MATHEMATICS

Textbook for Class X

1062

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Foreword

The National Curriculum Framework 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that, given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

The National Council of Educational Research and Training (NCERT) appreciates the hard work done by the textbook development committee responsible for this book. We wish to thank the Chairperson of the advisory group in Science and Mathematics, Professor J.V. Narlikar and the Chief Advisors for this book, Professor P. Sinclair of IGNOU, New Delhi and Professor G.P. Dikshit (Retd.) of Lucknow University, Lucknow for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organisations which have generously permitted us to draw upon their resources, material and personnel. We are especially grateful to the members of the National Monitoring Committee, appointed by the Department of Secondary and Higher Education, Ministry of Human Resource Development under the Chairpersonship of Professor Mrinal Miri and Professor G.P. Deshpande, for their valuable time and contribution. As an organisation committed to systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

Director
National Council of Educational
Research and Training

New Delhi
15 November 2006

Rationalisation of Content in the Textbooks

In view of the COVID-19 pandemic, it is imperative to reduce content load on students. The National Education Policy 2020, also emphasises reducing the content load and providing opportunities for experiential learning with creative mindset. In this background, the NCERT has undertaken the exercise to rationalise the textbooks across all classes. Learning Outcomes already developed by the NCERT across classes have been taken into consideration in this exercise.

Contents of the textbooks have been rationalised in view of the following:

- Overlapping with similar content included in other subject areas in the same class
- Similar content included in the lower or higher class in the same subject
- Difficulty level
- Content, which is easily accessible to students without much interventions from teachers and can be learned by children through self-learning or peer-learning
- Content, which is irrelevant in the present context

This present edition, is a reformatted version after carrying out the changes given above.

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a [1]**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the [2][unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Unity of the Nation" (w.e.f. 3.1.1977)

Preface

Through the years, from the time of the Kothari Commission, there have been several committees looking at ways of making the school curriculum meaningful and enjoyable for the learners. Based on the understanding developed over the years, a National Curriculum Framework (NCF) was finalised in 2005. As part of this exercise, a National Focus Group on Teaching of Mathematics was formed. Its report, which came in 2005, highlighted a constructivist approach to the teaching and learning of mathematics.

The essence of this approach is that children already know, and do some mathematics very naturally in their surroundings, before they even join school. The syllabus, teaching approach, textbooks etc., should build on this knowledge in a way that allows children to enjoy mathematics, and to realise that mathematics is more about a way of reasoning than about mechanically applying formulae and algorithms. The students and teachers need to perceive mathematics as something natural and linked to the world around us. While teaching mathematics, the focus should be on helping children to develop the ability to particularise and generalise, to solve and pose meaningful problems, to look for patterns and relationships, and to apply the logical thinking behind mathematical proof. And, all this in an environment that the children relate to, without overloading them.

This is the philosophy with which the mathematics syllabus from Class I to Class XII was developed, and which the textbook development committee has tried to realise in the present textbook. More specifically, while creating the textbook, the following broad guidelines have been kept in mind.

- The matter needs to be linked to what the child has studied before, and to her experiences.
- The language used in the book, including that for 'word problems', must be clear, simple and unambiguous.
- Concepts/processes should be introduced through situations from the children's environment.
- For each concept/process give several examples and exercises, but not of the same kind. This ensures that the children use the concept/process again and again, but in varying contexts. Here 'several' should be within reason, not overloading the child.
- Encourage the children to see, and come out with, diverse solutions to problems.

- As far as possible, give the children motivation for results used.
- All proofs need to be given in a non-didactic manner, allowing the learner to see the flow of reason. The focus should be on proofs where a short and clear argument reinforces mathematical thinking and reasoning.
- Whenever possible, more than one proof is to be given.
- Proofs and solutions need to be used as vehicles for helping the learner develop a clear and logical way of expressing her arguments.
- All geometric constructions should be accompanied by an analysis of the construction and a proof for the steps taken to do the required construction. Accordingly, the children would be trained to do the same while doing constructions.
- Add such small anecdotes, pictures, cartoons and historical remarks at several places which the children would find interesting.
- Include optional exercises for the more interested learners. These would not be tested in the examinations.
- Give answers to all exercises, and solutions/hints for those that the children may require.
- Whenever possible, propagate constitutional values.

As you will see while studying this textbook, these points have been kept in mind by the Textbook Development Committee. The book has particularly been created with the view to giving children space to explore mathematics and develop the abilities to reason mathematically. Further, two special appendices have been given — Proofs in Mathematics, and Mathematical Modelling. These are placed in the book for interested students to study, and are only optional reading at present. These topics may be considered for inclusion in the main syllabi in due course of time.

As in the past, this textbook is also a team effort. However, what is unusual about the team this time is that teachers from different kinds of schools have been an integral part at each stage of the development. We are also assuming that teachers will contribute continuously to the process in the classroom by formulating examples and exercises contextually suited to the children in their particular classrooms. Finally, we hope that teachers and learners would send comments for improving the textbook to the NCERT.

Parvin Sinclair
G.P. Dikshit
Chief Advisors
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Contents

Constitution of India

Part IV A (Article 51 A)

Fundamental Duties

It shall be the duty of every citizen of India —

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers, wildlife and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

*(k) who is a parent or guardian, to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

Note: The Article 51A containing Fundamental Duties was inserted by the Constitution (42nd Amendment) Act, 1976 (with effect from 3 January 1977).

*(k) was inserted by the Constitution (86th Amendment) Act, 2002 (with effect from 1 April 2010).

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1 REAL NUMBERS

1.1 Introduction

In Class IX, you began your exploration of the world of real numbers and encountered irrational numbers. We continue our discussion on real numbers in this chapter. We begin with very important properties of positive integers in Sections 1.2, namely the Euclid's division algorithm and the Fundamental Theorem of Arithmetic.

Euclid's division algorithm, as the name suggests, has to do with divisibility of integers. Stated simply, it says any positive integer *a* can be divided by another positive integer *b* in such a way that it leaves a remainder *r* that is smaller than *b*. Many of you probably recognise this as the usual long division process. Although this result is quite easy to state and understand, it has many applications related to the divisibility properties of integers. We touch upon a few of them, and use it mainly to compute the HCF of two positive integers.

The Fundamental Theorem of Arithmetic, on the other hand, has to do something with multiplication of positive integers. You already know that every composite number can be expressed as a product of primes in a unique way—this important fact is the Fundamental Theorem of Arithmetic. Again, while it is a result that is easy to state and understand, it has some very deep and significant applications in the field of mathematics. We use the Fundamental Theorem of Arithmetic for two main applications. First, we use it to prove the irrationality of many of the numbers you studied in Class IX, such as $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$. Second, we apply this theorem to explore when exactly the decimal expansion of a rational number, say $\frac{p}{q} (q \neq 0)$, is terminating and when it is non-terminating repeating. We do so by looking at the prime factorisation of the denominator q of $\frac{p}{q}$. You will see that the prime factorisation of q will completely reveal the nature of the decimal expansion of $\frac{p}{q}$.

So let us begin our exploration.

1.2 The Fundamental Theorem of Arithmetic

In your earlier classes, you have seen that any natural number can be written as a product of its prime factors. For instance, $2 = 2$, $4 = 2 \times 2$, $253 = 11 \times 23$, and so on. Now, let us try and look at natural numbers from the other direction. That is, can any natural number be obtained by multiplying prime numbers? Let us see.

Take any collection of prime numbers, say 2, 3, 7, 11 and 23. If we multiply some or all of these numbers, allowing them to repeat as many times as we wish, we can produce a large collection of positive integers (In fact, infinitely many). Let us list a few :

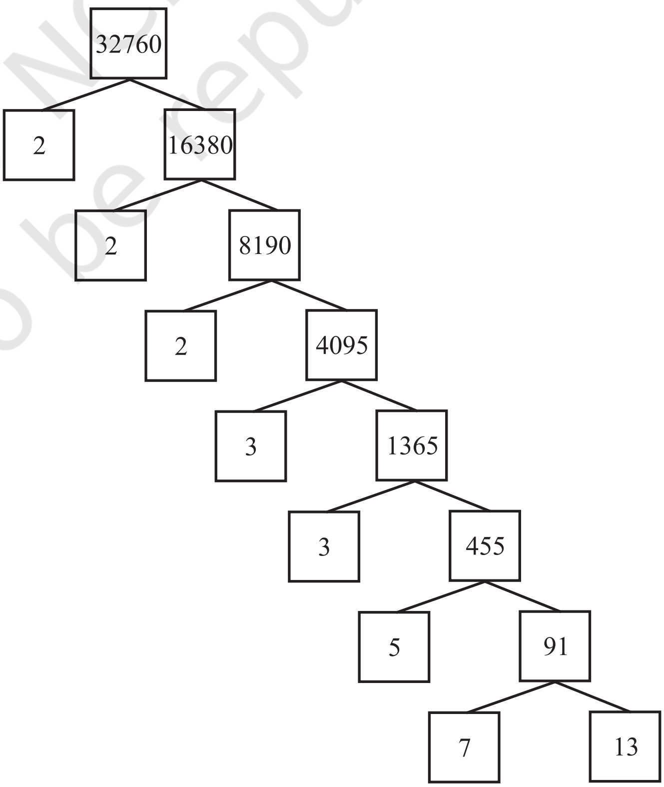
$7 \times 11 \times 23 = 1771$ $3 \times 7 \times 11 \times 23 = 5313$

$2 \times 3 \times 7 \times 11 \times 23 = 10626$ $2^3 \times 3 \times 7^3 = 8232$

$2^2 \times 3 \times 7 \times 11 \times 23 = 21252$

and so on.

Now, let us suppose your collection of primes includes all the possible primes. What is your guess about the size of this collection? Does it contain only a finite number of integers, or infinitely many? Infact, there are infinitely many primes. So, if we combine all these primes in all possible ways, we will get an infinite collection of numbers, all the primes and all possible products of primes. The question is – can we produce all the composite numbers this way? What do you think? Do you think that there may be a composite number which is not the product of powers of primes? Before we answer this, let us factorise positive integers, that is, do the opposite of what we have done so far.



We are going to use the factor tree with which you are all familiar. Let us take some large number, say, 32760, and factorise it as shown.

So we have factorised 32760 as $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 13$ as a product of primes, i.e., $32760 = 2^3 \times 3^2 \times 5 \times 7 \times 13$ as a product of powers of primes. Let us try another number, say, 123456789. This can be written as $3^2 \times 3803 \times 3607$. Of course, you have to check that 3803 and 3607 are primes! (Try it out for several other natural numbers yourself.) This leads us to a conjecture that every composite number can be written as the product of powers of primes. In fact, this statement is true, and is called the **Fundamental Theorem of Arithmetic** because of its basic crucial importance to the study of integers. Let us now formally state this theorem.

Theorem 1.1 (Fundamental Theorem of Arithmetic) : *Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.*

An equivalent version of Theorem 1.2 was probably first recorded as Proposition 14 of Book IX in Euclid's Elements, before it came to be known as the Fundamental Theorem of Arithmetic. However, the first correct proof was given by Carl Friedrich Gauss in his *Disquisitiones Arithmeticae*.

Carl Friedrich Gauss is often referred to as the 'Prince of Mathematicians' and is considered one of the three greatest mathematicians of all time, along with Archimedes and Newton. He has made fundamental contributions to both mathematics and science.

Carl Friedrich Gauss
(1777 – 1855)

The Fundamental Theorem of Arithmetic says that every composite number can be factorised as a product of primes. Actually it says more. It says that given any composite number it can be factorised as a product of prime numbers in a **'unique'** way, except for the order in which the primes occur. That is, given any composite number there is one and only one way to write it as a product of primes, as long as we are not particular about the order in which the primes occur. So, for example, we regard $2 \times 3 \times 5 \times 7$ as the same as $3 \times 5 \times 7 \times 2$, or any other possible order in which these primes are written. This fact is also stated in the following form:

The prime factorisation of a natural number is unique, except for the order of its factors.

In general, given a composite number x, we factorise it as $x = p_1 p_2 \ldots p_n$, where $p_1, p_2, \ldots, p_n$ are primes and written in ascending order, i.e., $p_1 \leq p_2 \leq \ldots \leq p_n$. If we combine the same primes, we will get powers of primes. For example,

$$32760 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 13 = 2^3 \times 3^2 \times 5 \times 7 \times 13$$

Once we have decided that the order will be ascending, then the way the number is factorised, is unique.

The Fundamental Theorem of Arithmetic has many applications, both within mathematics and in other fields. Let us look at some examples.

Example 1 : Consider the numbers 4^n, where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.

Solution : If the number 4^n, for any n, were to end with the digit zero, then it would be divisible by 5. That is, the prime factorisation of 4^n would contain the prime 5. This is not possible because $4^n = (2)^{2n}$; so the only prime in the factorisation of 4^n is 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorisation of 4^n. So, there is no natural number n for which 4^n ends with the digit zero.

You have already learnt how to find the HCF and LCM of two positive integers using the Fundamental Theorem of Arithmetic in earlier classes, without realising it! This method is also called the *prime factorisation method*. Let us recall this method through an example.

Example 2 : Find the LCM and HCF of 6 and 20 by the prime factorisation method.

Solution : We have : $6 = 2^1 \times 3^1$ and $20 = 2 \times 2 \times 5 = 2^2 \times 5^1$.

You can find HCF(6, 20) = 2 and LCM(6, 20) = $2 \times 2 \times 3 \times 5 = 60$, as done in your earlier classes.

Note that HCF(6, 20) = 2^1 = **Product of the smallest power of each common prime factor in the numbers.**

LCM (6, 20) = $2^2 \times 3^1 \times 5^1$ = **Product of the greatest power of each prime factor**, **involved in the numbers**.

From the example above, you might have noticed that HCF(6, 20) × LCM(6, 20) = 6 × 20. In fact, we can verify that **for any two positive integers *a* and *b*, HCF (*a*, *b*) × LCM (*a*, *b*) = *a* × *b*.** We can use this result to find the LCM of two positive integers, if we have already found the HCF of the two positive integers.

Example 3: Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.

Solution : The prime factorisation of 96 and 404 gives :

$$96 = 2^5 \times 3,\ \ 404 = 2^2 \times 101$$

Therefore, the HCF of these two integers is $2^2 = 4$.

Also, $$\text{LCM}(96, 404) = \frac{96 \times 404}{\text{HCF}(96, 404)} = \frac{96 \times 404}{4} = 9696$$

Example 4 : Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.

Solution : We have :

$$6 = 2 \times 3,\ \ 72 = 2^3 \times 3^2,\ 120 = 2^3 \times 3 \times 5$$

Here, 2^1 and 3^1 are the smallest powers of the common factors 2 and 3, respectively.

So, $$\text{HCF}(6, 72, 120) = 2^1 \times 3^1 = 2 \times 3 = 6$$

2^3, 3^2 and 5^1 are the greatest powers of the prime factors 2, 3 and 5 respectively involved in the three numbers.

So, $$\text{LCM}(6, 72, 120) = 2^3 \times 3^2 \times 5^1 = 360$$

Remark : Notice, $6 \times 72 \times 120 \neq \text{HCF}(6, 72, 120) \times \text{LCM}(6, 72, 120)$. So, the product of three numbers is not equal to the product of their HCF and LCM.

EXERCISE 1.1

1. Express each number as a product of its prime factors:

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

5. Check whether 6^n can end with the digit 0 for any natural number n.

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

1.3 Revisiting Irrational Numbers

In Class IX, you were introduced to irrational numbers and many of their properties. You studied about their existence and how the rationals and the irrationals together made up the real numbers. You even studied how to locate irrationals on the number line. However, we did not prove that they were irrationals. In this section, we will prove that $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and, in general, $\sqrt{p}$ is irrational, where p is a prime. One of the theorems, we use in our proof, is the Fundamental Theorem of Arithmetic.

Recall, a number 's' is called *irrational* if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Some examples of irrational numbers, with which you are already familiar, are :

$$\sqrt{2}, \sqrt{3}, \sqrt{15}, \pi, -\frac{\sqrt{2}}{\sqrt{3}}, 0.10110111011110\ldots, \text{ etc.}$$

Before we prove that $\sqrt{2}$ is irrational, we need the following theorem, whose proof is based on the Fundamental Theorem of Arithmetic.

Theorem 1.2 : *Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.*

***Proof :** Let the prime factorisation of a be as follows :

$a = p_1 p_2 \ldots p_n$, where $p_1, p_2, \ldots, p_n$ are primes, not necessarily distinct.

Therefore, $a^2 = (p_1 p_2 \ldots p_n)(p_1 p_2 \ldots p_n) = p_1^2 p_2^2 \ldots p_n^2$.

Now, we are given that p divides a^2. Therefore, from the Fundamental Theorem of Arithmetic, it follows that p is one of the prime factors of a^2. However, using the uniqueness part of the Fundamental Theorem of Arithmetic, we realise that the only prime factors of a^2 are $p_1, p_2, \ldots, p_n$. So p is one of $p_1, p_2, \ldots, p_n$.

Now, since $a = p_1 p_2 \ldots p_n$, p divides a.

We are now ready to give a proof that $\sqrt{2}$ is irrational.

The proof is based on a technique called 'proof by contradiction'. (This technique is discussed in some detail in Appendix 1).

Theorem 1.3 : $\sqrt{2}$ *is irrational.*

Proof : Let us assume, to the contrary, that $\sqrt{2}$ is rational.

* Not from the examination point of view.