2

ASSIGNMENT - 2

① An electronic company is testing two different types of resistors for their lifespan under constant voltage. The number of hours each resistor type lasts before failure is recorded.

Resistor X : 5, 6, 8, 1, 12, 4, 3, 9, 6, 10

Resistor Y : 2, 3, 6, 8, 1, 10, 2, 8

Test whether both resistors having same variance At 5% LOS with (7,9) dof $F_\alpha = 3.29$

Soln:- $n_1 = 10$ $n_2 = 8$

$H_0 = \sigma_1^2 = \sigma_2^2$

$H_1 = \sigma_1^2 \neq \sigma_2^2$

$\bar{x} = \frac{\sum x}{n} = \frac{64}{10} = 6.4$

$\bar{y} = \frac{\sum y}{n} = \frac{40}{8} = 5.0$

x	y	$(x-6.4)$	$(x-6.4)^2$	$(y-5.0)$	$(y-5.0)^2$
5	2	-1.4	1.96	-3	9
6	3	-0.4	0.16	-2	4
8	6	1.6	2.56	1	1
1	8	-5.4	29.16	3	9
12	1	5.6	31.36	4	16
4	10	-2.4	5.76	5	25
3	2	-3.4	11.56	-3	9
9	8	2.6	6.76	3	9
6		-0.4	0.16		
10		3.6	12.96		
$\sum x = 64$	$\sum y = 40$		$\sum 102.4$		$\sum 82$

$$S_1^2 = \frac{\sum(x-\bar{x})^2}{n_1-1} = \frac{102.4}{9} = 11.38$$

$$S_2^2 = \frac{\sum(y-\bar{y})^2}{n_2-1} = \frac{82}{7} = 11.71$$

At 3.29 : Table value :

$V_1 = n_2 - 1 = 7$ $V_2 = n_1 - 1 = 9$

$$|F| = \frac{S_2^2}{S_1^2} = \frac{11.71}{11.38} = 1.0289$$

$|F| = 1.0289 < 3.29$ [∴ Ho - accepted]

$\sigma_1^2 = \sigma_2^2$.

(2) A company keeps records of accidents. During a recent safety review, a random sample of 60 accidents was selected and classified by the day of the week on which they occured. [Table value is 9.488].

Day	Mon	Tue	Wed	Thu	Fri
No of accidents	8	12	9	14	17

Test whether there is any evidence that accidents are more likely on some days than others.

Soln

H_0: Accidents are equally likely to occur

H_1: Accidents are not equally likely to occur.

$$E_i = \frac{8+12+9+14+17}{5} = \frac{60}{5} = 12$$

(2)

O_i	E_i	$O_i - E_i$	$(O_i-E_i)^2/E_i$
8	12	-4	16/12
12	12	0	0
9	12	-3	9/12
14	12	2	4/12
17	12	5	25/12

$\Sigma = 54/12$

$$X^2 = \frac{\sum (O_i - E_i)^2}{E_i}$$

$$= \frac{54}{12} = 4.5$$

Degrees of freedom

$\Rightarrow n-1 = 4$

Table value = 9.488

$X^2 = 4.5 < 9.488$

$\therefore H_0$ - accepted

$\therefore$ Accidents are equally likely to occur.

(3) Out of 8000 graduates in a town 800 are females; out of 1600 graduate employees 120 are females. Use Chi-Square to determine if any distinction is made in appointment on the basis of sex. Value of Chi-square at 5% level for one degree of freedom is 3.84

	Male	Female	Total
Graduates in a town	7200	800	800
Graduate employees	1480	120	1600
Total	8680	920	9600

Soln. H_0: Appointment is independent of sex.

H_1 = Appointment is depends on sex.

Expected frequency:-

$$E(7200) = \frac{8000 \times 8680}{9600} = 7233.3$$

$$E(800) = \frac{8000 \times 920}{9600} = 766.6$$

$$E(1480) = \frac{1600 \times 8680}{9600} = 1446.6$$

$$E(120) = \frac{1600 \times 920}{9600} = 153.3$$

O_{ij}	E_{ij}	$\frac{(O_{ij} - E_{ij})^2}{E_{ij}}$
7200	7233.3	0.1533
800	766.6	1.4552
1480	1446.6	0.7711
120	153.3	7.2334
		Total = 9.613

$|\chi^2| > \chi^2_\alpha(\nu)$

$\nu = (2-1)(2-1) = 1$

$\alpha = 5\%$

$\boxed{\chi^2_{(5\%)} = 3.841}$

H_0 = is rejected

H_1 = is accepted

(4) In one sample of 10 observation, the sum of the squares of the deviations of the sample values from the sample mean was 120 and in another sample of 12 observation it was 314. Test whether this difference significant at 5% level of significance [Table value is 3.11].

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

$n_1 = 10 \qquad \Sigma(x-\bar{x})^2 = 120$

$n_2 = 12 \qquad \Sigma(y-\bar{y})^2 = 314$

$$S_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1 - 1} = \frac{120}{9} = 13.3333$$

$$S_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2 - 1} = \frac{314}{11} = 28.5455$$

$$F = \frac{S_2^2}{S_1^2} = \cancel{\frac{13.3333}{28.5455}} = \frac{28.5455}{13.3333} = 2.1409$$

Table value : At 5%

$V_1 = 10 - 1 = 9 \qquad V_2 = 12 - 1 = 11$

Table value = 3.11

$|F| = 2.1409 < 3.11 \qquad H_0 =$ accepted

$\therefore \sigma_1^2 = \sigma_2^2$

(5) Four machines A, B, C and D are used to produce a certain kind of cotton fabric. Samples of size 4 with each unit as 100 square meters are selected from the outputs of the machines at random and the number of flaws in each 100 squares meters is counted with the following result.

A	B	C	D
8	6	14	20
9	8	12	22
11	10	18	25
12	4	9	23

Qn:- Do you think that there is a significance difference in the performance of the four machines. Test 1% level of significance [F-table value is 5.29].

Soln:-

H_0: There is ~~no~~ no significant difference in the performance of 4 machines.

H_1: There is significant difference in the performance of 4 machines.

Correction Factor (C.F) : $\frac{G^2}{N} = \frac{(211)^2}{16}$ $\boxed{= 2782.56}$

SST = Total sum of square

$= 8^2 + 9^2 + 11^2 + 12^2 \ldots\ldots\ldots + 25^2 + 23^2 - C.F$

$= 3409 - 2782.56$ $\boxed{= 626.44.}$

R_1 = SSB = Sum of square between machines

$= \frac{40^2}{4} + \frac{28^2}{4} + \frac{53^2}{4} + \frac{90^2}{4} - C.F$

$\boxed{= 540.69.}$

Error R_2 = SSE = $626.44 - 540.69$ $\boxed{= 85.75}$

(4)

ANOVA TABLE

Source of Variation	Degrees of freedom	Sum of square	Mean square	F. Ratio
Between machines	$k-1$ $=4-1=3$	$R_1 = 540.69$	$\frac{540.69}{3} = 180.23$	$F = \frac{180.23}{7.14}$ $= 25.24$
Error	$N-k = 16-4$ $= 12$	$R_2 = 85.75$	$\frac{85.75}{12} = 7.14$	

At 1%

Table value :- 5.29

$F = 25.24 > 5.29$

H_0 - Rejected H_1 - accepted

∴ There is significant difference between performance of machine.

(5) An experiment was designed to study the performance of 4 different detergents for cleaning fuel injectors. The following cleaness readings were obtained with a specially designed equipment for 12 tanks of gas distributioned over 3 different models of engines.

	Engines			
Detergents	1	2	3	Total
A	45	43	51	139
B	47	46	52	145
C	48	50	55	153
D	42	37	49	128
Total	182	176	207	565

Test whether there is any significant difference (6)

(i) Among the four different engines and.
(ii) Among the detergents. [F_table value is $F_C = 5.14$ & $F_R = 8.94$].

Soln: H_0: There is no significant difference between engines & detergents.

H_1: There is significant difference between engines & detergents.

Correction factor (C.F) $= \frac{G^2}{N} = \frac{(565)^2}{12}$ $\boxed{= 26602.0833}$

Total sum of square (SST) $= 45^2 + 47^2 \ldots\ldots + 49^2 - C.F$

$= 26867 - 26602.0833$

$\boxed{= 264.9167}$

Sum of square between rows (or) Detergents $R_1 = SSR$ $= \frac{139^2}{3} + \frac{145^2}{3} + \frac{153^2}{3} + \frac{128^2}{3} - C.F$

$= 6440.33 + 7008.33 + 7803 + 5461.33 - C.F$

$= 26712.99 - 26602.0833$

$\boxed{= 110.9067}$

Sum of square between engines, $R_2 = SSC$ $= \frac{182^2}{4} + \frac{176^2}{4} + \frac{207^2}{4} - C.F$

$= 8281 + 7744 + 10712.25 - C.F$

$= 26737.25 - 26602.0833$

$\boxed{= 135.1667}$

Error (R_3) $= SST - R_1 - R_2 = 264.9167 - 110.9067 - 135.1667$ $\boxed{= 18.8433}$

ANOVA Table

Source of variation	Degrees of freedom	Sum of square	Mean square	F-test
Detergents (row)	$r-1 = 4-1 = 3$	110.9067	$\frac{110.9067}{3} = 36.9689$	$F_1 = \frac{36.9689}{3.1405} = 11.7716$
Engines (col)	$c-1 = 3-1 = 2$	135.1667	$\frac{135.1667}{2} = 67.5833$	$F_2 = \frac{67.5833}{3.1405} = 21.5199$
Error	$(r-1)(c-1)$ $3(2) = 6$	18.8433	$\frac{18.8433}{6} = 3.1405$	
Total	11	264.9167		

Table value :-

$F_C = 5.14$ $\qquad$ $F_R = 8.94$

$21.5199 > 5.14$ $\qquad$ $= 11.7716 > 8.94$

H_0 - Rejected $\qquad$ H_0 - Rejected

H_1 - Accepted $\qquad$ H_1 - Accepted

∴ There is significant difference between detergents and engines.

(7) Set up the analysis of variance for the following results of Latin Square Design at 5% level of significance

A12	C19	B10	D8
C18	B12	D6	A7
B22	D10	A5	C21
D12	A7	C27	B17

~~Sol~~

[F-table value is $F_C = F_R = F_K = 4.76$]

					Tot
	A12	C19	B10	D8	49
	C18	B12	D6	A7	43
	B22	D10	A5	C21	58
	D12	A7	C27	B17	63
Tot	64	48	48	53	213

A = 31
B = 61
C = 85
D = 36
213

H_0 :- There is no significant difference
H_1 :- There is significant difference

Corrected factor (C.F) $= \frac{G^2}{N} = \frac{(213)^2}{16}$ $\boxed{= 2835.5625}$

Total sum of square SST $\Big\}$ $= 12^2 + 18^2 + \dots\dots + 17^2 - C.F$
$= 3483 - 2835.5625$ $\boxed{= 647.4375}$

Sum of square between row $R_1 = SSR$ $\Big\}$ $= \frac{49^2}{4} + \frac{43^2}{4} + \frac{58^2}{4} + \frac{63^2}{4} - C.F$
$= 2895.75 - 2835.5625$
$\boxed{= 60.1875}$

Sum of square between column $R_2 = SSC$ $\Big\}$ $= \frac{64^2}{4} + \frac{48^2}{4} + \frac{48^2}{4} + \frac{53^2}{4} - C.F$
$= 1024 + 576 + 576 + 702.25 - C.F$
$\boxed{= 42.6875}$

Sum of square between treatments R_3 $\Big\}$ $= \frac{31^2}{4} + \frac{61^2}{4} + \frac{85^2}{4} + \frac{36^2}{4} - C.F$
$\boxed{= 465.1875}$

Error $= SST - R_1 - R_2 - R_3$
$= 647.4375 - 60.1875 - 42.6875 - 465.1875$ $\boxed{= 79.375}$

ANOVA TABLE

Source of variation	Degrees of freedom	Sum of square	Mean Square	F-Ratio
Between rows	$n-1 = 4-1 = 3$	60.1875	$= \frac{60.1875}{3} = 20.0625$	$F_1 = \frac{20.0625}{13.2292}$ $\boxed{= 1.5165}$
Between colums	$n-1 = 4-1 = 3$	42.6875	$= \frac{42.6875}{3} = 14.2292$	$F_2 = \frac{14.2292}{13.2292}$ $\boxed{= 1.0755}$
Between treatments	$n-1 = 4-1 = 3$	465.1875	$= \frac{465.1875}{3} = 155.0625$	$F_3 = \frac{155.0625}{13.2292}$ $\boxed{= 11.7212}$
Error	$(n-1)(n-2) = 3(2) = 6$	79.375	$= \frac{79.375}{6} = 13.2292$	

Tot : 15   647.4375

Table value : 4.76

$F_{E}' = 4.76$ $F_R = 4.76$ $F_K = 4.76$

$F_1 = 1.5165 < 4.76$ [H_0-accepted → there is no significant difference ~~between machines~~]

$F_2 = 1.0755 < 4.76$ [H_0- accepted → there is no significant difference].

$F_3 = 11.7212 > 4.76$ [H_0-rejected → there is significant difference].

(8) 20 pieces of cloth out of different rolls contained respectively 5, 4, 5, 6, 4, 4, 5, 6, 8, 7, 6, 5, 4, 7, 6, 5, 4, 6, 6, 6 imperfections. Ascertain whether the process is in a state of statistical control.

Soln:-

$$\bar{C} = CL = \frac{5+4+5+6+4+4+5+6+8+7+6+5+4+7+6+5+4+6+6+6}{20}$$

$= \frac{111}{20} = \boxed{5.55}$

$UCL = \bar{c} + 3\sqrt{\bar{c}} = 5.55 + 3\sqrt{5.55}$

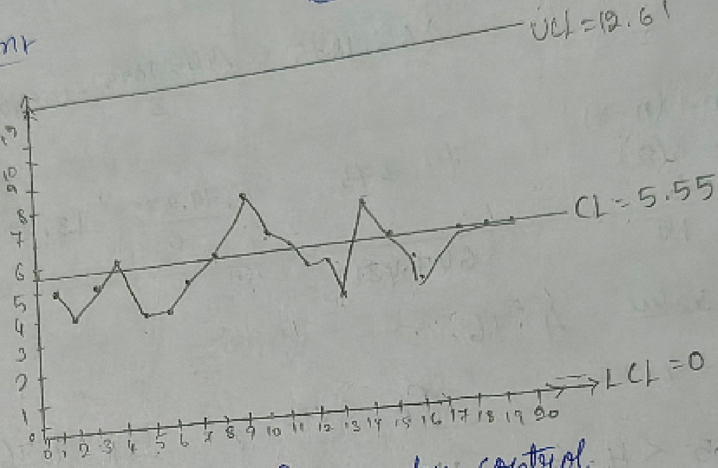
$= 5.55 + 3(2.3558)$

$\boxed{= 12.6174}$

$LCL = \bar{c} - 3\sqrt{\bar{c}} = 5.55 - 7.0674$

$\boxed{= -1.5174} \Rightarrow 0$ (counts can't be negative)

Graph Diagram:



$\therefore$ The process is under control.

(9) The following are the figures for the number of defectives of 10 samples each containing 100 items:-

Draw control chart for fraction defective (p chart) and np-chart and comment on the state of control of the process.

Sample no	1	2	3	4	5	6	7	8	9	10
No. of defectives	3	4	6	2	12	5	3	6	3	5

∴ n-p chart

$n = 100 \qquad N = 10$

$$CL = n\bar{p} = \frac{3+4+6+2+12+5+3+6+3+5}{10} \boxed{= 4.9}$$

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} \qquad \bar{p} = \frac{n\bar{p}}{n} = \frac{4.9}{100} = 0.049$$

$$= \cancel{100}\ 4.9 + 3\sqrt{4.9(1-0.049)}$$

$$= 4.9 + 3\sqrt{4.6599}$$

$$= 4.9 + 6.4760$$

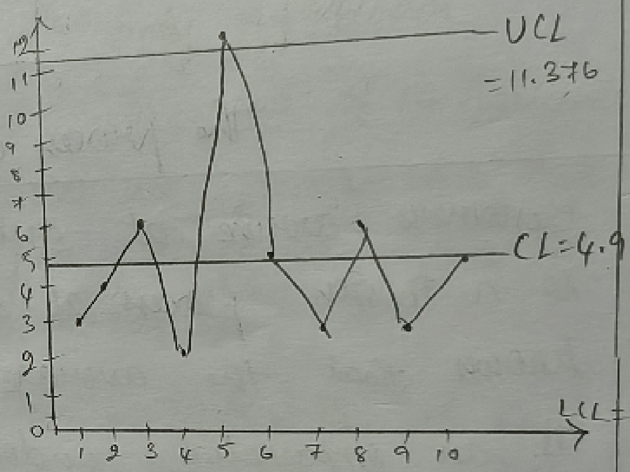
$$\boxed{= 11.376}$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$= 4.9 - 3\sqrt{4.9(1-0.049}$$

$$= 4.9 - 6.4760$$

$$\boxed{= -1.576} \Rightarrow 0$$



∴ The process is out of control.

p-chart

$$CL = \bar{p} = 0.049$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.049 + 3\sqrt{\frac{0.049(1-0.049)}{100}}$$

$$= 0.049 + 3\sqrt{0.00046599}$$

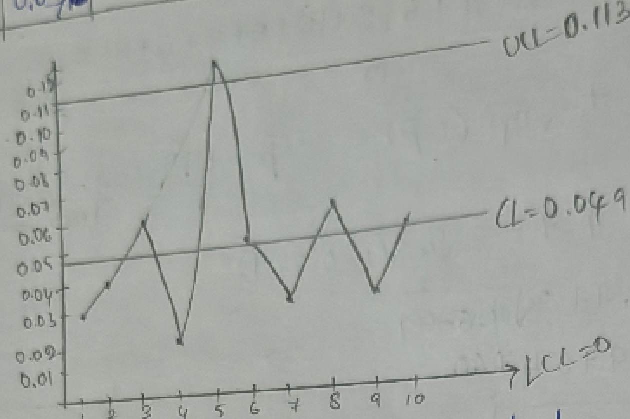
$$= 0.049 + 0.0647 \ \boxed{= 0.1137}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.049 + 3\sqrt{\frac{0.049(1-0.049)}{100}}$$

$$= 0.049 - 0.0647 = -0.0157$$

$$\boxed{= 0}$$

Sample no	1	2	3	4	5	6	7	8	9	10
proportion of defective	0.03	0.04	0.06	0.02	0.12	0.05	0.03	0.06	0.03	0.05



$\therefore$ The process out of control.

(10) Customers arrive at ATM machine of a bank according to a Poisson process at a rate of 15 per hour. It is known that the average time taken by each customer is an exponential random variable with mean 2 minutes.

(i) Find the average number of customers in the system and in the queue.

(ii) Find the average time a customers in the system and in the queue. Also find the chance that the ATM machine is idle?

Soln: $\lambda = 15$ per hour $\qquad \mu = \frac{60}{2} = 30$ per hour

i) $L_s = \frac{\lambda}{\mu - \lambda} = \frac{15}{30-15} = \frac{15}{15} = \boxed{1}$

$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{225}{450} \boxed{= 0.5}$

(8)

(ii) $W_s = \frac{1}{\mu - \lambda} = \frac{1}{30-15} = \frac{1}{15}$ $\boxed{= 0.06666}$

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{15}{30(30-15)} = \frac{15}{450} \boxed{= 0.03333}$$

P(system is ideal) = P(system is empty).

$$1 - \frac{\lambda}{\mu} = 1 - \frac{15}{30} = 1 - 0.5 \boxed{= 0.5}$$

(17) Patients arrive at a clinic with only one doctor according to Poisson distribution with mean of 15 patients per hour. the waiting room does not accommodate more than 7 patients. Service time per patient is exponential with a mean rate of 6 minutes. Find.

(i) Probability than an arriving patients will not wait.

(ii) Effective arrival rate at the clinic.

(iii) Expected time a patient spends at the queue and at the clinic.

(iv) Expected number of patients at the queue & the clinic.

Soln:-

$\lambda = 15$ per hour $\quad \mu = \frac{60}{6} = 10$ per hour

$k = 7$ (waiting) $+ 1$ (in service) $= 8$

(i) System empty $\rightarrow$ $P_0 = \frac{1 - \lambda/\mu}{1 - (\lambda/\mu)^{k+1}} = \frac{1 - 15/10}{1 - (15/10)^{8+1}}$

$$= \frac{\cancel{-}0.5}{\cancel{-}37.4433} \boxed{= 0.01335}$$

ii) $\lambda' = \mu(1-P_0)$

$= 10(1-0.01335)$

$\boxed{= 9.8665}$

iii) Expected time and iv) Expected number of patients

$$W_q = \frac{L_q}{\lambda'} \qquad W_s = \frac{L_s}{\lambda'}$$

$$L_q = L_s - \frac{\lambda}{\mu} \qquad L_s = \frac{\lambda/\mu}{1-\lambda/\mu} - \frac{(k+1)(\lambda/\mu)^{k+1}}{1-(\lambda/\mu)^{k+1}}$$

$$L_s = \frac{15/10}{1-15/10} - \frac{(8+1)(15/10)^{8+1}}{1-(15/10)^{8+1}}$$

$$= \frac{1.5}{1-1.5} - \frac{9(1.5)^9}{1-(1.5)^9}$$

$$= \frac{1.5}{-0.5} - \frac{345.9902}{-37.4433}$$

$$= -3 + 9.2403$$

$$\boxed{= 6.2403}$$

$$L_q = 6.2403 - 15/10 \boxed{= 4.7403}$$

$$W_q = \frac{L_q}{\lambda'} = \frac{4.7403}{9.8665} \boxed{= 0.4804}$$

$$W_s = \frac{L_s}{\lambda'} = \frac{6.2403}{9.8665} \boxed{= 0.6324}$$

(9)

A car service centre has 2 bays, which are operated simultaneously to service the cars. Due to space limitation, only 4 cars can wait in the queue for getting service, and other cars leave the centre without entering the queue. The cars arrive at the service centre at the average rate of 18 per day. The service time in both the bays is exponentially distributed with the average service rate of 12 cars per day.

(i) Find P_0 and effective arrival λ of cars at the car service centre.

(ii) Find the expected numbers of cars at the queue and at the car service centre.

(iii) Find the expected waiting of cars at the queue and at the cars service centre.

Soln:

$K = 2(\text{in service}) + 4(\text{waiting}) = 6$

$c = 2$

$c\mu = 2(0.5) = 1$

$\lambda = \frac{18}{24} = 0.75$ per hr $\quad \mu = \frac{12}{24} = 0.5$ per hr

i) $$P_0 = \frac{1}{\sum_{n=0}^{2-1} \frac{1}{n!}\left(\frac{0.75}{0.5}\right)^n + \frac{1}{2!}\left(\frac{0.75}{0.5}\right)^2 \sum_{n=2}^{6}\left(\frac{0.75}{1}\right)^{n-2}}$$

$$= \frac{1}{\sum_{n=0}^{1} \frac{1}{n!}(1.5)^n + \frac{(1.5)^2}{2} \sum_{n=2}^{6} (0.75)^{n-2}}$$

$$= \frac{1}{1+1.5+(1.125)(1+0.75+0.5625+0.4219+0.3164)}$$

$$= \frac{1}{5.93215} \quad \boxed{= 0.1685}$$

$$\lambda' = \lambda[1-P_k] = 0.75[1-P_k]$$

$$P_6 = \frac{1}{c!\,c^{n-c}}\left(\frac{\lambda}{\mu}\right)^n P_0 = \frac{1}{2!\,2^{6-2}}\left(\frac{0.75}{0.5}\right)^6 0.1685$$

$$= \frac{1}{32}(11.3906)0.1685$$

$$= \frac{1.9193}{32} = 0.0599$$

$$\lambda' = 0.75[1-0.0599]$$

$$\boxed{\lambda' = 0.7051}$$

$$\boxed{\rho = \frac{0.75}{2(0.5)} = 0.75}$$

ii) $$L_q = 0.1685(1.5)^2 \frac{0.75}{2(1-0.5625)}\left[1-(0.75)^4-(4)(1-0.75)(0.75)^4\right]$$

$$= 0.3791(0.8571)[1-0.3164-0.3164]$$

$$\boxed{= 0.1193}$$

$$L_s = 0.1193 + \frac{0.7051}{0.5} \quad \boxed{= 1.5295}$$

(iii) $$W_s = \frac{1}{0.7051} 1.5295 \quad \boxed{= 2.1691}$$

$$W_q = \frac{1}{0.7051} 0.1193 \quad \boxed{= 0.1691}$$