence, n = 929, p = 12</i>				
aorsf-fast	0.100 (0.017)	0.713 (0.016)	0.237	0.054
aorsf-cph	0.098 (0.017)	0.712 (0.015)	0.659	0.852
aorsf-net	0.095 (0.018)	0.713 (0.017)	47.589	0.049
cif-standard	0.091 (0.016)	0.701 (0.017)	0.717	3.703
obliqueRSF-net	0.087 (0.009)	0.711 (0.015)	249.790	16.701
aorsf-random	0.087 (0.012)	0.702 (0.015)	1.815	0.046
cif-rotate	0.084 (0.020)	0.694 (0.017)	13.154	3.977
cif-extension	0.081 (0.009)	0.706 (0.017)	8.446	4.066
rsf-standard	0.081 (0.020)	0.694 (0.015)	1.831	0.153
ranger-extratrees	0.079 (0.011)	0.700 (0.016)	0.636	0.254
glmnet-cox	0.073 (0.018)	0.706 (0.024)	0.125	0.003
xgboost-cox	0.059 (0.011)	0.695 (0.018)	3.136	0.003

A.2: Index of prediction accuracy, time-dependent concordance statistic, and computational time required to fit and compute predictions for several learning algorithms across 35 risk prediction tasks. (*continued*)

	Scaled Brier	C-Statistic	Model fitting	Risk prediction
nn-cox	-0.005 (0.006)	0.531 (0.054)	13.107	0.969
xgboost-aft	—	0.701 (0.019)	11.227	0.006
<i>Early breast cancer; recurrence or death, n = 614, p = 1692</i>				
obliqueRSF-net	0.072 (0.023)	0.751 (0.029)	1901.883	14.017
cif-rotate	0.070 (0.018)	0.747 (0.027)	6452.563	360.067
cif-standard	0.067 (0.019)	0.747 (0.030)	9.660	4.247
aorsf-cph	0.067 (0.029)	0.747 (0.026)	1.107	0.175
aorsf-fast	0.065 (0.028)	0.747 (0.027)	0.712	0.178
cif-extension	0.064 (0.016)	0.746 (0.028)	44.047	6.554
ranger-extratrees	0.061 (0.022)	0.742 (0.031)	0.228	0.169
glmnet-cox	0.041 (0.032)	0.724 (0.036)	5.780	0.006
aorsf-random	0.027 (0.015)	0.696 (0.038)	1.924	0.167
xgboost-cox	0.027 (0.034)	0.741 (0.030)	2.258	0.007
rsf-standard	0.024 (0.037)	0.695 (0.033)	0.366	0.743
aorsf-net	0.012 (0.062)	0.740 (0.025)	448.941	0.168
nn-cox	-0.006 (0.043)	0.669 (0.075)	19.374	1.808
xgboost-aft	—	0.744 (0.027)	9.708	0.010
<i>FCL; death, n = 541, p = 7</i>				
glmnet-cox	0.117 (0.028)	0.787 (0.037)	0.098	0.002
aorsf-cph	0.100 (0.039)	0.769 (0.033)	0.159	0.018
aorsf-fast	0.100 (0.037)	0.768 (0.032)	0.078	0.018
aorsf-net	0.097 (0.040)	0.760 (0.034)	12.491	0.018
obliqueRSF-net	0.089 (0.027)	0.758 (0.036)	97.158	5.447
cif-rotate	0.087 (0.048)	0.755 (0.027)	6.083	1.964
cif-extension	0.087 (0.036)	0.730 (0.034)	5.256	2.514
aorsf-random	0.087 (0.029)	0.757 (0.032)	1.061	0.018
cif-standard	0.084 (0.038)	0.743 (0.036)	0.347	1.132
ranger-extratrees	0.073 (0.016)	0.741 (0.037)	0.043	0.078
rsf-standard	0.072 (0.048)	0.732 (0.034)	0.156	0.040
xgboost-cox	0.029 (0.050)	0.679 (0.121)	0.266	0.002
nn-cox	-0.004 (0.014)	0.549 (0.110)	11.152	0.406
xgboost-aft	—	0.754 (0.038)	6.738	0.005
<i>FCL; relapse, n = 541, p = 7</i>				
glmnet-cox	0.029 (0.017)	0.620 (0.024)	0.100	0.002
ranger-extratrees	0.017 (0.016)	0.596 (0.025)	0.041	0.081
obliqueRSF-net	0.013 (0.016)	0.593 (0.024)	217.838	6.232
aorsf-random	0.013 (0.018)	0.595 (0.024)	1.166	0.020
xgboost-cox	0.010 (0.016)	0.598 (0.031)	1.331	0.002
cif-standard	0.008 (0.021)	0.594 (0.023)	0.315	1.056
aorsf-cph	0.007 (0.020)	0.595 (0.026)	0.256	0.022
aorsf-fast	0.007 (0.020)	0.594 (0.026)	0.114	0.021
aorsf-net	0.006 (0.020)	0.592 (0.026)	18.652	0.022
cif-extension	-0.005 (0.023)	0.580 (0.028)	6.164	2.309
nn-cox	-0.008 (0.023)	0.526 (0.055)	12.031	0.466
cif-rotate	-0.012 (0.025)	0.583 (0.030)	7.411	2.486

A.2: Index of prediction accuracy, time-dependent concordance statistic, and computational time required to fit and compute predictions for several learning algorithms across 35 risk prediction tasks. (*continued*)

	Scaled Brier	C-Statistic	Model fitting	Risk prediction
rsf-standard	-0.026 (0.032)	0.577 (0.024)	1.081	0.088
xgboost-aft	—	0.582 (0.034)	6.223	0.005
<i>GBSG II; recurrence or death, n = 686, p = 10</i>				
obliqueRSF-net	0.124 (0.016)	0.746 (0.017)	286.149	6.874
cif-standard	0.123 (0.020)	0.743 (0.020)	0.507	2.289
rsf-standard	0.120 (0.023)	0.738 (0.019)	1.437	0.115
aorsf-net	0.120 (0.024)	0.738 (0.020)	35.967	0.038
aorsf-cph	0.120 (0.025)	0.736 (0.018)	0.401	0.037
aorsf-fast	0.115 (0.024)	0.733 (0.017)	0.161	0.037
cif-extension	0.114 (0.017)	0.743 (0.019)	7.560	2.919
cif-rotate	0.107 (0.023)	0.729 (0.017)	11.241	3.023
aorsf-random	0.104 (0.023)	0.724 (0.025)	1.496	0.035
ranger-extratrees	0.094 (0.018)	0.736 (0.025)	0.102	0.143
glmnet-cox	0.090 (0.019)	0.728 (0.021)	0.106	0.002
xgboost-cox	0.083 (0.017)	0.730 (0.020)	2.538	0.003
nn-cox	-0.007 (0.012)	0.509 (0.061)	11.658	0.746
xgboost-aft	—	0.729 (0.021)	10.625	0.006
<i>GUIDE-IT; CVD death, n = 894, p = 59</i>				
aorsf-fast	0.075 (0.018)	0.745 (0.027)	0.161	0.035
aorsf-net	0.074 (0.019)	0.743 (0.027)	25.934	0.038
aorsf-cph	0.071 (0.018)	0.741 (0.028)	0.375	0.035
glmnet-cox	0.063 (0.041)	0.715 (0.091)	0.502	0.002
obliqueRSF-net	0.063 (0.013)	0.741 (0.023)	221.806	11.417
cif-rotate	0.059 (0.016)	0.721 (0.025)	35.190	4.947
cif-standard	0.058 (0.014)	0.738 (0.022)	1.564	3.334
ranger-extratrees	0.054 (0.013)	0.737 (0.029)	0.118	0.176
cif-extension	0.052 (0.011)	0.730 (0.022)	13.507	5.567
rsf-standard	0.046 (0.023)	0.705 (0.025)	0.195	0.064
xgboost-cox	0.039 (0.051)	0.747 (0.020)	3.831	0.003
aorsf-random	0.033 (0.012)	0.695 (0.030)	0.427	0.037
nn-cox	0.008 (0.018)	0.630 (0.123)	12.211	0.596
xgboost-aft	—	0.734 (0.020)	11.525	0.006
<i>GUIDE-IT; HF hospitalization, n = 894, p = 59</i>				
aorsf-net	0.082 (0.017)	0.722 (0.023)	50.238	0.056
aorsf-cph	0.081 (0.018)	0.722 (0.023)	0.668	0.051
aorsf-fast	0.080 (0.020)	0.722 (0.025)	0.248	0.051
ranger-extratrees	0.073 (0.010)	0.722 (0.022)	0.137	0.198
obliqueRSF-net	0.073 (0.010)	0.721 (0.023)	389.465	9.365
cif-standard	0.070 (0.010)	0.716 (0.023)	1.468	3.315
cif-rotate	0.067 (0.019)	0.708 (0.029)	41.655	5.185
cif-extension	0.064 (0.009)	0.714 (0.022)	15.396	6.113
glmnet-cox	0.058 (0.020)	0.699 (0.025)	0.416	0.003
rsf-standard	0.058 (0.022)	0.694 (0.026)	1.515	0.121
nn-cox	0.053 (0.028)	0.706 (0.032)	12.912	0.599
aorsf-random	0.049 (0.010)	0.682 (0.023)	0.803	0.051

A.2: Index of prediction accuracy, time-dependent concordance statistic, and computational time required to fit and compute predictions for several learning algorithms across 35 risk prediction tasks. (*continued*)

	Scaled Brier	C-Statistic	Model fitting	Risk prediction
xgboost-cox	0.038 (0.017)	0.698 (0.027)	2.955	0.003
xgboost-aft	—	0.697 (0.025)	11.409	0.006
<i>JHS; coronary heart disease, n = 3501, p = 80</i>				
aorsf-cph	0.040 (0.008)	0.778 (0.015)	2.036	0.153
obliqueRSF-net	0.039 (0.007)	0.780 (0.015)	185.659	130.539
aorsf-fast	0.039 (0.007)	0.776 (0.014)	0.586	0.151
aorsf-net	0.039 (0.009)	0.777 (0.016)	76.845	0.149
cif-standard	0.038 (0.006)	0.779 (0.017)	10.065	30.887
cif-extension	0.036 (0.004)	0.781 (0.019)	54.976	20.544
ranger-extratrees	0.035 (0.005)	0.777 (0.017)	3.892	1.677
cif-rotate	0.034 (0.010)	0.769 (0.018)	187.949	22.910
glmnet-cox	0.031 (0.010)	0.774 (0.020)	1.723	0.004
rsf-standard	0.031 (0.011)	0.752 (0.016)	2.099	0.226
nn-cox	0.030 (0.012)	0.759 (0.021)	10.787	5.787
aorsf-random	0.027 (0.004)	0.763 (0.019)	1.213	0.152
xgboost-cox	0.010 (0.023)	0.785 (0.022)	3.676	0.005
xgboost-aft	—	0.782 (0.017)	13.342	0.006
<i>JHS; stroke, n = 3639, p = 80</i>				
aorsf-cph	0.036 (0.006)	0.806 (0.016)	2.012	0.149
aorsf-net	0.035 (0.008)	0.807 (0.016)	70.506	0.144
aorsf-fast	0.035 (0.007)	0.806 (0.017)	0.581	0.147
obliqueRSF-net	0.030 (0.006)	0.807 (0.016)	161.865	165.270
glmnet-cox	0.028 (0.008)	0.798 (0.017)	2.325	0.004
cif-standard	0.028 (0.005)	0.803 (0.016)	10.613	32.291
rsf-standard	0.027 (0.010)	0.782 (0.018)	1.210	0.190
cif-extension	0.025 (0.003)	0.797 (0.018)	55.652	21.758
ranger-extratrees	0.023 (0.005)	0.791 (0.016)	2.997	2.557
cif-rotate	0.023 (0.009)	0.785 (0.017)	188.792	24.524
aorsf-random	0.022 (0.004)	0.775 (0.018)	1.115	0.153
nn-cox	0.016 (0.027)	0.772 (0.021)	10.445	6.338
xgboost-cox	0.001 (0.030)	0.776 (0.021)	2.953	0.005
xgboost-aft	—	0.784 (0.018)	13.017	0.006
<i>Lung cancer; death, n = 442, p = 24</i>				
aorsf-cph	0.063 (0.032)	0.691 (0.019)	0.312	0.030
aorsf-net	0.061 (0.030)	0.685 (0.019)	30.349	0.031
aorsf-fast	0.059 (0.033)	0.690 (0.019)	0.124	0.029
obliqueRSF-net	0.058 (0.020)	0.678 (0.020)	294.626	2.961
cif-extension	0.050 (0.018)	0.667 (0.019)	9.104	3.317
rsf-standard	0.050 (0.035)	0.673 (0.023)	0.941	0.073
cif-standard	0.050 (0.023)	0.667 (0.022)	0.315	0.846
ranger-extratrees	0.049 (0.016)	0.675 (0.019)	0.037	0.065
cif-rotate	0.047 (0.026)	0.664 (0.021)	17.129	3.089
glmnet-cox	0.041 (0.024)	0.664 (0.034)	0.123	0.002
aorsf-random	0.040 (0.022)	0.651 (0.023)	0.507	0.026
nn-cox	0.033 (0.025)	0.647 (0.029)	12.472	0.368

A.2: Index of prediction accuracy, time-dependent concordance statistic, and computational time required to fit and compute predictions for several learning algorithms across 35 risk prediction tasks. (*continued*)

	Scaled Brier	C-Statistic	Model fitting	Risk prediction
xgboost-cox	0.018 (0.019)	0.647 (0.027)	1.541	0.002
xgboost-aft	—	0.652 (0.026)	8.410	0.006
MESA; coronary heart disease, $n = 6785$, $p = 48$				
aorsf-fast	0.064 (0.009)	0.807 (0.010)	1.333	0.340
aorsf-net	0.062 (0.010)	0.805 (0.012)	169.694	0.379
obliqueRSF-net	0.062 (0.008)	0.808 (0.012)	488.662	265.171
aorsf-cph	0.060 (0.010)	0.801 (0.012)	4.935	0.352
cif-standard	0.059 (0.007)	0.803 (0.013)	24.446	98.767
cif-rotate	0.058 (0.009)	0.802 (0.013)	284.668	37.340
rsf-standard	0.057 (0.012)	0.795 (0.013)	3.426	1.202
ranger-extratrees	0.047 (0.004)	0.794 (0.011)	7.360	6.260
cif-extension	0.047 (0.003)	0.805 (0.013)	98.278	28.773
glmnet-cox	0.038 (0.017)	0.775 (0.016)	4.907	0.007
nn-cox	0.038 (0.017)	0.767 (0.021)	18.380	16.712
aorsf-random	0.031 (0.005)	0.735 (0.015)	2.853	0.387
xgboost-cox	0.015 (0.028)	0.802 (0.013)	4.763	0.009
xgboost-aft	—	0.802 (0.012)	18.127	0.009
MESA; death, $n = 6793$, $p = 48$				
aorsf-fast	0.145 (0.008)	0.792 (0.008)	1.868	0.487
aorsf-net	0.144 (0.008)	0.792 (0.009)	293.640	0.547
aorsf-cph	0.143 (0.008)	0.791 (0.009)	6.625	0.504
rsf-standard	0.140 (0.008)	0.784 (0.009)	4.818	0.502
obliqueRSF-net	0.139 (0.007)	0.791 (0.009)	1176.365	156.871
cif-standard	0.134 (0.007)	0.788 (0.009)	23.671	101.207
glmnet-cox	0.131 (0.026)	0.789 (0.012)	1.560	0.007
nn-cox	0.127 (0.020)	0.784 (0.016)	29.272	17.344
cif-rotate	0.126 (0.007)	0.783 (0.010)	319.953	37.290
ranger-extratrees	0.113 (0.004)	0.784 (0.008)	9.060	6.474
cif-extension	0.092 (0.003)	0.781 (0.009)	111.068	32.105
aorsf-random	0.068 (0.005)	0.725 (0.008)	5.755	0.541
xgboost-cox	0.056 (0.029)	0.794 (0.009)	8.021	0.009
xgboost-aft	—	0.793 (0.009)	20.287	0.010
MESA; heart failure, $n = 6785$, $p = 48$				
aorsf-fast	0.115 (0.011)	0.865 (0.012)	1.218	0.301
aorsf-net	0.114 (0.011)	0.863 (0.013)	147.441	0.338
aorsf-cph	0.109 (0.011)	0.858 (0.014)	4.549	0.315
rsf-standard	0.108 (0.012)	0.856 (0.012)	3.125	1.151
obliqueRSF-net	0.108 (0.008)	0.869 (0.011)	393.285	338.322
cif-rotate	0.105 (0.010)	0.869 (0.013)	260.914	38.177
cif-standard	0.102 (0.009)	0.864 (0.013)	24.344	101.534
cif-extension	0.077 (0.005)	0.864 (0.011)	94.628	30.361
ranger-extratrees	0.075 (0.005)	0.849 (0.015)	7.492	6.923
nn-cox	0.071 (0.024)	0.826 (0.016)	15.797	17.312
aorsf-random	0.064 (0.006)	0.795 (0.014)	2.377	0.355
glmnet-cox	0.043 (0.044)	0.767 (0.139)	3.777	0.006

A.2: Index of prediction accuracy, time-dependent concordance statistic, and computational time required to fit and compute predictions for several learning algorithms across 35 risk prediction tasks. (*continued*)

	Scaled Brier	C-Statistic	Model fitting	Risk prediction
xgboost-cox	-0.008 (0.019)	0.869 (0.011)	6.764	0.009
xgboost-aft	—	0.870 (0.012)	18.620	0.010
MESA; stroke, $n = 6783$, $p = 48$				
obliqueRSF-net	0.025 (0.004)	0.767 (0.016)	357.039	299.786
cif-rotate	0.025 (0.004)	0.764 (0.017)	268.402	37.899
cif-standard	0.025 (0.004)	0.762 (0.017)	23.450	98.166
aorsf-fast	0.025 (0.006)	0.764 (0.016)	1.186	0.296
aorsf-net	0.024 (0.006)	0.759 (0.017)	134.136	0.330
aorsf-cph	0.023 (0.005)	0.758 (0.016)	4.128	0.305
ranger-extratrees	0.022 (0.003)	0.759 (0.016)	7.610	6.738
glmnet-cox	0.021 (0.009)	0.765 (0.017)	3.876	0.007
cif-extension	0.021 (0.002)	0.768 (0.017)	94.913	29.730
rsf-standard	0.019 (0.009)	0.745 (0.018)	3.221	1.242
nn-cox	0.018 (0.007)	0.746 (0.028)	16.152	17.979
aorsf-random	0.013 (0.003)	0.714 (0.022)	2.275	0.330
xgboost-cox	0.000 (0.025)	0.762 (0.018)	4.347	0.008
xgboost-aft	—	0.764 (0.015)	16.257	0.009
Monoclonal gammopathy; death, $n = 1384$, $p = 8$				
cif-rotate	0.159 (0.019)	0.744 (0.014)	15.123	4.858
aorsf-cph	0.158 (0.016)	0.743 (0.011)	2.035	0.087
aorsf-fast	0.157 (0.016)	0.743 (0.011)	1.266	0.086
aorsf-net	0.155 (0.016)	0.741 (0.011)	84.984	0.088
obliqueRSF-net	0.155 (0.013)	0.743 (0.011)	232.467	12.924
cif-standard	0.151 (0.015)	0.738 (0.012)	1.537	5.829
rsf-standard	0.151 (0.017)	0.737 (0.011)	2.281	0.194
aorsf-random	0.146 (0.013)	0.735 (0.011)	2.457	0.082
cif-extension	0.143 (0.009)	0.747 (0.013)	11.222	4.744
glmnet-cox	0.137 (0.021)	0.726 (0.014)	0.136	0.003
xgboost-cox	0.122 (0.012)	0.733 (0.012)	3.919	0.003
ranger-extratrees	0.115 (0.005)	0.744 (0.012)	0.064	0.181
nn-cox	0.034 (0.057)	0.599 (0.112)	16.529	0.689
xgboost-aft	—	0.733 (0.013)	12.094	0.006
Monoclonal gammopathy; malignancy, $n = 1384$, $p = 8$				
glmnet-cox	0.015 (0.011)	0.651 (0.055)	0.118	0.003
aorsf-cph	0.011 (0.013)	0.644 (0.036)	0.568	0.040
aorsf-fast	0.010 (0.014)	0.642 (0.036)	0.193	0.040
ranger-extratrees	0.008 (0.006)	0.642 (0.030)	0.095	0.177
cif-extension	0.008 (0.010)	0.625 (0.028)	9.141	4.788
obliqueRSF-net	0.007 (0.010)	0.628 (0.033)	41.856	16.910
aorsf-net	0.007 (0.014)	0.641 (0.034)	22.236	0.041
aorsf-random	0.007 (0.013)	0.633 (0.033)	1.329	0.039
xgboost-cox	0.007 (0.017)	0.639 (0.039)	1.783	0.003
cif-standard	0.006 (0.011)	0.628 (0.033)	1.689	5.603
nn-cox	-0.003 (0.005)	0.510 (0.032)	11.229	0.597
rsf-standard	-0.009 (0.018)	0.616 (0.036)	0.824	0.073

A.2: Index of prediction accuracy, time-dependent concordance statistic, and computational time required to fit and compute predictions for several learning algorithms across 35 risk prediction tasks. (*continued*)

	Scaled Brier	C-Statistic	Model fitting	Risk prediction
cif-rotate	-0.024 (0.023)	0.553 (0.035)	12.975	4.603
xgboost-aft	—	0.629 (0.039)	10.748	0.006
<i>Movies released in 2015-2018; gross 1M USD, n = 551, p = 46</i>				
cif-rotate	0.636 (0.024)	0.943 (0.007)	19.879	3.571
glmnet-cox	0.618 (0.034)	0.940 (0.009)	0.204	0.002
nn-cox	0.534 (0.072)	0.910 (0.027)	17.266	0.663
aorsf-net	0.530 (0.028)	0.928 (0.010)	50.081	0.045
aorsf-cph	0.522 (0.024)	0.925 (0.011)	1.645	0.043
rsf-standard	0.519 (0.022)	0.922 (0.010)	1.631	0.106
aorsf-fast	0.517 (0.027)	0.923 (0.012)	1.090	0.042
xgboost-cox	0.512 (0.029)	0.932 (0.009)	13.972	0.004
cif-standard	0.472 (0.029)	0.902 (0.018)	0.453	1.910
cif-extension	0.454 (0.025)	0.920 (0.013)	8.854	3.924
ranger-extratrees	0.430 (0.025)	0.900 (0.019)	0.049	0.103
obliqueRSF-net	0.319 (0.022)	0.909 (0.017)	155.421	8.847
aorsf-random	0.300 (0.032)	0.849 (0.027)	1.681	0.039
xgboost-aft	—	0.927 (0.010)	33.545	0.008
<i>NCCTG Lung Cancer; death, n = 228, p = 9</i>				
aorsf-random	0.076 (0.030)	0.686 (0.026)	1.162	0.016
ranger-extratrees	0.062 (0.028)	0.675 (0.033)	0.023	0.031
aorsf-fast	0.061 (0.043)	0.672 (0.025)	0.066	0.016
obliqueRSF-net	0.058 (0.026)	0.676 (0.029)	106.084	1.468
aorsf-cph	0.058 (0.041)	0.670 (0.024)	0.153	0.016
cif-standard	0.055 (0.032)	0.670 (0.030)	0.130	0.252
cif-extension	0.051 (0.032)	0.664 (0.029)	3.799	1.502
aorsf-net	0.047 (0.040)	0.668 (0.026)	16.124	0.017
glmnet-cox	0.033 (0.031)	0.638 (0.059)	0.081	0.002
rsf-standard	0.023 (0.039)	0.642 (0.025)	0.087	0.037
cif-rotate	0.017 (0.041)	0.632 (0.032)	4.687	1.897
xgboost-cox	0.011 (0.023)	0.648 (0.032)	0.721	0.002
nn-cox	-0.051 (0.163)	0.512 (0.082)	10.516	0.209
xgboost-aft	—	0.637 (0.034)	6.271	0.005
<i>NKI 70 gene signature; death or metastasis, n = 144, p = 77</i>				
aorsf-net	0.142 (0.056)	0.804 (0.056)	10.521	0.014
aorsf-cph	0.124 (0.049)	0.802 (0.051)	0.076	0.013
aorsf-fast	0.121 (0.052)	0.802 (0.054)	0.049	0.014
cif-rotate	0.118 (0.059)	0.787 (0.049)	27.382	3.379
obliqueRSF-net	0.106 (0.051)	0.792 (0.061)	79.059	0.558
cif-extension	0.098 (0.055)	0.799 (0.061)	8.595	3.554
cif-standard	0.088 (0.051)	0.781 (0.065)	0.145	0.149
rsf-standard	0.087 (0.048)	0.755 (0.050)	0.067	0.025
ranger-extratrees	0.064 (0.044)	0.774 (0.054)	0.024	0.030
aorsf-random	0.053 (0.045)	0.741 (0.060)	0.136	0.014
nn-cox	0.051 (0.102)	0.715 (0.100)	11.692	0.121
glmnet-cox	0.049 (0.064)	0.726 (0.090)	0.265	0.002

A.2: Index of prediction accuracy, time-dependent concordance statistic, and computational time required to fit and compute predictions for several learning algorithms across 35 risk prediction tasks. (*continued*)

	Scaled Brier	C-Statistic	Model fitting	Risk prediction
xgboost-cox	-0.028 (0.029)	0.566 (0.093)	0.118	0.002
xgboost-aft	—	0.770 (0.056)	4.896	0.005
<i>Non-alcohol fatty liver disease; death, n = 17549, p = 24</i>				
aorsf-cph	0.213 (0.008)	0.868 (0.005)	16.251	2.106
aorsf-fast	0.212 (0.008)	0.868 (0.005)	4.412	2.014
aorsf-net	0.210 (0.008)	0.864 (0.006)	433.878	1.276
obliqueRSF-net	0.209 (0.008)	0.868 (0.006)	1428.698	1042.701
rsf-standard	0.207 (0.009)	0.860 (0.005)	10.516	1.205
glmnet-cox	0.207 (0.011)	0.860 (0.005)	1.345	0.012
cif-standard	0.205 (0.007)	0.863 (0.006)	67.597	624.624
cif-rotate	0.190 (0.008)	0.865 (0.005)	263.660	62.919
ranger-extratrees	0.181 (0.007)	0.860 (0.005)	39.632	80.768
cif-extension	0.166 (0.003)	0.866 (0.006)	125.288	53.237
aorsf-random	0.140 (0.006)	0.838 (0.007)	8.961	1.333
xgboost-cox	0.020 (0.015)	0.876 (0.005)	8.907	0.017
nn-cox	-0.002 (0.009)	0.565 (0.092)	20.937	106.131
xgboost-aft	—	0.875 (0.005)	27.908	0.015
<i>Primary biliary cholangitis; death, n = 276, p = 19</i>				
aorsf-fast	0.430 (0.032)	0.908 (0.021)	0.067	0.018
aorsf-cph	0.418 (0.034)	0.906 (0.021)	0.149	0.018
aorsf-net	0.413 (0.035)	0.905 (0.021)	14.298	0.020
cif-rotate	0.405 (0.040)	0.899 (0.022)	10.102	1.933
rsf-standard	0.392 (0.034)	0.895 (0.023)	0.113	0.038
obliqueRSF-net	0.369 (0.032)	0.907 (0.022)	111.656	1.787
aorsf-random	0.354 (0.031)	0.893 (0.020)	1.093	0.018
cif-standard	0.352 (0.034)	0.904 (0.025)	0.206	0.363
cif-extension	0.348 (0.033)	0.901 (0.023)	5.775	2.199
glmnet-cox	0.342 (0.044)	0.886 (0.028)	0.116	0.002
ranger-extratrees	0.277 (0.027)	0.894 (0.027)	0.029	0.038
xgboost-cox	0.256 (0.103)	0.881 (0.027)	4.960	0.003
nn-cox	-0.017 (0.018)	0.563 (0.124)	11.751	0.247
xgboost-aft	—	0.883 (0.024)	9.644	0.006
<i>Rotterdam tumor bank; death, n = 2982, p = 11</i>				
aorsf-net	0.166 (0.012)	0.762 (0.009)	145.278	0.191
obliqueRSF-net	0.163 (0.010)	0.761 (0.009)	439.210	37.683
aorsf-cph	0.163 (0.012)	0.759 (0.009)	3.388	0.195
aorsf-fast	0.160 (0.013)	0.757 (0.010)	1.684	0.194
cif-standard	0.159 (0.010)	0.759 (0.009)	4.629	22.523
rsf-standard	0.159 (0.014)	0.756 (0.009)	3.066	0.967
aorsf-random	0.153 (0.010)	0.752 (0.010)	3.554	0.181
cif-rotate	0.147 (0.011)	0.751 (0.011)	34.328	8.629
ranger-extratrees	0.139 (0.006)	0.749 (0.009)	3.498	2.152
xgboost-cox	0.130 (0.014)	0.753 (0.010)	4.068	0.004
cif-extension	0.129 (0.004)	0.751 (0.008)	22.740	8.982
glmnet-cox	0.118 (0.008)	0.731 (0.009)	0.278	0.004

A.2: Index of prediction accuracy, time-dependent concordance statistic, and computational time required to fit and compute predictions for several learning algorithms across 35 risk prediction tasks. (*continued*)

	Scaled Brier	C-Statistic	Model fitting	Risk prediction
nn-cox	-0.009 (0.042)	0.531 (0.050)	16.635	8.901
xgboost-aft	—	0.761 (0.009)	14.813	0.006
<i>Rotterdam tumor bank; recurrence, n = 2982, p = 11</i>				
obliqueRSF-net	0.148 (0.010)	0.737 (0.009)	520.813	39.774
aorsf-net	0.146 (0.011)	0.735 (0.009)	155.933	0.198
aorsf-cph	0.145 (0.012)	0.734 (0.008)	3.589	0.206
cif-standard	0.144 (0.011)	0.734 (0.009)	4.807	23.027
aorsf-fast	0.143 (0.011)	0.733 (0.009)	1.760	0.204
aorsf-random	0.140 (0.010)	0.730 (0.008)	3.803	0.185
rsf-standard	0.139 (0.012)	0.731 (0.008)	3.003	1.000
ranger-extratrees	0.135 (0.007)	0.734 (0.009)	3.564	2.555
cif-rotate	0.129 (0.010)	0.725 (0.009)	36.519	8.483
cif-extension	0.119 (0.006)	0.731 (0.008)	23.055	8.825
glmnet-cox	0.117 (0.008)	0.727 (0.008)	0.258	0.004
xgboost-cox	0.113 (0.008)	0.729 (0.009)	3.628	0.004
nn-cox	-0.007 (0.027)	0.511 (0.045)	17.722	7.825
xgboost-aft	—	0.735 (0.009)	14.317	0.006
<i>Serum free light chain; death, n = 7874, p = 10</i>				
aorsf-fast	0.250 (0.014)	0.825 (0.007)	2.956	0.566
aorsf-cph	0.250 (0.013)	0.825 (0.008)	7.220	0.575
aorsf-net	0.250 (0.012)	0.823 (0.008)	267.989	0.565
glmnet-cox	0.248 (0.012)	0.820 (0.007)	0.539	0.006
obliqueRSF-net	0.247 (0.011)	0.821 (0.008)	1113.284	151.041
ranger-extratrees	0.243 (0.009)	0.820 (0.007)	12.743	12.085
cif-standard	0.243 (0.011)	0.818 (0.008)	19.565	120.023
rsf-standard	0.243 (0.013)	0.815 (0.008)	5.399	0.585
aorsf-random	0.231 (0.012)	0.816 (0.008)	6.409	0.562
cif-rotate	0.228 (0.009)	0.819 (0.007)	65.379	21.402
cif-extension	0.201 (0.005)	0.820 (0.008)	41.381	20.976
xgboost-cox	0.094 (0.038)	0.824 (0.007)	5.927	0.008
nn-cox	0.002 (0.005)	0.616 (0.122)	26.319	26.895
xgboost-aft	—	0.823 (0.008)	18.504	0.009
<i>SPRINT; CVD death, n = 9361, p = 174</i>				
glmnet-cox	0.071 (0.011)	0.795 (0.011)	13.850	0.011
aorsf-net	0.070 (0.007)	0.796 (0.011)	330.532	0.613
aorsf-fast	0.070 (0.006)	0.797 (0.010)	2.561	0.604
aorsf-cph	0.069 (0.006)	0.797 (0.011)	8.283	0.620
obliqueRSF-net	0.068 (0.005)	0.798 (0.012)	1017.350	425.134
rsf-standard	0.065 (0.007)	0.788 (0.014)	3.894	1.314
cif-standard	0.061 (0.003)	0.798 (0.011)	50.694	181.616
cif-rotate	0.060 (0.005)	0.791 (0.012)	930.893	113.261
ranger-extratrees	0.054 (0.003)	0.791 (0.012)	7.314	7.943
nn-cox	0.039 (0.018)	0.764 (0.027)	20.414	24.555
cif-extension	0.034 (0.002)	0.789 (0.011)	122.779	33.342
aorsf-random	0.026 (0.002)	0.747 (0.016)	5.282	0.708

A.2: Index of prediction accuracy, time-dependent concordance statistic, and computational time required to fit and compute predictions for several learning algorithms across 35 risk prediction tasks. (*continued*)

	Scaled Brier	C-Statistic	Model fitting	Risk prediction
xgboost-cox	0.006 (0.017)	0.799 (0.011)	6.888	0.013
xgboost-aft	—	0.796 (0.012)	20.126	0.012
<i>SPRINT; death, n = 9361, p = 174</i>				
glmnet-cox	0.123 (0.012)	0.771 (0.009)	5.422	0.010
aorsf-cph	0.117 (0.008)	0.770 (0.008)	11.811	1.833
aorsf-fast	0.115 (0.009)	0.769 (0.008)	3.826	1.817
aorsf-net	0.113 (0.009)	0.769 (0.009)	587.892	0.933
obliqueRSF-net	0.112 (0.007)	0.767 (0.008)	2630.588	231.667
rsf-standard	0.110 (0.008)	0.763 (0.009)	6.407	0.663
cif-standard	0.106 (0.006)	0.764 (0.008)	49.842	190.804
nn-cox	0.097 (0.010)	0.757 (0.009)	34.063	33.278
ranger-extratrees	0.096 (0.005)	0.756 (0.009)	11.323	9.400
cif-rotate	0.090 (0.007)	0.745 (0.009)	1109.925	112.893
cif-extension	0.055 (0.002)	0.747 (0.009)	137.470	34.230
aorsf-random	0.052 (0.003)	0.720 (0.010)	8.360	0.992
xgboost-cox	0.030 (0.023)	0.772 (0.008)	9.057	0.014
xgboost-aft	—	0.772 (0.007)	23.335	0.013
<i>Systolic Heart Failure; death, n = 2231, p = 41</i>				
obliqueRSF-net	0.114 (0.012)	0.747 (0.012)	381.891	25.692
glmnet-cox	0.113 (0.013)	0.745 (0.012)	0.276	0.004
cif-rotate	0.113 (0.013)	0.741 (0.011)	71.714	10.523
aorsf-net	0.112 (0.013)	0.743 (0.012)	114.909	0.160
aorsf-cph	0.112 (0.013)	0.744 (0.012)	1.931	0.145
cif-standard	0.110 (0.011)	0.744 (0.011)	4.079	16.213
aorsf-fast	0.109 (0.015)	0.743 (0.012)	0.618	0.142
rsf-standard	0.105 (0.011)	0.735 (0.011)	2.510	0.252
cif-extension	0.094 (0.006)	0.744 (0.012)	29.918	10.109
ranger-extratrees	0.091 (0.008)	0.738 (0.012)	3.266	1.328
xgboost-cox	0.090 (0.009)	0.744 (0.010)	4.229	0.004
aorsf-random	0.080 (0.006)	0.731 (0.013)	2.025	0.143
nn-cox	0.074 (0.028)	0.705 (0.036)	18.945	5.181
xgboost-aft	—	0.741 (0.009)	13.087	0.006
<i>VA lung cancer trial; death, n = 137, p = 8</i>				
aorsf-net	0.201 (0.050)	0.797 (0.035)	9.901	0.012
aorsf-fast	0.200 (0.049)	0.795 (0.034)	0.047	0.012
cif-rotate	0.198 (0.065)	0.789 (0.036)	4.476	1.284
aorsf-cph	0.198 (0.052)	0.794 (0.035)	0.094	0.012
rsf-standard	0.176 (0.048)	0.787 (0.037)	0.078	0.025
cif-extension	0.174 (0.048)	0.795 (0.034)	3.676	1.195
glmnet-cox	0.160 (0.036)	0.788 (0.037)	0.087	0.002
aorsf-random	0.154 (0.044)	0.780 (0.037)	1.060	0.011
cif-standard	0.128 (0.040)	0.770 (0.037)	0.105	0.120
obliqueRSF-net	0.126 (0.034)	0.796 (0.029)	62.935	0.664
ranger-extratrees	0.092 (0.033)	0.778 (0.038)	0.020	0.026
xgboost-cox	0.067 (0.078)	0.750 (0.046)	1.408	0.002

A.2: Index of prediction accuracy, time-dependent concordance statistic, and computational time required to fit and compute predictions for several learning algorithms across 35 risk prediction tasks. (*continued*)

	Scaled Brier	C-Statistic	Model fitting	Risk prediction
xgboost-aft	—	0.754 (0.047)	5.530	0.005
nn-cox	-0.023 (0.033)	0.517 (0.114)	11.344	0.133

A.3: Discrimination of relevant versus irrelevant variables for several techniques to estimate variable importance.

Max correlation	No. observations	accelerated oblique RSF			xgboost		RSF
		Negation	ANOVA	Permutation	SHAP	Gain	Permutation
Overall	Overall	75.9	73.9	73.2	69.6	64.6	67.7
<i>Interactions</i>							
Overall	Overall	57.8	57.4	58.0	54.6	49.2	56.9
30	500	54.3	54.1	54.8	48.2	42.7	54.9
30	1,000	56.9	55.7	58.1	53.1	48.0	56.3
30	2,500	61.9	58.9	64.1	61.5	60.7	60.0
15	500	53.1	53.5	52.8	47.1	41.1	54.1
15	1,000	55.9	55.4	56.3	52.2	45.8	55.4
15	2,500	61.0	58.6	63.0	61.0	58.9	59.9
0	500	52.5	53.9	52.4	44.5	40.7	53.6
0	1,000	57.2	58.6	55.8	53.1	42.8	56.1
0	2,500	67.6	68.2	64.4	71.0	62.2	62.1
<i>Non-linear effects</i>							
Overall	Overall	71.7	69.3	67.9	66.1	60.1	61.8
30	500	58.8	58.3	57.8	53.4	48.5	55.5
30	1,000	61.1	59.4	59.0	57.1	52.0	56.3
30	2,500	62.1	60.2	61.1	60.0	56.4	57.9
15	500	63.8	61.5	60.7	55.3	49.4	57.7
15	1,000	67.5	65.1	64.6	62.5	56.0	59.8
15	2,500	70.2	67.2	69.1	66.8	62.3	62.3
0	500	75.5	72.3	68.5	60.1	55.8	61.1
0	1,000	88.3	83.9	78.0	81.5	68.6	67.6
0	2,500	98.4	96.3	91.8	97.7	91.6	78.3
<i>Combination effects</i>							
Overall	Overall	78.3	75.8	74.8	70.7	65.2	68.2
30	500	64.8	63.5	62.5	55.6	49.8	59.2
30	1,000	67.4	65.3	65.3	61.0	55.3	61.5
30	2,500	69.9	67.0	68.5	65.2	61.9	63.8
15	500	70.2	68.0	66.3	59.2	52.8	61.8
15	1,000	74.8	71.2	71.4	66.6	59.9	65.0
15	2,500	78.6	74.6	77.1	72.6	68.6	69.1
0	500	84.0	81.1	76.2	66.7	61.7	67.6
0	1,000	95.4	92.4	87.8	89.4	78.7	76.3

A.3: Discrimination of relevant versus irrelevant variables for several techniques to estimate variable importance. (*continued*)

Max correlation	No. observations	Negation	ANOVA	Permutation	SHAP	Gain	Permutation
0	2,500	99.8	99.3	97.8	99.7	97.9	89.0
<i>Main effects</i>							
Overall	Overall	91.0	88.9	88.7	85.0	82.6	83.2
30	500	79.3	77.3	75.5	70.3	66.5	71.2
30	1,000	83.5	80.5	80.8	76.8	73.9	74.9
30	2,500	86.5	83.5	85.1	81.7	80.4	79.3
15	500	86.3	83.3	81.8	75.7	71.3	75.3
15	1,000	91.3	88.1	88.5	84.6	81.3	81.1
15	2,500	94.5	91.6	93.7	90.2	89.0	86.5
0	500	97.8	96.3	94.0	86.5	83.4	85.9
0	1,000	100.0	99.7	99.4	99.4	98.0	95.2
0	2,500	100.0	100.0	100.0	100.0	100.0	99.8

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