Battle of the Sexes: Game Analysis

Game Theory Final Project

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1 Story

A man and a woman want to get together for an evening of entertainment, but they have no means of communication. They can either go to the ballet or the fight. The man prefers going to the fight, while the woman prefers going to the ballet. However, both prefer being together rather than being alone.

2 Game Tree

The extensive form of the game is depicted below, showing the sequential decision-making process. The dashed line from Woman's first to second decision node indicates simultaneous moves.

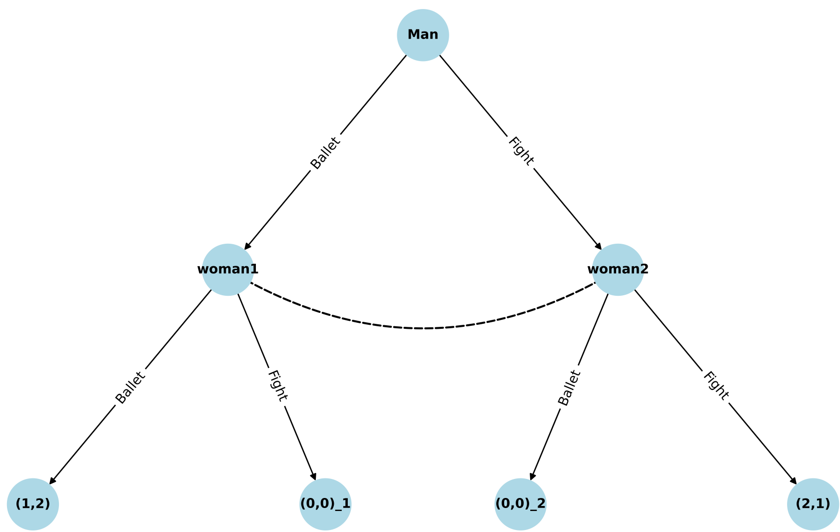

Figure 1: Game Tree for Battle of the Sexes

3 Normal Form Game

The normal form representation is shown below, summarizing the payoffs in a matrix format.

Man

Woman	Ballet	Fight
Ballet	(1,2)	(0,0)
Fight	(0,0)	(2,1)

Figure 2: Normal Form Payoff Matrix

4 Game Analysis

The payoff matrix for the Battle of the Sexes game is shown below, with payoffs given as (Man, Woman):

Man \ Woman	Ballet	Fight
Ballet	(1,2)	(0,0)
Fight	(0,0)	(2,1)

4.1 Dominated Strategies

To determine if any strategies are strictly dominated, we compare the payoffs for each player's strategies.

- **Man**: For Ballet, payoffs are 1 (if Woman plays Ballet) and 0 (if Fight). For Fight, payoffs are 0 (if Ballet) and 2 (if Fight). Since $1 > 0$ in one case and Fight yields $2 > 0$ in another, neither strategy is dominated.
- **Woman**: For Ballet, payoffs are 2 (if Man plays Ballet) and 0 (if Fight). For Fight, payoffs are 0 (if Ballet) and 1 (if Fight). Since $2 > 0$ and Fight yields $1 > 0$, neither strategy is dominated.

Result: No strictly dominated strategies exist.

4.2 Best Responses

We identify each player's best response to the other's strategies.

- **Man**:
  - If Woman plays Ballet: Payoffs are Ballet: 1, Fight: 0. Best response: Ballet $(1 > 0)$.

- If Woman plays Fight: Payoffs are Ballet: 0, Fight: 2. Best response: Fight ($2 > 0$).

- **Woman**:
  - If Man plays Ballet: Payoffs are Ballet: 2, Fight: 0. Best response: Ballet ($2 > 0$).
  - If Man plays Fight: Payoffs are Ballet: 0, Fight: 1. Best response: Fight ($1 > 0$).

4.3 Rationalizable Strategies

Rationalizable strategies are those that survive iterated elimination of strictly dominated strategies or are best responses to some beliefs about the opponent's strategies.

- **Check for strictly dominated strategies**:
  - For Man: Ballet yields payoffs 1 (if Woman plays Ballet) and 0 (if Fight), while Fight yields 0 (if Ballet) and 2 (if Fight). Since $1 > 0$ but $0 < 2$, neither strategy is strictly dominated.
  - For Woman: Ballet yields payoffs 2 (if Man plays Ballet) and 0 (if Fight), while Fight yields 0 (if Ballet) and 1 (if Fight). Since $2 > 0$ but $0 < 1$, neither strategy is strictly dominated.

  No strategies are eliminated in the first iteration, and since the strategy sets remain unchanged, no further eliminations occur.

- **Best response analysis**:
  - For Man: If Man believes Woman plays Ballet with probability q, his expected payoffs are:

    $$\text{Ballet} : q \cdot 1 + (1 - q) \cdot 0 = q$$
    $$\text{Fight} : q \cdot 0 + (1 - q) \cdot 2 = 2(1 - q)$$

    Ballet is a best response if $q \geq 2(1 - q)$, i.e., $q \geq \frac{2}{3}$. Fight is a best response if $2(1 - q) \geq q$, i.e., $q \leq \frac{2}{3}$. Both strategies are best responses for some $q \in [0, 1]$.
  - For Woman: If Woman believes Man plays Ballet with probability p, her expected payoffs are:

    $$\text{Ballet} : p \cdot 2 + (1 - p) \cdot 0 = 2p$$
    $$\text{Fight} : p \cdot 0 + (1 - p) \cdot 1 = 1 - p$$

    Ballet is a best response if $2p \geq 1 - p$, i.e., $p \geq \frac{1}{3}$. Fight is a best response if $1 - p \geq 2p$, i.e., $p \leq \frac{1}{3}$. Both strategies are best responses for some $p \in [0, 1]$.

Result: Since no strategies are eliminated and all strategies are best responses to some beliefs, the rationalizable strategies are {Ballet, Fight} for Man and {Ballet, Fight} for Woman.

4.4 Expected Payoffs

Let Man play Ballet with probability p and Fight with $1-p$, and Woman play Ballet with probability q and Fight with $1-q$.

- **Man's expected payoff**:

$$E_M = q \cdot 1 + (1-q) \cdot 0 + (1-p) \cdot [q \cdot 0 + (1-q) \cdot 2] = q + 2(1-p)(1-q)$$

- **Woman's expected payoff**:

$$E_W = p \cdot 2 + (1-p) \cdot 0 + (1-q) \cdot [p \cdot 0 + (1-p) \cdot 1] = 2p + (1-p)(1-q)$$

4.5 Pure Strategy Nash Equilibria

A pure strategy Nash equilibrium occurs where no player can improve their payoff by unilaterally deviating.

- **(Ballet, Ballet)**: Man gets 1 (vs. 0 if Fight), Woman gets 2 (vs. 0 if Fight). Stable.
- **(Ballet, Fight)**: Man gets 0 (prefers Fight for 2), Woman gets 0 (prefers Ballet for 1). Unstable.
- **(Fight, Ballet)**: Man gets 0 (prefers Ballet for 1), Woman gets 0 (prefers Fight for 2). Unstable.
- **(Fight, Fight)**: Man gets 2 (vs. 0 if Ballet), Woman gets 1 (vs. 0 if Ballet). Stable.

Result: Pure Nash equilibria are (Ballet, Ballet) and (Fight, Fight).

4.6 Mixed Strategy Nash Equilibrium

In a mixed strategy Nash equilibrium, each player chooses probabilities to make the other indifferent.

- **Man's indifference**: Woman chooses q such that Man's expected payoffs from Ballet and Fight are equal.

$$\begin{aligned}
&\text{Ballet} : q \cdot 1 + (1-q) \cdot 0 = q \\
&\text{Fight} : q \cdot 0 + (1-q) \cdot 2 = 2(1-q) \\
&q = 2(1-q) \\
&q = 2 - 2q \\
&3q = 2 \\
&q = \frac{2}{3}
\end{aligned}$$

- **Woman's indifference**: Man chooses p such that Woman's expected payoffs from

Ballet and Fight are equal.

$$
\begin{aligned}
&\text{Ballet} : p \cdot 2 + (1-p) \cdot 0 = 2p \\
&\text{Fight} : p \cdot 0 + (1-p) \cdot 1 = 1 - p \\
&2p = 1 - p \\
&3p = 1 \\
&p = \frac{1}{3}
\end{aligned}
$$

At the mixed NE, Man plays Ballet with $p = \frac{1}{3}$, Fight with $1 - p = \frac{2}{3}$; Woman plays Ballet with $q = \frac{2}{3}$, Fight with $1 - q = \frac{1}{3}$. Expected payoffs are:

- Man: $E_M = q = \frac{2}{3}$
- Woman: $E_W = 2p = 2 \cdot \frac{1}{3} = \frac{2}{3}$