Ch. 1 D.C. circuits

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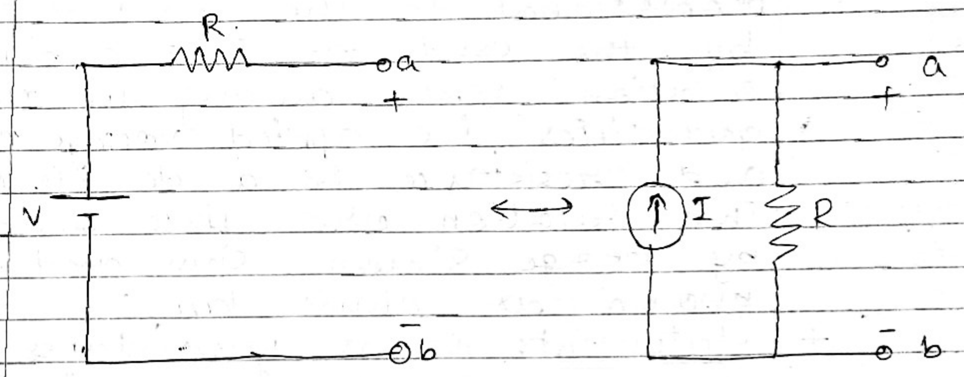
* Source Transformation :-

=> A voltage source with a series resistance can be converted into an equivalent current source with a parallel resistance.



(a) (b)

=> conversely, a current source with a parallel resistance can be converted into a voltage source with a series resistance.

=> Source conversion can be applied to controlled source as well. The controlling variable, however, must not be tampered with any way since the operation of the controlled source depends on it.

* Ohm's law :-

=> As the rate of flow of water through a pipe is directly proportional to the effective pressure and inversely proportional to the frictional resistance similarly the current flowing through a conductor is directly proportional to the potential difference across the ends of the conductor and inversely proportional to the resistance offered by the conductor. Thus a definite relation exists among the three quantities i.e. applied voltage, current and resistance in a dc circuit. This relation was first discovered by George Simon Ohm, and it is known as Ohm's law.

* **Statement:** Ohm's law states that the current flowing betn any two points of a conductor is directly proportional to the potential difference across them, provided the physical condn's do not change.

⇒ Mathematically,

$I \propto V$ or $\frac{V}{I}$ = constant

or $\frac{V_1}{I_1} = \frac{V_2}{I_2} = \dots = \frac{V_n}{I_n}$ = constant.

⇒ In other words Ohm's law can also be stated as:

→ The ratio of potential difference across any two points of a conductor to the current flowing betn them is always constant, provided the physical condn's do not change. This constant is called the resistance of the conductor.

∴ $\frac{V}{I} = R$

It can also be stated as

$V = IR$ or $I = V/R$

* **Limitations:-**

⇒ Ohm's law does not apply to all non-metallic conductors.

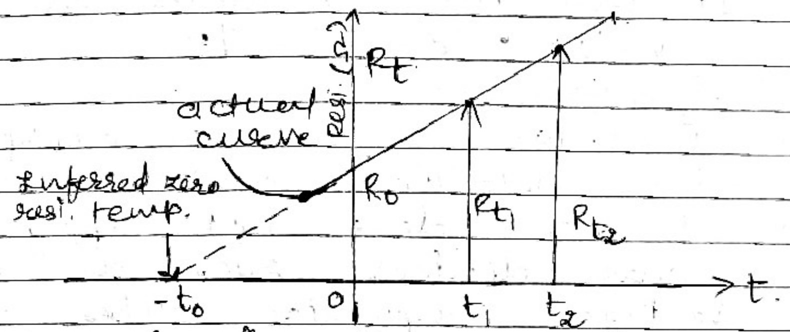
⇒ It also does not apply to non-linear devices such as zener diode, voltage regulator tubes.

⇒ It is true for metal conductors at constant temperature. If the temp. changes, the law is not applicable.

* **Temp. co-efficient of Resistance:-**

⇒ The resistance of most electrical changes with the change in temperature. The resistance of a pure metallic conductor increases with increase in temperature and decrease with decrease in temperature. The variation of resistance with temperature is governed by a property of the material called the temp. co efficient of resistance.

⇒ If the resistance of any pure metal is plotted on a temp. base Rt is found that over the normal range of temperatures, the graph is practically a straight line.



⇒ If this line is extended backward, it cuts the temp. axis at some temp. $-t_0°c$. It means that theoretically, the resi. of the pure metal is zero at $-t_0°c$. This temp. is known as inferred zero resi. temp.

⇒ From fig., using the similarity of triangles.

$$\frac{R_{t_2}}{R_{t_1}} = \frac{t_0+t_2}{t_0+t_1} \quad \text{—— (1)}$$

⇒ where R_{t_1} and R_{t_2} are the resistances at temperatures $t_1°c$ and $t_2°c$ respectively. Thus, if the resistance R_{t_1} for any temperature $t_1°c$ is known, then resistance for any other temp. t_2 can be computed from above eqⁿ provided that t_0 for that particular material is known.

⇒ let a metallic conductor having a resistance of R_0 at 0°c be heated to t°c and let its resistance at this temp. be R_t.

⇒ From eqⁿ (1), we have

$$\frac{R_t}{R_0} = \frac{t_0+t}{t_0+0}$$

$$\text{or } R_t = \left(\frac{t_0+t}{t_0}\right)R_0$$

$$= \left(1+\frac{t}{t_0}\right)R_0 = R_0 + \frac{1}{t_0}R_0 t$$

$$\text{or } R_t - R_0 = \frac{1}{t_0}R_0 t \quad \text{—— (2)}$$

⇒ change in resistance, $\Delta R = R_t - R_0$

$$= \frac{1}{t_0}R_0 t$$

$$= \alpha_0 R_0 t \quad \text{—— (3)}$$

where $\alpha_0 = 1/t_0$ is called the temp. co-efficient of resi. of the material at 0°c. The eqⁿ (3) may be written as

$$\Delta R = \alpha_0 R_0 t \quad \text{—— (4)}$$

* **Computation of resistance at dif. temp.:**

⇒ (1) If R_0 and α_0 are given, then the resistance R_t at any other temp., t°c can be calculated as under:

From eqⁿ (3),

$$\Delta R = R_t - R_0 = \alpha_0 R_0 t$$

$\therefore$ $R_t = R_0 + \alpha_0 R_0 t$

$= R_0[1+\alpha_0 t]$ —— (4).

(ii) In case R_0 is not given but α_0 is given, then relation betn the known resistance R_{t_1} at t_1°c and the unknown resistance R_{t_2} at t_2°c can be found as follows:

$\Rightarrow$ Eqn (4) can be rewritten as

$R_{t_1} = R_0[1+\alpha_0 t_1]$ —— (5).

$R_{t_2} = R_0[1+\alpha_0 t_2]$ —— (6)

$\Rightarrow$ Taking the ratio of the eqns. (5) & (6) we get

$$\frac{R_{t_2}}{R_{t_1}} = \frac{R_0[1+\alpha_0 t_2]}{R_0[1+\alpha_0 t_1]} = \frac{1+\alpha_0 t_2}{1+\alpha_0 t_1}$$

Thus, $\dfrac{R_{t_2}}{R_{t_1}} = \dfrac{1+\alpha_0 t_2}{1+\alpha_0 t_1}$ —— (7)

or $\dfrac{R_{t_2}}{R_{t_1}} \cong [(1+\alpha_0 t_2)[(1+\alpha_0 t_1)]^{-1}$

$\cong (1+\alpha_0 t_2)(1-\alpha_0 t_1)$

$\cong 1+\alpha_0 t_2 - \alpha_0 t_1 - \alpha_0^2 t_1 t_2$

$\cong 1+\alpha_0(t_2-t_1)$

or $R_{t_2} \cong R_{t_1}[1+\alpha_0(t_2-t_1)]$ —(8).

* Computation of α at diff. temp:-

(i) Relation betn α_0 and α_{t_1}:-

$\Rightarrow$ Consider a conductor of resistance R_0 at 0°c. when its temp. is raised to t_1°c its res. increases to R_{t_1}.

$\therefore$ $R_{t_1} = R_0[1+\alpha_0 t_1]$ —— (1)

$\Rightarrow$ Let us suppose that the conductor of resistance R_{t_1} at t_1°c be now cooled down to 0°c to give a resistance of the value R_0.

Then, $R_0 = R_{t_1}[1+\alpha_{t_1}(0-t_1)]$ ——

$= R_{t_1}[1-\alpha_{t_1} t_1]$ — (2).

From eqn (2),

$R_0 = R_{t_1} - R_{t_1}\cdot\alpha_{t_1}\cdot t_1$

or $\alpha_{t_1} = \dfrac{R_{t_1} - R_0}{t_1 \times R_{t_1}}$ —— (3)

Substituting the value of R_{t_1} from eqn (1) we get

$$\alpha_{t_1} = \frac{R_0[1+\alpha_0 t_1] - R_0}{t_1 \times R_{t_1}}$$

$$= \frac{R_0 + R_0\alpha_0 t_1 - R_0}{t_1 \times R_0(1+\alpha_0 t_1)}$$

$$= \frac{\alpha_0}{1+\alpha_0 t_1} \quad \text{—— (4)}$$

(ii) Relation betn α_{t_1} and α_{t_2}:-

$\Rightarrow$ From eqn (4) $\alpha_{t_1} = \dfrac{\alpha_0}{1+\alpha_0 t_1}$

$\Rightarrow$ Similarly, if the final temp. is t_2°c and the corresponding value of α is α_{t_2} then $\alpha_{t_2} = \dfrac{\alpha_0}{1+\alpha_0 t_2}$ —— (5)

$\Rightarrow$ From eqns (4) & (5),

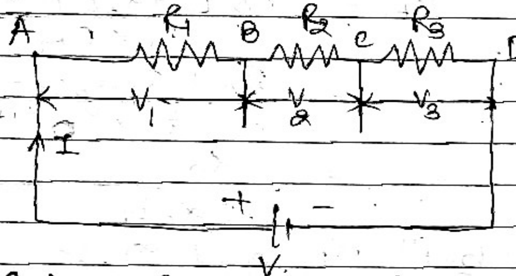
$$\frac{1}{\alpha_{t_2}} - \frac{1}{\alpha_{t_1}} = \frac{1+\alpha_0 t_2}{\alpha_0} - \frac{1+\alpha_0 t_1}{\alpha_0}$$

$\therefore \frac{1}{\alpha_{t_2}} = \frac{1}{\alpha_{t_1}} + (t_2 - t_1)$

or $\alpha_{t_2} = \frac{(t_2 - t_1)}{\frac{1}{\alpha_{t_1}} + (t_2 - t_1)}$

* Resistances in series:-

⇒ When resistors are connected end to end, so that they form one path for the flow of current, then resistors are said to be connected in series and such circuits are known as series circuits.



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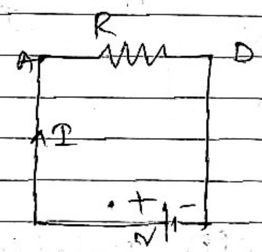


(a) series circuit (b) Equivalent ckt.

⇒ Let three resistors R_1, R_2 and R_3 be connected in series across a battery of V volts as shown in fig. (a). Obviously this p.d. across causes the flow of current of I amp. throughout all the resistors R_1, R_2 and R_3. Now, according to Ohm's law, Voltage drop across resi. R_1, $V_1 = IR_1$
Voltage drop across resi. R_2, $V_2 = IR_2$
Voltage drop across resi. $V_3 = IR_3$

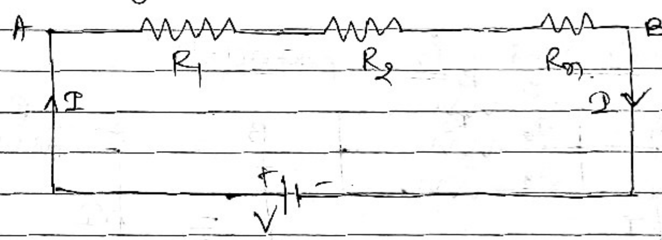
⇒ Now voltage drop across whole ckt,
$V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$
or $\frac{V}{I} = R_1 + R_2 + R_3$

⇒ According to Ohm's law the ratio V/I is R.

∴ Equivalent resi. of the series circuit
$R = R_1 + R_2 + R_3$ —— (1).

⇒ Thus, when no. of resistances are connected in series, the equivalent resistance is given by the arithmetic sum of their individual resistances
i.e. $R = R_1 + R_2 + R_3 + \ldots + R_n$.
or $R = \sum_{i=1}^{n} R_i$

* Voltage division in series circuit:-



⇒ Let 'n' resistances $R_1, R_2, \ldots R_n$ be connected in series across a battery of V Volts as shown in fig. The current flowing through all the resistance I and is same as shown in fig.

⇒ The equivalent or total resistance,

$$R = R_1 + R_2 + \dots + R_n = \sum_{i=1}^{n} R_i \quad \text{—(1)}$$

⇒ According to ohm's law, the current flowing in the series ckt, $I = V/R$ —(2)

⇒ Voltage drop across each resi. is

$V_1 = IR_1 \quad V_2 = IR_2, \dots \quad V_n = IR_n$ —(3)

⇒ Substituting the value of I in eqn (2) we get,

$$V_1 = \frac{V}{R} R_1 = \frac{R_1 V}{R}$$

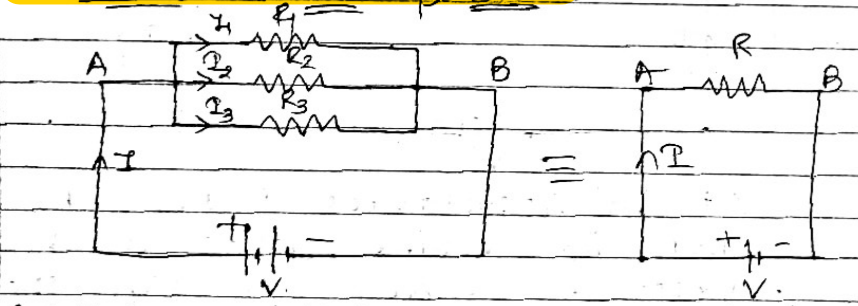
$$V_2 = \frac{V}{R} R_2 = \frac{R_2 V}{R}$$

$$V_n = \frac{V}{R} R_n = \frac{R_n V}{R} \quad \text{—(4)}$$

∴ Voltage drop across any resistance R_x in the series circuit,

$$\boxed{V_x = \frac{R_x}{R} V} \quad \text{—(5)}$$

* Resistances in parallel:-



(a) Resi connected in parallel (b) Equi. circuit

⇒ Let three resistances R_1, R_2 and R_3 be connected in parallel as shown in fig.

and the p.d. of V volts be applied across the circuit. The total current I divides into three parts : I_1 flowing through R_1, I_2 flowing through R_2 and I_3 flowing through R_3. Since the potential difference across each resi. is same and equal to the p.d. applied to the parallel circuit i.e. V volts

⇒ According in ohm's law.

current in resi. R_1, $I_1 = \frac{V}{R_1}$ —(1)

current in resi. R_2, $I_2 = \frac{V}{R_2}$ —(2)

current in resi. R_3, $I_3 = \frac{V}{R_3}$ —(3)

since $I = I_1 + I_2 + I_3$

$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

∴ Total current flowing through the ckt

$$I = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] \quad \text{—(4)}$$

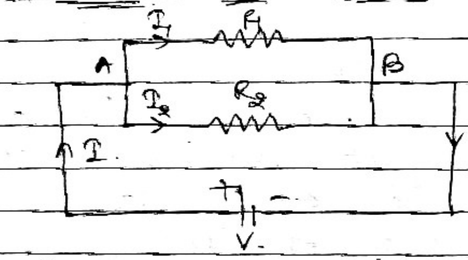
or $\frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$$\therefore \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{—(5)}$$

⇒ In general, if n resistors of resi. $R_1, R_2, R_3, \dots, R_n$ are connected in parallel, then equivalent resi. R of the ckt. is given by the eqn

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}} \quad \text{——(6)}$$

* Division of current in parallel ckt.:



⇒ Fig. shows two resistors having resistances R_1 and R_2 connected in parallel across a supply voltage of V volts. Let the current in each branch be I_1 and I_2 respectively.

⇒ According to Ohm's law, current flowing through resi. R_1,

$$I_1 = \frac{V}{R_1} \quad \text{——(i)}$$

and current flowing through resi. R_2,

$$I_2 = \frac{V}{R_2} \quad \text{——(ii)}$$

From eq^{n}s. (i) & (ii)

$$V = I_1R_1 = I_2R_2 \quad \text{——(iii)}$$

$\therefore$ $I_1R_1 = I_2R_2$

$\therefore$ $\frac{I_1}{I_2} = \frac{R_2}{R_1}$ ——(iv)

⇒ It is obvious that

$$I = I_1 + I_2 \quad \text{——(v)}$$

$\therefore$ $I_2 = I - I_1$

⇒ Substituting the value of I_2 in eq^n (iv) we get,

$$\frac{I_1}{I - I_1} = \frac{R_2}{R_1}$$

$\therefore$ $I_1R_1 = R_2(I - I_1) = R_2I - R_2I_1$

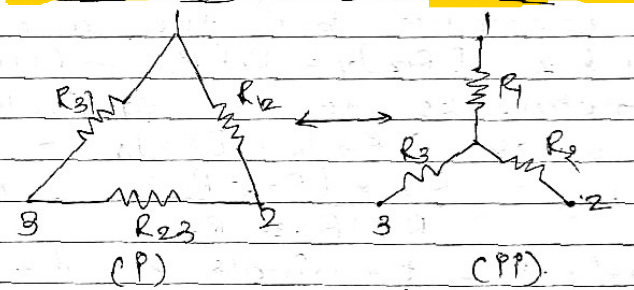
$\therefore$ $(R_1 + R_2)\, I_1 = R_2 I$

$\therefore$ $I_1 = \frac{R_2}{R_1 + R_2} I$

Similarly, $\boxed{I_2 = \frac{R_1}{R_1 + R_2} I}$

* Delta-star and star-Delta Transformation

(i) Delta-star Transformation:-



⇒ Consider a configuration shown in fig. (i) where three resistances R_{12}, R_{23} and R_{31} are connected in delta bet^n the terminals 1, 2 and 3. So far as the respective terminals are concerned, these three given resistances can be replaced by three resistances R_1, R_2 and R_3 connected in star as shown in fig. (ii).

⇒ These two arrangements will be electrically

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equivalent if the resistance as measured bet^n any pair of terminals is the same for both the arrangements. i.e. $[R_{12}]_Y = [R_{12}]_\Delta$ — (i)

* First consider delta connection:-

⇒ Bet^n terminals 1 and 2, there are two parallel paths; one having a resistance of R_{12} and the other having a resistance of $(R_{23}+R_{31})$.

∴ Resistance bet^n terminals 1 & 2, $[R_{12}]_\Delta$

$$= \frac{R_{12} \times (R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}} \quad \text{—— (ii)}$$

* Now consider star connection:-

⇒ The resistance bet^n the same terminals 1 & 2 $[R_{12}]_Y = R_1 + R_2$ — (iii)

For networks to be equivalent of each other, $[R_{12}]_Y = [R_{12}]_\Delta$

$$\text{or } R_1+R_2 = \frac{R_{12} \times (R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}} \quad \text{—— (iv)}$$

similarly for terminals 2 and 3 and 3 and 1, we get,

$$R_2+R_3 = \frac{R_{23} \times (R_{12}+R_{31})}{R_{12}+R_{23}+R_{31}} \quad \text{—— (v)}$$

$$\& \quad R_3+R_1 = \frac{R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} \quad \text{—— (vi)}$$

⇒ Now

$$R_1+R_2+R_3 = \frac{R_{12}R_{23}+R_{23}R_{31}+R_{31}R_{12}}{R_{12}+R_{23}+R_{31}} \quad \text{—— (vii)}$$

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⇒ Subtraction of eq^n (v) from eq^n (vii) give

$$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}} \quad \text{—— (1)}$$

⇒ Subtraction of eq^n (vi) from eq^n (vii) give

$$R_2 = \frac{R_{23}R_{12}}{R_{12}+R_{23}+R_{31}} \quad \text{—— (2)}$$

⇒ Subtraction of eq^n (iv) from eq^n (vii) gives

$$R_3 = \frac{R_{31}R_{23}}{R_{12}+R_{23}+R_{31}} \quad \text{—— (3)}$$

⇒ Equations (1) to (3) are the formulas for delta - star transformation.

* Star - Delta transformation:-

⇒ The transformation can be easily obtained using the eq^{n}s (1) to (3) given above

⇒ Multiplying the eq^{n}s (1) and (2), (2) & (3) and (3) & (1) and adding them together we get.

$$R_1R_2+R_2R_3+R_3R_1 = \frac{R_{12}^2R_{23}R_{31} + R_{23}^2R_{31}R_{12} + R_{31}^2R_{12}R_{13}}{(R_{12}+R_{23}+R_{31})^2}$$

$$= \frac{R_{12}R_{23}R_{31}(R_{12}+R_{23}+R_{31})}{(R_{12}+R_{23}+R_{31})^2}$$

$$= \frac{R_{12}R_{23}R_{31}}{R_{12}+R_{23}+R_{31}} \quad \text{—— (4)}$$

Now,
Division of eq^n (4) by eq^n (3) gives

$$\frac{R_1R_2+R_2R_3+R_3R_1}{R_3} = R_{12}$$

$$\therefore R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = R_1 + R_2 + \frac{R_1R_2}{R_3} \quad \text{---(5)}$$

Similarly division of eq^n (4) by eq^n (5) gives

$$\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = R_{23}$$

$$\text{Or } R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = R_2 + R_3 + \frac{R_2R_3}{R_1} \quad \text{---(6)}$$

Similarly division of eq^n (4) by eq^n (2)

$$\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = R_{31}$$

$$\therefore R_{31} = R_3 + R_1 + \frac{R_3R_1}{R_2} \quad \text{---(7)}$$

* Kirchhoff's laws :-

(1) Kirchhoff's first law :-

⇒ It is also known as Kirchhoff's current law (KCL) or point law. It states that the algebraic sum of all currents terminating at a point is zero at any instant of time.

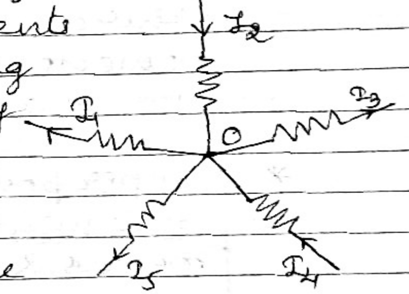
⇒ Mathematically, $\sum_{n=1}^{n} I_n = 0$ —(1)

where n denotes the no. of branches meeting at the point.

* Explanation: An algebraic sum is one in which the sign of the qty. is taken into account. If the currents entering a node are assigned +ve sign, then the currents leaving the node will be assigned −ve sign or vice versa.

⇒ The choice of the sign convention is arbitrary, but once a sign convention is chosen, it should not be changed w.r.t. any particular node while writing KCL eq^n. In applying KCL the current directed towards a node are considered positive and those directed away from a node are considered −ve.

⇒ Consider a portion of some network as shown in fig. currents I_2 and I_4 are entering the node O. Hence they are assigned +ve sign. currents I_3, I_3 and I_5 are leaving the node O and hence they are assigned −ve sign.



Applying KCL, at node O, we get

$$(-I_1) + (+I_2) + (-I_3) + (+I_4) + (-I_5) = 0$$

$$\text{or } I_2 + I_4 = I_1 + I_3 + I_5 \quad \text{---(2)}$$

or Sum of incoming currents = Sum of outgoing currents.

(2) Kirchhoff's Second law :-

⇒ It is also known as Kirchhoff's voltage law or mesh law.

⇒ It states as follows: At any instant

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of time the algebraic sum of all branch voltages around any closed loop of a electric network is zero. In other words, the algebraic sum of emfs around a closed loop of a equals the algebraic sum of ohmic or IR drops around the loop. This law can be mathematically stated as

$$\sum_{j=1}^{n} V_j = 0 \quad \text{--- (3)}$$

where V_j is the voltage in the j^{th} element of a closed loop having n branches.

* Superposition Theorem:-

⇒ statement: "In a linear bilateral network containing more than one sources of voltage or current, the total current or voltage in any branch of the network is equal to the algebraic sum of the currents or voltages in that branch caused by each source acting separately at a time while all other sources set to zero leaving behind their internal resistances in the network."

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* Application:-

⇒ Select any one of energy in the ckt.

⇒ Set all other independent sources equal to zero. That is, replace all other ideal voltage sources by short circuits and all other ideal current sources by open circuit.

⇒ Keep the dependent sources in the circuit undistributed.

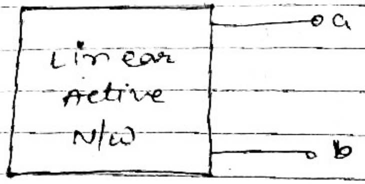
⇒ Determine the magnitude and direction of current through the desired branch as a result of a single source selected in step 1.

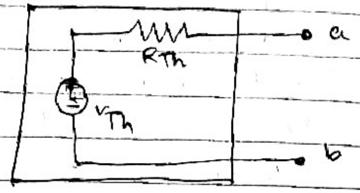
⇒ Repeat steps 1 to 4 for each source in turn until the component currents through the desired branch has been calculated due to each and every source acting alone at a time.

⇒ Algebraically and all the component currents to obtain the desired branch current. This sum is the actual current when all sources are acting simultaneously i.e. when all sources are present.

* Thevenin's Theorem:-

⇒ statement: Any linear, active, bilateral n/w consisting of independent voltages and current sources and resistors, can be





(a) Linear Active Network

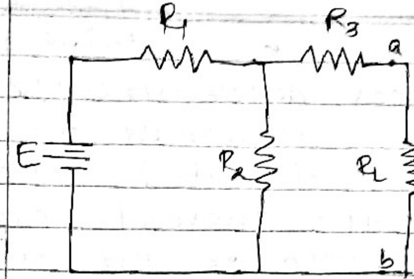
(b) Thevenin's Equivalent Network

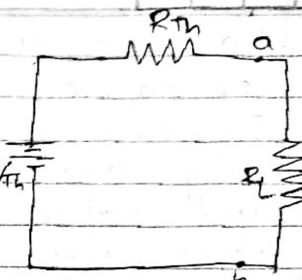
replaced at a pair of terminals a-b by a simple equivalent network consisting of a single voltage source V_{Th} in series with a single resistance R_{Th} as shown in fig. (b). The single voltage source V_{Th} is the voltage across the terminal a-b when they are open circuited. This voltage is called the Thevenin equivalent voltage, V_{Th}. The series resistance R_{Th} is given by the ratio of open circuit voltage V_{Th} and the short ckt. current at the terminals a-b of the original network. That is,

$R_{Th} = \frac{V_{Th}}{I_{sc}}$

* Application:-

⇒ Remove the resistance R_L thus creating an open circuit at terminals a-b with respect to which the Thevenin's equivalent network is desired.





(a) Network

(b) Thevenin's Equi. n/w

⇒ Find the voltage across the terminals a-b when they are open circuited. This voltage is called Thevenin voltage V_{Th} or V_{oc}.

In fig. (a) $V_{Th} = \frac{R_2}{R_1+R_2} E$ —— (i)

⇒ (i) If all the ~~resistances~~ sources in the original n/w are independent, the equivalent Thevenin resistance R_{Th} is equal to the total resistance at open circuited terminals a-b looking back into n/w when all ideal voltages are replaced by short circuits and all ideal current sources are replaced by open circuit. All non-ideal independent sources are replaced by their internal resistance.

⇒ In fig. (a) short circuit the battery and find the R_{Th} of the network as seen from the terminals a-b.

$R_{Th} = R_3 + (R_1 || R_2)$

$= R_3 + \frac{R_1 R_2}{R_1+R_2}$ —— (ii).

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(ii) In case the network contains the dependent sources determine the current I_{sc} through the terminals a-b when they are short circuited. Determine the value of the Thevenin resistance R_{Th} with the help of the relation

$$R_{Th} = \frac{V_{Th}}{I_{sc}} \quad \text{——— (iii)}$$

⇒ Draw the Thevenin's equivalent n/w betn the terminals a-b by connecting the voltage source V_{Th} in series with R_{Th} as shown in fig. (b).

⇒ The resistance R_L that was removed in step 1 is now reconnected across the terminals a-b. It is to be noted that the polarity of V_{Th} is such that it produces the current in the resistance R_L in the same direction as the original network produced it.

In fig. (b) the current through R_L is

$$I_{R_L} = \frac{V_{Th}}{R_{Th}+R_L} \quad \text{——— (iv)}.$$

* Norton's Theorem:-

⇒ statement: "Any linear, active, bilateral network consisting of voltage and current sources and resistors can be replaced at a pair of terminals A-B

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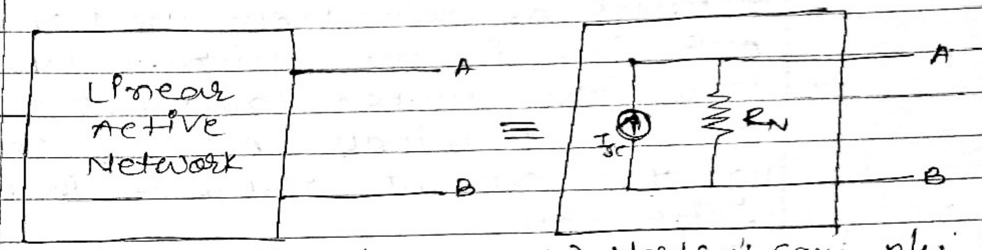


(a) Linear Active N/W (b) Norton's equi. n/w

by a single equivalent n/w consisting of a single current source I_N in parallel with a single resistance R_N.

* Application:-

⇒ Remove the resistance R_L thus creating an open circuit betn terminals A and B. Short circuit the terminals A and B w.r.t. which the Norton's equivalent network is desired.

⇒ Determine the short circuit current at these terminals. This current is I_N or I_{sc}. There is an alternative method of determining I_N from the following relation:

$$I_N = \frac{V_{Th}}{R_{Th}}$$

⇒ Redraw the network with each ideal voltage source replaced by a short ckt. and each ideal current source replaced by an open circuit. Non-ideal sources are replaced by their internal resistance

⇒ Calculate the resistance of the redrawn network as seen from the terminals A–B. This is the resistance R_N.

⇒ Draw the Norton equivalent network at the terminals A–B by connecting the current source I_N in parallel with R_N.

⇒ The resistance R_L that was removed in step 1 is now reconnected across the terminals A–B. It is to be noted that the direction of I_N is such that it circulates the current in the resistance R_L in the same direction as the original circuit circulated it.

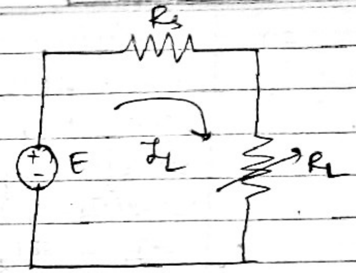
⇒ The current through the resistance R_L is calculated from the formula using current division rule.

$$I_L = \frac{R_N}{R_N + R_L} I_N$$

* Maximum power transfer theorem:-

⇒ Statement: A resistive load connected to a d.c. source receives max. power when the load resistance is equal to the source resistance. i.e. $R_L = R_s$

⇒ Proof: Consider a d.c. source with internal resistance R_s. It is supplying a current to a variable load resi. R_L as shown in fig.



⇒ The load current is given by $I_L = \frac{E}{R_s + R_L}$

The power delivered to the load is

$$P = I_L^2 R_L$$

$$= \left(\frac{E}{R_s+R_L}\right)^2 R_L$$

$$= \frac{E^2 R_L}{(R_s+R_L)^2} \quad \text{——— (1)}$$

⇒ For P to be maximum $\frac{dP}{dR_L}$ should be zero.

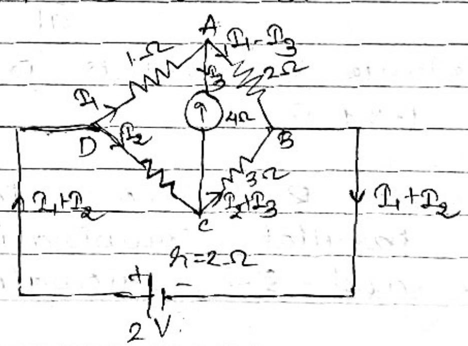
$\therefore \frac{dP}{dR_L} = E^2\left[\frac{(R_s+R_L)^2 \times 1 - 2R_L(R_s+R_L)}{(R_L+R_s)^4}\right] = 0$

$\therefore (R_s+R_L)^2 - 2R_L(R_s+R_L) = 0$

$\therefore (R_s+R_L) = 2R_L$

$\therefore \boxed{R_L = R_s}$ ——— (2)

Ex: Determine the currents in the unbalanced bridge crt. of fig. Also determine the p.d. across BD and the resistance from B to D.



⇒ Applying KVL to the closed path DACD, we get

$-I_1 - 4I_3 + 2I_2 = 0$

$\therefore I_1 - 2I_2 + 4I_3 = 0$ —— (i)

similarly, for closed path ABCA, we get

$-2(I_1 - I_3) + 3(I_2 + I_3) + 4I_3 = 0$

$\therefore 2I_1 - 3I_2 - 9I_3 = 0$ —— (ii)

similarly, for closed path DABED, we get

$-I_1 - 2(I_1 - I_3) - 2(I_1 + I_2) + 2 = 0$

$\therefore 5I_1 + 2I_2 - 2I_3 = 2$ —— (iii)

writing above eqⁿs in matrix form,

$$\begin{bmatrix} 1 & -2 & 4 \\ 2 & -3 & -9 \\ 5 & 2 & -2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

Using cramer's rule

$I_1 = \frac{30}{91} A$, $I_2 = \frac{17}{91} A$ and $I_3 = \frac{1}{91} A$

⇒ P.D. across D and B $= 2 - 2(I_1 + I_2)$

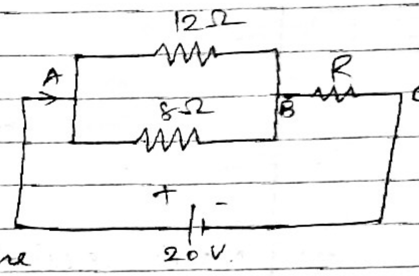
$= 2 - 2\left(\frac{30}{91} + \frac{17}{91}\right)$

$= \frac{88}{91} V.$

⇒ equi. resistance betⁿ points D and B

$= \frac{88/91}{47/91} = 1.87\ \Omega.$

Ex: A resistance R is connected in series with a parallel combination of two resi. 12 Ω and 8 Ω. Calculate R if the power dissipated in the circuit is 70 W when the applied voltage is 20 V across the circuit.



⇒ equivalent resi. of the parallel combination betⁿ AB

$R_{AB} = \frac{12 \times 8}{12 + 8} = 4.8\ \Omega.$

⇒ Total resistance of the circuit $= (R + 4.8)\ \Omega$ —— (i)

⇒ Power dissipated in the circuit = 70 W

⇒ P.D. across the circuit V = 20 V.

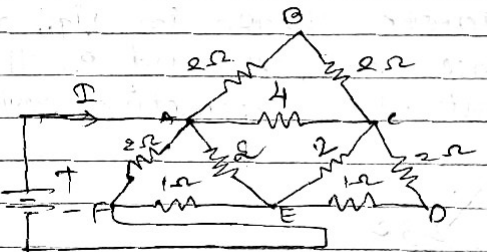
⇒ current through the ckt. $I = \frac{P}{V} = \frac{70}{20} = 3.5 A$

⇒ Total resistance of the ckt. $= V/I = \frac{20}{3.5} = 5.714$ —— (ii)

⇒ From eqⁿ (i) & (ii), we get

$R + 4.8 = 5.714 \quad \therefore R = 0.914\ \Omega.$

Ex: Determine the current supplied by the source in the circuit of fig.

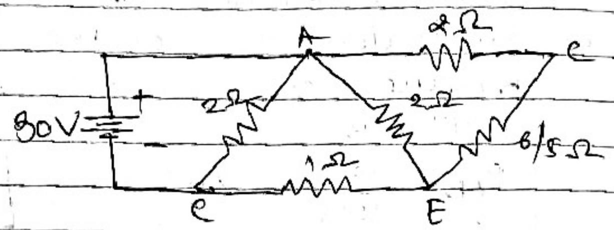


⇒ Resistance betⁿ A & C $= 4 \| (2+2)$

$= \frac{4 \times 4}{4 + 4} = 2\ \Omega.$

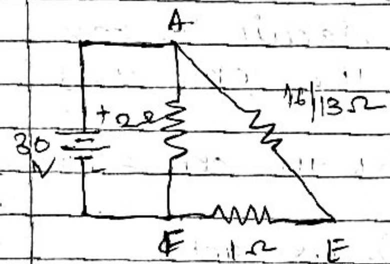
⇒ Similarly the resistance betⁿ C & E

$= 2 \| (2+1) = \frac{2\times 3}{2+3} = \frac{6}{5}\ \Omega.$



⇒ Now resistance betⁿ A & E

$= 2 \| \left(2+\frac{6}{5}\right) = 2 \| \frac{16}{5} = \frac{16}{13}\ \Omega$



Resi. betⁿ A & F

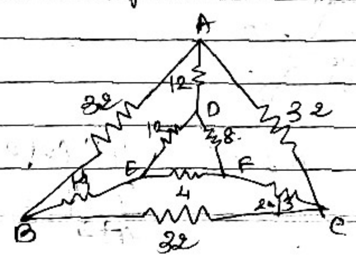
$= 2 \| \left(1+\frac{16}{13}\right)$

$= \frac{58}{55}\ \Omega$

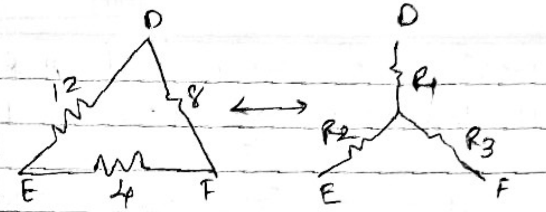
current supplied by source,

$I = \frac{30}{58/55} = 28.44$ Amp.

Ex: In the network shown in fig, determine the resistance betⁿ A and B. The numbers represent the respective resi. in ohm.



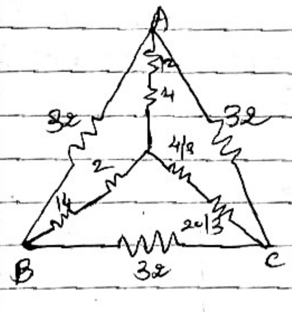
⇒

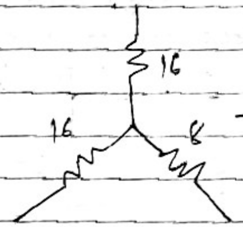


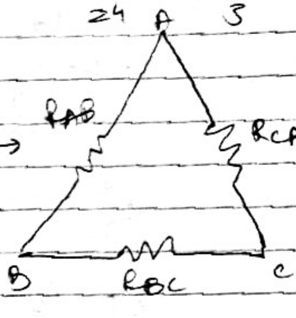
$R_1 = \frac{12\times 8}{12+8+4} = 4\ \Omega$

$R_2 = \frac{12\times 4}{12+8+4} = 2\ \Omega$

& $R_3 = \frac{4\times 8}{24} = \frac{4}{3}\ \Omega$



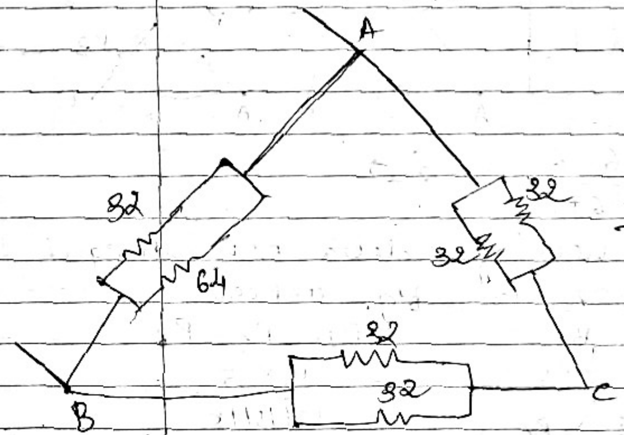


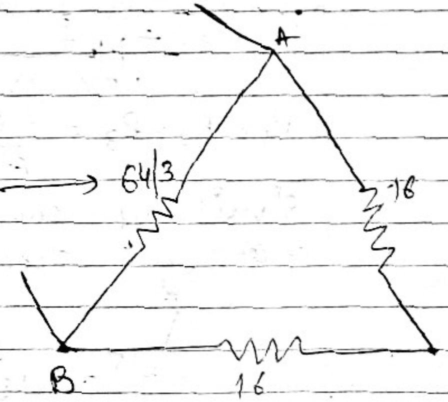


$R_{AB} = 16+16+\frac{16\times 16}{8} = 64\ \Omega$

$R_{BC} = 16+8+\frac{16\times 8}{16} = 32\ \Omega$

$R_{CA} = 16+8+\frac{16\times 8}{16} = 32\ \Omega$

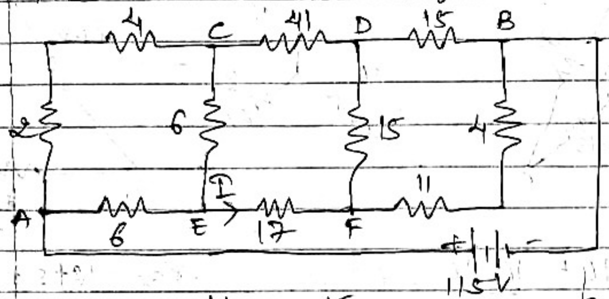




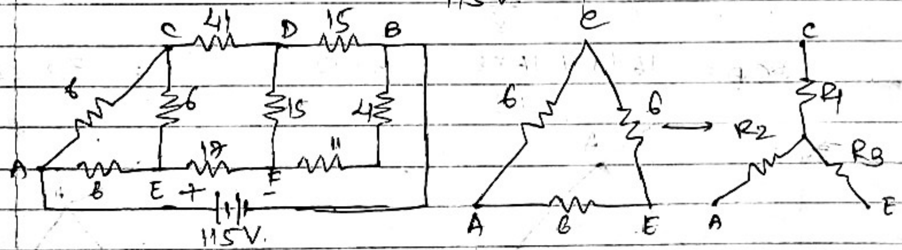
⇒ Effective resi. bet^n terminals A & B

$= \frac{64}{3} \parallel (16+16) = \frac{64}{3} \parallel 32 = 12.8\ \Omega$

* Find the current in 17 Ω resistor in the network shown in fig. using star delta conversion. The numbers indicate the resistance of each member in ohms.

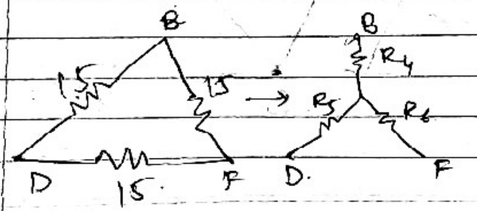


⇒



$R_1 = R_2 = R_3 = \frac{6 \times 6}{6+6+6} = 2\ \Omega$

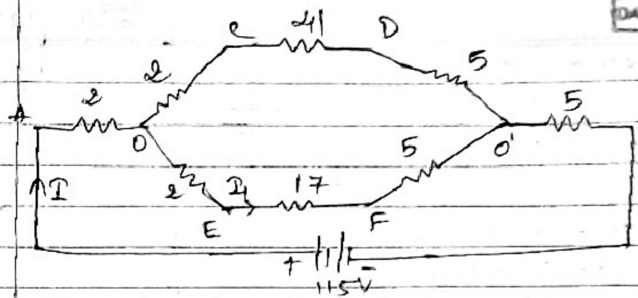
⇒ Similarly connecting delta DBF into equi. star as shown in fig. where



$R_4 = R_5 = R_6$

$= \frac{15 \times 15}{15+15+15}$

$= 5\ \Omega$



⇒ The total circuit resistance,

$R = 2+5+(2+41+5) \parallel (2+17+5)$

$= 2+5+(48)\parallel 24 = 2+5+16 = 23\ \Omega$

∴ current supplied by the supply

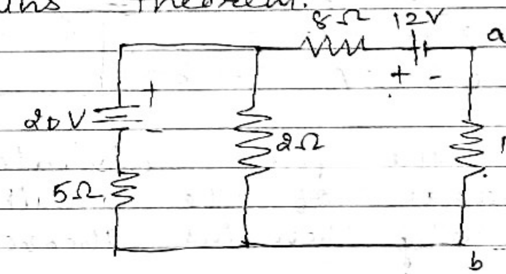
$I = \frac{V}{R} = \frac{115}{23} = 5A$

⇒ current in 17 Ω resi. $I_1 = 5 \times \frac{48}{48+24}$

$= 5 \times \frac{48}{72}$

$= 10/3 = 3.333\ A$

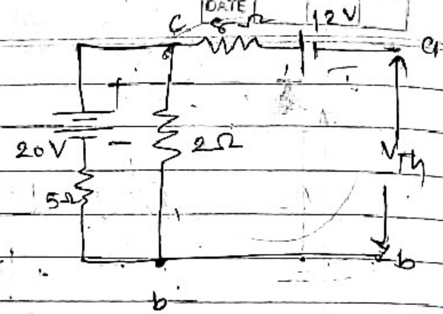
Ex: Find the current in the 10 Ω resi. of the circuit shown in fig. use Thevenin's theorem.



⇒ (i) Determination of V_{Th}:-

$\Rightarrow$ $V_{cb} = 20 \times \frac{2}{2+5}$

$= \frac{40}{7}$ V.



Now, $V_{Th} = V_{ac} + V_{cb}$

$= -12 + \frac{40}{7}$

$= -6.2857$ V.

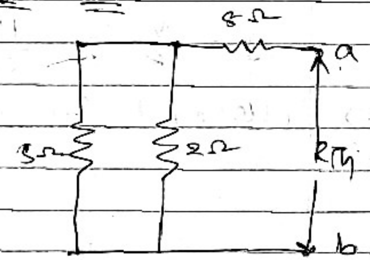
i.e. the point b is at higher potential

c(ii) Determination of R_{Th}:-

$\Rightarrow$ $R_{Th} = 8 + (2 \| 5)$

$= 8 + \frac{10}{7}$

$= 9.4286\ \Omega$.

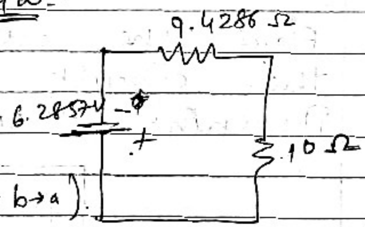


c(iii) Thevenin's equi. n/w:-

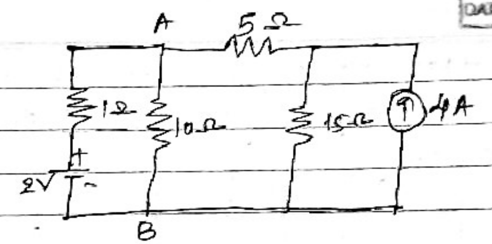
$\Rightarrow$ $I = \frac{V_{Th}}{R_{Th} + R_L}$

$= \frac{6.2857}{9.4286 + 10}$

$= 0.3235$ A (from b→a).



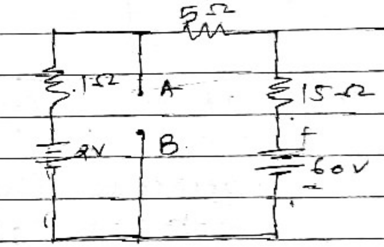
Ex: Find the power dissipated in the 10Ω resistance of the circuit shown in fig. using Thevenin's theorem.



$\Rightarrow$ Converting 4A current source into voltage source and removing the 10Ω resistance the ckt. is shown in fig.

$\Rightarrow$ current in 1Ω resi.

$= \frac{60-2}{15+5+1} = \frac{58}{21}$ A.



$V_{Th} = 2 + \frac{58}{21} \times 1$

$= 4.762$ V.

$R_{Th} = 1 \| (5+15)$

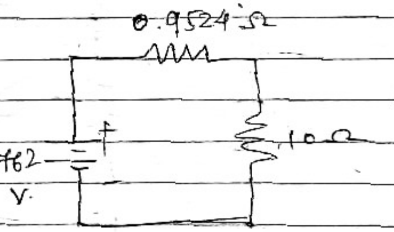
$= \frac{1 \times 20}{1+20} = 0.9524\ \Omega$

$\Rightarrow$ current in 10Ω resistance

$I = \frac{V_{Th}}{R_{Th} + R_L}$

$= \frac{4.762}{0.9524 + 10}$

$= 0.4348$ A.

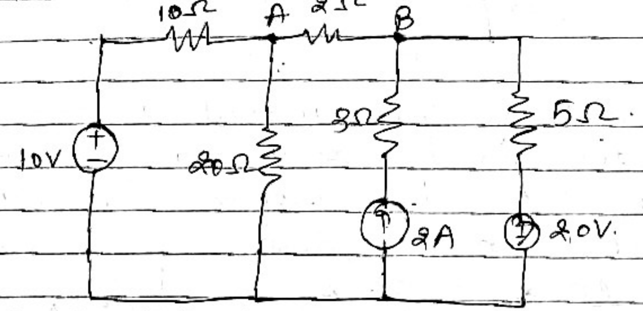


Power in 10Ω resistance $= I^2 R_L$

$= (0.4348)^2 \times 10$

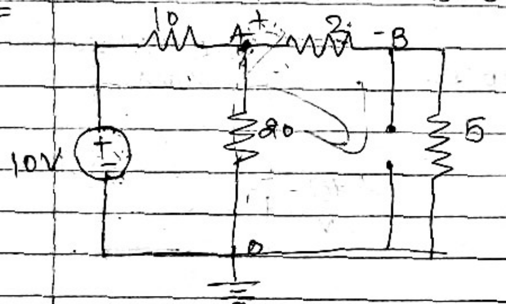
$= 1.89$ W

Ex: Find the voltage across the 2Ω resistor in fig. by using the superposition theorem.



$\Rightarrow$ Let us find the voltage across the 2Ω resistor due to individual sources. The algebraic sum of these voltages gives the total voltage across the 2Ω resistor.

Step-1: Consider 10 V source



Assume a voltage V at node A. Applying KCL we get

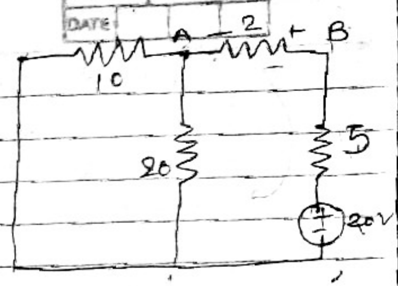
$$\frac{V}{20} + \frac{V}{7} + \frac{V-10}{10} = 0$$

$\therefore V[0.1 + 0.05 + 0.143] = 1$

$\therefore \boxed{V = 3.41 V}$

$\Rightarrow$ The voltage across 2Ω resistor due to 10 V source is $V_1 = 3.41 \times \frac{2}{7} = 0.97 V$

Step-2: Consider 20 V source to find the voltage across 2Ω resistor while the other sources are set to zero.



$\Rightarrow$ Assuming voltage V at node A as shown in fig, the KCL eq^n is

$$\frac{V-20}{7} + \frac{V}{20} + \frac{V}{10} = 0.$$

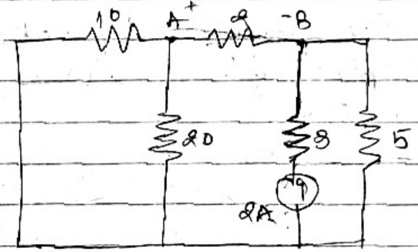
$\therefore \boxed{V = 9.76 V}$

$\Rightarrow$ The voltage across the 2Ω resistor due to 20 V source is

$$V_2 = \left(\frac{20-V}{7}\right) \times 2$$

$= 2.92$ V opposite polarity

Step-3: Consider 2A source to find the voltage across 2Ω resistor while the other sources are set equal to zero.



The current for the 2Ω resistor

$= 2 \times \frac{5}{5+8.67}$

$= 0.73$ A

$\Rightarrow$ The voltage across 2Ω resistor,

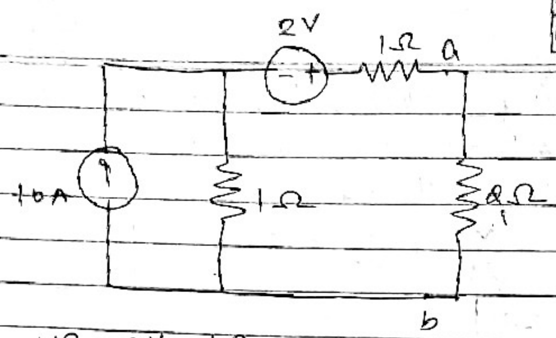
$V_3 = 2 \times 0.73 = 1.46$ V (opposite polarity)

$\Rightarrow$ Applying superposition theorem

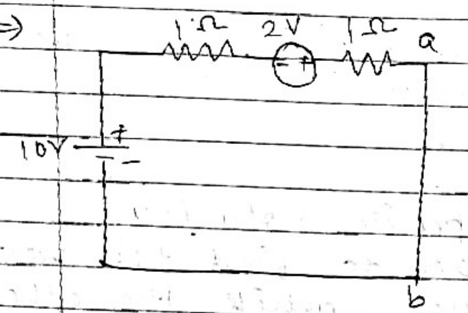
$V_{AB} = V_1 + V_2 + V_3 = 0.97 - 2.92 - 1.46$

$= -3.41$ V (B→A)

Ex: Determine the current through the 2Ω resistor of fig by using Norton's theorem.



$\Rightarrow$

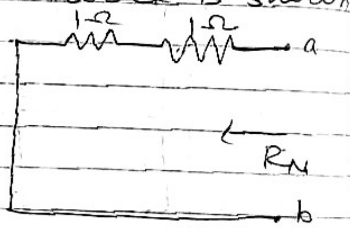


$\Rightarrow$ The terminals a and b are short ckted. The current source is also converted into the voltage source.

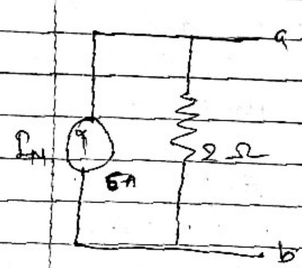
$\Rightarrow$ In fig.

$$I_N = \frac{10+2}{1+1} = 6 \text{ Amp}$$

$\Rightarrow$ To determine R_N, the network is shown in fig. after replacing voltage source by short-ckt. across the terminals a and b.



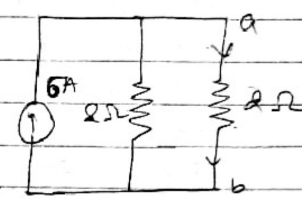
$R_N = 1+1 = 2\,\Omega$.



$\Rightarrow$ The Norton's equivalent network betn terminals a-b is drawn by connecting the current source I_N in parallel with R_N as shown in fig.

$\Rightarrow$ Now, connect the $2\,\Omega$ resistor that was removed is now reconnected across the terminals a-b as shown in fig.

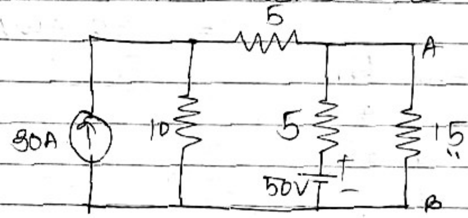
$\Rightarrow$ In fig., by current division rule.



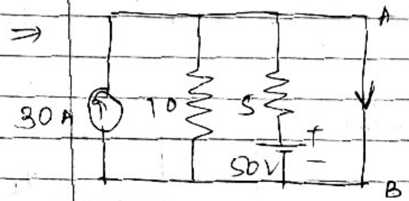
$$I = \frac{2}{2+2} \times 6$$
$$= 3\,A.$$

$\therefore$ current through $2\,\Omega$ resistor $= 3\,A$.

Ex: For the network shown in fig. draw a Norton's equivalent network and determine the current through $15\,\Omega$ resistor. All resistances are in Ohms



$\Rightarrow$

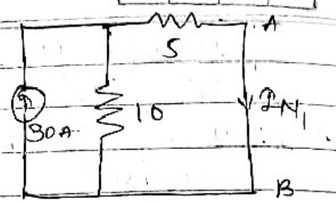


$\Rightarrow$ Since there are two sources due to which current will flow betn A and B therefore applying superposition theorem,

(i) considering only current source the ckt. is reduced as shown in fig.

In fig. $I_{N_1} = 30 \times \frac{10}{10+5} = 20\,A$

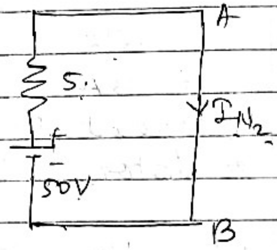
⇒ Considering only Voltage source, the ckt will be as shown in fig below.



From the fig.

$I_{N_2} = \frac{50}{5}$

$= 10$ Amp.

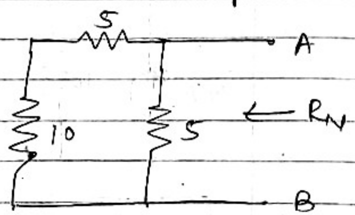


⇒ According to Superposition theorem, Norton current $I_N = I_{N_1} + I_{N_2}$

$= 20 + 10$

$= 30$ Amp.

⇒ To calculate the R_N, the ckt is redrawn bet^n the terminals A and B with sources set to zero.



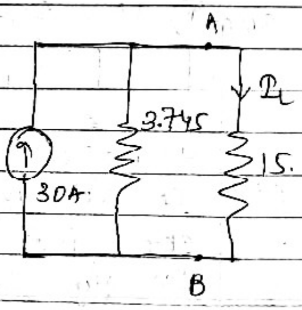
∴ $R_N = 5 \,||\, (5+10)$

$= 3.75\ \Omega$

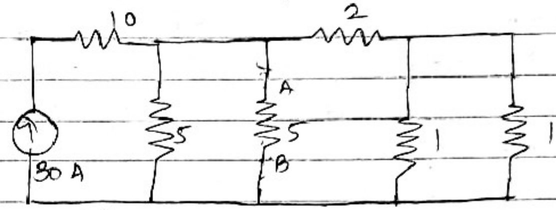
⇒ Norton's equivalent n/w is shown in fig

⇒ From fig. current through $15\ \Omega$ resistor, $I_L = 30 \times \frac{3.75}{3.75+15}$

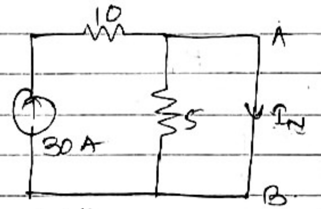
$= 6$ Amp.



Ex: Determine the current flowing through the $5\ \Omega$ resistor in the ckt of fig using Norton's theorem.



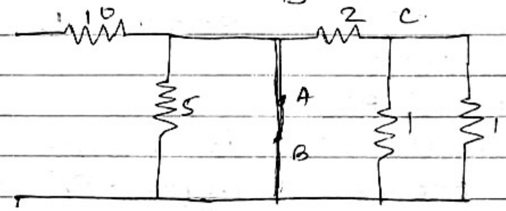
⇒



⇒ In fig.

$I_N = 30$ Amp

⇒



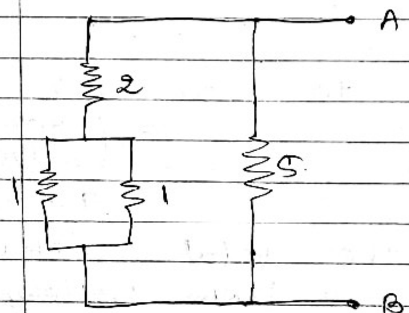
$R_N = 5 \,||\, (2 + 1 \,||\, 1)$

$= 5 \,||\, \left(2 + \frac{1 \times 1}{1+1}\right)$

$= 5 \,||\, 2.5$

$= \frac{5 \times 2.5}{5 + 2.5}$

$= 1.67\ \Omega$

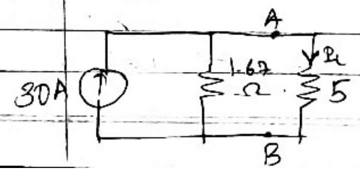


⇒ Norton's equi ckt in fig.

⇒ current through $5\ \Omega$ resi.

$I_L = 30 \times \frac{1.67}{1.67 + 5}$

$= 7.5$ A



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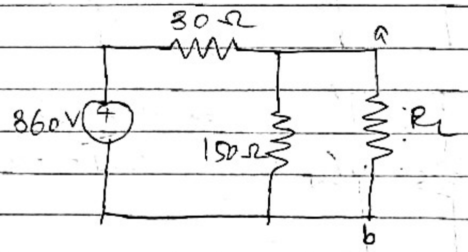
Ex: Fig. shows a circuit feeding a load resi R_L. (a) Find the value of R_L so that it draws max. power calculate this power.
(b) when R_L is adjusted for max. power transfer, what percentage of power delivered by the battery reaches R_L?



(a) ⇒ $V_{Th} = 860 \times \frac{150}{150+30} = 800 V$

$R_{Th} = 30 || 150 = 25 \Omega$.

⇒ Thevenin's equi. n/w is shown in fig.
For max. power transfer R_L should be equal to R_{Th}
i.e. $R_L = R_{Th} = 25\Omega$.
Power delivered to load $= I^2 R_L$

$$= \left(\frac{800}{25+25}\right)^2 \times 25$$

$$= 900 W.$$

(b) when $R_L = 25\Omega$, the total resistance of the ckt. as seen from battery terminals

$$= (30 + (25) || 150)$$

$$= 30 + \frac{25 \times 150}{25 + 150} = 51.428 \Omega$$

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⇒ current supplied by battery $= \frac{360}{51.428}$

$= 7A$

⇒ Power supplied by battery $= 360 \times 7$

$= 2520 W$

⇒ Percentage of power delivered to the load $= \frac{900}{2520} \times 100 = 35.7\%$.

× × ×

Ch. 8 Work, Power and Energy

* **Work**: Work is said to be done by a force when it moves a body through a certain distance. It is measured in terms of the product of the force and the distance moved in the direction of the force.

Work = Force × distance

$W = F \times d$

$1 Nm = 1 J$.

* **Power**: Power is defined as the rate of doing work or the amount of work done in unit time.

i.e. Power $= \frac{\text{Work}}{\text{Time}}$

Mathematically $P = \frac{W}{t}$.

1 watt = 1 joule/second.