CT 303: Digital Communications Lab 5: Exercises

Name: Dhrumil Mevada ID: 201901128

Problem 1:

(a) Write a Matlab function signalx that evaluates the following signal at an arbitrary set of points:

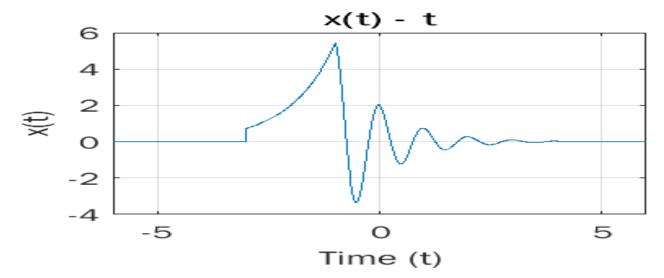
$$x(t) = 2et+2$$
 , $-3 \le t \le -1$
 $2e-t \cos 2\pi t$, $-1 \le t \le 4$
 0 . else

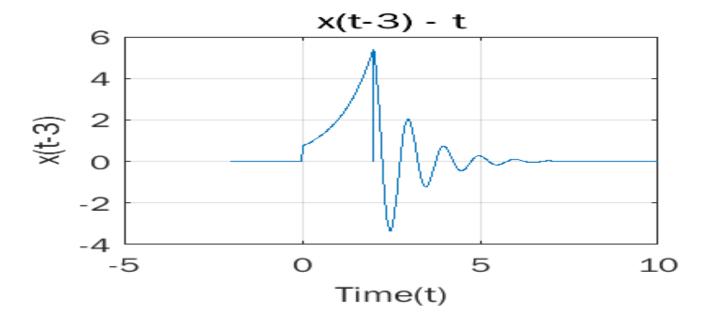
That is, given an input vector of time points, the function should give an output vector with the values of x evaluated at those time points. For time points falling outside [-3, 4], the function should return the value zero.

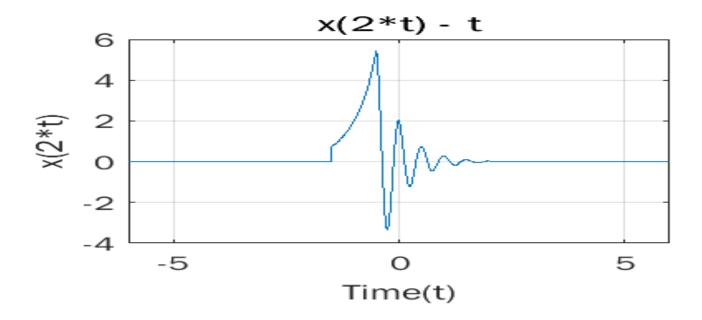
- (b) Use the function signalx to plot x(t) versus t, for $-6 \le t \le 6$. To do this, create a vector of sampling times spaced closely enough to get a smooth plot. Generate a corresponding vector using signalx. Then plot one against the other.
- (c) Use the function signalx to plot x (t 3) versus t.
- (d) Use the function signalx to plot x (3 t) versus t.
- (e) Use the function signalx to plot x(2t) versus t.
 - > Codes:-
 - %1 a and 1 b syms f(x);

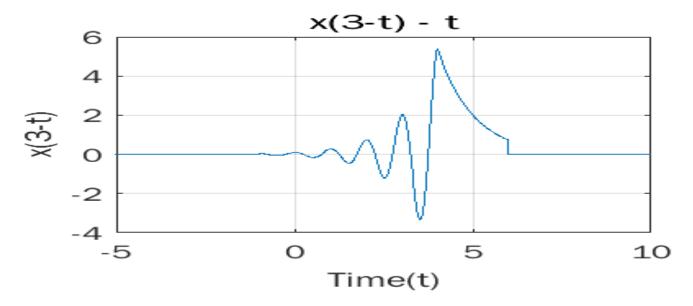
```
f(x) = piecewise(-3 < x < -1, 2*exp(x+2), -1 < x < 4, 2*exp(-1)
      x)*cos(2*pi*x), 0);
      y=linspace(-6,6,1000);
      answer_of_b = f(y);
      figure;
      plot(y,answer_of_b);
      title('x(t) - t');
      xlabel('Time (t)');
      ylabel('x(t)');
      grid on;
> %1 c
       y=linspace(-2,10,1000);
       answer_c = f(y-3);
       figure;
       plot(y,answer_c);
       title('x(t-3) - t');
       xlabel('Time(t)');
       ylabel('x(t-3)');
       grid on;
> %1 d
       y=linspace(-5,10,1000);
       answer_d = f(3-y);
       figure;
       plot(y,answer_d);
       title('x(3-t) - t');
       xlabel('Time(t)');
       ylabel('x(3-t)');
       grid on;
> %1 e
      y=linspace(-6,6,1000);
      answer_e = f(2*y);
      figure;
      plot(y,answer_e);
      title('x(2*t) - t');
      xlabel('Time(t)');
```

ylabel('x(2*t)'); grid on;









Observations:

- **1.** 1st pic -3 to -1 it increases exponentially, and then from -1 to 4, it behaves as an exponentially decreasing sinusoidal wave.
- **2.** 2^{nd} x(t-3) is right shifted x(t) by t=3.
- **3.** x(3-t) is also a shifted version of x(t). First, x(t) is time-reversed and then shifted rightwards by t=3.
- **4.** x(2t) is the compressed version of x(t) by 2 times.

Problem 2:

Suppose that we want to compute the Fourier transform of the sine pulse $u(t) = \sin \pi t I[0,1](t)$. The Fourier transform for this can be computed analytically to be

$$U(f) = (2 \cos \pi f / \pi (1 - 4f^{*}(2)) * e^{-j\pi f} (2.63)$$

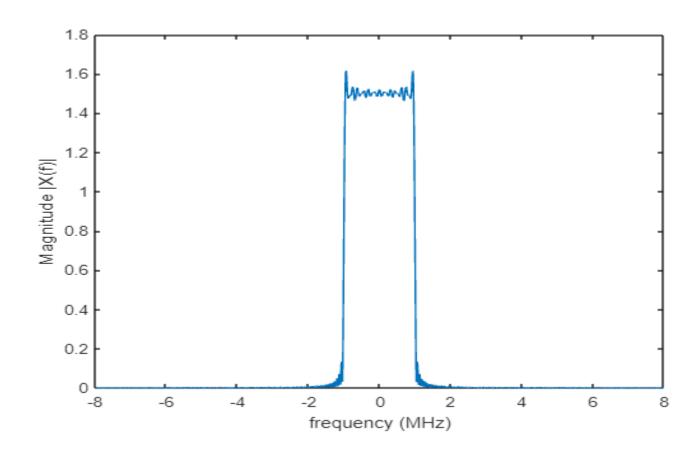
Note that U(f) has a 0/0 form at f = 1/2, but using L'Hospital's rule, we can show that U(1/2) != 0. Thus, the first zeros of U(f) are at f = $\pm 3/2$. This is a timelimited pulse and hence cannot be bandlimited, but U(f) decays as 1/f2 for f large, so we can capture most of the energy of the pulse within a suitably chosen finite frequency interval. Let us use the DFT to compute U(f) over f \in (-8, 8). This means that we set 1/(2ts) = 8, or ts = 1/16, which yields about 16 samples over the interval [0, 1] over which the signal u(t) has support. Suppose now that we want the frequency granularity to be at least fs = 1/160. Then we must use a DFT with N \geq 1/tsfs = 2560 = Nmin. In order to efficiently compute the DFT using the FFT, we choose N = 4096, the next power of 2 at least as large as Nmin.

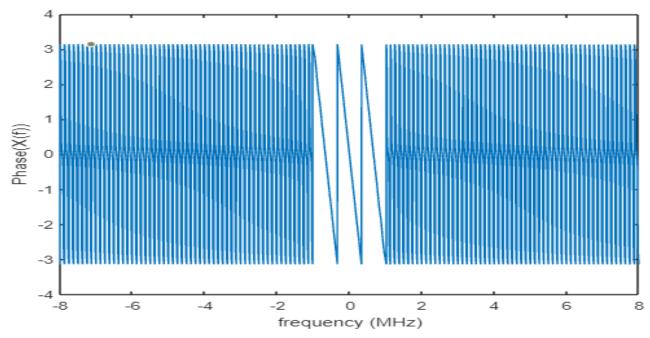
Code :-

```
clear;
close all;
clear vars;
t = -8:1/16:8;
s = 3*sinc(2*t-3);
[X,f,df] = contFT(s,-8,1/16,1e-3);
figure(1);
title('Magnitude Response of s(t)');
plot(f,abs(X));
ylabel('Magnitude |X(f)|');
xlabel('frequency (MHz)');
figure(2);
title('Phase Response of s(t)');
plot(f,angle(X));
ylabel('Phase(X(f))');
xlabel('frequency (MHz)');
```

Function:-

```
function [X,f,df] = contFT(x,tstart,dt,df_desired)
Nmin=max(ceil(1/(df_desired*dt)),length(x));
%choose FFT size to be the next power of 2
Nfft = 2^(nextpow2(Nmin));
%compute Fourier transform, centering around DC
X=dt*fftshift(fft(x,Nfft));
%achieved frequency resolution
df=1/(Nfft*dt);
%range of frequencies covered
f = ((0:Nfft-1)-Nfft/2)*df; %same as f=-1/(2*dt):df:1/(2*dt) - df
%phase shift associated with start time
X=X.*exp(-1i*2*pi*f*tstart);
end
```





Observations: -

- 1. 1st pic shows the magnitude response of 3*sinc(2t-3). It is a rectangular pulse. However, due to the sampling and truncation of the time domain sinc pulse, the rectangular pulse has some ripples at the corner. This effect is known as **Gibbs Phenomenon**.
- **2.** 2^{nd} pic shows the Phase response of 3*sinc(2t-3). The Phase plot has a meaning in the interval f = [-1,1].

Problem 3:

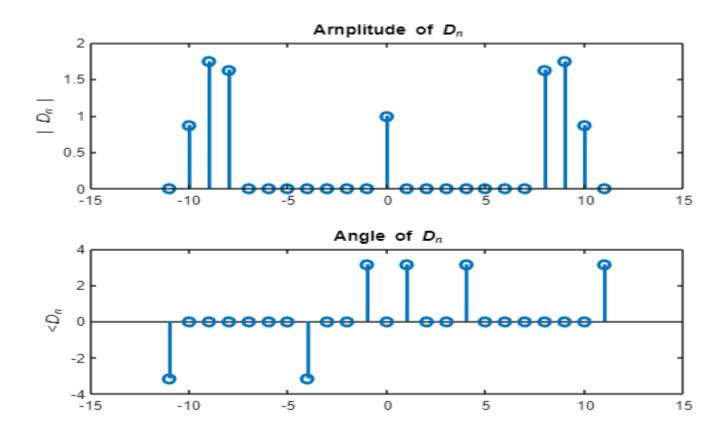
Find the exponential Fourier series for the periodic square wave w(t).

Code:-

```
clc;
clear;
close all;
clear vars;
```

```
i = sqrt(-1);
                  % Define i for complex algebra
b = 2; a = -2;
                   %determine one signal period
tol = exp(-5);
                   % set integration error tolerance
                  % length of the period
T = b-a;
                   %Number of FS coefficients on each side of
N = 11;
zero frequency
Fi = (-N:N)*2*pi/T; % Set frequency range
% now calculate D O and store it in D(N+I);
Func = @(t) funct_sqr(t/2);
D(N+1) = (1/T)^*(quad(Func,a,b,tol));
                                          % Using quad.m
integration
for i = 1 : N
%Calculate Dn for n=1,...,N(stored in D(N+2)...D(2N+I)
Func = @(t)\exp(-j^2t^*)^*t^*i/T).*funct_sqr(t/2);
D(i+N+1)=quad(Func,a,b,tol);
% Calculate Dn for n=-N,...,-1(stored in D(I)...D(N)
Func=@(t)exp(j*2*pi*t*(N+1-i)/T).*funct_sqr(t/2);
D(i)=quad(Func,a,b,tol);
end
figure (1);
subplot(211);
s1 = stem((-N:N),abs(D));
set(s1,'Linewidth',2);
ylabel('| {\itD}_{\itn} |');
title('Arnplitude of {\itD}_{\itn}');
subplot(212);
s2 = stem((-N:N), angle(D));
set(s2, 'Linewidth',2); ylabel('<\\itD\_{\itn\}');
title('Angle of {\itD}_{\itn}');
```

Plots:



Observations:-

- **1.** 1st pic shows the Fourier Series coefficients of Square Wave. The magnitude Spectrum of Dn is symmetric for a Square Pulse. Phase Spectrum is additive inverse on both the sides of origin for a Square Pulse.
- **2.** 2nd pic shows the Fourier Series coefficients of a Triangular Wave. Both the Phase and Magnitude of Dn are symmetric about the origin of a triangular Pulse.

Problem 4:

```
Code:-
clc; clear; close all; clear vars;
n = [-80:80];
%Fourier Series coefficient of x(t)
x=(1/2)*(sinc(n/2));
%Sampling Interval
ts = 1/40; %time vector
t = [-0.5:ts:4.5]; h_t = exp(-t(21:181)./2);
%Impulse Response
fs = 1/ts; h = [zeros(1,20) h_t zeros(1,20)];
%Transfer Function
H = fft(h)/fs;
%Frequency Resolution
df = fs/80;
f = [0:df:fs] - fs/2;
%Rearrange H
H1 = fftshift(H); y = x.*H1(21:181);
figure(1);
stem(n,x,'-o','linewidth',1.75,'color','red');
title('Discrete Spectrum of x(t)');
figure(2);
stem((-600:6:600),abs(H1),'o','linewidth',1.75,'color','black');
title('Magnitude of H(n/6)');
figure(3);
stem(n,abs(y),'-o','linewidth',1.75,'color','blue');
title('Discrete Spectrum of Output');
```

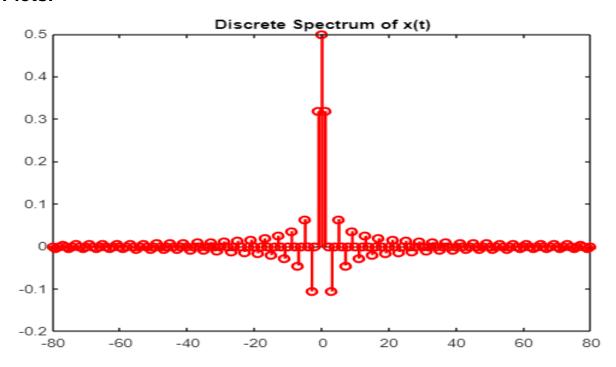


Figure 1:Fourier series co-efficient of message signal

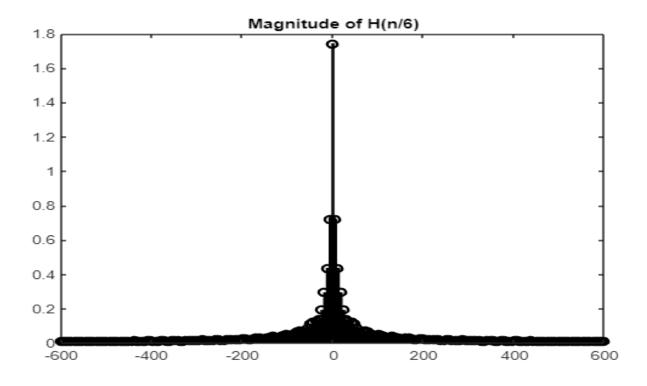


Figure 2:Fourier series co-efficient H(n/6)

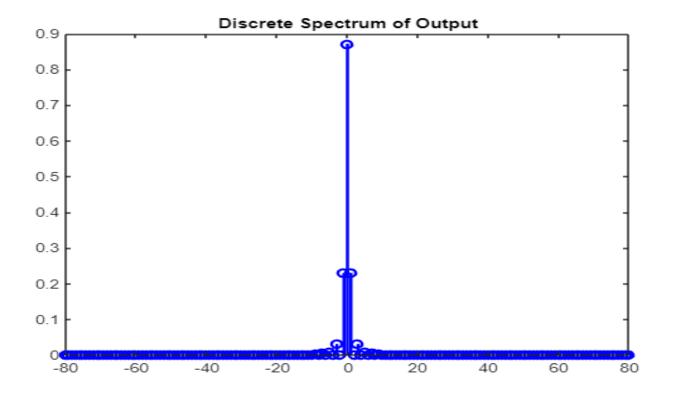


Figure 3:Fourier series co-efficient of output signal

Observations:-

- **1.** $Xn = (\frac{1}{2}) \times sinc(\frac{n}{2})$ which is plotted in figure 1.
- **2.** H(n/T), where T = 6; is plotted in figure 2.
- **3.** Yn = $(\frac{1}{2})$ x sinc($\frac{n}{2}$) x H($\frac{n}{6}$) which is plotted in figure 3.

Lab 6

Problem 1:-

a) 4.13 The periodic signal x () has a period of 2 and in the interval [0, 2] is defined as

$$x(t) = t$$
 0<=t<1
-t + 2 1<= t < 2

- a. Design an 8-level uniform PCM quantizer for this signal and plot the quantized output of this system.
- b. Plot the quantization error for this system.
- c. By calculating the power in the error signal, determine the SQNR for this system in decibels.
- d. Repeat parts a, b, and c using a 16-level uniform PCM system.

> Function:

```
function [sqnr,a_quan,code]=u_pcm(a,n) 

amax=max(abs(a)); 

a_quan=a/amax; 

b_quan=a_quan; 

d=2/n; 

q=d.*(0:n-1); 

q=q-((n-1)/2)*d; 

for i=1:n 

a_quan(find((q(i)-d/2 <= a_quan) & (a_quan <= q(i)+d/2)))=... 

q(i).*ones(1,length(find((q(i)-d/2 <= a_quan) & (a_quan <= q(i)+d/2))); 

b_quan(find( a_quan==q(i) ))=(i-1) *ones(1,length(find( a_quan==q(i)))); 

end 

a quan=a quan*amax;
```

```
nu=ceil(log2(n));
code=zeros(length(a),nu);
for i=1:length(a)
for j=nu:-1:0
if (fix(b_quan(i)/(2^j))==1)
code(i,(nu-j)) = 1;
b_quan(i) = b_quan(i) - 2^j;
end
end
end
sqnr=20*log10(norm(a)/norm(a-a_quan));
❖ Code:-
%for 8 level and 16 level quantized output:
% MATLAB script for Illustruive Problem 9. Chapter +
clear;
clc;
echo on
t = 0:0.01:2;
yval = zeros(1,length(t));
f1 = @(x)(x); %Function
f2 = @(x)(-x+2);
for i = 1:length(t)
  p = t(i);
  if (p >= 0) & (p < 1)
     yval(i) = f1(p);
  elseif (p >= 1) && (p < 2)
     yval(i) = f2(p);
  else
     yval(i) = 0;
  end
end
a = yval;
```

```
[sqnr8,aquan8,code8]=u_pcm(a,8);
[sqnr16,aquan16,code16]=u_pcm(a,16);
%Press a key to see the SQNR for N = 8
%pause
sqnr8
%pause
% Press a key to see the SONR for N = 16
sanr16
% Press a key to see the plot of the signal and its quantized
versions
%pause
figure;
plot(t,a,t,aquan8,'k-','linewidth',0.8);
legend('Original function', '8 level PCM Quantized
output', 'Location', 'south');
xlabel('Time (t)');
ylabel('f(t) and f^{-}(t));
title('8 level quantized output');
grid on;
figure;
plot(t,a,t,aquan16,'k-','linewidth',0.8);
legend('Original function', '16 level PCM Quantized
output', 'Location', 'south');
xlabel('Time (t)');
ylabel('f(t) and f^{-}(t)');
title('16 level quantized output');
grid on;
figure;
plot(t, (a-aquan8), 'k-', 'linewidth', 0.8);
xlabel('Time (t)');
ylabel('Quantization Error for 8 level');
grid on;
figure;
plot(t, (a-aquan16), 'k-', 'linewidth', 0.8);
xlabel('Time (t)');
```

```
ylabel('Quantization Error for 16 level');
grid on;
fprintf('SQNR for 8-level Quantization %f\n\n', (sqnr8));
fprintf('SQNR for 16-level Quantization %f\n\n', (sqnr16));
```

- \triangleright For 8 level quantizer the number of bits is n = 3.
- ➤ So according to the known formula of SQNR, SQNR = (6)*(n) + 1.8
- ➤ So, for the 8 level quantizer calculated SQNR is 19.8 and observed SQNR is 17.9837.
- ➤ Similarly for the 16 level quantizer the number of bits is n=4. So, for the 16 level quantizer calculated SQNR is 25.8 and observed SQNR is 24.0043.

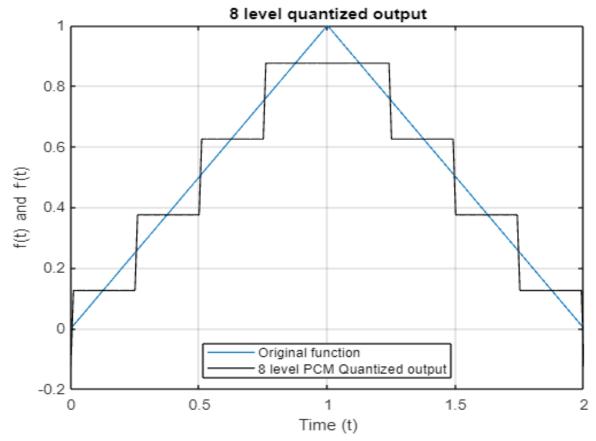


Figure 1:8 level Quantized Output

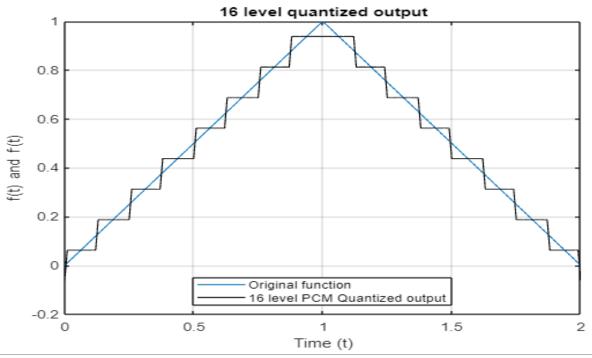


Figure 2:16 level Quantized Output

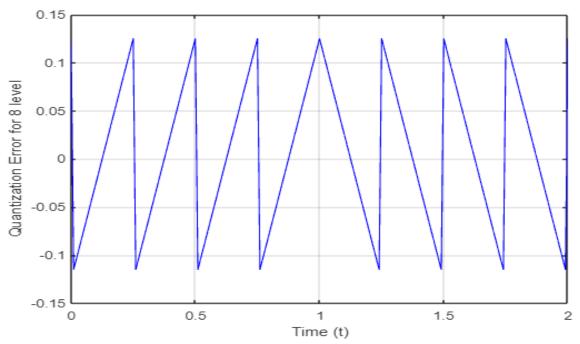


Figure 3:Quantized Error Of 8 level

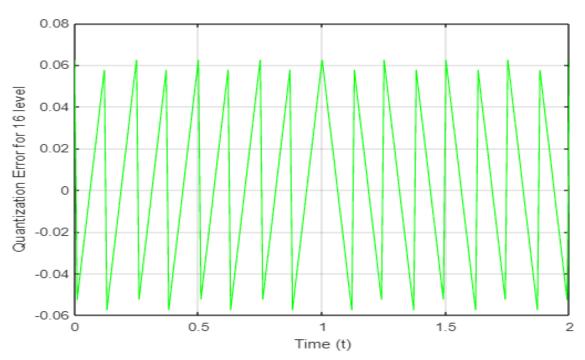


Figure 4: Quantized Error Of 8 level

Observations:

- 1. SQNR = 8.3898 dB.
- 2. Non uniform Quantizer. Step size increases as we move away form the origin.
- 3. Quantization Error is non Uniform. It is less for sample values nearer to the origin where as it increases gradually for higher sample values.

Problem 4.16:-

For 8 level non-uniform u-law PCM with u=255.

> Main Code:-

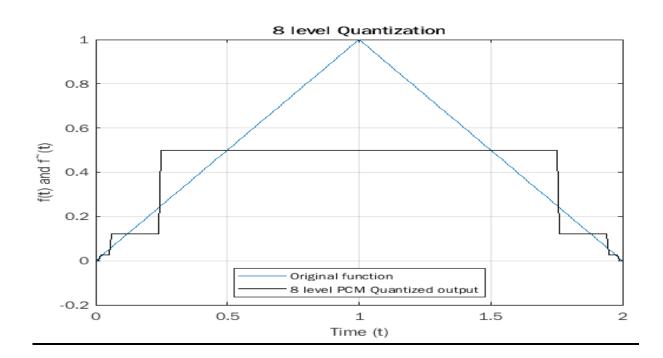
```
clear;
clc;
close all;
t = 0:0.01:2;
yval = zeros(1,length(t));
f1 = @(x)(x); %Function
f2 = @(x)(-x+2);
for i = 1:length(t)
  p = t(i);
  if( p \ge 0) && ( p < 1)
     yval(i) = f1(p);
  elseif (p >= 1) && (p < 2)
     yval(i) = f2(p);
  else
     yval(i) = 0;
  end
end
a = yval;
mew = 255;
[sqnr8,aquan8,code8]=mula_pcm(a,8,mew);
[sqnr16,aquan16,code16]=mula_pcm(a,16,mew);
sqnr8;
```

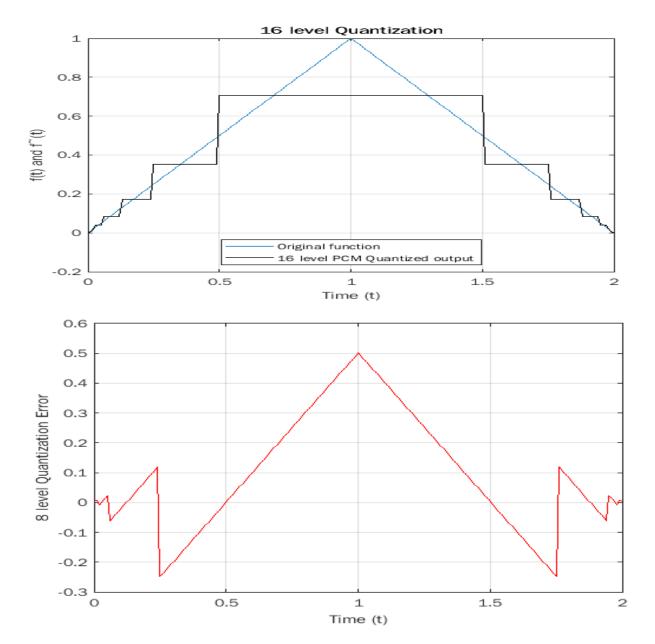
```
sqnr16;
figure;
plot(t,a,t,aquan8,'k-','linewidth',0.8);
legend('Original function', '8 level PCM Quantized
output' 'Location' 'south');
xlabel('Time (t)');
ylabel('f(t) and f^{(-)}(t));
title('8 level Quantization');
grid on;
saveas(gcf, Lab6 Q1 NonUni 8.png');
figure;
plot(t,a,t,aquan16,'k-','linewidth',0.8);
legend('Original function', '16 level PCM Quantized
output', 'Location', 'south');
xlabel('Time (t)');
ylabel('f(t) and f^{(-)}(t));
title('16 level Quantization');
grid on;
saveas(gcf,'Lab6_Q1_NonUni_16.png');
figure:
plot(t, (a-aquan8), 'r-', 'linewidth', 0.8);
xlabel('Time (t)');
ylabel('Quantization Error');
ylabel('8 level Quantization Error');
grid on;
saveas(gcf,'Lab6_Q1_NonUniError_8.png');
figure;
plot(t, (a-aquan16), 'b-', 'linewidth', 0.8);
xlabel('Time (t)');
ylabel('16 level Quantization Error');
grid on;
saveas(gcf, 'Lab6_Q1_NonUniError_16.png');
fprintf('SQNR for 8-level Quantization %f\n\n', (sqnr8));
```

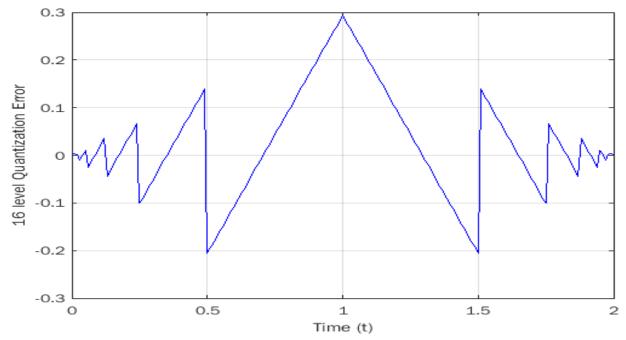
> Function:-

```
function [sqnr,a_quan,code]=mula_pcm(a,n,mu)
  [y,maximum]=mulaw(a,mu);
  [sqnr,y_q,code]=u1_pcm(y, n);
     a quan=invmulaw(y q,mu);
     a_quan=maximum*a_quan;
     sqnr=20*log10(norm(a)/norm(a-a_quan));
function [sqnr,a_quan,code]=u1_pcm(a,n)
  amax=max(abs(a));
     a_quan=a/amax;
      b_quan=a_quan;
     d=2/n;
     q=d.*(0:n-1);
     q=q-((n-1)/2)*d;
     for i=1:n
        a_quan(find((q(i)-d/2 \le a_quan) & (a_quan \le a_quan))
        q(i)+d/2))=...
        q(i).*ones(1,length(find((q(i)-d/2 <= a_quan) & (a_quan)
        \neq q(i)+d/2)));
        b_quan(find( a_quan==q(i) ))=(i-1) *ones(1,length(find(
        a_quan==q(i)));
      end
     a_quan=a_quan*amax;
     nu=ceil(log2(n));
     code=zeros(length(a),nu);
     for i=1:length(a)
     for j=nu:-1:0
     if (fix(b quan(i)/(2^i))==1)
     code(i,(nu-j)) = 1;
     b quan(i) = b quan(i) - 2^i;
      end
```

```
end
     end
     sqnr=20*log10(norm(a)/norm(a-a_quan));
function [y,a]=mulaw(x,mu)
     %MULAW mu-law nonlinearity for nonuniform PCM
     % Y=MULAW(X,MU).
     % X=input vector.
     a=max(abs(x));
     y=(log(1+mu*abs(x/a))./log(1+mu)).*sign(x);
end
function x=invmulaw(y,mu)
     %INVMULAW the inverse of mu-law nonlinearity
  %X=INVMULAW(Y,MU) Y=normalized output of the mu-law
  nonlinearity.
     x=(((1+mu).^{(abs(y))-1})./mu).*sign(y);
end
```







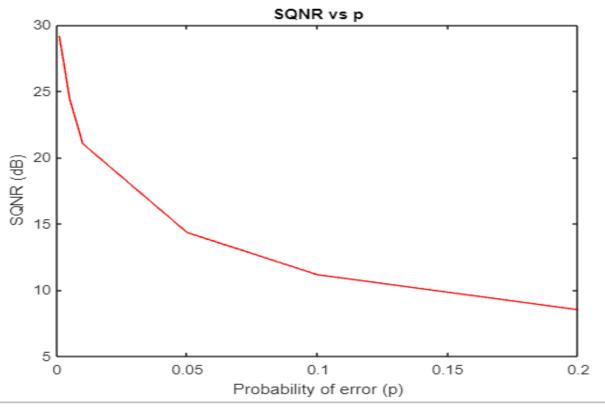
Observations:

- 1. SQNR = 14.0273 dB. SQNR increase as number of level increase.
- 2. Non Uniform Quantizer. Step size increases as we move away from the origin.
- Quantization Error is non Uniform. It is less for sample values nearer to the origin where as it increases gradually for higher sample values.
- 4. However, quantization error for a 16 level Non Uniform PCM system is less as compared to a 8 level Non Uniform PCM System.

Problem 4.15:-

> Main Code:-

```
a = randn(1,1000);
[sqnr64,aquan64,code64] = pcm(a,64);
p = [0.001 0.005 0.01 0.05 0.1 0.2];
SNR = zeros(1,6);
for i=1:6
noise = rand(1000,6) < p(i);
y = mod(code64 + noise,2);
SNR(i)=20*log10(norm(code64)/norm(code64-y));
end
figure(1);
plot(p,SNR);
title('SQNR vs p');
xlabel('Probability of error (p)');
ylabel('SQNR (dB)');
```



➤ For 6-bit-per-symbol Non-Uniform PCM with u=255.

Problem 4.18:-

➤ Main Code:-

```
a = randn(1,1000);
[sqnr64,aquan64,code64] = mula_pcm(a,64,255);
p = [0.001 \ 0.005 \ 0.01 \ 0.05 \ 0.1 \ 0.2];
SNR = zeros(1,6);
for i=1:6
noise = rand(1000,6) < p(i);
y = mod(code64 + noise, 2);
SNR(i)=20*log10(norm(code64)/norm(code64-y));
end
figure(1);
plot(p,SNR,'r');
title('SQNR vs p','r');
xlabel('Probability of error (p)');
ylabel('SQNR (dB)');
                                SQNR vs p
     30
     25
    20
  SQNR (dB)
     10
                    0.05
                                    0.1
                                                  0.15
       0
                                                                 0.2
```

Probability of error (p)

Observations: -

1. SQNR (dB) decreases as the probability of error increases as shown above.

Problem 2:-

sampling and reconstructing sum of two cosine functions of duration 2 seconds and frequencies 5 Hz and 8 Hz, respectively.

> main code:

```
clear:
clc;
td=0.002:
%original sampling rate 500 Hz
t = [0:td:2.];
%time interval of 1 second
% 1Hz+3Hz sinusoids
xsig=cos(10*pi*t) + cos(16*pi*t);
Lsig=length(xsig);
ts=0.02;
%inew sampling rate = 50Hz.
Nfactor=ts/td;
%send the signal through a 16-level uniform quantizer
[s out,sq out,sqh out,Delta,SQNR] =
sampandquant(xsig,16,td,ts);
% calculate the Fourier transforms
Lfft=2\ceil(log2(Lsig)+1);
Fmax=1/(2*td);
Faxis=linspace (-Fmax, Fmax, Lfft);
Xsig=fftshift (fft (xsig, Lfft) );
S_out=fftshift (fft (s_out, Lfft));
```

```
%Examples of sampling and reconstruction using
%a) ideal impulse train through LPF
%b) flat top pulse reconstruction through LPF
%plot the original signal and the sample signals in time
%and frequency domain
figure (1);
subplot (311); sfigla=plot (t, xsig, 'k');
hold on; sfiglb=plot (t, s_out (1:Lsig), 'b'); hold off;
set (sfigla, 'Linewidth', 2); set (sfiglb, 'Linewidth', 2.);
xlabel ( 'time ( sec) ' );
title ('Signal {\itg}_T({\itt}) and its uniform samples');
subplot (312); sfiglc=plot (Faxis, abs (Xsig));
xlabel ( ' frequency (Hz) ');
axis ([-150 150 0 300])
set (sfiglc, 'linewidth', 1); title('Spectrum of {\itg}_T({\itt})');
subplot (313); sfigld=plot (Faxis, abs (S_out));
xlabel ('frequency(Hz)');
axis ([-150 150 0 300/Nfactor])
set(sfiglc, 'linewidth', 1);
title ( ' Spectrum of {\itg}_T({\itt})' );
BW= 10:
%Bandwidth i s no I arger than I 0H z.
H_{pf} = zeros(1, Lfft); H_{pf}(Lfft/2 - BW : Lfft/2 + BW - 1) = 1;
% i deal LPF
S_recv=Nfactor * S_out .* H_lpf;
% ideal f i I ter ing
s recv= real(ifft ( fftshift ( S_recv ) ) );
% recons t ructed £ - domain
s_recv= s_recv (1 : Lsig );
figure (2)
subplot(211); sfig2a=plot (Faxis, abs (S_recv));
xlabel ( 'frequency (Hz)');
axis ([-150 150 0 300]);
title ( 'Spectrum of ideal filtering (reconstruction) ');
```

```
subplot(212);
sfig2b=plot (t, xsig, 'k-.', t, s_recv (1: Lsig), 'b');
legend ( ' original signal ' , ' reconstructed signal ' );
xlabel ( 'time(sec)');
title ( ' original signal versus ideally reconstructed signal ' );
set (sfig2b, 'linewidth', 2);
%non-ideal reconstruction
ZOH=ones (1, Nfactor);
s_ni=kron (downsample (s_out, Nfactor), ZOH);
S ni=fftshift (fft (s ni, Lfft));
S recv2=S ni.*H lpf;
%ideal filtering
s_recv2=real (ifft (fftshift (S_recv2) ) );
% reconstructed f-domain
s_recv2=s_recv2 (1: Lsig);
% reconstructed t-domain
% plot the ideally reconstructed signal in time and frequency
domain
figure(3)
subplot(211); sfig3a=plot (t, xsig, 'b', t, s_ni (1:Lsig), 'b');
xlabel ('time (sec)');
title ('original signal versus flat-top reconstruction');
subplot (212);
sfig3b=plot (t, xsig, 'b', t, s_recv2 (1:Lsig), 'b--');
legend ('original signal', 'LPF reconstruction');
xlabel ('time (sec) ');
set (sfig3a, 'Linewidth', 2); set (sfig3b, 'Linewidth', 2);
title ('original and flat-top reconstruction after LPF');
> functions :
function [ s_out , sq_out , sqh_out , Delta , SQNR ] =
sampandquant(sig in,L,td,ts)
```

```
if ( rem(ts/td, 1 ) == 0 )
nfac=round (ts/td);
p_zoh=ones (1,nfac);
s_out=downsample (sig_in, nfac) ;
[sq_out, Delta, SQNR] = uniquan(s_out, L);
s_out=upsample (s_out , nfac) ;
sqh_out=kron (sq_out, p_zoh) ;
sq_out=upsample (sq_out, nfac);
else
warning ( 'Error! ts/td is not an integer! ');
s out=[]; sq out=[]; sqh out=[]; Delta=[]; SQNR=[];
end
end
function [ q_out , Delta , SQNR ] = uniquan(sig_in , L)
sig_pmax=max(sig_in);
sig nmax=min(sig in);
Delta= (sig_pmax-sig_nmax) /L;
q_level=sig_nmax+Delta/2: Delta: sig_pmax-Delta/2; % define
Q-levels
L_sig=length(sig_in);
sigp= (sig_in-sig_nmax) /Delta+1/2;
qindex=round (sigp);
qindex=min (qindex, L);
q_out=q_level (qindex);
SQNR=20*log10 (norm(sig_in)/norm(sig_in-q_out));
end
```

