Program: Computing and Data Science
Level: First **Term: Fall 2021/2022**
Course Code: **Course Title: Statistic and Probability 2**
Time Allowed:2 hrs. **Total points: 60**
Professor(s) name(s): Ibrahim Galal

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Question 1

[1] **A medical research worker intends to use the mean of a random sample of size 80 to estimate the mean blood pressure of women in their fifties. If , based on experience,he knows that σ=7.5 mm of mercury, what can he assert with probability 0.98 about the maximum error?**

[2] **The average amount of money μ for a hospital's accounts receivable must be estimated. The population variance σ^2 is known to be 100. The sample size neededto be 96% confident that the error of estimating the mean μ will not exceed 2 L.E.is**

a. 77 **b.** 96 **c.** 106 **d.** None of the above.

[4] **A 90% confidence interval estimate for the mean μ with n=30, x =79.1 s=16.8, assume that the population has a normal distribution, is:**

a. $72.83 < \mu < 85.37$ **b.** $73.89 < \mu < 84.31$
c. $73.92 < \mu < 84.28$ **d.** $70.65 < \mu < 87.5$

[5] **The width of a 100(1-α)% confidence interval for the mean μ decreases when**

a. α increases **b.** n decreases **c.** The S.D. increases **d.** None of theabov

[6] **Sixteen measurements of the reaction time of an individual to certain stimuli were recorded and gave mean of 0.296 seconds and standard deviation of 0.1 seconds. A 98% confidence interval estimate for the actual mean reaction time isgiven by**

a. (0.270,0.322) **b.** (0.231,0.361) **c.** (0.238,0.364) **d.** None of the above

Question 2

[1] **The yield of a chemical process is being studied. From previous experience yield is known tobe normally distributed and $\sigma = 3$. The past five days of plant operation have resulted in the following percent yields: 91.6, 88.75, 90.8, 89.95, and 91.3.**

a- Find a 95% confidence interval on the true mean yield.

b- Is there evidence that the mean yield is not 90%? Use $\alpha = 0.05$

C- What is the P-value for this test?

[2] An airline institutes a new system for waiting line to try to reduce the average waiting time. The mean waiting time with the old system was 6.1 minutes. A sample of 14 waiting lines is taken. The resulting sample mean is 5.423 and the standard deviation is 2.797. Test the null hypothesis of no change against an appropriate research hypothesis at $\alpha = 0.05$

One-Sample T: C1

Variable	N	Mean	StDev	SE Mean	95% Upper Bound	T	P
C1	14	......	....	0.748	6.747	......	0.191

a- $H_0: \mu$ .... vs $H_1 : \mu$ ....

b- T = ...

c- P-value=...........................

d- Decision ....

e- Find a 95% confidence interval on the true mean yield.

Question 3

[1] Two methods for producing gasoline from crude oil are investigated. The yields (%) of bothprocesses are assumed to be normally distributed. The following yield (%) data have been obtained from the pilot plant

Process	Sample size	Sample Mean	Sample variance
A	15	25.8	0.64
B	12	21.2	0.81

a- Find a 98% confidence interval for the true mean yield of process A.

b- Test at the 0.02 significant level, whether it is reasonable to assume that the two populationsof yields (%) of both processes A and B have equal variances.

c- Test at the 0.05 level of significance whether the difference between the true means yield of processes A and B is significant (state clearly H_0, H_a, decision and conclusion).

[2] The following are the average weekly losses of man-hours due to accidents in 9 industrialplants before and after a certain safety program were put into operation.

Before the safety program	45	73	46	124	33	57	83	34	17
After the safety program	44	65	44	119	35	56	80	31	11

Use the 0.05 level of significance to test whether the safety program is effective. State,clearly, H_0 and H_a , your decision and the conclusion.

Question 4

[1] **(a)** Complete the following ANOVA table. .05

(b) How many treatments were compared?

(c) How many observations were analyzed?

(d) At the $\alpha = 0.05$ level of significance, can one conclude that the treatmentshave different effects? Why?

Source	SS	d.f.	MS	F	P
Treatments	154.999	4	*	*	*
Error	*	*	*		
Total	200.4773	39			

[2] Answer (T/F) true or false:

a- The alternative hypothesis is stated in terms of a sample statistic.

b- a large P-value indicates strong evidence against H_o.

c- If there is sufficient evidence to reject H_o at α=0.10, then there is sufficient evidenceto reject it also at α=0.05.

d- If the population mean is known, there is no reason to run a hypothesis test on thepopulation mean .

e- The P-value is usually chosen before an experiment is conducted.

f- A well-planned test of significance should result in a statement either that H_o is true orthat it is false.

BEST WISHES

D.IBRAHIM GALAL

Program: general
Level: 3rd
Course Code: 02-24-00201
Time Allowed: 2 hrs.
Professor name: Dr Ibrahim Galal and Dr Ahmed Tayel

Final Exam
Term: Fall 2022
Course Title Probability and Statistics 2
Total points: 60

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- الامتحان يتكون من 6 أسئلة في 4 ورقات (7 صفحات).
- يرجى كتابة البيانات على جميع الأوراق.

Question 1 (8 pts):

The life time of an electronic device is known to be approximately Normal with standard deviation of **25** months.

(a) [3 pts] What sample size should be taken in order to be **90**% confident that the error in estimating the mean lifetime is less than **10** months?

(b) [2 pts] Given $\overline{X} = \mathbf{100}$ months, construct the **90**% confidence interval of the mean life time of the device, assuming the sample size of **20**?

(c) [3 pts] Test the hypothesis that the device can last -in average- for longer than **90** months? Use $\alpha = \mathbf{0,1}$

[Hint: $\phi(1.28) = 0.9$, $\phi(1.65) = 0.95$, $\phi(1.79) = 0.96$]

Question 2 (6 pts):

Suppose the IQ levels among individuals in two different cities are known to be normally distributed each with population standard deviations of **15**. A scientist wants to know if the mean IQ level between individuals in city A and city B are different, so she selects a random sample of **20** individuals from each city and records their IQ levels. Suppose she collects two simple random samples with the following information:

$\bar{X}_1$ (sample 1 from city A mean IQ) = $\mathbf{100.65}$, $\bar{X}_2$ (sample 2 from city B mean IQ) = $\mathbf{108.8}$

(a) [1 pt] Define the Null and alternative hypothesis?
(b) [3 pts] Calculate the p-value of the test statistic? [Hint: $\phi(1.718) = 0.957$]
(c) [2 pts] Draw conclusion about the hypothesis for $\alpha = 0.01$, $\alpha = 0.05$, and $\alpha = 0.1$?

48

Question 3 (9 pts):

If we wish to compare a new teaching technique with the conventional classroom procedure. In a sample of 10 students, a test is given to assess the new teaching technique before and after applying the new teaching technique. The pre-post test scores of the test are provided

Student	1	2	3	4	5	6	7	8	9	10
Pre-test score	18	21	16	22	19	24	17	21	23	18
Post-test score	22	25	18	24	16	29	21	23	19	20

di

Test the hypothesis that the new learning technique leads to improvement in the students' skills.

(a) [2 pt] State the Null and alternative hypothesis?
(b) [4 pts] Calculate the test statistic?
(c) [3 pts] Calculate the P-value and make a decision? Use $\alpha = 0.05$
(d) [1 pt] what is the type of error you may be committed in (c)?

[Hint: $t_{0.046,9} = 1.89$, $t_{0.044,10} = 1.89$, $\phi(1.89) = 0.971$]

...m: general
Level: 3rd
Course Code: 0...
Tim...

Question 4 (13 pts):

An apple farm owner wants to compare his two farms to see if there are any weight difference in the apples. Assume the apple weights follow normal distributions. Given the following statistical summaries of the samples taken from the two farms.

Farm A	$n_1 = 15$	$\bar{X}_1 = 86$	$s_1 = 7$
Farm B	$n_2 = 10$	$\bar{X}_2 = 80$	$s_2 = 8$

(a) [3 pts] Test the hypothesis of equal variance, assume $\alpha = 0.05$?

(b) [6 pts] Construct the 95% confidence interval for the difference of means of the apple weights from the two farms? [Assume equal variance]

(c) [4 pts] Test the hypothesis that the two farms have different apple weights ($\alpha = 0.05$)? [Assume equal variance]

[Hint: $F_{0.69,14,9} = 0.766$, $F_{0.025,14,9} = 3.798$, $F_{0.975,14,9} = 0.31$, $t_{0.025,23} = 2.069$, $t_{0.03,23} = 1.986$]

am: general
Level: 3rd
Course -

Question 5 (7 pts):

Weights in kg of 10 students are given as $38, 40, 45, 53, 47, 43, 55, 48, 52, 49$.

Test the hypothesis that the variance is equal to 20 against the alternative that it is not equal to 20.

(a) [4 pts] Calculate the test statistic?

(b) [2 pts] Calculate the P-value? [Hint: $\chi^2_{0.12,9} = 14$, $\chi^2_{0.17,10} = 14$, $\chi^2_{0.18,9} = 12.6$, $\chi^2_{0.25,10} = 12.6$]

(c) [1 pt] Determine the range of significance for which the null hypothesis is rejected?

Question 1 (8 pts):

The life time of an electronic devic...
months.

Question 6 (17 pts):

For the given ANOVA table

Source of variation	Sum of squares	Degrees of freedom	Mean sum of squares	Fc	P
Treatments	100	B	C	E	G
Error	A	54	D		
Total	600	59			

(a) [6 pts] Find the values of A, B, C, D, E, and G? [Hint: $F_{0.07,B,54} = E$]

(b) [2 pts] How many treatments were compared?

(c) [2 pts] How many observations were analyzed?

(d) [1 pt] Find an estimate for the experimental error?

(e) [2 pts] At $\alpha = 0.05$, can we conclude that the treatment has different effects? Why?

(f). [2 pts] If $\overline{X}_{1.} = 30$ and $\overline{X}_{i.} = 28$ for the remaining treatments, then find the value of $\overline{X}_{..}$?

(g) [2 pts] Find an estimate for the treatment effect of the first treatment?

Model answer

Program: Intelligent Systems
Level: two
Course Code: 02-24-00201
Time Allowed: 1 hrs.
Professor(s) name(s): Dr Mervat Mikhail

Term: Fall 2021
Course Title Probability and Statistics II
Total points: 20

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Question 1 (5 Marks):

Let $X_1, X_2, \dots, X_n$ be a random sample from an exponential population

$$f(X;\lambda) = \frac{1}{\lambda} e^{\frac{-X}{\lambda}} \quad \lambda > 0, X > 0$$

$$E(X) = \frac{1}{\lambda} \quad V(X) = \frac{1}{\lambda^2}$$

Find MLE $\hat{\lambda}$ for the parameter λ and use the invariance property of MLE to get MLE of $E(X)$ and MLE of $V(X)$

$$(\lambda) = \frac{1}{\lambda} e^{-\frac{X_1}{\lambda}} \cdot \frac{1}{\lambda} e^{-\frac{X_2}{\lambda}} \cdots \frac{1}{\lambda} e^{-\frac{X_n}{\lambda}} \quad 1$$

$$= \left(\frac{1}{\lambda}\right)^n e^{-\frac{\sum X_i}{\lambda}} \quad 1$$

$$(\lambda) = -n \ln \lambda - \frac{\sum X_i}{\lambda} \quad 1$$

$$\frac{L^*(\lambda)}{d\lambda} = 0$$

$$-\frac{n}{\lambda} + \frac{\sum X_i}{\lambda^2} = 0$$

$$\frac{\sum X_i}{\lambda^2} = \frac{n}{\lambda}$$

$$\boxed{\hat{\lambda} = \bar{X}} \quad 1$$

$$\text{MLE of } E(X) = \frac{1}{\hat{\lambda}} = \frac{1}{\bar{X}} \quad \frac{1}{2}$$

$$\text{MLE of } V(X) = \frac{1}{\hat{\lambda}^2} = \frac{1}{\bar{X}^2} \quad \frac{1}{2}$$

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Question 2 (4 Marks):

Suppose that X, Y, U are independent random variables $X \sim N(3,16)$, $Y \sim N(0,2)$, $U \sim \chi^2(11)$. What is the distribution of the random variable $W = \frac{4U+2Y^2}{3(x-3)^2}$

$\frac{X-3}{4} \sim N(0,1)$ $\quad \left(\frac{X-3}{4}\right)^2 \sim \chi^2(1)$ 1

$\frac{Y}{\sqrt{2}} \sim N(0,1)$ $\quad \left(\frac{Y}{\sqrt{2}}\right)^2 + U \sim \chi^2(12)$ 1

$U \sim \chi^2(11)$

let $W = \frac{\left(U + \left(\frac{Y}{\sqrt{2}}\right)^2\right)/12}{\left(\frac{X-3}{4}\right)^2/1} \sim F(12,1)$ 1

$= \frac{\cancel{16}^{\,4}\left(U + \frac{Y^2}{2}\right)}{\cancel{12}_{\,3}\,(X-3)^2}$

$= \frac{4U + 2Y^2}{3(X-3)^2}$

Question 3 (4 Marks):

A sample of 30 people is randomly selected from city A, where the average height of adults is 160 centimeters with a standard deviation of 9 centimeters.

A second random sample of size 40 is selected form city B, where the average height of adults is 158 centimeters with a standard deviation of 10 centimeters.

Find that probability that the sample mean computed from the samples selected from city A will exceed the sample mean computed from the samples selected from city B by at least 2.6 centimeters but less than 4.8 centimeters.

Assume the difference of the means to be measured to the nearest tenth.

$\varphi(0.1) = 0.54$	$\varphi(0.3) = 0.62$	$\varphi(0.5) = 0.69$	$\varphi(0.8) = 0.70$	$\varphi(1.2) = 0.88$	$\varphi(1.4) = 0.92$

City A

$n_A = 30$

$\mu_A = 160$

$\sigma_A = 9$

City B

$n_B = 40$

$\mu_B = 158$

$\sigma_B = 10$

$\bar{X}_A \sim N\left(160, \frac{9^2}{30}\right)$ 1

$\bar{X}_B \sim N\left(158, \frac{100}{40}\right)$

$\bar{X}_A - \bar{X}_B \sim N\left(2, \frac{81}{30} + \frac{100}{40}\right)$ 1

$Z = \frac{(\bar{X}_A - \bar{X}_B) - (\mu_A - \mu_B)}{\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}}$

$P(4.8 \geq \bar{X}_A - \bar{X}_B \geq 2.6)$

$P\left(\frac{2.8}{\sqrt{\frac{81}{30} + \frac{100}{40}}} \geq Z \geq \frac{0.6}{\sqrt{\frac{81}{30} + \frac{100}{40}}}\right)$ 1

$P(1.2 \geq Z \geq 0.3)$

$= \varphi(1.2) - \varphi(0.3)$ 1

$= 0.88 - 0.62$

$= 0.26$

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Question 4 (9 Marks): Choose the correct answer

1. The difference between the expected value of the estimator and the true value of the parameter is called

   Contradiction   Error   Difference   Bias

2. Which of the following is an example of a point estimate

   The population S.D.   The sample mean   The population Variance   The population mean

3. If $\hat{\theta}_1, \hat{\theta}_2$ are two estimators for population parameter θ then $\hat{\theta}_2$ is said to be more efficient than $\hat{\theta}_1$ if

   $MSE(\hat{\theta}_1) > MSE(\hat{\theta}_2)$ $Var(\hat{\theta}_1) > Var(\hat{\theta}_2)$

   $E(\hat{\theta}_1) > E(\hat{\theta}_2)$ None of the above

4. If a random variable $U \sim \chi^2(6)$ then $E(U) =$

   1   12   6   None of the above

5. If $X_1, X_2, \dots, X_9$ are i.i.d N(0,25), then the constant K so that $W = K(\sum_{i=1}^{9} X_i)^2$ has χ^2 distribution is

   $\frac{1}{9}$ $\frac{1}{15}$ $\frac{1}{5}$ $\frac{1}{225}$

If $S^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{X})^2}{n-1}$ is an estimator for population variance σ^2 and $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

6. $E(S^2) =$

   $n-1$ $\frac{\sigma^2}{n}$ $2(n-1)$ σ^2

7. $Bias(S^2) =$

   $\frac{-\sigma^2}{n}$ 0 $\frac{\sigma^2}{n}$ $\frac{-\sigma^2}{n-1}$

8. $MSE(S^2) =$

   $\frac{2\sigma^4}{n-1}$ $\frac{2\sigma^4}{n}$ $\frac{2(n-1)\sigma^4}{n^2}$ $\frac{\sigma^4}{n}$

9. S^2 is ....estimator for population variance σ^2

   biased -Consistent   Unbiased -Consistent

   biased -Inconsistent   Unbiased -Inconsistent

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Model answer

Program: AI, Business, Health, Media
Level: two
Term: Fall 2022
Course Code: 02-24-00201
Course Title Probability and Statistics II
Time Allowed: 45 mins.
Total points: 20
Professor(s) name(s): Dr Mervat Mikhail

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Student Name (in Arabic):

ID :

program:

Question 1 (5 Marks):

Suppose $X_1, X_2, \dots, X_n$ are i.i.d random variables with density function

$$f(X;\theta) = \theta^2 X e^{-\theta X} \quad \theta > 0, X > 0, \qquad E(X) = \frac{2}{\theta}$$

Find MLE $\hat{\theta}$ for the parameter θ and use the invariance property of MLE to get MLE of $E(X)$

$$L(\theta) = \theta^{2n} \prod X_i \, e^{-\theta \sum X_i}$$ 1

$$\ln L(\theta) = 2n \ln\theta + \sum \ln X_i - \theta \sum X_i$$ 1

$$\frac{\partial L}{\partial \theta} = 0$$

$$\frac{2n}{\theta} - \sum X_i = 0$$ 1

$$\hat{\theta} = \frac{2n}{\sum X_i} = \frac{2}{\bar{X}}$$ 1

$$\hat{E}(X) = \frac{2}{\hat{\theta}} = \frac{2\bar{X}}{2} = \bar{X}$$ 1

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Question 2 (6 Marks):

Suppose that $X_1, X_2, \dots, X_9$ are i.i.d N(0,25). Let W_1, W_2 are two random variables where

$$W_1 = \left(\frac{\sum_{i=1}^{9} X_i}{15}\right)^2 \qquad , W_2 = \sum_{i=1}^{9}\left(\frac{X_i}{5}\right)^2$$

Complete the missing parts in the following table

$W_1 \sim \chi^2_1$	$E(W_1) = 1$	$V(W_1) = 2$
$W_2 \sim \chi^2_9$	$E(W_2) = 9$	$V(W_2) = 18$

Write your Steps here

$$\sum X_i \sim N(0, 9(25))$$

$$\frac{\sum X_i}{3(5)} \sim N(0,1)$$

$$\left(\frac{\sum X_i}{15}\right)^2 \sim \chi^2_1$$

$$\frac{X_i}{5} \sim N(0,1)$$

$$\sum_{i=1}^{9}\left(\frac{X_i}{5}\right)^2 \sim \chi^2_9$$

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Question 3 (4 Marks):

A population has mean 75 and standard deviation 12. A random sample of size 121 is taken

(a) what is the mean and standard deviation of the sample mean?
(b) What is the probability that the sample mean falls between 73.2 and 76.8?

$\varphi(0.8) = 0.791$	$\varphi(1.65) = 0.950$	$\varphi(1.89) = 0.960$	$\varphi(2.65) = 0.996$

$$\bar{X} \sim N\left(75, \frac{12^2}{121}\right)$$

a)

$$E(\bar{X}) = 75$$

$$V(\bar{X}) = \frac{12^2}{121} \quad \therefore \text{st. dev of } \bar{X} = \frac{12}{\sqrt{121}} = \frac{12}{11}$$

b) $P(73.2 < \bar{X} < 76.8)$

$$= P\left(\frac{73.2-75}{\frac{12}{11}} < Z < \frac{76.8-75}{\frac{12}{11}}\right)$$

$$= P(-1.65 < Z < 1.65)$$

$$= \Phi(1.65) - (1 - \Phi(1.65))$$

$$= 2\Phi(1.65) - 1$$

$$= 2(0.95) - 1$$

$$= 0.9$$

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Question 4 (5 Marks):

Let $X_1, X_2, \ldots, X_n$ be a random sample from EXP(θ) and define $\hat{\theta}_1 = \bar{X}$ and $\hat{\theta}_2 = n\bar{X}/(n+1)$

Complete the missing parts in the following table

$\text{Bias}(\hat{\theta}_1) =$	0 ½			
$\text{Bias}(\hat{\theta}_2) =$	$-\frac{1}{n+1}\theta$ ½			
$\text{Var}(\hat{\theta}_1) =$	$\frac{\theta^2}{n}$ $\frac{1}{2}$	$\text{Var}(\hat{\theta}_1)\vert_{n=2} =$	$\frac{\theta^2}{2}$	$\frac{1}{2}$
$\text{Var}(\hat{\theta}_2) =$	$n\theta^2/(n+1)^2$ $\frac{1}{2}$	$\text{Var}(\hat{\theta}_2)\vert_{n=2} =$	$\frac{2}{9}\theta^2$	$\frac{1}{2}$
$\text{MSE}(\hat{\theta}_1) =$	θ^2/n $\frac{1}{2}$	$\text{MSE}(\hat{\theta}_1)\vert_{n=2} =$	$\frac{\theta^2}{2}$	$\frac{1}{2}$
$MSE(\hat{\theta}_2) =$	$\theta^2/n+1$ $\frac{1}{2}$	$\text{MSE}(\hat{\theta}_2)\vert_{n=2} =$	$\frac{\theta^2}{3}$	$\frac{1}{2}$

Write your Steps here

$E(\hat{\theta}_1) = E(\bar{X}) = \theta$ unbiased

$E(\hat{\theta}_2) = E\left(\frac{n}{n+1}\bar{X}\right) = \frac{n}{n+1}\theta$ biased

$\text{Bias}(\hat{\theta}_2) = \frac{n}{n+1}\theta - \theta = \frac{-1}{n+1}\theta$

$V(\hat{\theta}_1) = V(\bar{X}) = \frac{\theta^2}{n}$

$V(\hat{\theta}_2) = V\left(\frac{n}{n+1}\bar{X}\right) = \left(\frac{n}{n+1}\right)^2 \frac{\theta^2}{n} = \frac{n\theta^2}{(n+1)^2}$

$MSE(\hat{\theta}_1) = var(\hat{\theta}_1) = \frac{\theta^2}{n}$

$MSE(\hat{\theta}_2) = var(\hat{\theta}_2) + (bias(\hat{\theta}_2))^2 = \frac{1}{n+1}\theta^2$

$\frac{n}{(n+1)^2}\theta^2 + \frac{1}{(n+1)^2}\theta^2$

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Program: general
Level: 3rd
Course Code: 02-24-00201
Time Allowed: 1 hr.
Professor name: Dr Ibrahim Galal and Dr Ahmed Tayel

Midterm Exam
Term: Fall 2022
Course Title Probability and Statistics 2
Total points: 30

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02-24-00201

Name: **ID:**

Useful values: $\phi(1.8) = 0.964$, $\phi(2.55) = 0.9945$, $\phi(1.697) = 0.955$

Question 1 (5 Marks):

The mean GPA for students in School A is 3.0; the mean GPA for students in School B is 2.8. The standard deviation in both schools is 0.25. The GPAs of both schools are normally distributed. If 9 students are randomly sampled from each school, what is the probability that:

(a) the sample mean for School A will exceed that of School B by 0.5 or more?

(b) the sample mean for School B will be greater than the sample mean for School A?

$\mu_A = 3 \quad \mu_B = 2.8$

$\sigma_A = 0.25 \quad \sigma_B = 0.25$

$n_A = 9 \quad n_B = 9$

(a) $P(\bar{X}_A - \bar{X}_B \geq 0.5) = P\left(Z \geq \dfrac{0.5 - (3-2.8)}{\sqrt{\frac{(0.25)^2}{9} + \frac{(0.25)^2}{9}}}\right)$

$P(Z \geq 2.5456) = 1 - \Phi(2.55) = 1 - 0.9945 =$
0.0055

(b) $P(\bar{X}_B > \bar{X}_A) = P(\bar{X}_B - \bar{X}_A > 0) = P\left(Z < \dfrac{0 - (3-2.8)}{\sqrt{\frac{(0.25)^2}{9} + \frac{(0.25)^2}{9}}}\right)$

$P(Z < -1.697) = 1 - \Phi(1.697)$

$= 1 - 0.9955 = 0.045$

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Question 2 (5 Marks):

Suppose that a random sample is taken from a Normal population with mean **100** and standard deviation **10**.

(a) For a sample of size **36**, what is the probability that the sample mean falls in the interval from **97** to **103** ?

(b) What is the minimum sample size required for the standard deviation (error) of the sample mean to be less than or equal to **2** ?

(a) $P(97 < \bar{X} < 103)$.

$$P\left(\frac{97-100}{10/\sqrt{36}} < Z < \frac{103-100}{10/\sqrt{36}}\right)$$

$$P(-1.8 < Z < 1.8)$$

$$= \Phi(1.8) - \Phi(-1.8) = 2\Phi(1.8) - 1$$

$$= 2 * 0.964 - 1 = 0.928$$

(b) $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{n}} \leq 2.$

$$2\sqrt{n} \geq 10$$

$$\sqrt{n} \geq 5$$

$$n \geq 25$$

minimum Value $\boxed{n = 25}$

Question 3 (5 Marks):

Let X be a random variable having a gamma distribution with unknown parameters α and β. Find the moment estimators of the unknown parameters.

(Hint: $M_X(t) = (1 - \beta t)^{-\alpha}$)

$E(x) = \alpha\beta$

$Var(x) = \alpha\beta^2$

$\alpha\beta^2 = E(x^2) - (E(x))^2$

$\alpha\beta^2 = E(x^2) - \alpha^2\beta^2$

$E(x^2) = \alpha\beta^2 + \alpha^2$

the first Population moment $\boxed{1}$ $E(x) = \frac{dM_x(t)}{dt}\Big|_{t=0} = -\alpha(1-\beta t)^{-\alpha-1} * (-\beta)$

$$= \alpha\beta(1-\beta t)^{-\alpha-1}\Big|_{t=0} = \alpha\beta$$

the second Population moment $\boxed{2}$ $E(x^2) = \frac{d}{dt}E(x)\Big|_{t=0} = (-\alpha-1)\alpha\beta(1-\beta t)^{-\alpha-2} * -\beta$

$$= \alpha\beta^2(\alpha+1) = \alpha^2\beta^2 + \alpha\beta^2$$

$\boxed{3}$ the First Sample moment, the second sample moment

$m_1 = \bar{X}$ $\quad$ $m_2 = \frac{1}{n}\sum x_i^2$

$m_1 = \mu_1'$ & $m_2 = \mu_2'$.

$\bar{X} = \alpha\beta$

$\boxed{\hat{\alpha} = \frac{\bar{X}}{\hat{\beta}}}$ #

$\alpha\beta^2 + \alpha^2\beta^2 = \frac{1}{n}\sum x_i^2$

$\bar{X}\beta + \bar{X}^2 = \frac{1}{n}\sum x_i^2$

$\boxed{\hat{\beta} = \frac{\frac{1}{n}\sum x_i^2 - (\bar{X})^2}{\bar{X}}}$ #

$\boxed{\hat{\beta} = \frac{\frac{1}{n}\sum x_i^2 - (\bar{X})^2}{\bar{X}}}$

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Question 4 (8 Marks):

Consider $X_1, X_2, \dots, X_{10}$ i.i.d. random variables with $N(\mu, \sigma^2)$ where $\mu = 0$ and $\sigma^2 = 4$.
What is the probability distribution of the following:

(a) $2X_2 - 3X_4 + 1$

(b) $\dfrac{cX_1}{\sqrt{\sum_{i=2}^{4} X_i^2}}$; and what is the value of c?

(c) $\dfrac{c\sum_{i=1}^{4} X_i^2}{\sum_{i=8}^{10} X_i^2}$; and what is the value of c?

$\boxed{a}$ linear combination from Normal.

$$E(2X_2 - 3X_4 + 1) = 2E(X_2) - 3E(X_4) + 1 = 1$$

$$Var(2X_2 - 3X_4 + 1) = 4\,var(X_2) + 9\,var(X_4) = 52$$

$$\therefore 2X_2 - 3X_4 + 1 \sim N(1, 52)$$

$\boxed{2}$

$$\frac{cX_1}{\sqrt{\sum_{i=2}^{4} X_i^2}} = c\,\frac{\frac{X_1 - 0}{2}}{\sqrt{\sum_{i=2}^{4}\left(\frac{X_i - 0}{2}\right)^2}}$$

أقسم البسط والمقام على 2

$$= c\,\frac{Z}{\sqrt{\sum_{i=2}^{4} Z_i^2}}$$

ناقص ان المقام يكون مقسوم على الـ degree of freedom.

$$= c\,\frac{Z}{\sqrt{\sum_{i=2}^{4} Z_i^2/3}} = \overset{\sqrt{3}}{\boxed{C}}\,\frac{Z}{\sqrt{\frac{Z_2 + Z_3 + Z_4}{3}}}$$

$\boxed{\therefore C = \sqrt{3}}$

T dist^{n} with 3 degree of freedom.

$\boxed{3}$

$$\frac{c\sum_{i=1}^{4} X_i^2}{\sum_{i=8}^{10} X_i^2} = c\,\frac{\sum_{i=1}^{4}\left(\frac{X_i - 0}{2}\right)^2}{\sum_{i=8}^{10}\left(\frac{X_i - 0}{2}\right)^2} = \frac{c\sum_{i=1}^{4} Z_i^2}{\sum_{i=8}^{10} Z_i^2} = \frac{Z_i^2/4}{Z_i^2/3} = \frac{\frac{1}{4}\chi^2_{(4)}/4}{\frac{1}{3}\chi^2_{(3)}/3} =$$

$$= \frac{3}{4}\,\frac{\chi^2_{(4)}/4}{\chi^2_{(3)}/3}$$

$\therefore C = \frac{3}{4}$

F-dist^{n} with $(4,3)$ degree of freedom.

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Question 5 (8 Marks):

Consider $X_1, X_2, \dots, X_n$ a random sample from a normal population with mean μ and variance σ^2.

(a) Find the maximum likelihood estimator (MLE) for μ and σ^2?

(Hint: $f_X(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$)

(b) Let the estimator of μ in part (a) called $\hat{\mu}_1$. Compare $\hat{\mu}_1$ with the estimator function $\hat{\mu}_2 = \frac{X_1+X_2}{2}$ in terms of biasness and efficiency?

$\boxed{a}$ $\ell(\mu,\sigma^2) = \prod_{i=1}^{n} \left(\frac{1}{\sqrt{2\pi}\sqrt{\sigma^2}}\right)^n e^{\frac{-1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2} = (2\pi)^{\frac{-n}{2}} (\sigma^2)^{\frac{-n}{2}} e^{\frac{-1}{2\sigma^2}\left(\sum_{i=1}^{n}(x_i-\mu)^2\right)}$

$$\boxed{\ell^* = \ln(\ell) = \frac{-n}{2}\ln(2\pi) - \frac{n}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}$$

$\frac{d\ell^*(\mu,\sigma^2)}{d\mu} = \frac{-(-2)\sum_{i=1}^{n}(x_i-\mu)}{2\sigma^2} = 0.$ $\qquad \sum_{i=1}^{n}(x_i-\mu) = 0.$

$\sum x_i - \sum\mu = 0$ $\qquad \because \bar{X} = \frac{\sum x_i}{n}. \quad \therefore \sum x_i = n\bar{X}$

$n\bar{X} - n\mu = 0.$

$\boxed{\mu = \bar{X}} \Rightarrow \boxed{1}$

$\frac{\partial\ell^*(\mu,\sigma^2)}{\partial\sigma^2} = \frac{-n}{2} * \frac{1}{\sigma^2} + \frac{\sum(x_i-\mu)^2}{2\sigma^4} = 0$

$\frac{d\ell^*(\mu,\sigma^2)}{d\sigma^2} = \frac{-n}{2} * \frac{1}{\sigma^2} + \frac{\sum(x_i-\mu)^2}{2\sigma^4} = 0.$

$\frac{-n}{2}\sigma^2 + \frac{1}{2}\sum(x_i-\mu)^2 = 0.$

$\frac{-n}{2}\sigma^2 + \frac{1}{2}\sum(x_i-\mu)^2 = 0.$

$\boxed{\hat{\sigma}^2 = \frac{1}{n}\sum(x_i-\mu)^2}$ # 2

$\boxed{b}$ $\hat{\mu}_1 = \bar{X} \Rightarrow E(\hat{\mu}_1) = E(\bar{X}) = \mu \quad \therefore \hat{\mu}_1$ is unbaised estimator.

$\hat{\mu}_2 = \frac{X_1+X_2}{2.} \Rightarrow E(\hat{\mu}_2) = \frac{1}{2}E(\mu X_1) + E(X_2) = \frac{1}{2}(\mu+\mu) = \mu$

$\therefore \hat{\mu}_2$ is Unbaised estimator.

$var(\hat{\mu}_1) = var(\bar{X}) = \frac{\sigma^2}{n}, \quad var(\hat{\mu}_2) = \frac{1}{4}(\sigma^2+\sigma^2) = \frac{\sigma^2}{2}$

efficiency $= eff(\hat{\mu}_2/\hat{\mu}_1) = \frac{var(\hat{\mu}_1)}{var(\hat{\mu}_2)} = \frac{\sigma^2/n}{\sigma^2/2} = \frac{2}{n} \Rightarrow \hat{\mu}_1$ is more efficient than. $\hat{\mu}_2$ when $n>2$.

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Program: general
Level: 3rd
Course Code: 02-24-00201
Time Allowed: 1 hr.
Professor name: Dr Ibrahim Galal and Dr Ahmed Tayel

Midterm Exam
Term: Spring 2023
Course Title Probability and Statistics 2
Total points: 30

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Name: Model Answer **ID:**

Useful values: $\phi(0.8) = 0.788$, $\phi(1.65) = 0.951$, $\phi(1.8) = 0.964$, $\phi(1.89) = 0.971$, $\phi(2.65) = 0.996$

Question 1 (6 Marks):

In a city, working hours of males is **15** hrs with standard deviation of **7** hrs. For females, the working hours is **10** hrs with standard deviation of **6**. If a sample of **100** men and **50** women is taken. What is the probability that men are working at most **3** hours more than women in average?

Req: $Pr(\bar{X}_1 - \bar{X}_2 \leq 3)$

Popⁿ 1: $\mu_1 = 15$, $\sigma_1 = 7$, $n_1 = 100$

Popⁿ 2: $\mu_2 = 10$, $\sigma_2 = 6$, $n_2 = 50$

$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_{\bar{X}_1} - \mu_{\bar{X}_2} = \mu_1 - \mu_2 = 15 - 10 = 5$

$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{49}{100} + \frac{36}{50}} = \sqrt{\frac{121}{100}} = 1.1$

$Pr(\bar{X}_1 - \bar{X}_2 \leq 3) = Pr\left(Z \leq \frac{3-5}{1.1}\right) = Pr(Z \leq -1.81) = 1 - \varphi(1.8)$

$= 1 - 0.964 = \boxed{0.036}$

Question 2 (6 Marks):

A population has mean **75** and standard deviation **12** . A random sample of size **121** is taken

(a) What is the mean and standard deviation of the sample mean?

(b) What is the probability that the sample mean falls between **73.2** and **76.8**?

$\mu = 75$, $\sigma = 12$, $n = 121$

(a) $\mu_{\bar{X}} = \mu = 75$, $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{121}} = \frac{12}{11} = 1.09$

(b) $Pr(73.2 < \bar{X} < 76.8) = Pr\left(\frac{73.2-75}{1.09} < Z < \frac{76.8-75}{1.09}\right)$

$= Pr(-1.65 < Z < 1.65) = \varphi(1.65) - \varphi(-1.65)$

$= \varphi(1.65) - [1 - \varphi(1.65)] = 2\varphi(1.65) - 1 = 2 * 0.951 - 1$

$= \boxed{0.902}$

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Question 3 (6 Marks):

Let $X_1, X_2, \dots, X_n$ be a random sample from the distribution

$f(X;\theta) = \frac{1}{\theta} e^{-X/\theta}$, $X \geq 0$, where $E[X] = \theta$ and $Var(X) = \theta^2$

Define $\hat{\theta}_1 = \bar{X}$ and $\hat{\theta}_2 = n\bar{X}/(n+1)$.

Compare between the two estimators in terms of **bias**, **efficiency**, and **consistency**.

$$E(\hat{\theta}_1) = E(\bar{X}) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \frac{1}{n}\sum_{i=1}^{n}\theta = \frac{1}{n}\cdot n\theta = \theta$$

$$E(\hat{\theta}_2) = \frac{n}{n+1} E(\bar{X}) = \frac{n}{n+1}\theta$$

$$Var(\hat{\theta}_1) = Var(\bar{X}) = \frac{1}{n^2}\sum_{i=1}^{n} Var(X_i) = \frac{1}{n^2}\sum_{i=1}^{n}\theta^2 = \frac{1}{n^{\cancel{2}}}\cdot \cancel{n}\theta^2 = \frac{1}{n}\theta^2$$

$$Var(\hat{\theta}_2) = \frac{n^2}{(n+1)^2} Var(\bar{X}) = \frac{n^2}{(n+1)^2}\cdot\frac{1}{n}\theta^2 = \frac{n}{(n+1)^2}\theta^2$$

Bias: $\hat{\theta}_1$ is unbiased while $\hat{\theta}_2$ is biased

$MSE = \text{variance} + \text{bias}^2$

Efficiency: $MSE(\hat{\theta}_1) = \frac{1}{n}\theta^2 + 0^2$

$$MSE(\hat{\theta}_2) = \frac{n}{(n+1)^2}\theta^2 + \left(\frac{n}{n+1}\theta - \theta\right)^2$$

$$= \frac{n}{(n+1)^2}\theta^2 + \left(\frac{\cancel{n}-\cancel{n}-1}{n+1}\theta\right)^2 = \frac{n}{(n+1)^2}\theta^2 + \frac{1}{(n+1)^2}\theta^2$$

$$= \frac{(n+1)}{(n+1)^2}\theta^2 = \frac{1}{n+1}\theta^2$$

$$eff(\hat{\theta}_2/\hat{\theta}_1) = \frac{MSE(\hat{\theta}_1)}{MSE(\hat{\theta}_2)} = \frac{\frac{1}{n}\theta^2}{\frac{1}{n+1}\theta^2} = \frac{n+1}{n} > 1$$

$\therefore \hat{\theta}_2$ is more efficient than $\hat{\theta}_1$

Consistency $\lim_{n\to\infty} MSE(\hat{\theta}_1) = 0$, $\lim_{n\to\infty} MSE(\hat{\theta}_2) = 0$

$\therefore$ both $\hat{\theta}_1$ & $\hat{\theta}_2$ are consistent

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Name: **ID:**

Question 4 (6 Marks):

Consider $X_1, X_2, \ldots, X_9$ i.i.d. random variables with $N(\mu, \sigma^2)$ where $\mu = 0$ and $\sigma^2 = 25$. Let W_1 and W_2 are two random variables where

$$W_1 = \left(\frac{\sum_{i=1}^{9} X_i}{15}\right)^2 \quad , \quad W_2 = \sum_{i=1}^{9}\left(\frac{X_i}{5}\right)^2$$

(a) What is the distribution of W_1 and W_2 ?

(b) Find the expectation and variance of W_1 and W_2?

$\boxed{W_1}$ $\sum_{i=1}^{9} X_i \sim$ Normal

$$E\left[\sum_{i=1}^{9} X_i\right] = \sum_{i=1}^{9} E[X_i] = 0$$

$$\operatorname{var}\left[\sum_{i=1}^{9} X_i\right] = \sum_{i=1}^{9} \operatorname{Var}(X_i) = \sum_{i=1}^{9} 25 = 25 * 9$$

std. dev. $\sqrt{25*9}$

$5*3 = 15$

$$\therefore \frac{\sum_{i=1}^{9} X_i - 0}{15} \sim N(0,1)$$

$$\left(\frac{\sum_{i=1}^{9} X_i}{15}\right)^2 \sim \chi^2_1$$

$E(W_1) = 1$

$V(W_1) = 2$

$\boxed{W_2}$ $\frac{X_i - 0}{5} \sim N(0,1)$

$$\sum_{i=1}^{9}\left(\frac{X_i - 0}{5}\right)^2 \sim \chi^2_9$$

$E(W_2) = 9$

$V(W_2) = 18$

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Question 5 (6 Marks):

Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. random variables with density function

$$f(X;\theta) = \theta^2\, X\, e^{-\theta X} \quad ; \quad \theta > 0\,, X > 0$$

(a) Find the maximum likelihood estimator (MLE) for θ?

(b) Use the invariance property of MLE to get the MLE of $E[X] = \frac{1}{\theta}$?

$n\bar{X}$

(a) $$L = \prod_{i=1}^{n} \left(\theta^2 x_i\, e^{-\theta x_i}\right) = \theta^{2n} \left(\prod_{i=1}^{n} x_i\right) e^{-\theta \sum x_i}$$

$$L^* = \ln(L) = 2n \ln\theta + \underbrace{\ln\left(\prod_{i=1}^{n} x_i\right)}_{\text{Constant}} - n\bar{x}\theta$$

$$\frac{dL^*}{d\theta} = \frac{2\cancel{n}}{\theta} - \cancel{n}\bar{X} = 0 \Rightarrow \theta\bar{X} = 2 \Rightarrow \boxed{\hat{\theta} = \frac{2}{\bar{X}}}$$

(b) $$\text{MLE}(E[X]) = \text{MLE}\left(\frac{1}{\theta}\right)$$

$$= \frac{1}{\hat{\theta}} = \boxed{\frac{\bar{X}}{2}}$$

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(14)

Question 1: [4 marks]

Consider the three independent random variables X, Y and V such that $X \sim N(0,16)$, $Y \sim N(4,6)$, and $V \sim \chi^2(16)$.

a) State the distribution, the mean, and the variance of the random variable $3X - 2Y$

b) State the distribution and the mean of the random variable $\frac{X^2}{16} + \frac{(Y-4)^2}{6}$ $T = \frac{z}{\sqrt{u/n}}$

c) State the distribution and the mean of the random variable $\frac{X}{\sqrt{V}}$ $u = \left(\frac{u_1/n_1}{u_2/n_2}\right)$

a)

iid $\therefore$ cov = 0

$3X \sim N(0, 144)$

$2Y \sim N(8, 24)$

$3X - 2Y \sim N(-8, 168)$

$\mu = -8$ Var = 168

$\rightarrow$ Normal distribution

(4)

b)

$\cancel{\frac{X_1^2}{16} + \frac{X_1 + 16 - 8Y}{6}}$

$\frac{X^2}{16} + \frac{Y^2 \cancel{+16-4Y}}{6}$

$\sqrt{\frac{X^2 - 0}{16}}$ $\qquad \sqrt{\frac{(Y-4)^2}{6}}$

$\frac{X - 0}{4} = Z$ or $\frac{X - \mu}{\sigma}$ $\qquad = \frac{Y-4}{3} \rightarrow \sim Z$

$\therefore \frac{X^2}{16} + \frac{(Y-4)^2}{6}$

$= Z_1^2 + Z_2^2 = \chi_2^2$

Chi-squared of n = 2 $\mu = 2$ Var = 4

c)

$\frac{\frac{1}{16} X}{\sqrt{V/16}}$ $\qquad \frac{1}{16} X \sim N(0,16)$

$\therefore X' \sim (0,1)$

$\therefore \frac{X}{\sqrt{V}}$ $\therefore$ t-distribution of n = 16

$\mu = 0$ $\quad$ Var = $\frac{n}{n-2} = \frac{16}{14}$

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Question 2: [4 marks]

Consider the following table of a t-distribution

Degrees of freedom (V)	Amount of area in one tail (α)							
	0.0005	0.001	0.005	0.010	0.025	0.050	0.100	0.200
1	636.6192	318.3088	63.65674	31.82052	12.70620	6.313752	3.077684	1.376382
2	31.59905	22.32712	9.924843	6.964557	4.302653	2.919986	1.885618	1.060660
3	12.92398	10.21453	5.840909	4.540703	3.182446	2.353363	1.637744	0.978472
4	8.610302	7.173182	4.604095	3.746947	2.776445	2.131847	1.533206	0.940965
5	6.868827	5.893430	4.032143	3.364930	2.570582	2.015048	1.475884	0.919544
6	5.958816	5.207626	3.707428	3.142668	2.446912	1.943180	1.439756	0.905703
7	5.407883	4.785290	3.499483	2.997952	2.364624	1.894579	1.414924	0.896030

a) If $T \sim t(n)$, for n=4. find the value of the constant k such that

$$P(k < T < 7.173182) = 0.099$$

$\alpha_1 - 0.001 = 0.099$
$\alpha_1 = 0.1$
$\alpha_2 = 0.001$

b) If $U \sim t(6)$, find $P(U > t_{0.025,6})$

c) Find the values of $t_{0.025,6}$ and $t_{0.975,6}$

a) $\alpha_1 = 0.1$, $n=4$

$\therefore k = 1.533206$

b) $P(u > t_{0.025,6})$

$= \alpha = 0.025$

c) $t_{0.025,6} = 2.446912$

$t_{0.975,6} = \quad \alpha \approx 1$

$1-\alpha = 0.025$

$t_{-0.975,6} = t_{0.025,6} = 2.446912$

$\therefore t_{0.975,6} = -t_{0.-975,6} = -2.446912$

4

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Question 3: [5 marks]

An electronics company manufactures resistors that have a mean resistance of 100 ohms and a standard deviation of 10 ohms. The distribution of resistance is normal. If we consider a sample of 25 resistors

a. What is the mean and the standard deviation of the sample mean?

b. What is the probability that the sample will have an average resistance of fewer than 95 ohms?

c. If the sample have an average resistance greater than 100 ohms, what is the probability that it will have resistance fewer than 104 ohms?

d. For such population, what is the minimum sample size required for the standard deviation of the sample mean to be less than or equal to 1?

[Hint: $\phi(1) = 0.8413, \phi(1.25) = 0.8944, \phi(2) = 0.9772, \phi(2.5) = 0.9938$]

$\mu = 100$, $\sigma = 10$, $n = 25$

$\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$

$\bar{x} \sim N(100, 4)$

a. mean = 100

Standard deviation $= \sqrt{4} = 2$

b. $P(\bar{x} < 95)$

$P\left(\frac{\bar{x}-100}{2} < \frac{95-100}{2}\right)$

$P(Z < -2.5)$

$= \phi(-2.5)$

$= 1 - \phi(2.5)$

$= 1 - 0.9938$

$= 0.0062$

c. $P(100 < \bar{x} < 104) =$

$P\left(\frac{100-100}{2} < Z < \frac{104-100}{2}\right) =$

$P(0 < Z < 2) =$

$\phi(2) - \phi(0) =$

$0.9772 - 0.5 = 0.4772$

d. $\sqrt{\frac{\sigma^2}{n}} \leq 1$, $\frac{\sigma^2}{n} \leq 1$, $\sigma^2 \leq n$, n minumum value is $\sigma^2 = 100$

~~3~~ 4.5

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Question 4: [3 marks]

Let X be a random variable having a gamma distribution with unknown parameters α and β, where the PDF $f(x) = \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)}$, $x \geq 0, \alpha > 0, \text{and } \beta > 0$

Find the moment estimators of the unknown parameters.

[Hint: for gamma distribution $E[X] = \alpha\,\beta$, and $Var(X) = \alpha\,\beta^2$]

$M_1 = M_{(t)}' \; E(x)$
$= \alpha\beta$

$m_1 = \frac{1}{n}\Sigma X_i$
$= \bar{X}$

1

$M_2 = E(X^2)$
$= \alpha \quad (\text{w.r.t } \beta)$

$M_2 = \beta \quad (\text{w.r.t } \alpha)$

$m_2 = \frac{\Sigma X_i^2}{n}$

$\boxed{\hat{\alpha} = \frac{\Sigma X_i^2}{n}}$

$\boxed{\hat{\beta} = \frac{\Sigma X_i^2}{n}}$

Question 5: [4 marks]

Given a random sample of size n from a population with

$$f(x) = \frac{1}{\theta^2} x\, e^{-\frac{x}{\theta}}$$

Find the MLE of its parameter θ.

$L(\theta) = \theta^{-2n} \Sigma X_i\, e^{-\frac{\Sigma X_i}{\theta}}$

$\ln L(\theta) = -2n \ln\theta + \ln \Sigma X_i - \frac{\Sigma X_i}{\theta} \cancel{\ln e}$

$d' = \frac{-2n}{\theta} + \frac{\Sigma X_i}{\theta^2}$

(let $d = 0$)

$\frac{2n}{\theta} = \frac{\Sigma X_i}{\theta^2}$

$2n\theta^2 = \Sigma X_i\, \theta$

$\theta = \frac{\Sigma X_i}{2n}$

$\hat{\theta} = \frac{1}{2}\bar{X}$

4

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16.5

Program:
Level: Two
Course Code: 00201-24-02
Time Allowed: 1 hrs.
Professor: Dr Ahmed Yehia

Term: second 2023/2024
Course Title Probability and Statistics 2

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Student Name: Nadeen Ahmad Nabeel ID 220594.

[Hint: $\phi(1) = 0.8413, \phi(1.2) = 0.8849, \phi(1.25) = 0.8944, \phi(1.697) = 0.955, \phi(1.8) = 0.964, \phi(2) = 0.9772, \phi(2.4) = 0.9918, \phi(2.5) = 0.9938], \phi(2.55) = 0.9945]$

Question 1: [5 marks]

Consider the three independent random variables X, Y and V such that $X \sim N(0,18)$, $Y \sim N(4,6)$, and $V \sim \chi^2(16)$.

a) State the distribution, the mean, and the variance of the random variable $3X - 7Y$

b) State the distribution and the mean of the random variable $\frac{X^2}{18} + \frac{(Y-4)^2}{6}$

c) Find the value of the constant k, so that $k\frac{X}{\sqrt{V}}$ is a valid t distribution $k = 4/9$

a)
$E(3X-7Y)$
$3E(X) - 7E(Y)$
$3(0) - 7(4)$
-28
$var(3X-7Y)$
$9\,var(X) + 49\,var(Y)$
$9(18) + 49(6)$
456
$N \sim (-28, 456)$
mean $= -28$
var $= 456$

3.5

$\frac{1}{9} = 4$

b) $X \sim N(0,18)$ $\quad Y \sim N(4,6)$
$\left(\frac{X-0}{9}\right)^2 + \left(\frac{Y-4}{\sqrt{6}}\right)^2$
$\frac{X^2}{18} \sim \frac{(Y-4)^2}{6}$ $\quad Z^2 + Z^2$
$\chi^2_{(1)} + \chi^2_{(1)} = \chi^2_{(2)}$ mean $= 2$ Chi dist

c) t-dis $= \frac{Z}{\sqrt{U/n}}$ or $\frac{Z}{\sqrt{X^2_n/n}}$ $\quad t_{16}$
$k = \frac{1}{3}$
4
$k \frac{\left(\frac{X-0}{9}\right)^{\frac{4}{9}\frac{9}{4}}}{36\,\frac{1}{9}\sqrt{\chi^2_{(16)}/16}}$ $\quad \frac{1}{4}$ $\quad 2.25$
$3 \quad 81 \quad 1.16$
$\frac{1}{9}(x) = 16$
$x = 4$
$4\,\frac{1}{x} \cdot \frac{1}{9} \quad 4 \quad \frac{1}{81}$
0.0277 $\boxed{k = 4/9}$

Question 2 (2 Marks): complete the following.

If $X_1, X_2, \dots, X_n$ is a random sample from a normal population $N(\mu, \sigma^2)$ and $\bar{X}$ is the sample mean, then $Var(X_1) = Var(X_2) = \dots \sigma^2 \dots$, $E(X_n) = \dots \mu \dots$, $Var(\bar{X}) = \dots \frac{\sigma^2}{n} \dots$ and $E(\bar{X}) = \dots \mu$

$E\left(\frac{1}{n}\sum X_i\right)$ $\quad n\sigma^2$ $\quad 2$ $\quad \mu$

16.0

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Question 3: [3 marks]

Find the first, second, and the third quartiles (Q1, Q2, and Q3) and the Interquartile range for the following scores: 64, 65, 80, 72, 85, 78, 94, 92, 80, 70, 68.

arrange

64, 65, 68, 70, 72, 78, 80, 80, 85, 92, 94

median $= Q_2 = \frac{11+1}{2} = 6^{th} = 78$

$Q_3 = \frac{80+85}{2}$

82.5

$IQR = Q_3 - Q_1$

13.5

$Q_1 = \frac{68+70}{2}$

$= 69$

1

Question 4: [5 marks]

The mean GPA for students in School A is **3.0** and the mean GPA for students in School B is **2.8**. The standard deviation in both schools is **0.25**. The GPAs of both schools are normally distributed. If **19** students are randomly sampled from each school, what is the probability that:
(a) the sample mean for School A will exceed that of School B by **0.3** or more?
(b) the sample mean for School B will be greater than the sample mean for School A?

a)

$\mu_1 = 3$ $N\sim(3, \frac{\sigma^2}{n})$

$\sigma = 0.25$ $N\sim(3, \frac{(0.25)^2}{19})$

$n = 19$

$N\sim(2.8, \frac{0.25^2}{19})$

$\mu_B = 2.8$

$\sigma = 0.25$

$n = 19$

$z \geq \frac{\bar{X}_A - \bar{X}_B - (\mu_A - \mu_B)}{\sqrt{\frac{\sigma_A^2}{n} + \frac{\sigma_B^2}{n}}}$

$P(\bar{X}_A - \bar{X}_B \geq 0.3)$

$P(Z \geq \frac{0.3 - (3 - 2.8)}{\sqrt{\frac{(0.25)^2}{19} + \frac{(0.25)^2}{19}}})$

$P(Z \geq 1.232)$

$1 - 0.8849 = 0.1151$

b)

$P(\bar{X}_B > \bar{X}_A)$

$P(\bar{X}_B - \bar{X}_A > 0)$

$P(\bar{X}_B - \bar{X}_A > \frac{0 - (2.8 - 3)}{\sqrt{\frac{(0.25)^2}{19} + \frac{(0.25)^2}{19}}})$

$P(Z > 2.4657)$

$\Phi\ 2.4657$

$1 - 0.9918$

5

$\frac{4}{5}$

Question 5 (5 Marks):

Suppose that a random sample is taken from a Normal population with mean **100** and ...
(a) For a sample of size **36**, what is the probability that the sample mean falls in the ...
(b) What is the minimum sample size required for the standard deviation of the sa... equal to **2**?

a)

$\mu = 100$ $Z\sim(\mu, \frac{\sigma}{\sqrt{n}})$

$\sigma = 10$ $N\sim(100, \frac{\sigma^2}{n})$

$n = 36$ $N\sim(100, \frac{100}{36})$

$P(97 < \bar{X} < 103)$

$P(\frac{97-100}{\frac{10}{\sqrt{36}}} < \bar{X} < \frac{103-100}{\frac{10}{\sqrt{36}}})$

$-1.8 < Z < 1.8$

$\Phi\ 1.8 - (1 - \Phi(1.8))$

$0.964 - (1 - 0.9...$

b) $n \leq$

$P(2...$

try

Question 6 (6 Marks):

Let $x_1 = 1, x_2 = 3, x_3 = 7, x_4 = 9$ is a random sample ... $[\alpha, 6]$. Use method of moments to find an estimator ...

$m_2 = (\bar{X})^2 = \frac{1}{n}$

$\mu_2 = E(X) =$

$\mu_1 = \bar{x} \mid \hat{\alpha} = \alpha$

$\frac{6}{6-\alpha}$

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Question 5 (5 Marks):

Suppose that a random sample is taken from a Normal population with mean **100** and standard deviation **10**.
(a) For a sample of size **36**, what is the probability that the sample mean falls in the interval from **97** to **103** ?
(b) What is the minimum sample size required for the standard deviation of the sample mean to be less than or equal to **2** ?

a) $\mu = 100$ $Z \sim (\mu, \frac{\sigma}{\sqrt{n}})$

$\sigma = 10$ $N \sim (100, \frac{\sigma^2}{n})$

$n = 36$ $N \sim (100, \frac{100}{36})$

$P(97 < \bar{X} < 103)$

$P\left(\frac{97-100}{\frac{10}{\sqrt{36}}} < \bar{X} < \frac{103-100}{\frac{10}{\sqrt{36}}}\right)$

$-1.8 < Z < 1.8$

$\Phi 1.8 - (1 - \Phi(1.8))$

$0.964 - (1 - 0.964) = 0.928$

b) $n \leq 2$

$P\left(2 \geq \frac{X-100}{\frac{10}{\sqrt{n}}}\right)$

3

try $\frac{103-100}{\frac{10}{\sqrt{36}}} = 1.8$ $(2 \geq 1.8)$

try $\frac{104-100}{\frac{10}{\sqrt{36}}} = 2.4$ X

Question 6 (6 Marks):

Let $x_1 = 1, x_2 = 3, x_3 = 7, x_4 = 9$ is a random sample drawn from uniform distribution on the interval $[\alpha, 6]$. Use method of moments to find an estimator for α, then find an estimate for the parameter α.

$\frac{1}{1-\alpha}$

$m_1 = \bar{X} = \frac{1}{n}\sum x_i$

$\mu_1 = E(X) = \sum_{1,3,7,9} x \cdot \frac{1}{1-\alpha}$

$m_2 = (\bar{X})^2 = \frac{1}{n}\sum x_i^2$

$\mu_2 = E(X) = \int$

$\mu_1 = \bar{X} \mid \hat{\alpha} = \alpha$ or diff

var $- E(X)^2 = E(X)$

$\sigma^2 - \mu_2 = \frac{1}{n}\sum x_i^2$

$\frac{6}{6-\alpha} =$ Zero

$\frac{6}{6-\alpha} + 3\frac{6}{6-\alpha} + 7\frac{6}{6-\alpha} + 9\frac{6}{6-\alpha} = \frac{1}{n}\sum X_i$

$\frac{6}{6-\alpha}(1+3+7+9)$

Question 7 (4 Marks):

If $T \sim t(n)$, for n=7, find the value of the constant k such that $P(0.89603 < T < k) = 0.199$

$df = 7$

$t_{\alpha, 7}$

$t_{0.200, 7} < T < K = 0.199$

$0.89603 - K = 0.199$

$\boxed{K = 0.69703}$

$0.200 - \alpha =$

$K = 0.69703$

2

0.69703

$-t_{\alpha} = t_{1-\alpha, 7}$

$0.69703 = t_{1-\alpha, 7}$

$t_{\alpha, 7} = 0.69703$

Degrees of freedom (ν)	Amount of area in one tail (α)							
	0.0005	0.001	0.005	0.010	0.025	0.050	0.100	0.200
1	636.6192	318.3088	63.65674	31.82052	12.70620	6.313752	3.077684	1.376382
2	31.59905	22.32712	9.924843	6.964557	4.302653	2.919986	1.885618	1.060660
3	12.92398	10.21453	5.840909	4.540703	3.182446	2.353363	1.637744	0.978472
4	8.610302	7.173182	4.604095	3.746947	2.776445	2.131847	1.533206	0.940965
5	6.868827	5.893430	4.032143	3.364930	2.570582	2.015048	1.475884	0.919544
6	5.958816	5.207626	3.707428	3.142668	2.446912	1.943180	1.439756	0.905703
7	5.407883	4.785290	3.499483	2.997952	2.364624	1.894579	1.414924	0.896030
8	5.041305	4.500791	3.355387	2.896459	2.306004	1.859548	1.396815	0.888890
9	4.780913	4.296806	3.249836	2.821438	2.262157	1.833113	1.383029	0.883404
10	4.586894	4.143700	3.169273	2.763769	2.228139	1.812461	1.372184	0.879058
11	4.436979	4.024701	3.105807	2.718079	2.200985	1.795885	1.363430	0.875530
12	4.317791	3.929633	3.054540	2.680998	2.178813	1.782288	1.356217	0.872609
13	4.220832	3.851982	3.012276	2.650309	2.160369	1.770933	1.350171	0.870152
14	4.140454	3.787390	2.976843	2.624494	2.144787	1.761310	1.345030	0.868055
15	4.072765	3.732834	2.946713	2.602480	2.131450	1.753050	1.340606	0.866245
16	4.014996	3.686155	2.920782	2.583487	2.119905	1.745884	1.336757	0.864667
17	3.965126	3.645767	2.898231	2.566934	2.109816	1.739607	1.333379	0.863279

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