BASICS OF CALCULUS

BY-PARTH SHARMA

8460442462

Basics

* **Divisibility** → (i) $\frac{0}{X} = 0$ (ii) $\frac{X}{0} = \infty$ {X= any value}

(iii) $\frac{\infty}{X} = \infty$ (iv) $\frac{X}{\infty} = 0$

* **Power** → (i) $(x)^0 = 1$ (ii) $(x)^{-0} = \frac{1}{(x)^0} = 1$

(iii) $(x)^{\infty} = \infty$ (iv) $(x)^{-\infty} = \frac{1}{(x)^{\infty}} = \frac{1}{\infty} = 0$

* **Laws of indices** →

(i) $(X)^x \cdot (X)^y = (X)^{x+y}$

(ii) $(X^x)^y = a^{xy}$

(iii) $(ab)^x = a^x \cdot b^x$

(iv) $\frac{X^x}{X^y} = (X)^{x-y}$

* **Rules of logarithm** →

(i) $\log_{10}(X) = \log(x)$ {common log}

(ii) $\log_e(X) = \ln(x)$ {natural log}

(iii) $\ln(x) = 2.303 \log(x)$ { Rel^n between natural log & common log }

(iv) $\log_{10}(0) = \infty$

(v) $\log_{10}(1) = 0$

(vi) $\log_{10}(10) = 1$ ⇐ $\{\log_x(x) = 1\}$

(vii) $\log_{10}(\infty) = \infty$

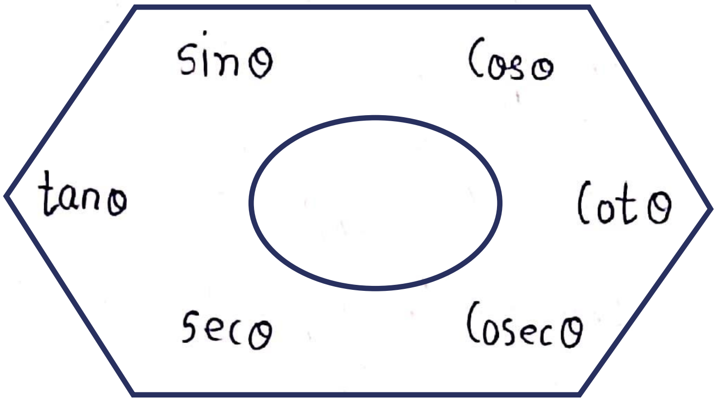
(viii) $\log_{10}(m.n) = \log_{10}(m) + \log_{10}(n)$

(ix) $\log_{10}\left(\frac{m}{n}\right) = \log_{10}(m) - \log_{10}(n)$

(x) $\log_{10}(x)^n = n.\log_{10}(x)$

(xi) $\log_a(b) = \frac{\log_{10}(b)}{\log_{10}(a)}$

* Basics of Trigonometry →



$\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$ or $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\cos\theta = \dfrac{1}{\sec\theta}$ or $\sec\theta = \dfrac{1}{\cos\theta}$

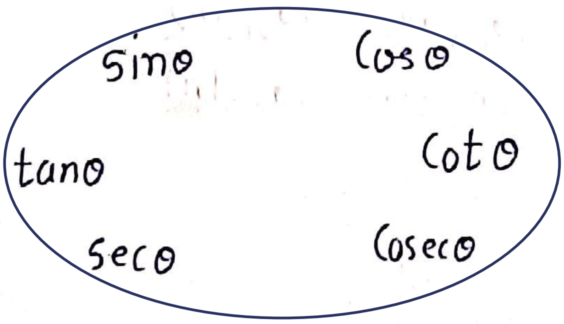
$\tan\theta = \dfrac{1}{\cot\theta}$ or $\cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

* Angle table →

θ ⇒	0°	$30^\circ\left(\frac{\pi}{6}\right)$	$45^\circ\left(\frac{\pi}{4}\right)$	$60^\circ\left(\frac{\pi}{3}\right)$	$90^\circ\left(\frac{\pi}{2}\right)$
short-cut ⇒	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$
$\sin\theta$ ⇒	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$ ⇒	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$ ⇒	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\cot\theta$ ⇒	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\operatorname{cosec}\theta$ ⇒	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1
$\sec\theta$ ⇒	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞

* Complementary angles →



$\sin(90-\theta) = \cos\theta$ or $\cos(90-\theta) = \sin\theta$

$\tan(90-\theta) = \cot\theta$ or $\cot(90-\theta) = \tan\theta$

$\sec(90-\theta) = \operatorname{cosec}\theta$ or $\operatorname{cosec}(90-\theta) = \sec\theta$

* Supplementary angles →

$\sin(180-\theta) = \sin\theta$, $\sin(180+\theta) = -\sin\theta$

$\cos(180-\theta) = -\cos\theta$, $\cos(180+\theta) = -\cos\theta$

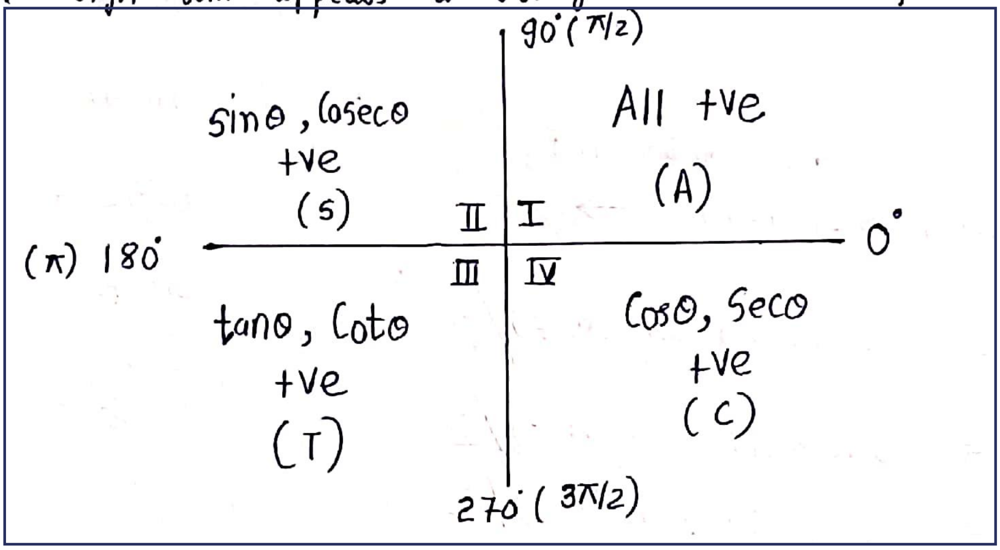
$\tan(180-\theta) = -\tan\theta$, $\tan(180+\theta) = \tan\theta$

$\cot(180-\theta) = -\cot\theta$, $\cot(180+\theta) = \cot\theta$

$\mathrm{cosec}(180-\theta) = \mathrm{cosec}\,\theta$, $\mathrm{cosec}(180+\theta) = -\mathrm{cosec}\,\theta$

$\sec(180-\theta) = -\sec\theta$, $\sec(180+\theta) = -\sec\theta$

note: + & − sign will appears according to the trignometric Qudrantes.



* Even and odd Trigonometric functions →

Even functions

$f(-x) = f(x)$

$\Downarrow$

$\cos(-x) = \cos x$

$\sec(-x) = \sec x$

odd functions

$f(-x) = -f(x)$

$\Downarrow$

$\sin(-x) = -\sin x$

$\tan(-x) = -\tan x$

$\mathrm{cosec}(-x) = -\mathrm{cosec}\,x$

$\cot(-x) = -\cot x$

note: ① Even functions are symmetric to y- axis

② odd functions are symmetric to origin.

** Double angle formule →

general formule

1. $\cos 2x = 2\cos^2 x - 1$
2. $\cos 2x = 1 - 2\sin^2 x$
3. $\cos 2x = \cos^2 x - \sin^2 x$
4. $\sin 2x = 2\sin x \cos x$

Converted to most commonly used form

Double the angle.

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

Ex → $\cos^2 2x = \frac{1+\cos 4x}{2}$

$\sin^2 4x = \frac{1-\cos 8x}{2}$

$\cos^2 5x = \frac{1+\cos 10x}{2}$

$\sin^2 7x = \frac{1-\cos 14x}{2}$

$\cos^2 \frac{x}{2} = \frac{1+\cos x}{2}$

** Conversion of multiply function into addition / subtractions →

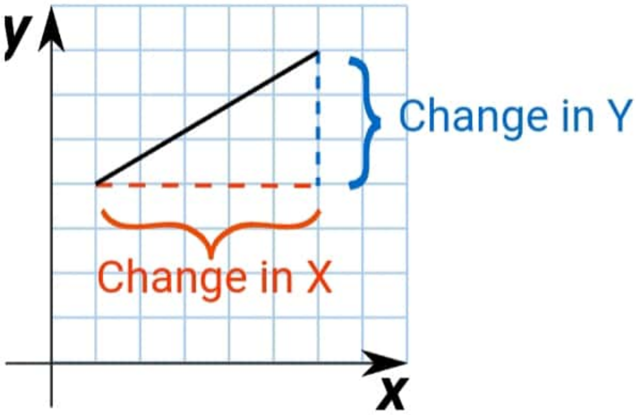
1. $\sin\alpha \cdot \cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)/2$
2. $\cos\alpha \cdot \sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)/2$
3. $\cos\alpha \cdot \cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)/2$
4. $\sin\alpha \cdot \sin\beta = -\cos(\alpha+\beta) + \cos(\alpha-\beta)/2$

** Types of functions →

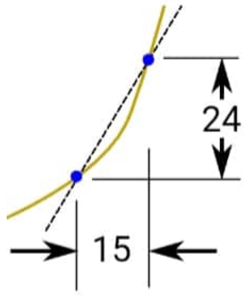
1. algebric functions ⇒ x^n
2. Trigonometric functions ⇒ $\sin x, \cos x, \tan x$ etc
3. Exponential functions ⇒ e^x
4. Inverse - Trigonometric functions ⇒ $\sin^{-1}x, \cos^{-1}x, \tan^{-1}x$ etc
5. Logarithmic functions ⇒ $\log_{10} x, \log_e x$
6. hyper bolic functions ⇒ $\sinh x, \cosh x, \tanh x$ etc.

It is all about slope!

$$\text{Slope} = \frac{\text{Change in Y}}{\text{Change in X}}$$



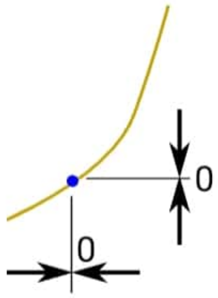
We can find an **average** slope between two points.



average slope = $\frac{24}{15}$

But how do we find the slope **at a point**?

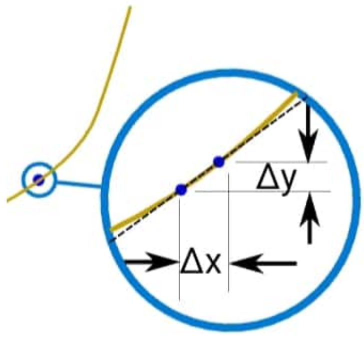
There is nothing to measure!



slope = $\frac{0}{0}$ = ???

But with derivatives we use a small difference ...

... then have it **shrink towards zero**.

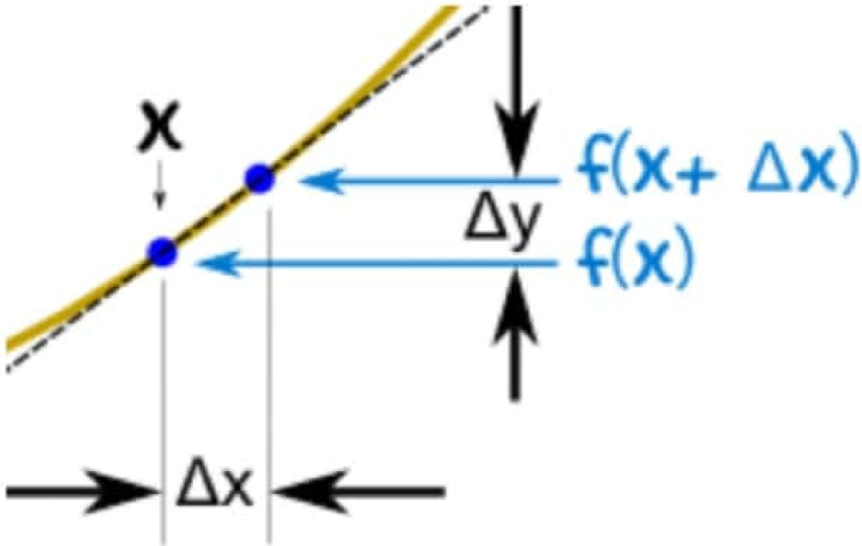


Let us Find a Derivative!

To find the derivative of a function $y = f(x)$ we use the slope formula:

$$\text{Slope} = \frac{\text{Change in Y}}{\text{Change in X}} = \frac{\Delta y}{\Delta x}$$

And (from the diagram) we see that:



x changes from x to $x+\Delta x$

y changes from $f(x)$ to $f(x+\Delta x)$

Now follow these steps:

- Fill in this slope formula: $\dfrac{\Delta y}{\Delta x} = \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$
- Simplify it as best we can
- Then make **Δx** shrink towards zero.

Example: the function $f(x) = x^2$

We know $f(x) = x^2$, and we can calculate $f(x+\Delta x)$:

Start with: $f(x+\Delta x) = (x+\Delta x)^2$

Expand $(x + \Delta x)^2$: $f(x+\Delta x) = x^2 + 2x\,\Delta x + (\Delta x)^2$

The slope formula is: $\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

Put in $f(x+\Delta x)$ and $f(x)$: $\dfrac{x^2 + 2x\,\Delta x + (\Delta x)^2 - x^2}{\Delta x}$

Simplify (x^2 and $-x^2$ cancel): $\dfrac{2x\,\Delta x + (\Delta x)^2}{\Delta x}$

Simplify more (divide through by Δx): $= 2x + \Delta x$

Then **as Δx heads towards 0** we get: $= 2x$

Result: the derivative of x^2 is $2x$

In other words, the slope at x is **2x**

We write **dx** instead of "**Δx heads towards 0**".

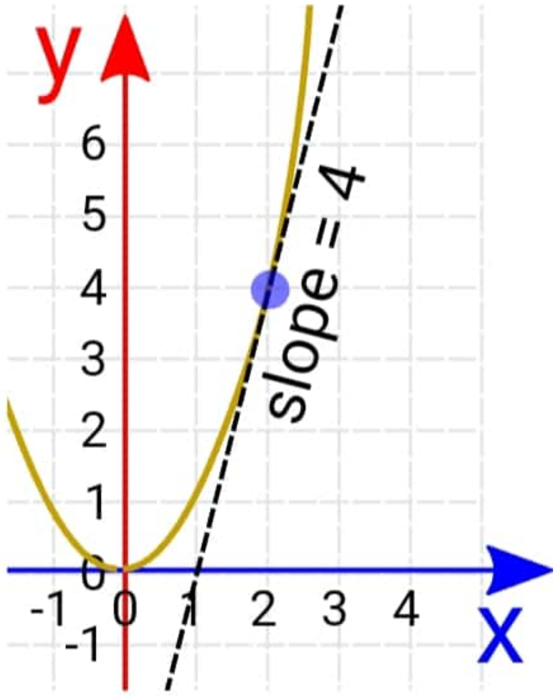
And "the derivative of" is commonly written $\frac{d}{dx}$:

$$\frac{d}{dx}x^2 = 2x$$

"The derivative of $\mathbf{x^2}$ *equals* **2x***"*

or simply *"d dx of* $\mathbf{x^2}$ *equals* **2x***"*

What does $\frac{d}{dx}x^2 = 2x$ mean?



It means that, for the function x^2, the slope or "rate of change" at any point is **2x**.

So when **x=2** the slope is **2x = 4**, as shown here:

Or when **x=5** the slope is **2x = 10**, and so on.

Note: sometimes $f'(x)$ is also used for "the derivative of":

$$f'(x) = 2x$$

"The derivative of f(x) equals 2x"

or simply *"f-dash of x equals 2x"*

Notation

"Shrink towards zero" is actually written as a limit like this:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

"The derivative of **f** equals **the limit as Δx goes to zero** of f(x+Δx) - f(x) over Δx"

Or sometimes the derivative is written like this (explained on Derivatives as dy/dx):

$$\frac{dy}{dx} = \frac{f(x+dx) - f(x)}{dx}$$

The process of finding a derivative is called "differentiation".

You **do** differentiation ... to **get** a derivative.

* Derivative formule →

① $$\boxed{\frac{d}{dx}(x^n) = (n)(x)^{n-1}}$$ {algebric function} { Power function}

Ex: ① $\frac{d}{dx}(x^2) = 2x^1 = 2x$ ② $\frac{d}{dx}(x^{-4}) = (-4)(x)^{-5} = \frac{-4}{x^5}$

③ $\frac{d}{dx}(2x^3) = (2)(3)(x)^2 = 6x^2$ ④ $\frac{d}{dx}(4x^{-2}) = (4)(-2)(x)^{-3} = \frac{-8}{x^3}$

⑤ $\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x)^{1/2} = \left(\frac{1}{2}\right)(x)^{\frac{1}{2}-1} = \left(\frac{1}{2}\right)(x)^{-1/2} = \frac{1}{2(x)^{1/2}} = \frac{1}{2\sqrt{x}}$

⑥ $\frac{d}{dx}(\sqrt[4]{x}) = \frac{d}{dx}(x)^{1/4} = \left(\frac{1}{4}\right)(x)^{\frac{1}{4}-1} = \left(\frac{1}{4}\right)(x)^{-3/4} = \frac{1}{4(x)^{3/4}}$

② $$\boxed{\frac{d}{dx}(K) = 0}$$ {Where K=Constant}

Ex: ① $\frac{d}{dx}(5) = \frac{d}{dx}(5x^0) = (5)(0)(x)^{-1} = 0$

③ $$\boxed{\frac{d}{dx}(\log x) = \frac{1}{x}}$$

④ $$\boxed{\frac{d}{dx}(a^x) = a^x \log a}$$ {lograthmic function}

Ex: ① $\frac{d}{dx}(\log 2) = 0$

② $\frac{d}{dx}(2^x) = 2^x \log 2$

⑤ $$\boxed{\frac{d}{dx}(e^x) = e^x}$$ { Exponential function}

⑥ $\frac{d}{dx}(\sin x) = \cos x$

⑦ $\frac{d}{dx}(\cos x) = -\sin x$

⑧ $\frac{d}{dx}(\tan x) = \sec^2 x$

⑨ $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

⑩ $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

⑪ $\frac{d}{dx}(\text{cosec}\, x) = -\text{cosec}\, x \cdot \cot x$

{Trignometric function}

(12) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

(13) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

(14) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

(15) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

Inverse Trignometric function.

** hyperbolic function →

$\boxed{\sinh x = \frac{e^x - e^{-x}}{2}}$ $\boxed{\cosh x = \frac{e^x + e^{-x}}{2}}$ $\boxed{\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}}$

① $\frac{d}{dx}(\sinh x) = \cosh x$

② $\frac{d}{dx}(\cosh x) = \sinh x$

** chain Rule for multiple function → (Composite function)

⇒ multiple function means a single term contain multiple function.

Ex ① $\sin(5x)$ = sin is trignometric function which contain $5x$ which is a algebric function.

② e^{2x^2} = e is Exponential function which contain $2x^2$ which is a algebric function.

$\frac{d}{dx}\{\text{multiple function}\}$ = [Derivative of OUTSIDE function or main function, leaving inside function untouch] • [Derivative of INSIDE function] ----

Ex: ① $\frac{d}{dx}(2x+1)^2 = 2(2x+1)\cdot(2+0) = 4(2x+1)$

② $\frac{d}{dx}\sqrt{4x^2+2} = \frac{d}{dx}(4x^2+2)^{1/2} = \frac{1}{2}(4x^2+2)^{-1/2}\cdot(8x+0) = \frac{8x}{2\sqrt{4x^2+2}}$

③ $\frac{d}{dx}(\log x^2) =$ here log is main function which contain x^2.

$$= \underbrace{\left(\frac{1}{x^2}\right)}_{\substack{\text{derivative}\\ \text{of}\\ \log}}\cdot\underbrace{(2x)}_{\substack{\text{derivative}\\ \text{of}\\ x^2}} = \frac{2}{x}$$

④ $\frac{d(a^{x^2})}{dx} = (a^{x^2}\log a)\cdot(2x) = 2x\,a^{x^2}\log a$.

⑤ $\frac{d}{dx}(e^{-2x^3}) =$ here e is main function which contain $-2x^3$

$$= \underbrace{(e^{-2x^3})}_{\substack{\text{derivative}\\ \text{of}\\ e}}\cdot\underbrace{(\{-2\}(3)(x)^2)}_{\substack{\text{derivative}\\ \text{of}\\ -2x^3}} = -6x^2e^{-2x^3}$$

⑥ $\frac{d}{dx}\log(\sin 3x) =$ here log is main function which contain $\sin 3x$ & sin is containing $3x$ function

$$= \underbrace{\left(\frac{1}{\sin 3x}\right)}_{\substack{\text{derivative}\\ \text{of}\\ \log}}\underbrace{(\cos 3x)}_{\substack{\text{derivative}\\ \text{of}\\ \sin}}\underbrace{(3)}_{\substack{\text{derivative}\\ \text{of}\\ 3x}} = 3\cot 3x$$

⑦ $\frac{d}{dx}\sin(x^2) = \{\cos(x^2)\}\cdot(2x) = 2x\cos x^2$

⑧ $\frac{d}{dx}\sin^2 x = \frac{d}{dx}(\sin x)^2 = \underbrace{2(\sin x)^1}_{\substack{\text{derivative}\\ \text{of}\\ \text{Power}}}\cdot\underbrace{(\cos x)}_{\substack{\text{derivative}\\ \text{of}\\ \sin x}} = \sin 2x$

(9) $\frac{d}{dx}(\sec 4x) = \{\sec 4x \cdot \tan 4x\} \cdot (4) = 4 \sec 4x \cdot \tan 4x$.

(10) $\frac{d}{dx}(\cot 2x^2) = (-\operatorname{cosec} 2x^2) \cdot (4x) = -4x \cdot \operatorname{cosec} 2x^2$

(11) $\frac{d}{dx}\tan(e^{x^2}) = (\sec(e^{x^2})) \cdot (e^{x^2}) \cdot (2x) = 2x e^{x^2} \cdot \sec(e^{x^2})$

(12) $\frac{d}{dx}\sinh 4x = (\cosh 4x) \cdot (4) = 4\cosh 4x$.

(13) $\frac{d}{dx}\cos^{-1} 2x = \left(-\frac{1}{\sqrt{1-4x^2}}\right) \cdot (2) = \frac{-2}{\sqrt{1-4x^2}}$

(14) $\frac{d}{dx}\tan^{-1} 3x = \left(\frac{1}{1+9x^2}\right)(3) = \frac{3}{1+9x^2}$

** Working Rules for Derivatives →

(1) $\boxed{\frac{d}{dx}\{k f(x)\} = k \frac{d f(x)}{dx}}$

Ex → $\frac{d}{dx} 5x^2 = 5 \frac{d}{dx} x^2 = (5)(2)(x) = 10x$

(2) $\boxed{\frac{d}{dx}\{f(x) \pm g(x)\} = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)}$

Ex → $\frac{d}{dx}\{x^2 + \sin 3x - e^{2x}\} = \frac{d}{dx} x^2 + \frac{d}{dx}\sin 3x - \frac{d}{dx} e^{2x} = 2x + 3\cos 3x - 2e^{2x}$

(3) $\boxed{\frac{d}{dx}\{f(x) \cdot g(x)\} = g(x) \cdot \frac{d f(x)}{dx} + f(x) \frac{d g(x)}{dx}}$

Ex → (1) $\frac{d}{dx}(x^2 \cdot e^{3x}) = (e^{3x}) \cdot \frac{d}{dx} x^2 + (x^2) \frac{d}{dx}(e^{3x}) = (e^{3x})(2x) + (x^2)(3e^{3x})$

$= 2xe^{3x} + 3x^2 e^{3x}$

(2) $\frac{d}{dx}(\sin 2x \cdot \cos 4x) = (\cos 4x) \frac{d}{dx}\sin 2x + (\sin 2x) \frac{d}{dx}\cos 4x$

$= (\cos 4x)(2\cos 2x) + (\sin 2x)(-4\sin 4x)$

(4) $$\boxed{\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x)\cdot\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{\{g(x)\}^2}}$$

Ex:- ① $$\frac{d}{dx}\left(\frac{2x}{1+x^2}\right) = \frac{(1+x^2)\frac{d}{dx}(2x) - (2x)\frac{d}{dx}(1+x^2)}{(1+x^2)^2} = \frac{(1+x^2)(2)-(2x)(2x)}{(1+x^2)^2}$$

② $$\frac{d}{dx}\left(\frac{e^{2x}}{x^2}\right) = \frac{(x^2)\frac{d}{dx}(e^{2x}) - (e^{2x})\frac{d}{dx}(x^2)}{(x^2)^2} = \frac{(x^2)(2e^{2x}) - (e^{2x})(2x)}{x^4}$$

** Variable dependency →

- Let us consider a Equation $y = x^2 + 2x$
- now derivate this Eqn w.r.t x variable $\Rightarrow$ $\frac{dy}{dx}$

dependent variable

independent variable.

$$\frac{dy}{dx} = \frac{d}{dx}\{x^2+2x\} = 2x+2$$

Note → means when dependent variable will be differentiated w.r.t independent variable, it will be always have dy/dx term.

Ex → ① $x^2 + y^2 + 4 = 0$ differentiate w.r.t x.

$$\Rightarrow 2x + 2y\cdot\frac{dy}{dx} + 0 = 0$$

$$\Rightarrow 2x = -2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

② $\sin 3x + \log y + e^{3x} = 0$ differentiate w.r.t x

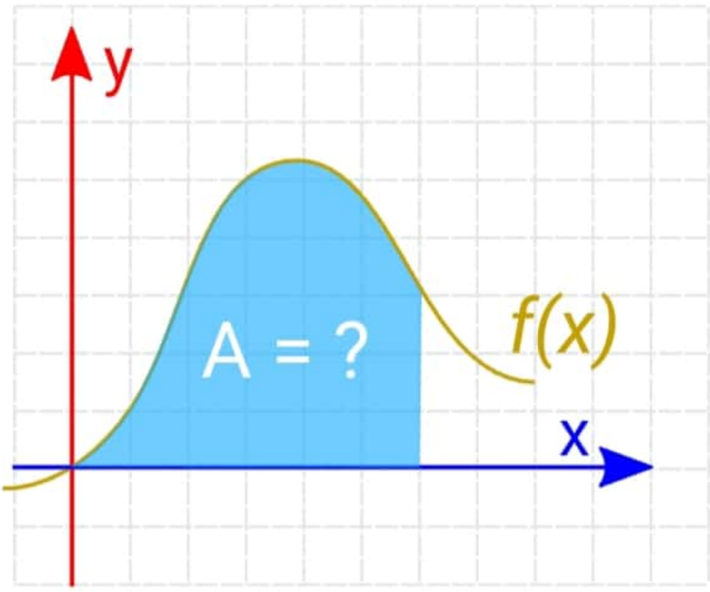
$$\Rightarrow 3\cos 3x + \left(\frac{1}{y}\right)\frac{dy}{dx} + 3e^{3x} = 0$$

$$\left(\frac{1}{y}\right)\frac{dy}{dx} = -(3e^{3x} + 3\cos 3x)$$

$$\frac{dy}{dx} = -y(3e^{3x} + 3\cos 3x)$$

Integration is a way of adding slices to find the whole.

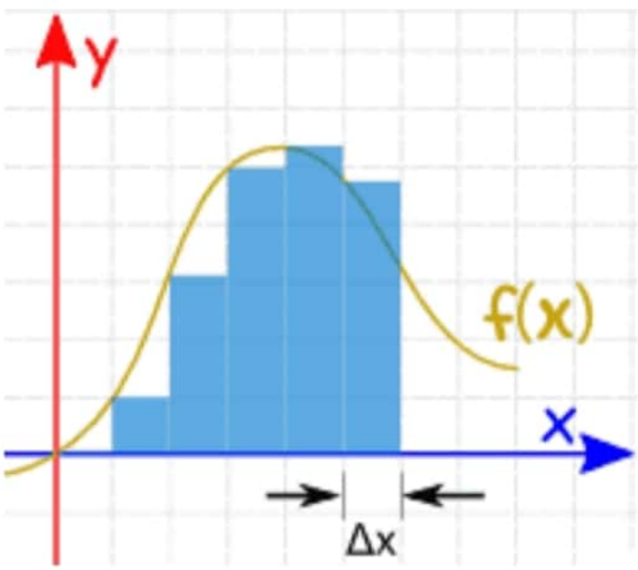
Integration can be used to find areas, volumes, central points and many useful things. But it is easiest to start with finding the **area under the curve of a function** like this:



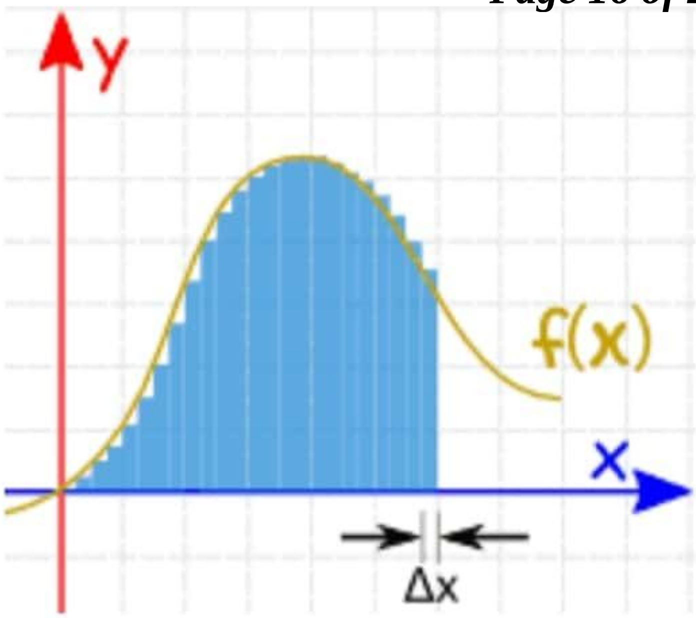
What is the area under **y = f(x)** ?

Slices

We could calculate the function at a few points and **add up slices of width Δx** like this (but the answer won't be very accurate):

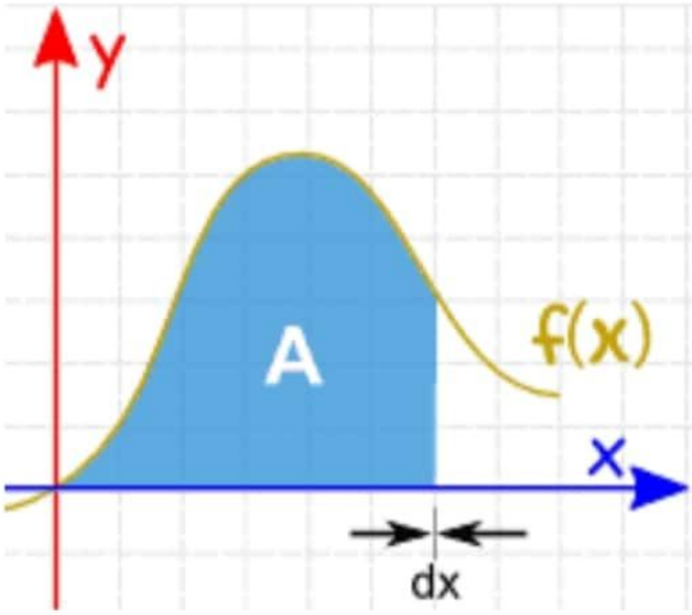


We can make **Δx** a lot smaller and **add up many small slices** (answer is getting better):



And as the slices **approach zero in width**, the answer approaches the **true answer**.

We now write **dx** to mean the **Δx** slices are approaching zero in width.



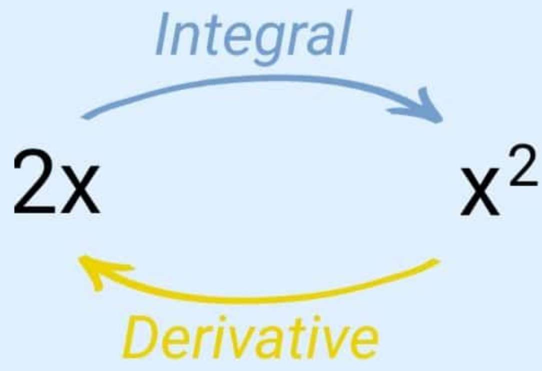
That is a lot of adding up!

But we don't have to add them up, as there is a "shortcut". Because ...

... finding an Integral is the **reverse** of finding a Derivative.

Example: What is an integral of 2x?

We know that the derivative of x^2 is $2x$ ...

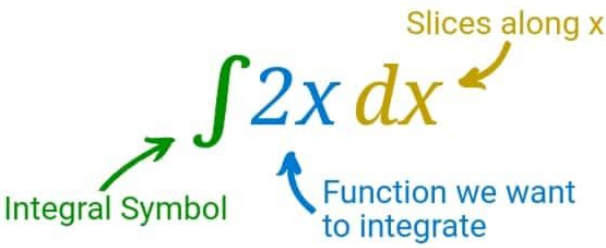


... so an integral of $2x$ is x^2

You will see more examples later.

Notation

The symbol for "Integral" is a stylish "S"
(for "Sum", the idea of summing slices):



After the Integral Symbol we put the function we want to find the integral of (called the Integrand),

and then finish with **dx** to mean the slices go in the x direction (and approach zero in width).

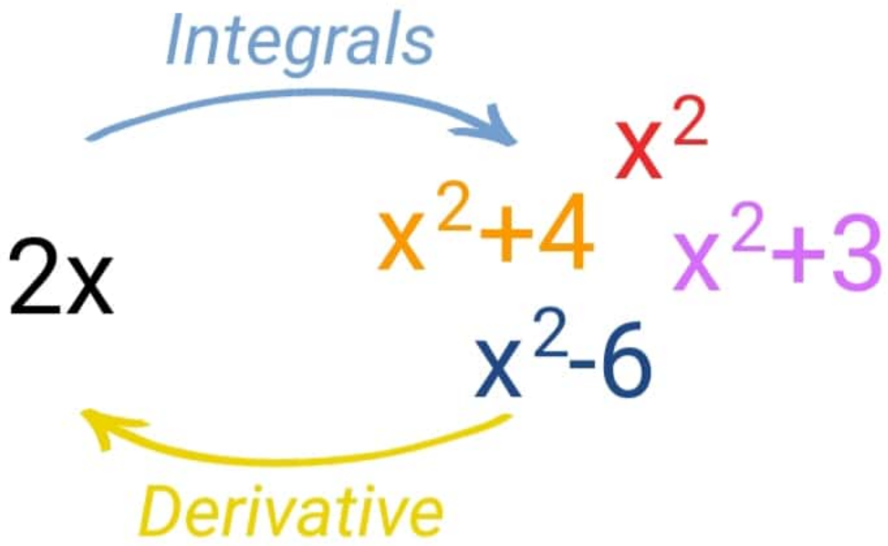
And here is how we write the answer:

$$\int 2x \, dx = x^2 + C$$

Plus C

We wrote the answer as x^2 but why + C ?

It is the "Constant of Integration". It is there because of **all the functions whose derivative is 2x**:

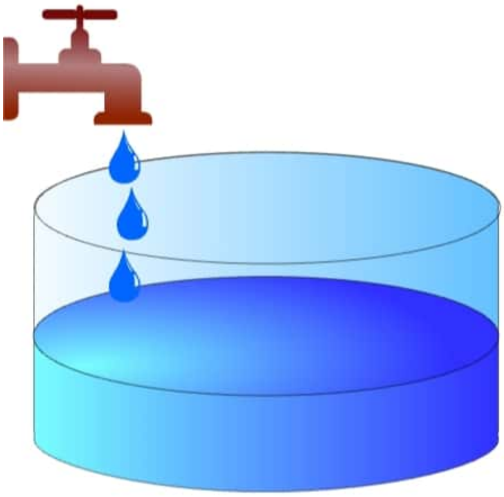


The derivative of $\mathbf{x^2+4}$ is **2x**, and the derivative of $\mathbf{x^2+99}$ is also **2x**, and so on! Because the derivative of a constant is zero.

So when we **reverse** the operation (to find the integral) we only know **2x**, but there could have been a constant of any value.

So we wrap up the idea by just writing + C at the end.

Tap and Tank

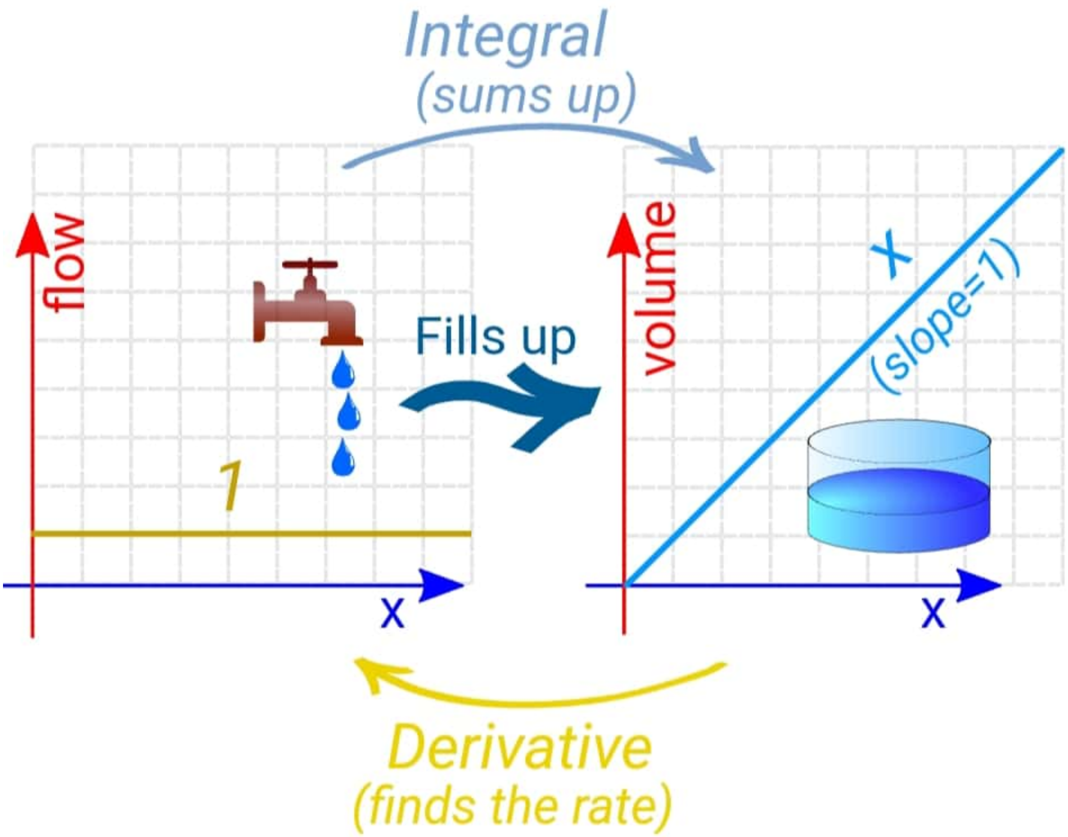


Integration is like filling a tank from a tap.

The input (before integration) is the **flow rate** from the tap.

Integrating the flow (adding up all the little bits of water) gives us the **volume of water** in the tank.

Simple Example: Constant Flow Rate



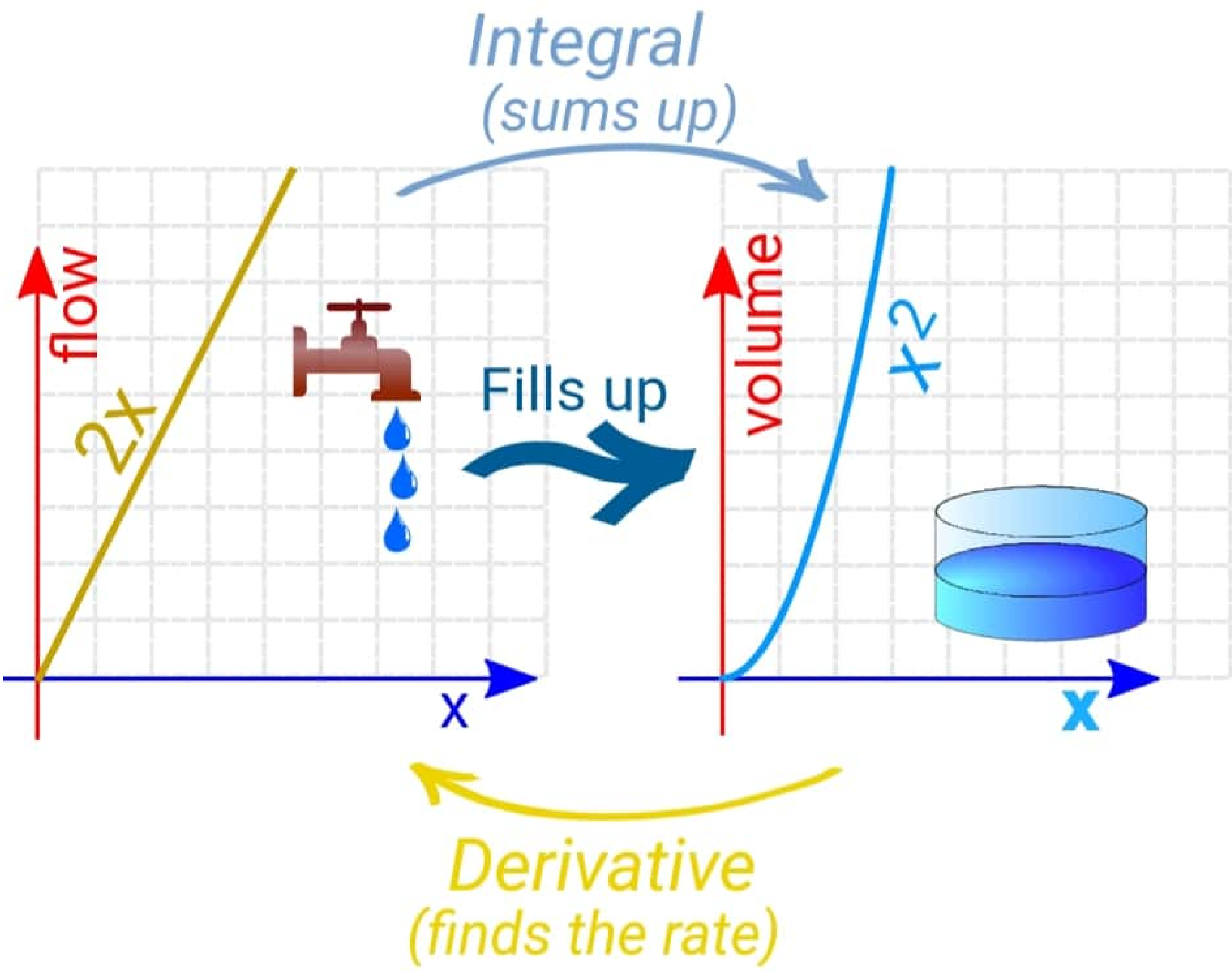
Integration: With a flow rate of **1**, the tank volume increases by **x**

Derivative: If the tank volume increases by **x**, then the flow rate is **1**

This shows that integrals and derivatives are opposites!

Now For An Increasing Flow Rate

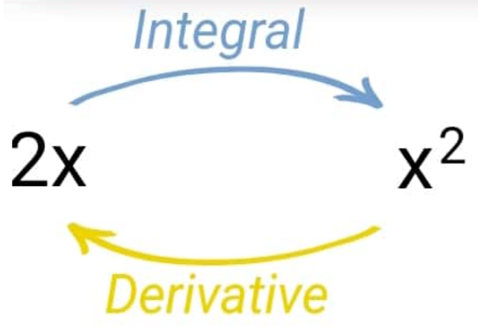
Imagine the flow starts at 0 and gradually increases (maybe a motor is slowly opening the tap).



As the flow rate increases, the tank fills up faster and faster.

Integration: With a flow rate of **2x**, the tank volume increases by $\mathbf{x^2}$

Derivative: If the tank volume increases by $\mathbf{x^2}$, then the flow rate must be **2x**

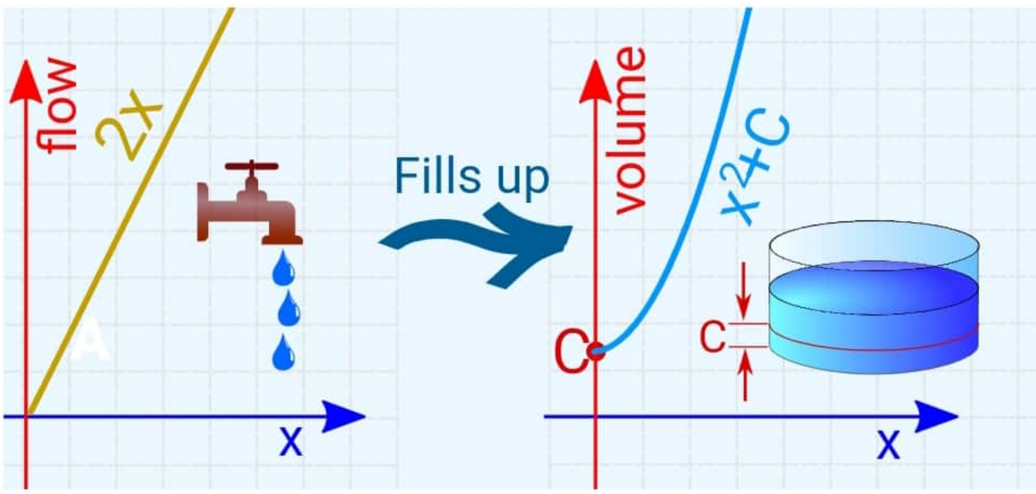


So Integral and Derivative **are** opposites.

We can write that down this way:

The integral of the flow rate **2x** tells us the volume of water: $\int 2x\,dx = x^2 + C$

And the slope of the volume increase **x^2+C** gives us back the flow rate: $\frac{d}{dx}(x^2 + C) = 2x$



And hey, we even get a nice explanation of that "C" value ... maybe the tank already has water in it!

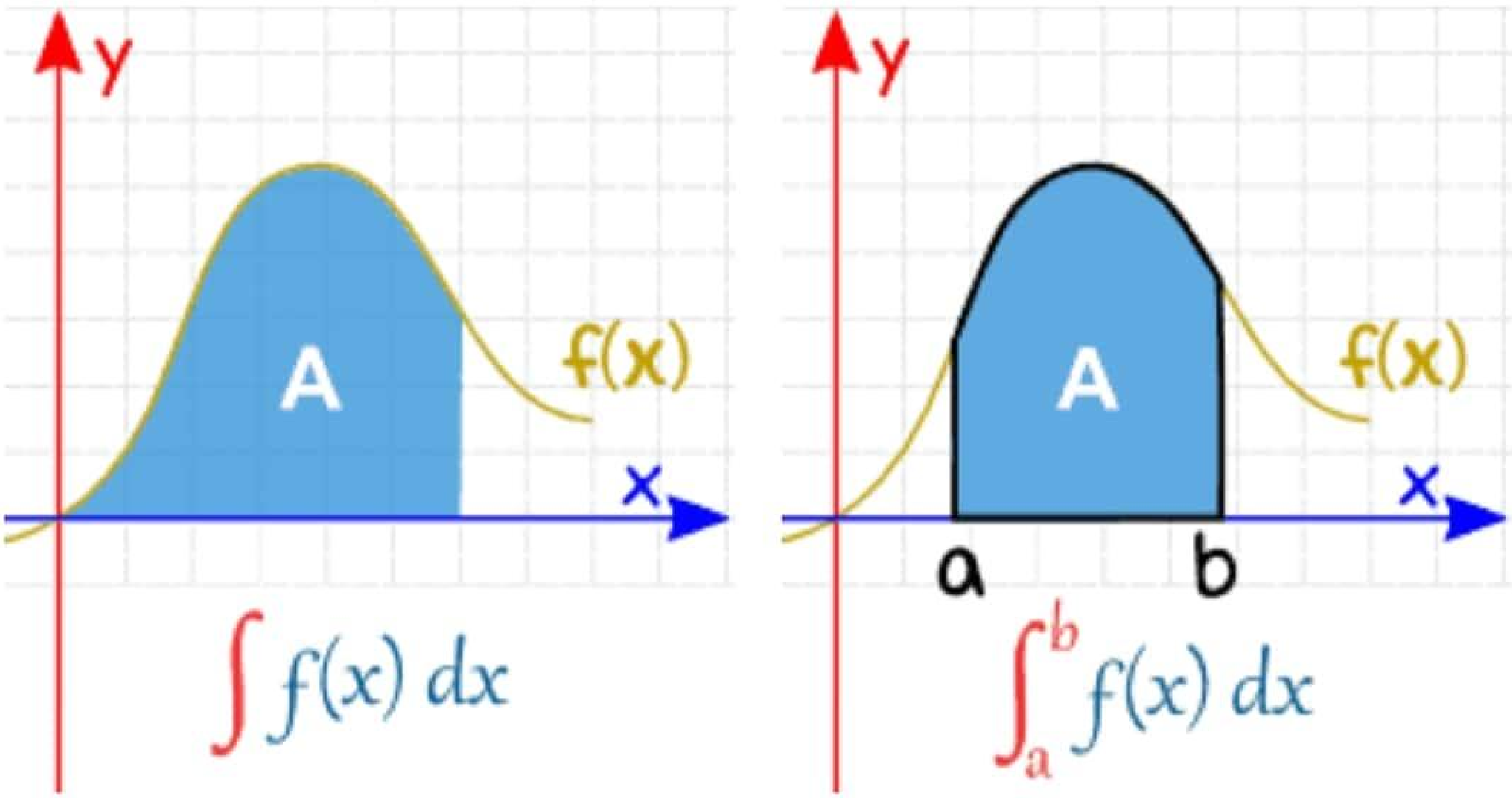
- The flow still increases the volume by the same amount
- And the increase in volume can give us back the flow rate.

Which teaches us to always add "+C".

Definite vs Indefinite Integrals

We have been doing **Indefinite Integrals** so far.

A **Definite Integral** has actual values to calculate between (they are put at the bottom and top of the "S"):



Indefinite Integral | **Definite** Integral

** Integration Formule →

① $$\boxed{\int x^n dx = \frac{x^{n+1}}{n+1} + c}$$ (algebric function or power function)

Ex – i) $\int x^2 dx = \frac{x^3}{3} + c$

ii) $\int x^{-3} dx = \frac{x^{-2}}{-2} + c$

(iii) $\int \frac{1}{2x^2} dx = \frac{1}{2}\int x^{-2} dx = \left(\frac{1}{2}\right)\frac{(x)^{-1}}{-1} = -\frac{1}{2x} + c$

② $$\boxed{\int k\,dx = kx + c}$$ (Constant no)

Ex→ i) $\int 2\,dx = 2x + c$

ii) $\int -5\,dx = -5x + c$

iii) $\int (ab)\,dx = abx + c$

③ $$\boxed{\int \frac{1}{x} dx = \log|x| + c}$$ and ④ $$\boxed{\int a^x dx = \frac{a^x}{\log a} + c}$$ {lograthmic function}

- $\int \frac{1}{x \pm a} dx = \log|x \pm a| + c$
- $\int \frac{1}{a+x} dx = \log|a+x| + c$
- $\int \frac{1}{a-x} dx = -\log|a-x| + c$

Ex → i) $\int \frac{2}{x} dx = 2\log|x| + c$

ii) $\int -\frac{1}{3x} dx = -\frac{1}{3}\log|x| + c$

iii) $\int 5^x dx = 5^x/\log 5 + c$

(iv) $\int \frac{2}{x-5} dx = 2\log|x-5| + c$

(v) $\int \frac{1}{2-x} dx = -\log|2-x| + c$

⑤ $$\int e^{ax} dx = \frac{e^{ax}}{a} + c$$ (Exponential function)

Note: In integration (composite function or multiple function will not be solved. only variable having coefficient will be divided after integration.

Ex→ i) $\int e^{2x} dx = \frac{e^{2x}}{2} + c$

ii) $\int e^{-5x} dx = \frac{e^{-5x}}{-5} + c$

Ex→ i) $\int e^{x^2} dx$ = not possible because of composite function.

ii) $\int e^{-5x} dx$ = -5 is coefficient of x variable.

$= \frac{e^{-5x}}{-5} + c$

iii) $\int \frac{2}{5x+4} dx = 2\int \frac{1}{5x+4} dx = \frac{2\log|5x+4|}{5} + c$

⑥ $\int \sin x\, dx = -\cos x + c$

⑦ $\int \cos x\, dx = \sin x + c$

⑧ $\int \sec^2 x\, dx = \tan x + c$

⑨ $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

⑩ $\int \sec x . \tan x\, dx = \sec x + c$

⑪ $\int \operatorname{cosec} x . \cot x\, dx = -\operatorname{cosec} x + C$

} Trigonometric function

Ex→ i) $\int \cos 5x\, dx = \frac{\sin 5x}{5} + c$

{ x is having coefficient 5 hence it will divided }

ii) $\int \sin 2x\, dx = -\frac{\cos 2x}{2} + c$

iii) $\int \sec^2 3x\, dx = \frac{\tan 3x}{3} + c$

iv) $\int \sec 2x . \tan 2x\, dx = \frac{\sec 2x}{2} + c$

⑫ $\int \tan x\, dx = \log|\sec x| + c$

⑬ $\int \cot x\, dx = \log|\sin x| + c$

⑭ $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| + c$

⑮ $\int \sec x\, dx = \log|\sec x + \tan x| + c$

} standard trigonometric function.

⑯ $\int \frac{1}{x^2+a^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

⑰ $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$

} inverse trigonometric function.

** Working Rule for Integration →

① $\boxed{\int K f(x)\,dx = K\int f(x)\,dx}$

② $\boxed{\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx}$

③ $\boxed{\int [f(x)\cdot g(x)]\,dx = f(x)\cdot g(x)_1 - \int f'(x)\cdot g(x)_1\,dx}$

where $= f'(x) = \frac{d f'(x)}{dx}$

$g(x)_1 = \int g(x)\,dx$

$f(x) \Leftarrow$ L − I A T E $\Rightarrow g(x)$

L → logarithmic

I → inverse

A → algebaic

T → Trignometry

E → Exponential.

Choice of $f(x)$ & $g(x)$

Ex → ① $\int 2x \cdot e^{3x}\,dx$

$2x =$ algebaic function which is near to $f(x)$

$e^{3x} =$ Exponential function which is near to $g(x)$

$= f(x)\cdot g(x)_1 - \int f'(x)\,g(x)_1\,dx$

$= (2x)\left\{\frac{e^{3x}}{3}\right\} - \int (2)\left\{\frac{e^{3x}}{3}\right\}dx = \frac{2xe^{3x}}{3} - \left(\frac{2}{3}\right)\left\{\frac{e^{3x}}{3}\right\}$

(ii) $\int e^{3x}\cdot \sin 4x\,dx$

$e^{3x} = g(x)$

$\sin 4x = f(x)$

$= (\sin 4x)\left\{\frac{e^{3x}}{3}\right\} - \int (4\cos 4x)\left\{\frac{e^{3x}}{3}\right\}dx$ (This is carried out further also ...)

(iii) $\int 4x^2 \cdot \sin 2x\,dx$

$4x^2 = f(x)$

$\sin 2x = g(x)$

$= (4x^2)\left\{-\frac{\cos 2x}{2}\right\} - \int (8x)\left\{-\frac{\cos 2x}{2}\right\}dx$

$= (4x^2)\left\{-\frac{\cos 2x}{2}\right\} - \left[(8x)\left\{-\frac{\sin 2x}{4}\right\} - \int (8)\left\{-\frac{\sin 2x}{4}\right\}dx\right]$

$= -2x^2\cos 2x + 2\sin 2x + \cos 2x + C$

Short cut for $\int f(x)\cdot g(x)dx \rightarrow$

① In $\int x^n \cdot e^x dx$ or $\int x^n \cdot \sin x\, dx$ or $\int x^n \cdot \cos x\, dx$

$$\boxed{\int u.v dx = (u)\{v_1\} - (u')\{v_2\} + (u'')\{v_3\} - (u''')\{v_4\} \ldots\ldots}$$

where u', u'', u''' = first, second, third order derivative of u.
v_1, v_2, v_3 = first, second, third order integral of v.

Ex→ i) $\int x^2 e^{3x} dx = (x^2)\left\{\frac{e^{3x}}{3}\right\} - (2x)\left\{\frac{e^{3x}}{9}\right\} + (2)\left\{\frac{e^{3x}}{27}\right\} + c$

$u = x^2$
$v = e^{3x}$

ii) $\int 4x^2 \sin 2x dx = (4x^2)\left\{-\frac{\cos 2x}{2}\right\} - (8x)\left\{-\frac{\sin 2x}{4}\right\} + (8)\left\{+\frac{\cos 2x}{8}\right\}$

$u = 4x^2$ $\quad = -2x^2\cos 2x + 2x\sin 2x + \cos 2x + c$
$v = \sin 2x$

iii) $\int 3x \cdot \cos 4x dx = (3x)\left\{+\frac{\sin 4x}{4}\right\} - (3)\left\{-\frac{\cos 4x}{16}\right\} + c$

$u = 3x$
$v = \cos 4x$

②
$$\boxed{\begin{aligned} \int e^{ax}\cdot \cos bx\, dx &= \frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right) + c \\ \int e^{ax}\cdot \sin bx\, dx &= \frac{e^{ax}}{a^2+b^2}\left(a\sin bx - b\cos bx\right) + c \end{aligned}}$$

Ex→ i) $\int e^{2x}\cdot \cos 5x dx = \frac{e^{2x}}{4+25}\left(2\cos 5x + 5\sin 5x\right) + c$

ii) $\int e^{-3x}\cdot \sin 2x dx = \frac{e^{-3x}}{9+4}\left(-3\sin 2x - 2\cos 2x\right) + c$

DIFFERENTIATION &INTEGRATION FORMULAS

DIFFERENTIATION	INTEGRATION
1. $\frac{d}{dx}(x^n)$=n$x^{(n-1)}$	1. $\int x^n dx$=$\frac{x^{(n+1)}}{(n+1)}$+c
2. $\frac{d}{dx}(e^{ax})$=ae^{ax}	2. $\int e^{ax} dx$=$\frac{e^{ax}}{a}$+c
3.$\frac{d}{dx}\log(x)$=$\frac{1}{x}$	3.$\int \frac{1}{x} dx$=$\log(x) + c$
4.$\frac{d}{dx}\{\log(x+a)\}$=$\frac{1}{(x+a)}$	4.$\int \frac{1}{(x+a)} dx$=$\log(x+a) + c$
5.$\frac{d}{dx}\{\log(x-a)\}$=$\frac{1}{(x-a)}$	5.$\int \frac{1}{(x-a)} dx$=$\log(x-a) + c$
6.$\frac{d}{dx}a^x$=$a^x \log_e a$	6.$\int a^x dx$=$\frac{a^x}{\log_e a} + c$
7.$\frac{d}{dx}\sin(ax)$=$a.\cos(ax)$	6.$\int \sin(ax)\, dx$=$-\frac{\cos(ax)}{a} + c$
8.$\frac{d}{dx}\cos(ax)$=$-a.\sin(ax)$	7.$\int \cos(ax) dx$=$\frac{\sin(ax)}{a} + c$
9.$\frac{d}{dx}\tan(ax)$=$a.sec^2(ax)$	8.$\int \tan(ax) dx$=$\frac{\log\{\sec(ax)\}}{a} + c$
10.$\frac{d}{dx}\cot(ax)$=$-a.cosec^2(ax)$	9.$\int \cot(ax)\, dx$=$\frac{\log\{\sin(ax)\}}{a} + c$
11.$\frac{d}{dx}\sec(ax)$=$a.\sec(ax).\tan(ax)$	10.$\int \sec(ax) dx$=$\frac{\log\{\sec(ax)+\tan(ax)\}}{a} + c$
12.$\frac{d}{dx}cosec(ax)$=$-a.cosec(ax).\cot(ax)$	11.$\int cosec(ax)\, dx$=$\frac{\log\{cosec(ax)-\cot(ax)\}}{a} + c$

PARTH SHARMA Contact No. +91-8460442462