

Session 1

DC Circuits

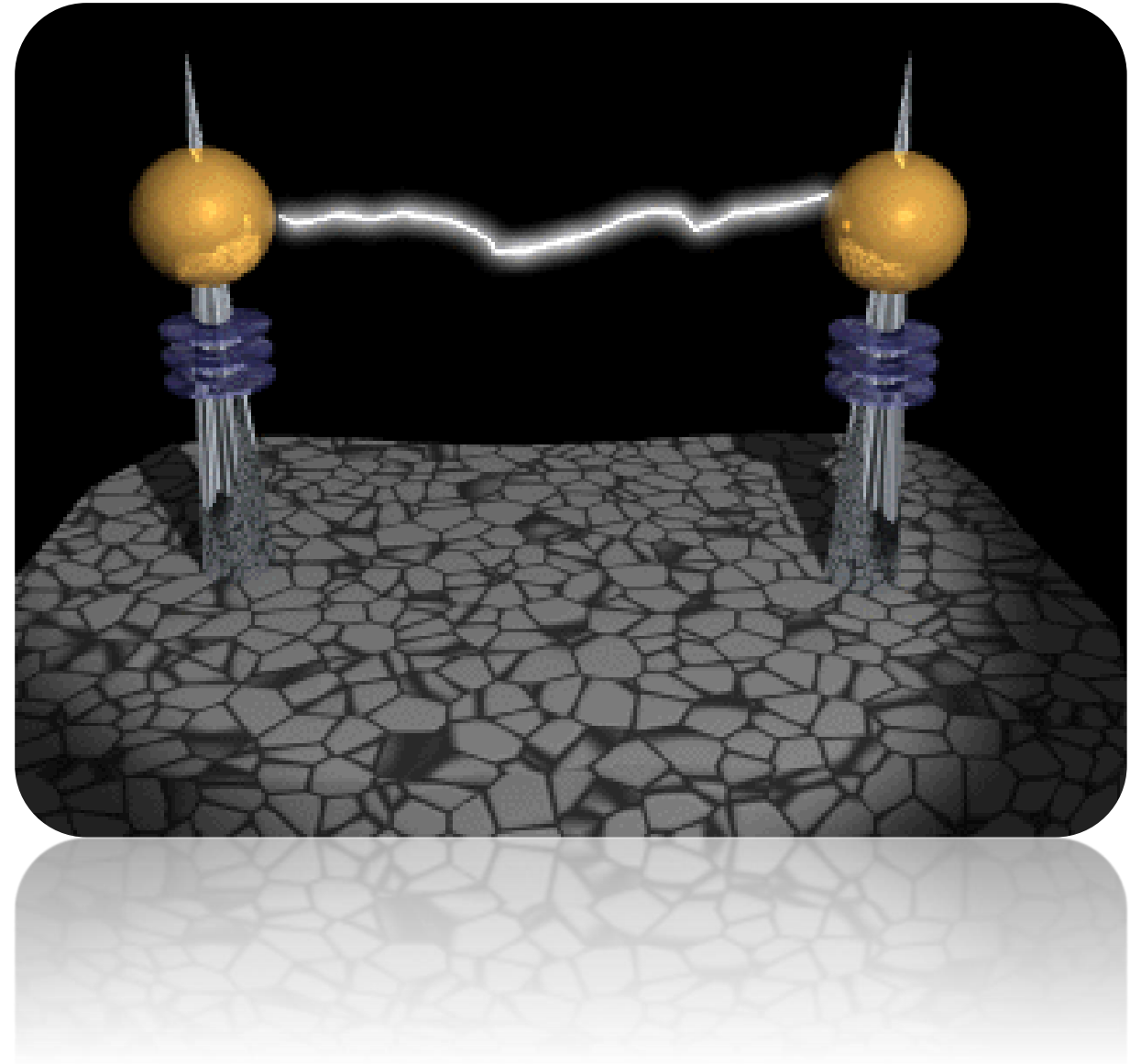
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Contents :

- Voltage and current Sources,
- Source Transformation,
- Star-Delta Transformation,
- Application of Kirchhoff's Law,
- Superposition Theorem,
- Thevenin's Theorem
- Norton's Theorem.



Ohm's Law



- Discovered in 1825
- Relates 3 key quantities in electrical circuits
- Voltage (V)
- Current (I)
- Resistance (R)

$$V = I \times R$$

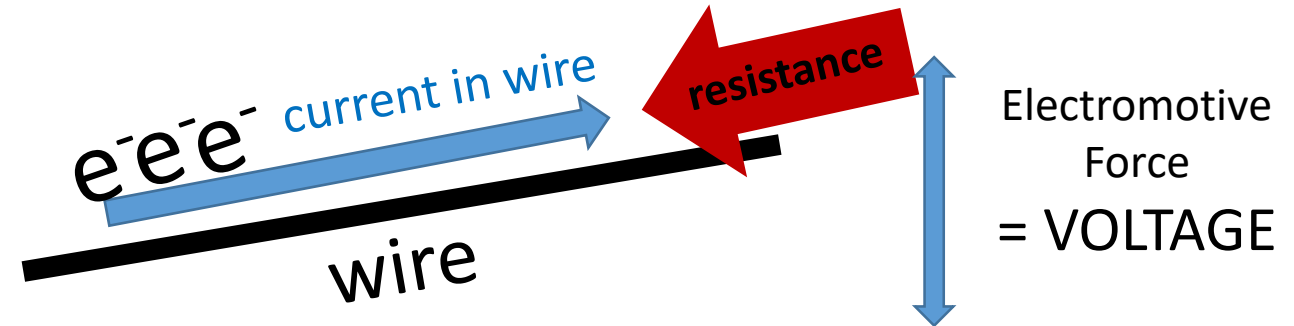
Voltage = Current x Resistance

In scientific units: Volts = Amperes x Ohms

Think of the voltage as the FORCE which is DRIVING the total electrical flow rate (current), *against* the resistance encountered in a portion of an electrical circuit.

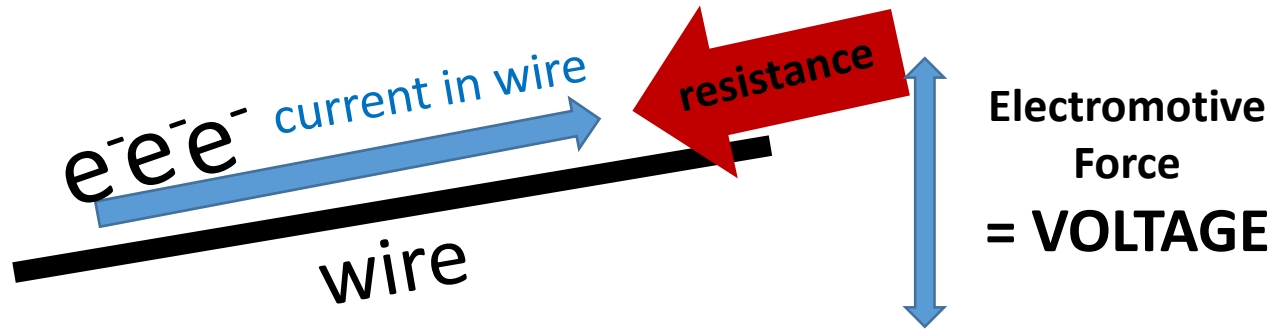
$$\text{Voltage} = (\text{electrical}) \text{ Current} \times (\text{electrical}) \text{ Resistance}$$

Compare to pushing or cycling a bike up a hill

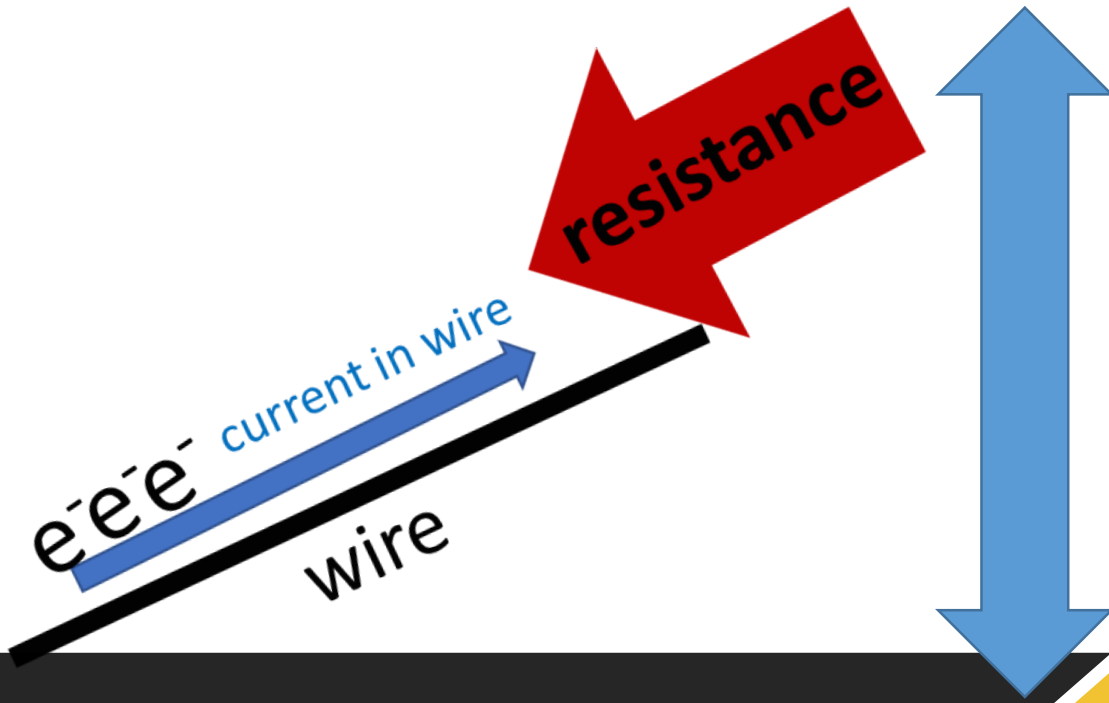


- 1) The force is your capacity for work to push or cycle the bike (or to 'drive' it); **that is like the Voltage in a circuit.**
- 2) The **resistance** is like the friction force on the tyres, the stiffness of the bike components, and the steepness of the hill; **all these factors work together to determine the rate of progress for a given force.**
- 3) **The rate of progress (up the hill) – is similar to the “current” in a circuit, which measures the total passage of electricity in a given time through a particular point.**

Suppose a wire has twice the resistance



Doubling the resistance of the circuit wire will mean twice the electromotive force (voltage) required to drive the same current through the circuit.



The greater the electrical resistance, the greater the applied voltage V needs to be to drive the same current I

How do we use Ohm's law?



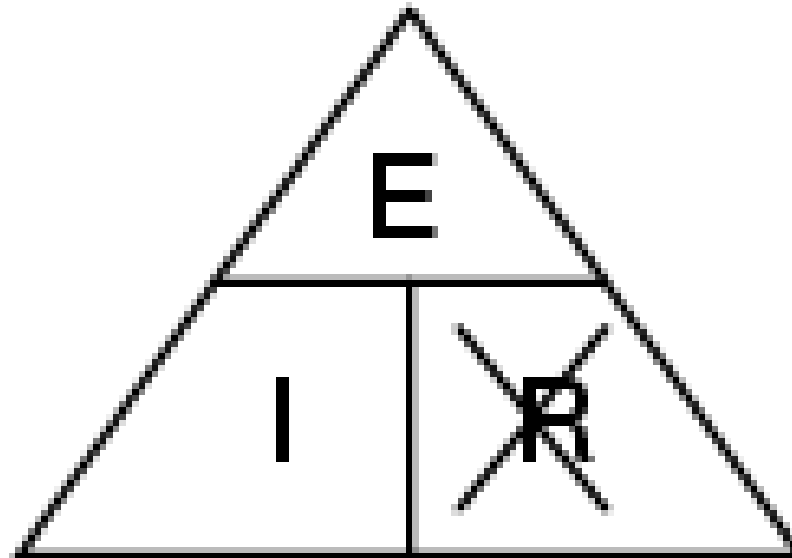
- $V (E) = I \times R$

- $I = \frac{V}{R}$

$$R = \frac{V}{I}$$

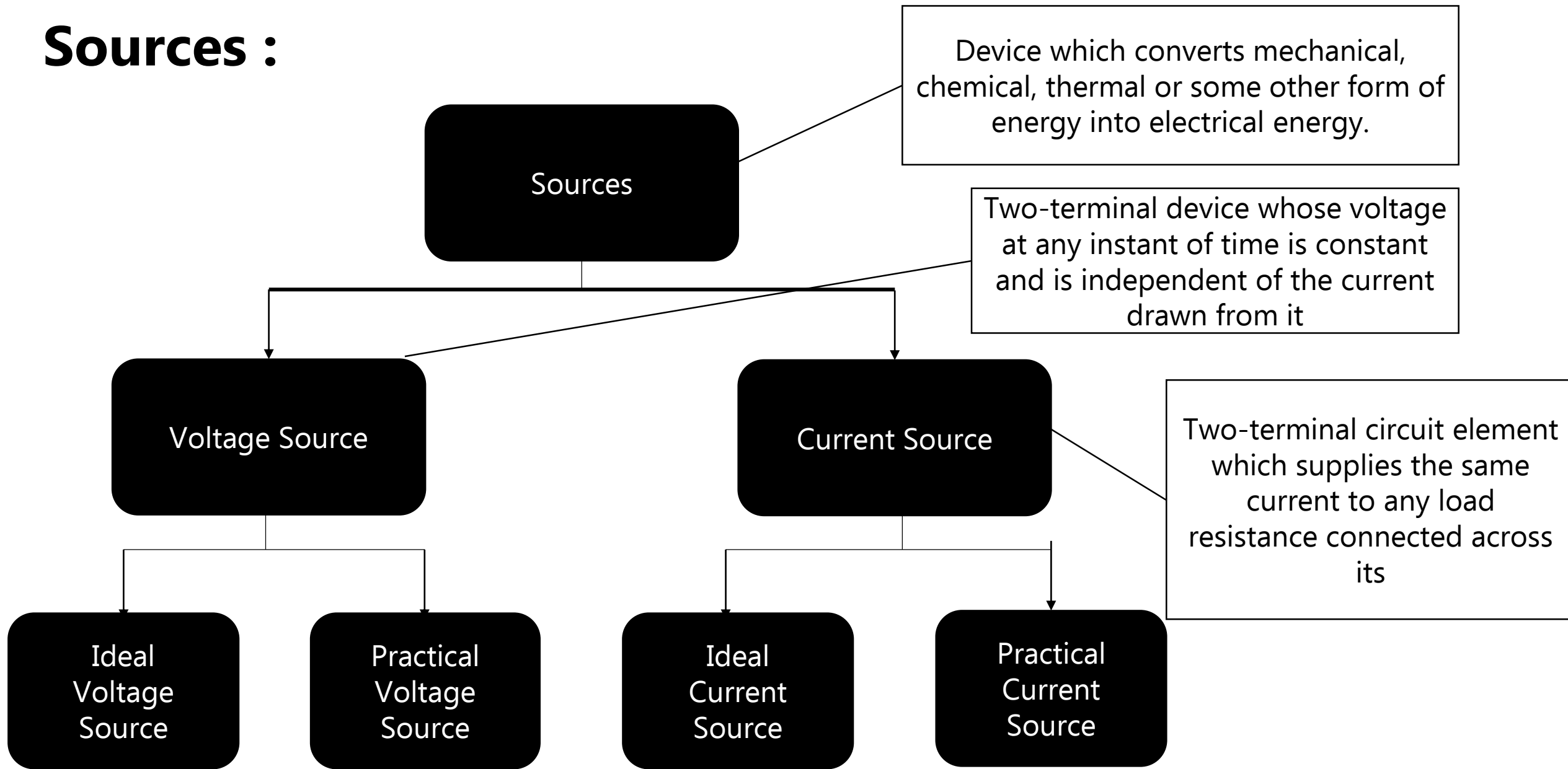
How do we calculate?

- Battery voltage is 12V
- Current is 4Amps
- Resistance 200 Ohms ?

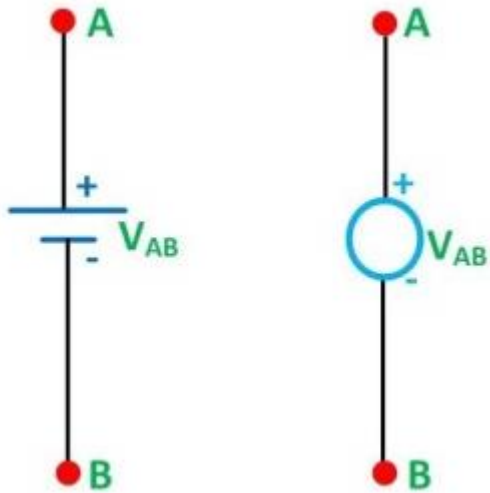


$$R = \frac{E}{I}$$

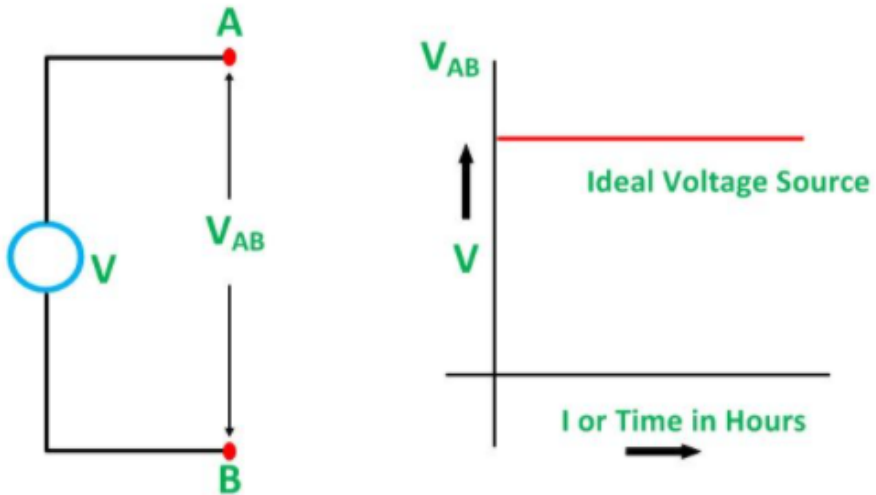
Sources :



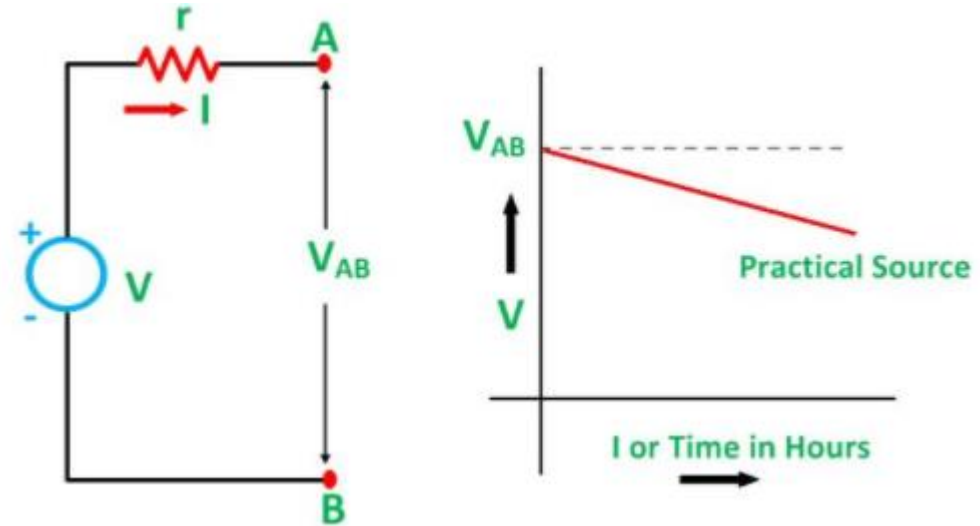
Voltage Source :



Ideal Characteristics:



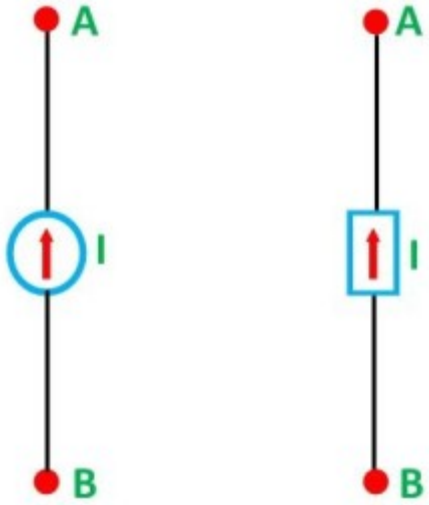
Practical Characteristics:



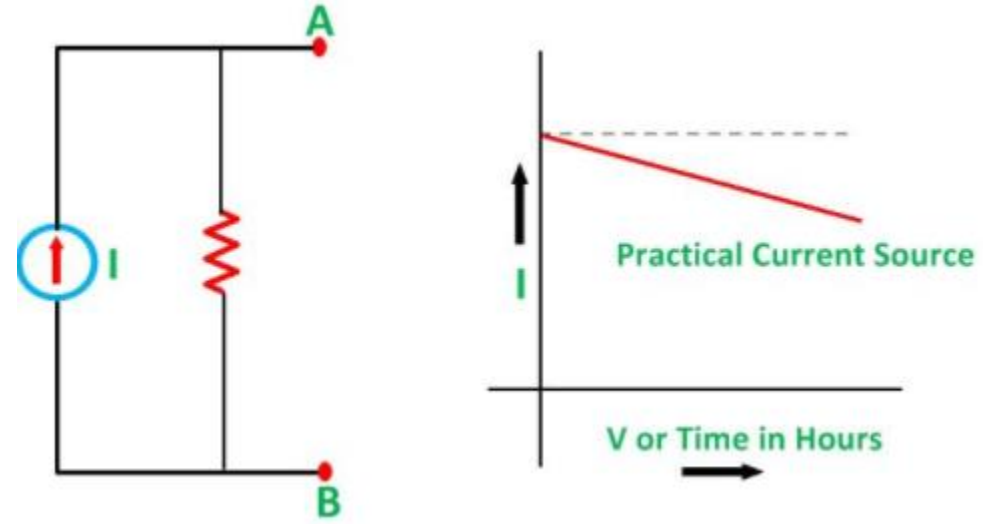
Examples:

- Batteries
- Alternators

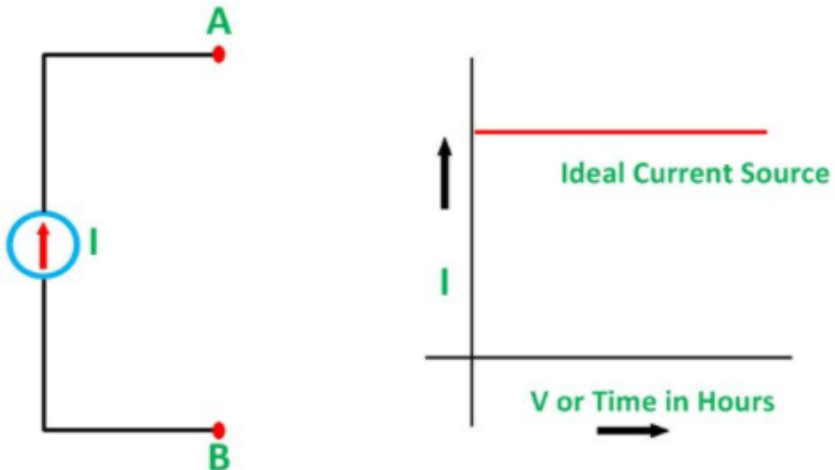
Current Source :



Practical Characteristics:



Ideal Characteristics:



Examples:

- Photoelectric Cells
- Collectors current of Transistor

Resistor: (R)

- It is property of the material to oppose the flow of the current.
- Unit: Ω (Ohm)
- Resistor depend upon
 - Length
 - Cross Section
 - Material
 - Temperature

$$R \propto \frac{l}{a}$$

$$R = \rho \frac{l}{a}$$

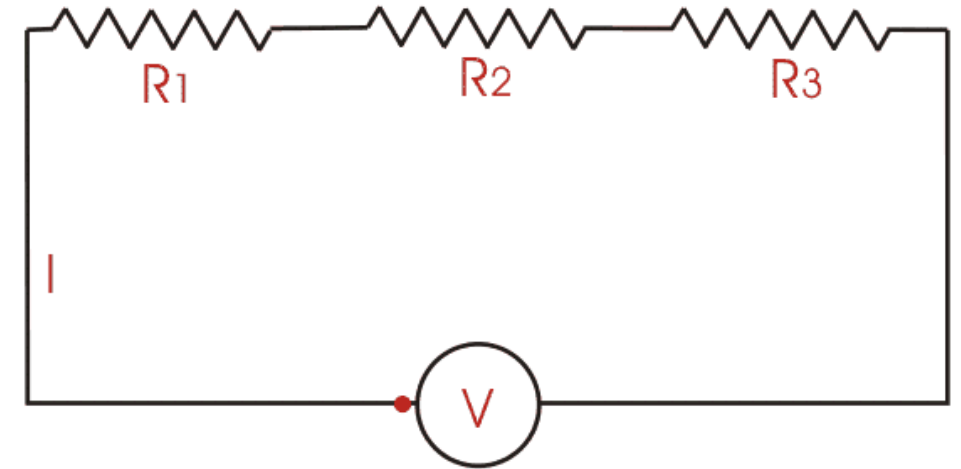
Where ρ = resistivity

Unit: $\Omega \cdot m$

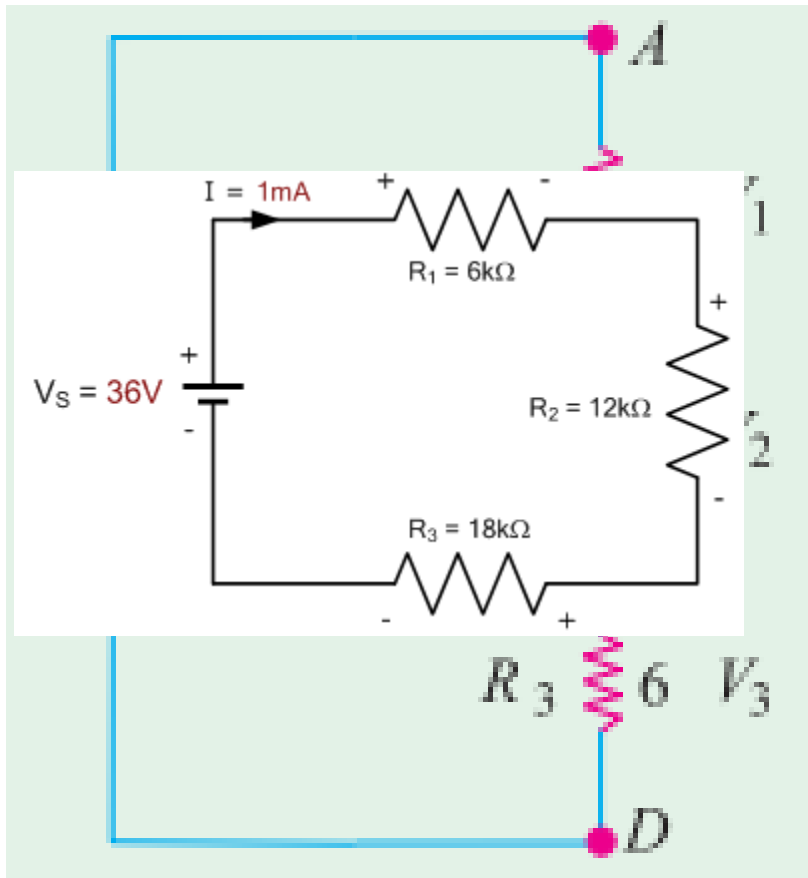
Resistance in Series :

The equivalent resistance R_e of a number of resistors connected in series is equal to the sum of the individual resistances

- $V_T = V_1 + V_2 + V_3$; ($V = IR$)
- $I_T R_e = I_1 R_1 + I_2 R_2 + I_3 R_3$
- *But . .* $I_T = I_1 = I_2 = I_3$
- $R_e = R_1 + R_2 + R_3$



Voltage Divider Rule



$$V = V_1 + V_2 + V_3$$

$$V_1 = I R_1 \quad V_2 = I R_2 \quad V_3 = I R_3$$

$$V = I R_1 + I R_2 + I R_3$$

$$V = I (R_1 + R_2 + R_3)$$

$$I = \frac{V}{(R_1 + R_2 + R_3)}$$

$$V_1 = R_1 \frac{V}{(R_1 + R_2 + R_3)}$$

$$V_2 = R_2 \frac{V}{(R_1 + R_2 + R_3)}$$

$$V_3 = R_3 \frac{V}{(R_1 + R_2 + R_3)}$$

$$V_{R(x)} = V_S \left(\frac{R_x}{R_T} \right)$$

Resistance in Parallel :

- *Resistors are said to be connected in parallel when there is more than one path for current.*
- In other words the currents in the branches of a parallel circuit add up to the supply current.

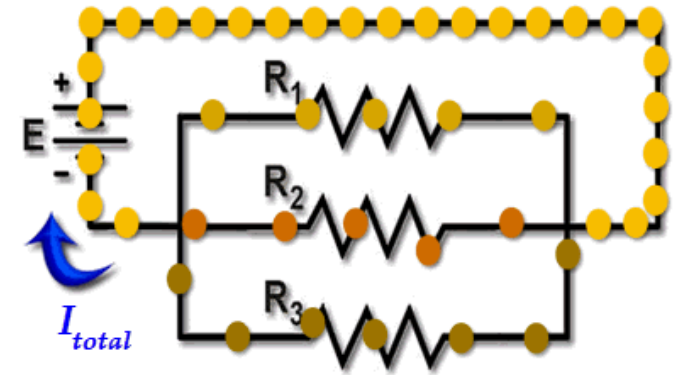
- $V_T = V_1 = V_2 = V_3$

- $I_T = I_1 + I_2 + I_3$

Ohm's law : $I = \frac{V}{R}$

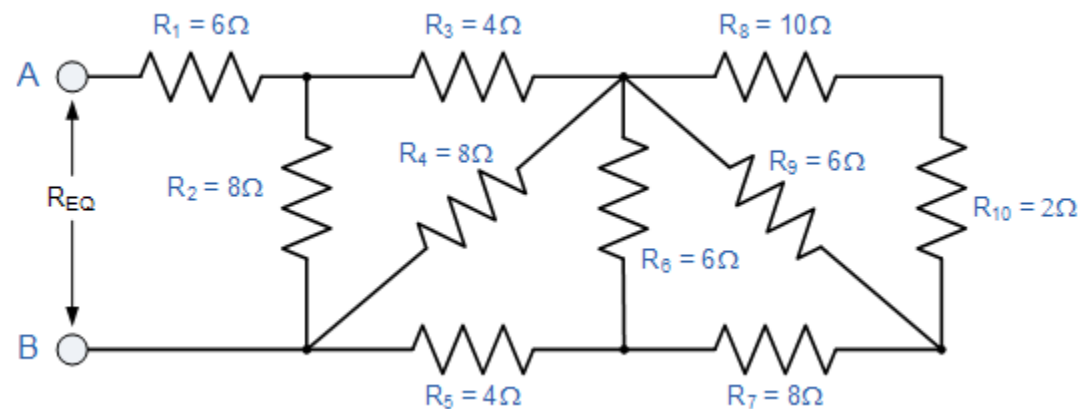
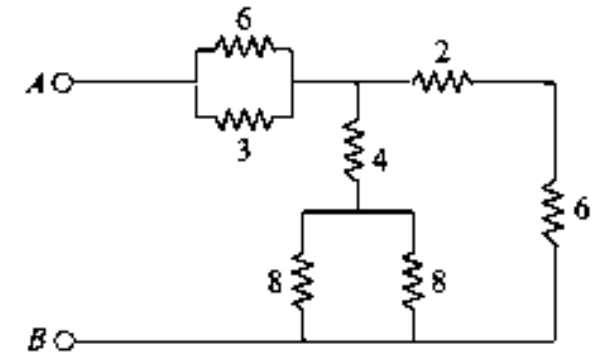
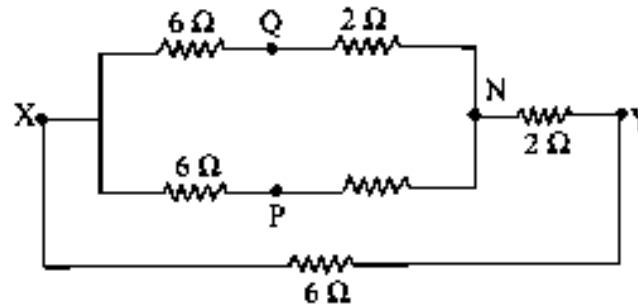
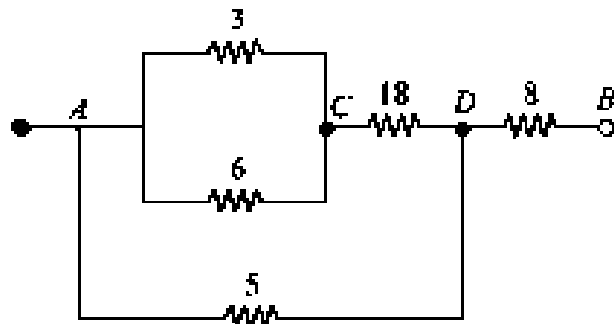
- $\frac{V_T}{R_e} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$

- $\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

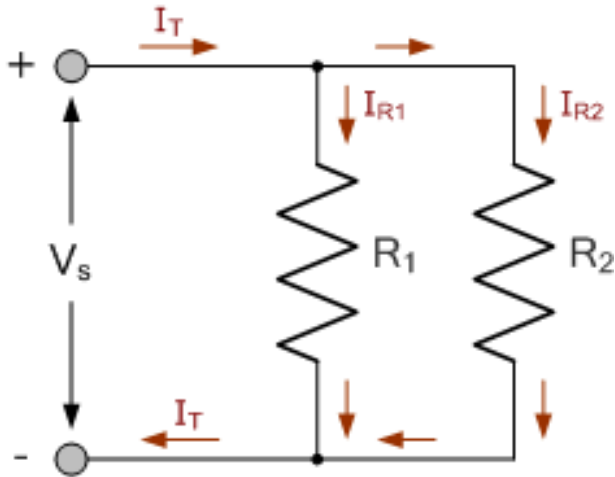


Examples:

- Calculate the Effective Resistance Between two terminals.



Current Divider Rule



$$I_T = I_{R1} + I_{R2}$$

$$I_{R1} = \frac{V}{R_1}$$

$$I_{R2} = \frac{V}{R_2}$$

$$I_T = \frac{V}{R_1} + \frac{V}{R_2}$$

$$I_T = V \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \Rightarrow V = I_T \frac{R_1 R_2}{R_1 + R_2}$$

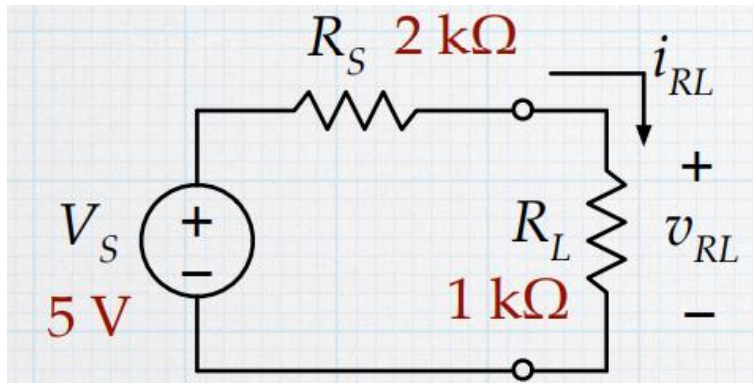
$$I_{R1} = \frac{1}{R_1} I_T \frac{R_1 R_2}{R_1 + R_2} = I_T \frac{R_2}{R_1 + R_2}$$

$$I_{R2} = \frac{1}{R_2} I_T \frac{R_1 R_2}{R_1 + R_2} = I_T \frac{R_1}{R_1 + R_2}$$

Source Transmission :

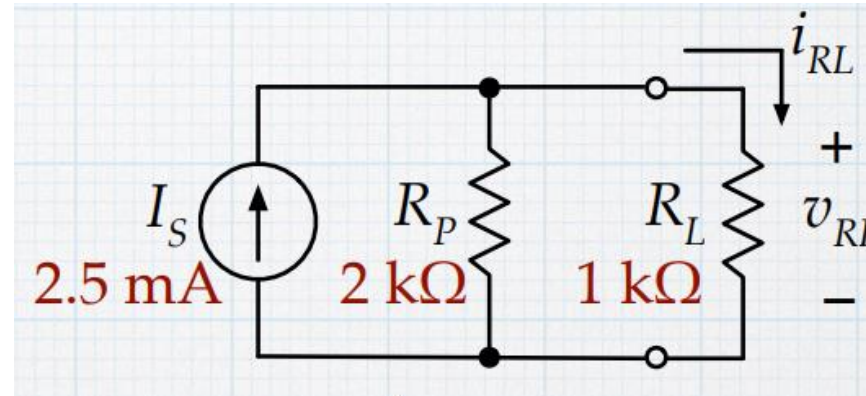
Voltage Sources

Current Sources



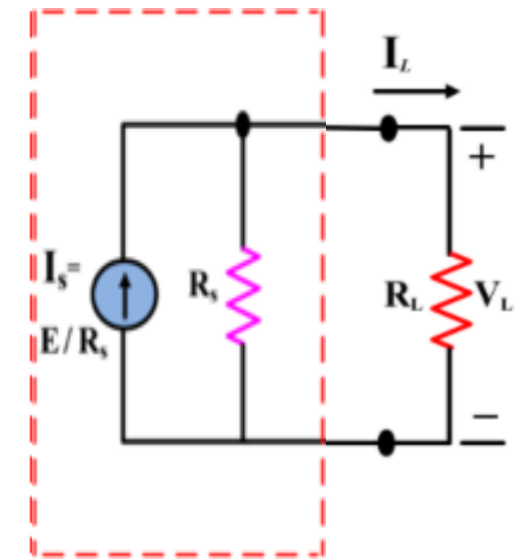
$$v_{RL} = \frac{R_L}{R_L + R_S} V_S$$
$$= \frac{1 \text{ k}\Omega}{1 \text{ k}\Omega + 2 \text{ k}\Omega} (5 \text{ V}) = 1.67 \text{ V}$$

$$i_{RL} = 1.67 \text{ mA}, P = 2.78 \text{ mW}$$



$$i_{RL} = \frac{\frac{1}{R_L}}{\frac{1}{R_L} + \frac{1}{R_P}} I_S$$
$$= \frac{\frac{1}{1 \text{ k}\Omega}}{\frac{1}{1 \text{ k}\Omega} + \frac{1}{2 \text{ k}\Omega}} (2.5 \text{ mA}) = 1.67 \text{ mA}$$

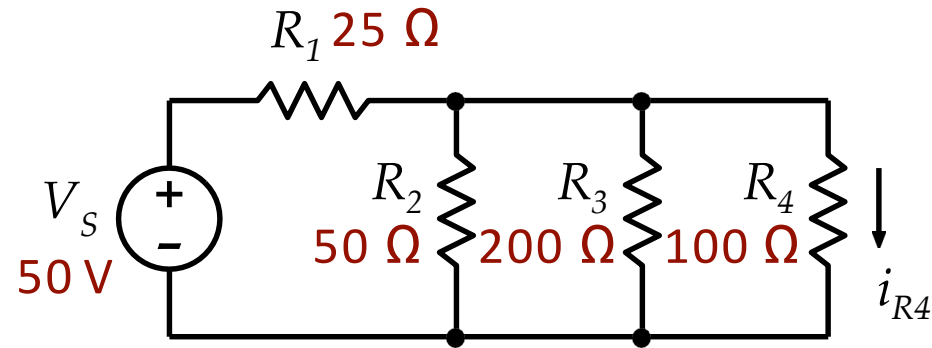
$$v_{RL} = 1.67 \text{ V}, P = 2.78 \text{ mW}$$



(b) Current source

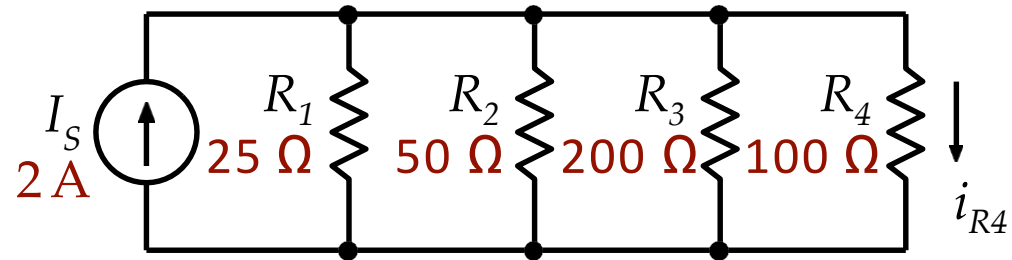
Example 1

Find i_{R4} in the circuit at right.



Use a source transformation to put everything in parallel.

$$I_S = V_S / R_1 = 50\text{ V} / 25\ \Omega = 2\text{ A}.$$

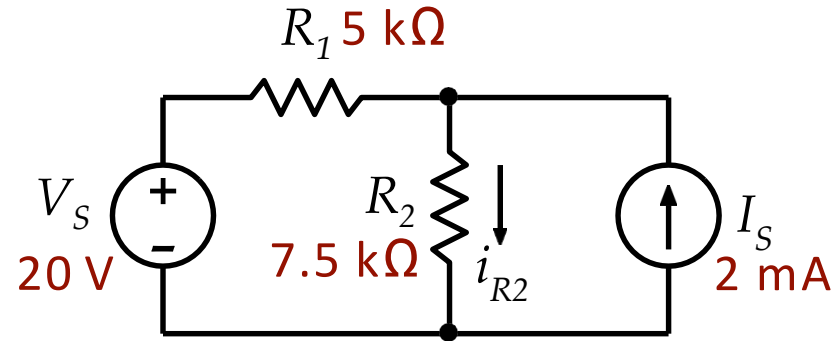


Then use a current divider:

$$\begin{aligned} i_{R4} &= \frac{\frac{1}{R_4}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} I_S \\ &= \frac{\frac{1}{100\ \Omega}}{\frac{1}{25\ \Omega} + \frac{1}{50\ \Omega} + \frac{1}{200\ \Omega} + \frac{1}{100\ \Omega}} (2\text{ A}) = 0.267\text{ A} \end{aligned}$$

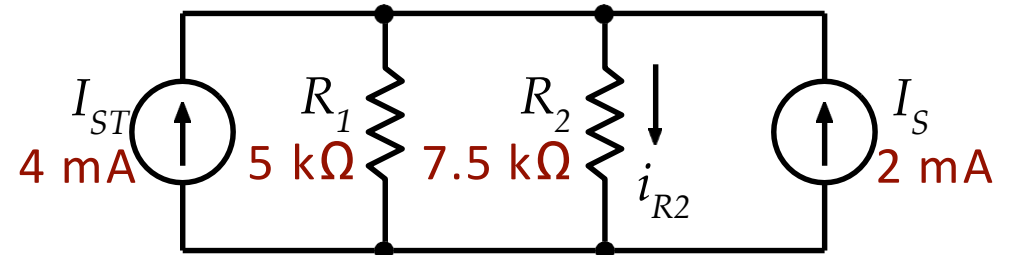
Example 2

Find i_{R2} in the circuit at right.

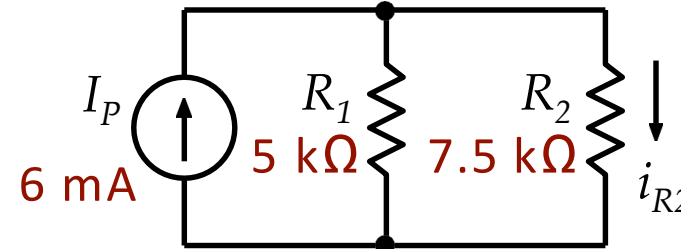


Transform the voltage source / resistor combo.

$$I_{ST} = V_S / R_1 = 20\text{ V} / 5\text{ k}\Omega = 4\text{ mA}.$$



Combine the two current sources, $I_P = I_{ST} + I_S = 6\text{ mA}$...

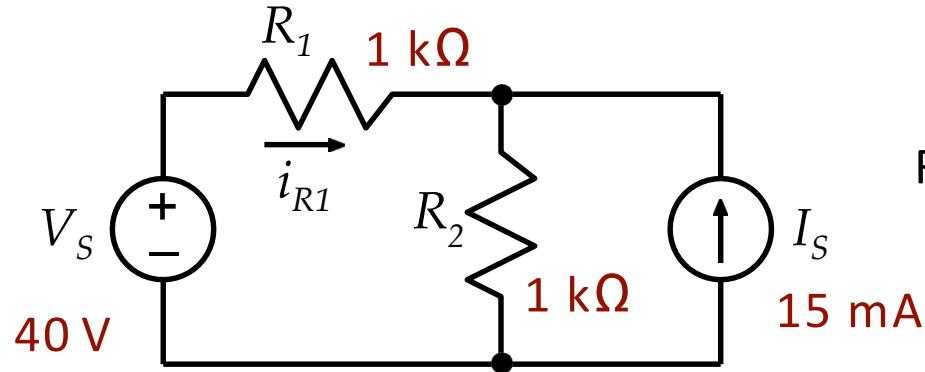


...and use the current divider then once again.

$$i_{R2} = \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} I_P$$

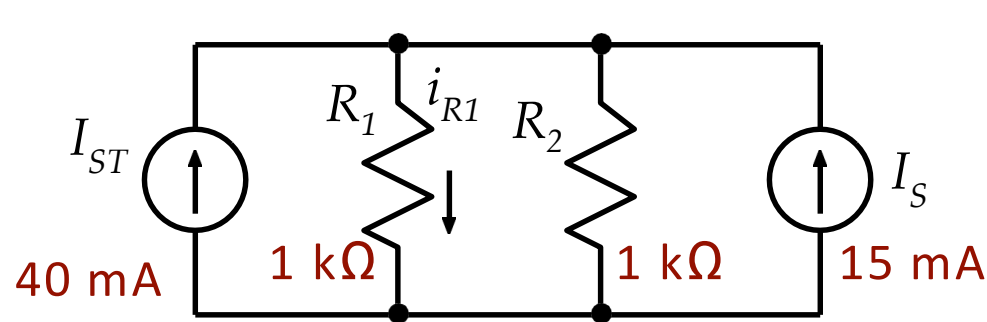
$$= \frac{\frac{1}{7.5\text{ k}\Omega}}{\frac{1}{7.5\text{ k}\Omega} + \frac{1}{5\text{ k}\Omega}} (6\text{ mA}) = 2.4\text{ mA}$$

Example 3 (An incorrect application)



Find i_{R1} in the circuit at left.

The previous example worked nicely so use the same method. Transform V_S & R_1 , and use current divider with the total current.

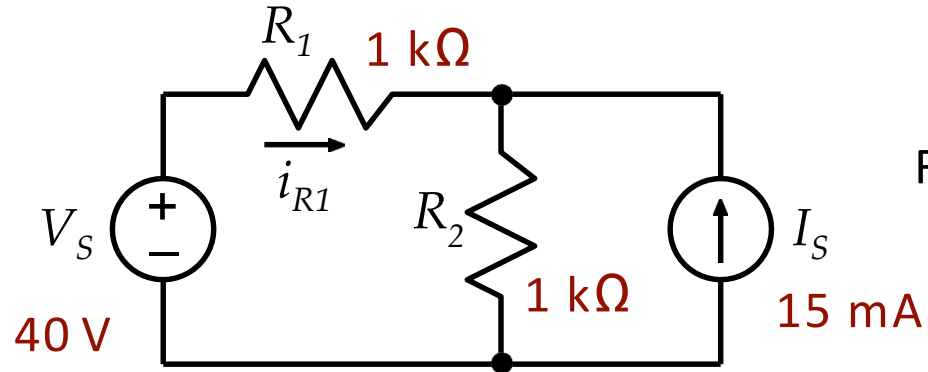


$$i_{R1} = \frac{\frac{1}{1k\Omega}}{\frac{1}{1k\Omega} + \frac{1}{1k\Omega}} (I_{ST} + I_S)$$

$$= \frac{\frac{1}{1k\Omega}}{\frac{1}{1k\Omega} + \frac{1}{1k\Omega}} (55\text{mA}) = 27.5\text{mA}$$

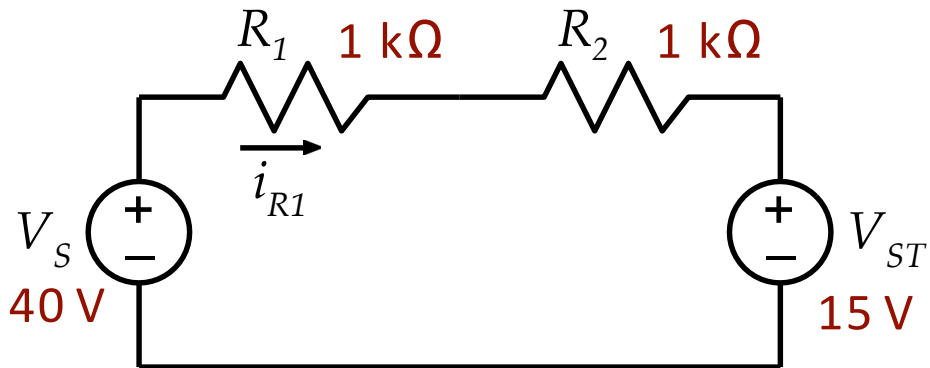
It seems nice, but it is wrong because you cannot transform the component for which you are trying to find voltage or current. (To see that it is wrong, insert $i_{R1} = 27.5\text{ mA}$ in the original circuit and show that there are serious inconsistencies with the currents and voltages.)

Example 3 (Redo it correctly.)



Find i_{R1} in the circuit at left.

Transform I_S & R_2 .

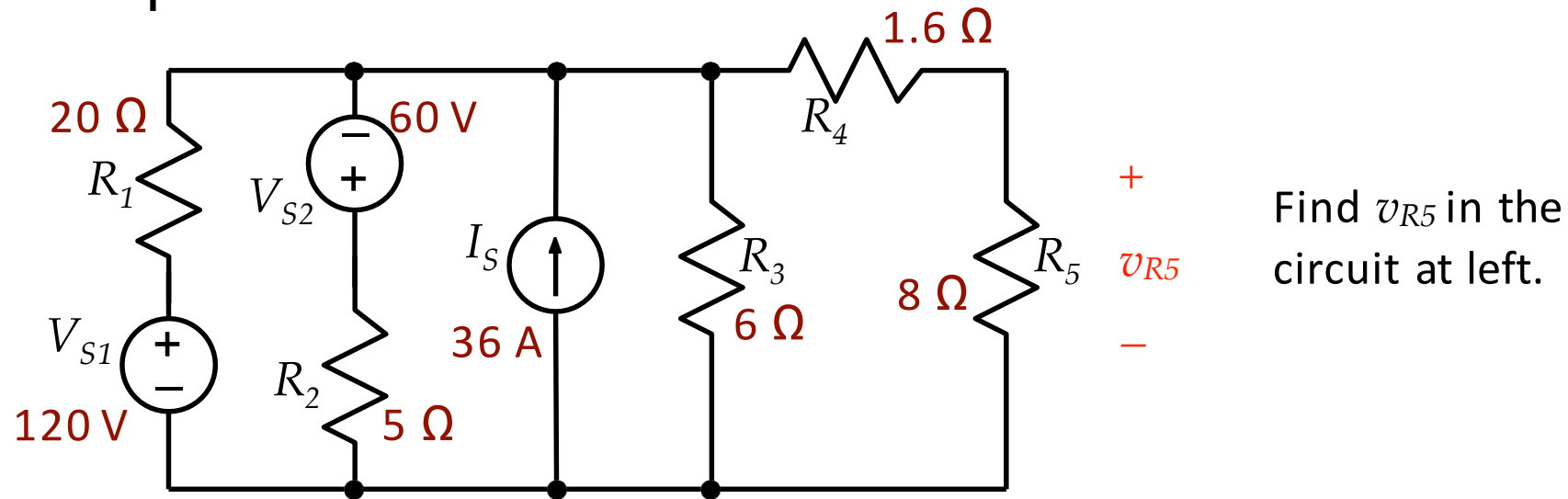


This is the correct answer.

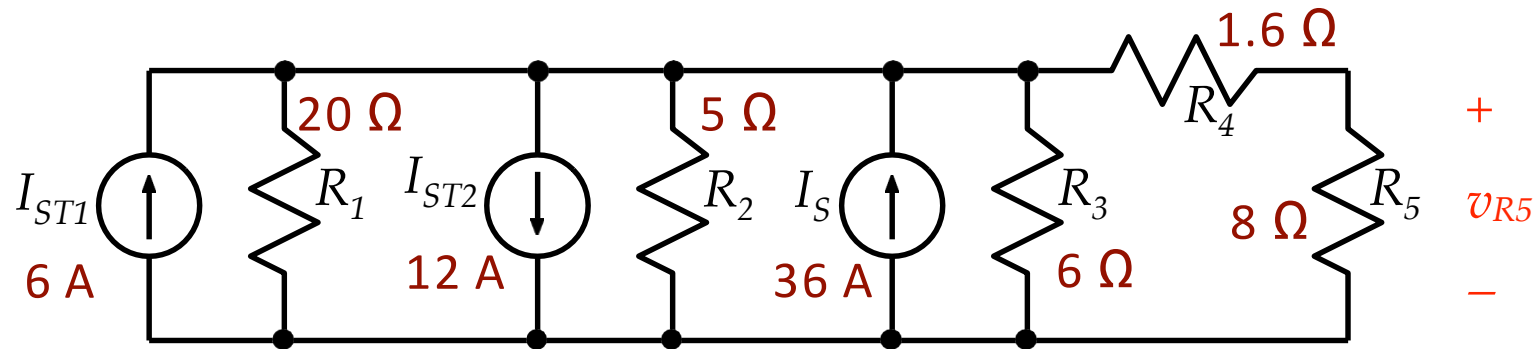
Writing a KVL loop equation and solving for i_{R1} gives

$$\begin{aligned} i_{R1} &= \frac{V_S - V_{ST}}{R_1 + R_2} \\ &= \frac{40V - 15V}{1k\Omega + 1k\Omega} = 12.5mA \end{aligned}$$

Example 4

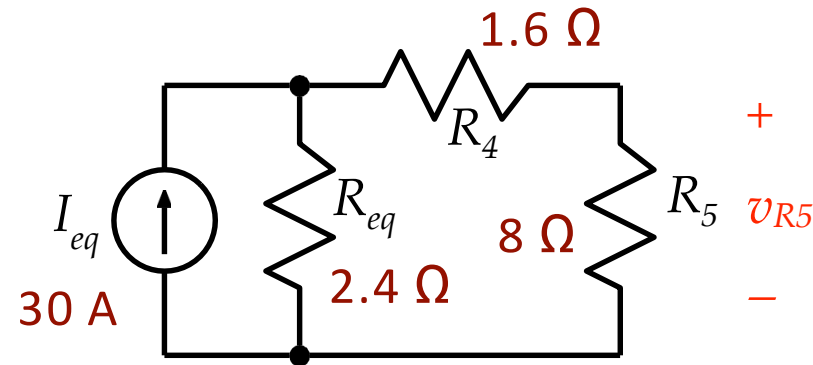


Transform two voltage sources to current sources. (Pay attention to polarity.)



Example 4 (cont.)

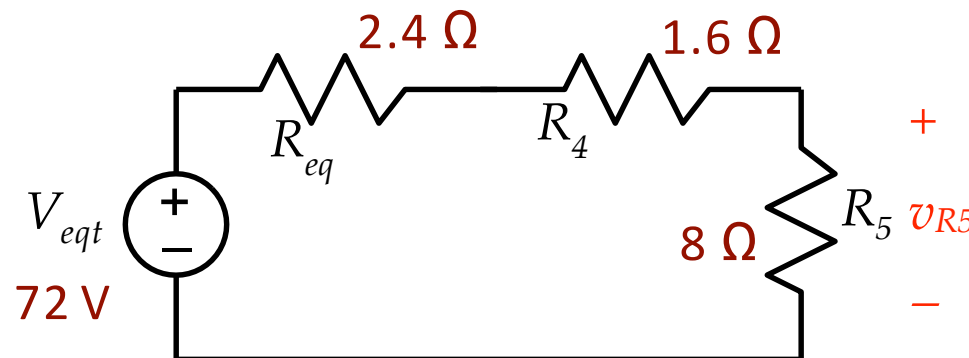
Add the parallel current sources into one. Combine the parallel resistors into one.



$$I_{eq} = 6\text{ A} - 12\text{ A} + 36\text{ A} = 30\text{ A}.$$

$$R_{eq} = 20\ \Omega \parallel 5\ \Omega \parallel 6\ \Omega = 2.4\ \Omega.$$

Transform I_{eq} & R_{eq} :



Use voltage divider:

$$\begin{aligned} V_{f/5} &= \frac{f_{l5}}{f_{l_{eq}} + f_{l4} + f_{l5}} V_{eqt} \\ &= \frac{8\text{ fi}}{2.4\text{ fi} + 1.6\text{ fi} + 8\text{ fi}} (72\text{ V}) \\ &= 48\text{ V} \end{aligned}$$

Kirchhoff 's law :

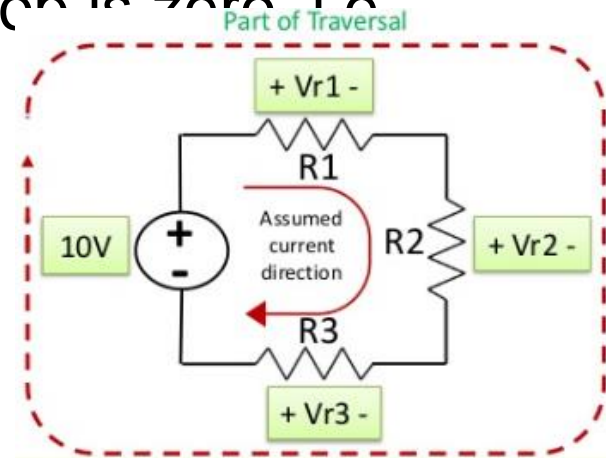
TYPES OF KIRCHHOFF'S LAW

✓ KIRCHHOFF'S VOLTAGE
LAW

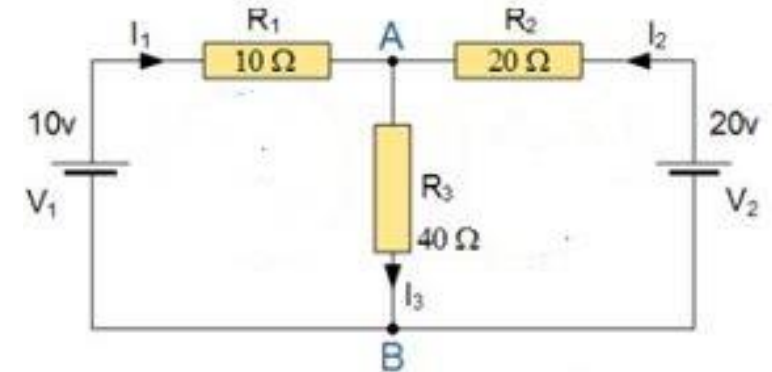
✓ KIRCHHOFF'S CURRENT
LAW

- **Kirchhoff's Voltage Law** (KVL) states that the algebraic sum of the voltages across any set of branches in a closed loop is zero i.e.

$$\sum V_{across branches} = 0$$



- Resulting KVL Equation : $V_{r1} + V_{r2} + V_{r3} - 10 = 0$



Loop :1

$$+V_1 - I_1 R_1 - (I_1 - I_2) R_3 = 0$$

$$+10 - I_1 (10) - (I_1 - I_2) (40) = 0$$

$$50 I_1 - 40 I_2 = 10$$

Loop :2

$$-(I_2 - I_1) R_3 - I_2 R_2 - V_2 = 0$$

$$-(I_2 - I_1) 40 - I_2 (20) - 20 = 0$$

$$40 I_1 - 60 I_2 = 20$$

$$I_1 = -0.143$$

$$I_2 = -0.429$$

Example :1

Find the current passing through R1 Resistor using KVL, if the Values are gives as follows:

$$V_1 = 24 \text{ V}$$

$$V_2 = 16 \text{ V}$$

$$V_3 = 8 \text{ V}$$

$$R_1 = 2 \Omega$$

$$R_2, R_3, R_4 = 4 \Omega$$

For Loop 1:

$$+ V_1 - V_3 - R_2 (I_1 - I_3) - R_4 (I_1 - I_2) = 0$$

$$+ 24 - 8 - 4 (I_1 - I_3) - 4 (I_1 - I_2) = 0$$

For Loop 2:

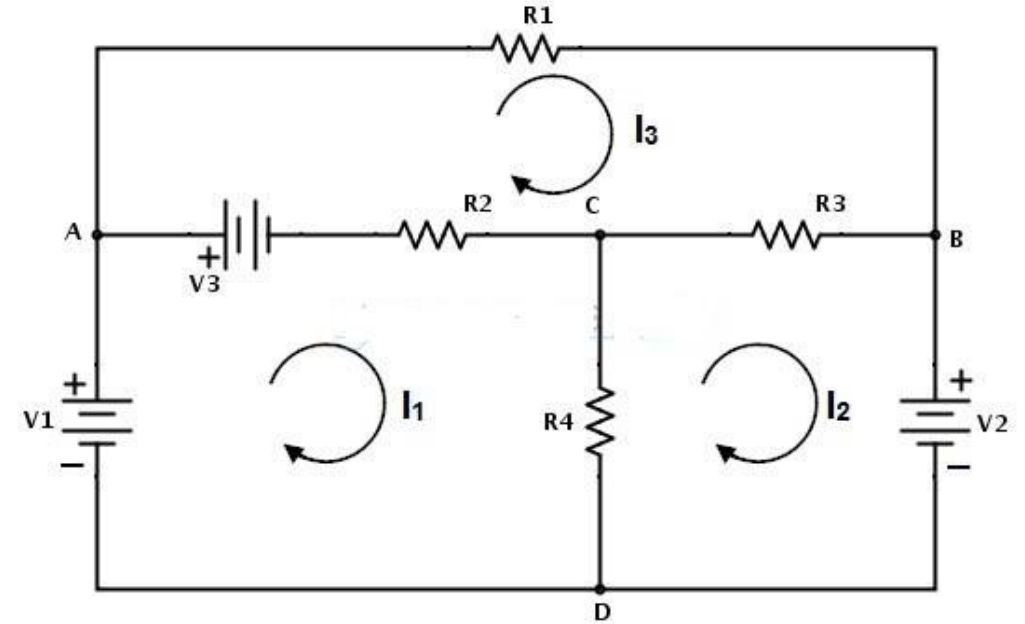
$$-R_4 (I_2 - I_1) - R_3 (I_2 - I_3) - V_2 = 0$$

$$-4 (I_2 - I_1) - 4 (I_2 - I_3) - 16 = 0$$

For Loop 3:

$$-R_1 (I_3) - R_3 (I_3 - I_2) - R_2 (I_3 - I_1) + V_3 = 0$$

$$-2(I_3) - 4 (I_3 - I_2) - 4 (I_3 - I_1) + 8 = 0$$



$$+8I_1 - 4I_2 - 4I_3 = 16$$

$$-4I_1 + 8I_2 - 4I_3 = -16$$

$$-4I_1 - 4I_2 + 10I_3 = 8$$

$$I_1 = 5.33 \text{ A}$$

$$I_2 = 2.66 \text{ A}$$

$$I_3 = 4 \text{ A}$$

Example :2

Find the current passing through R1 Resistor using KVL.

For Loop 1:

$$+ V_1 - R_2 (I_1 - I_3) - R_1 (I_1 - I_2) = 0$$

For Loop 2:

$$-R_1 (I_2 - I_1) - R_3 (I_2 - I_3) + V_2 = 0$$

For Loop 3:

$$-R_4 (I_3) - R_3 (I_3 - I_2) - R_2 (I_3 - I_1) = 0$$

$$+8I_1 - 6I_2 - 2I_3 = 50$$

$$-6I_1 + 14I_2 - 8I_3 = 10$$

$$-2I_1 - 8I_2 + 20I_3 = 0$$

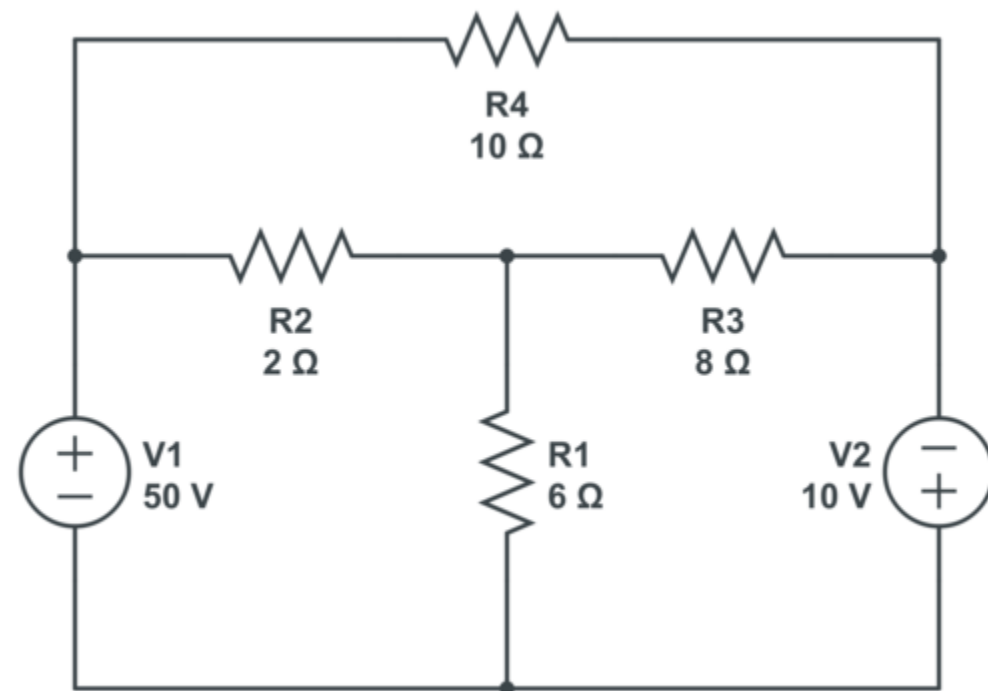
$$I_1 = 16 \text{ A}$$

$$I_2 = 11 \text{ A}$$

$$I_3 = 6 \text{ A}$$

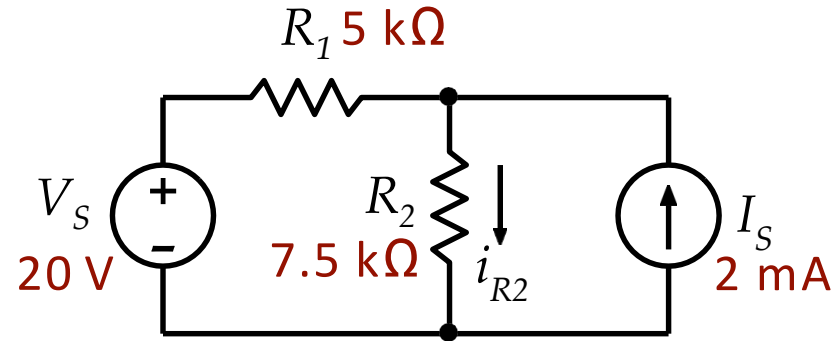
Current passing through R1:

$$I = I_1 - I_2 = 16 - 11 = 5 \text{ Amp}$$



Example 3

Find i_{R2} in the circuit at right.
By KVL and KCL



By KVL

For Loop 1:

$$+20 - 5k I_1 - 7.5k (I_1 - I_2) = 0 \quad \text{--- 1}$$

For Loop 2:

$$I_2 = -2 \text{ mA} \quad \text{--- 2}$$

Put the Value of 2 in 1

$$+20 - 5k I_1 - 7.5k (I_1 + 2 \text{ m}) = 0$$

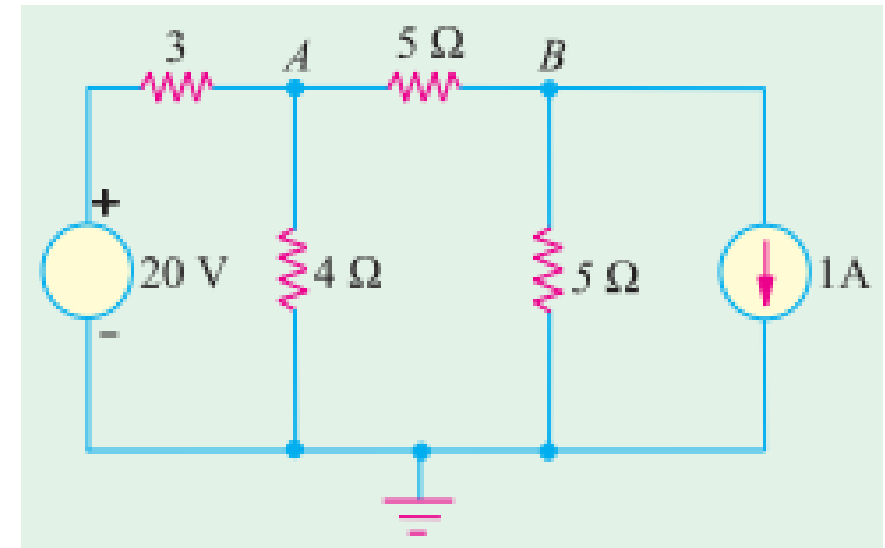
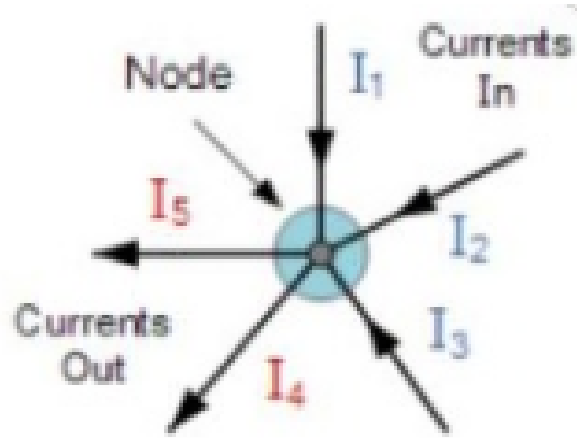
$$+20 - 5k I_1 - 7.5k (I_1) - 15 = 0$$

$$I_1 = \frac{5}{12.5k} = 0.4 \text{ mA}$$

Current Passing through R2 is

$$I = I_1 - I_2 = 0.4 - (-2) = 2.4 \text{ mA}$$

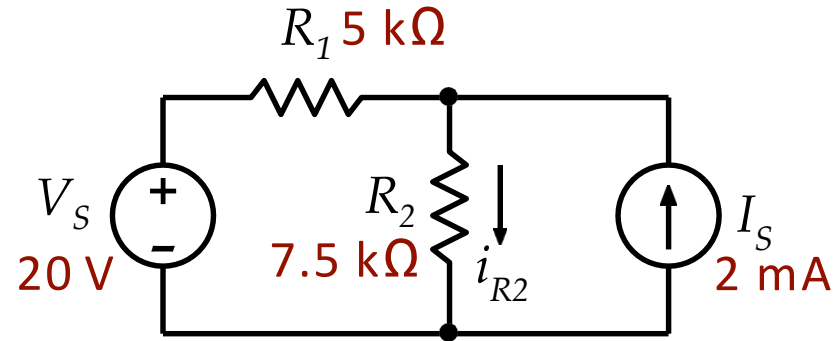
- **Kirchhoff's Current Law** (KCL) states that the algebraic sum of all currents entering and leaving any point in the circuit is zero. i.e.,



- Resulting KCL Equation : $I_1 + I_2 + I_3 - I_4 - I_5 = 0$

Example 2

Find i_{R2} in the circuit at right.
By KVL and KCL



By KVL

For node 1:

$$\frac{V_1 - V_s}{R_1} + \frac{0 - V_1}{R_2} + I_s = 0$$

————— 1

$$\frac{V_1 - 20}{5} + \frac{V_1 - 0}{7.5} - 2 = 0$$

$$12.5 V_1 - 150 - 75 = 0$$

$$V_1 = \frac{225}{12.5} = 18\text{ V}$$

Current Passing through R_2 is

$$I = \frac{18}{7.5} = 2.4\text{ mA}$$

Example :3

Find the current passing through R1 Resistor using KVL.

For node b:

$$\frac{V_b - V_a}{2} + \frac{V_b - 0}{6} + \frac{V_b - (-V_c)}{8} = 0$$

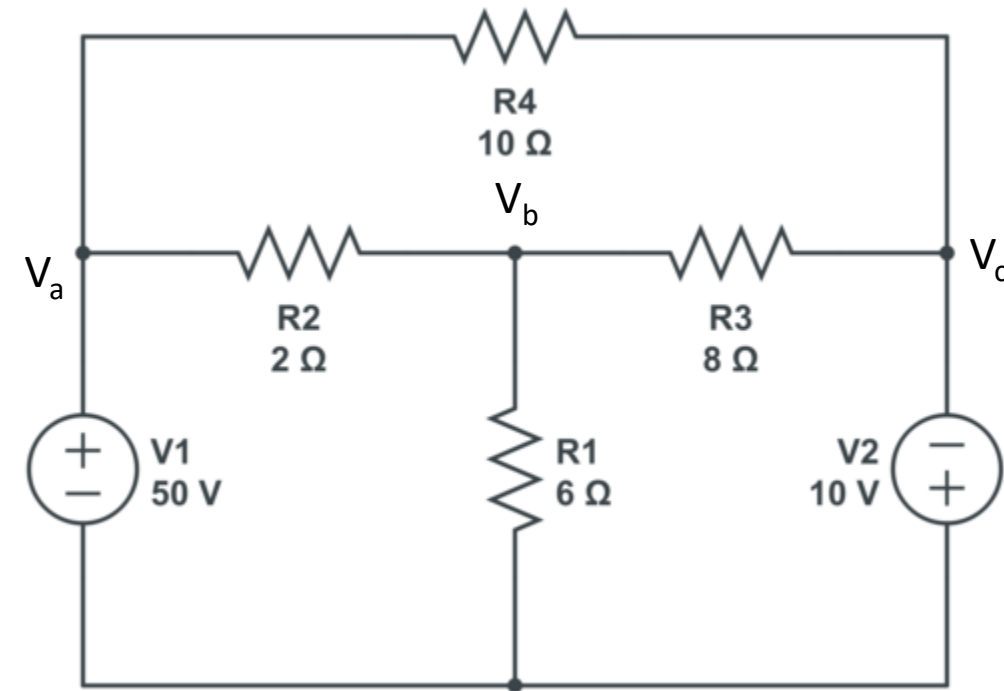
$$\frac{V_b - 50}{2} + \frac{V_b}{6} + \frac{V_b + 10}{8} = 0$$

$$12V_b - 600 + 4V_b + 3V_b + 30 = 0$$

$$19V_b = 570$$

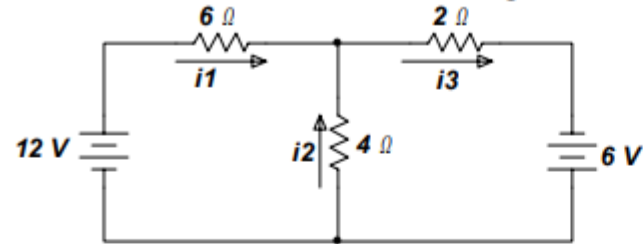
$$V_b = 30 \text{ V}$$

$$\text{Current passing through 6 ohm resistor} = \frac{V_b}{6} = \frac{30}{6} = 5 \text{ Amp}$$

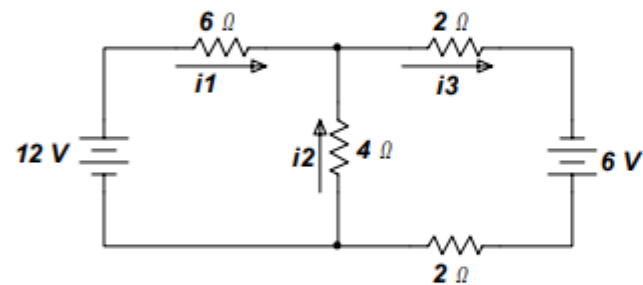


Examples

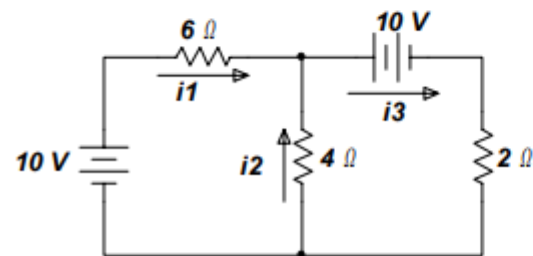
Determine the currents in the following circuits with reference to the indicated direction.



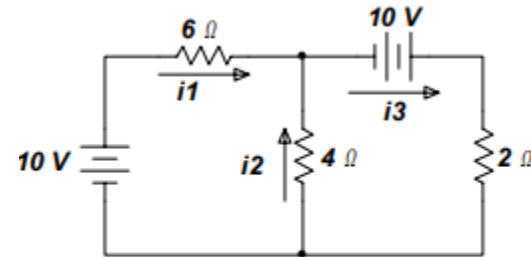
Answer: $i_1 = 2.180\text{ A}$, $i_2 = 0.270\text{ A}$, $i_3 = 2.450\text{ A}$



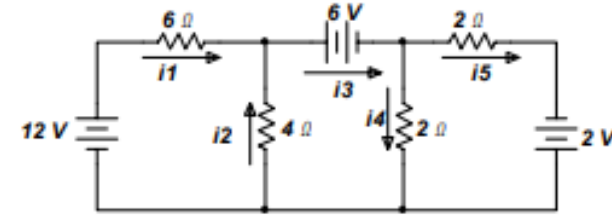
Answer: $i_1 = 1.877\text{ A}$, $i_2 = -0.187\text{ A}$, $i_3 = 1.690\text{ A}$



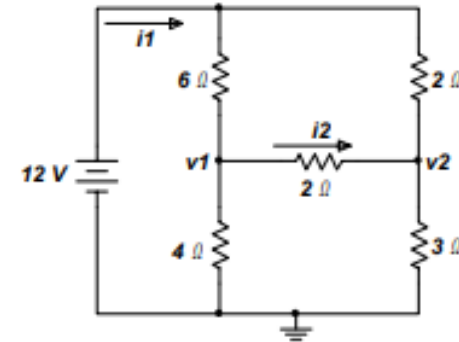
Answer: $i_1 = 0.455\text{ A}$, $i_2 = -1.820\text{ A}$, $i_3 = -1.36\text{ A}$



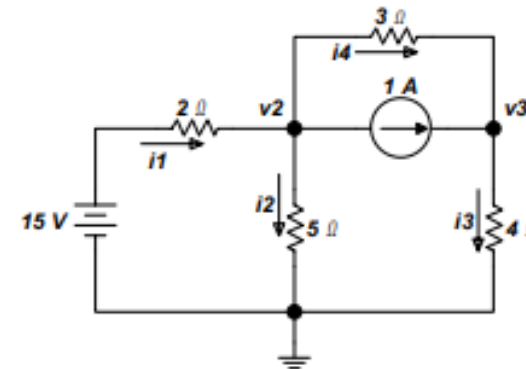
Answer: $i_1 = 2.270\text{ A}$, $i_2 = 0.909\text{ A}$, $i_3 = 3.18$



Answer: $i_1 = 1.180\text{ A}$, $i_2 = -1.240\text{ A}$, $i_3 = -0.058\text{ A}$, $i_4 = -0.529\text{ A}$, $i_5 = 0.471\text{ A}$



Answer: $i_1 = 3.690\text{ A}$, $i_2 = -0.429\text{ A}$, $v_1 = 5.83\text{ V}$, $v_2 = 6.69\text{ V}$



Answer: $i_1 = 3.31\text{ A}$, $i_2 = 1.68\text{ A}$, $i_3 = 1.63\text{ A}$, $i_4 = 0.627\text{ A}$, $v_2 = 8.39\text{ V}$, $v_3 = 6.51\text{ V}$

Delta – Star Transformation :

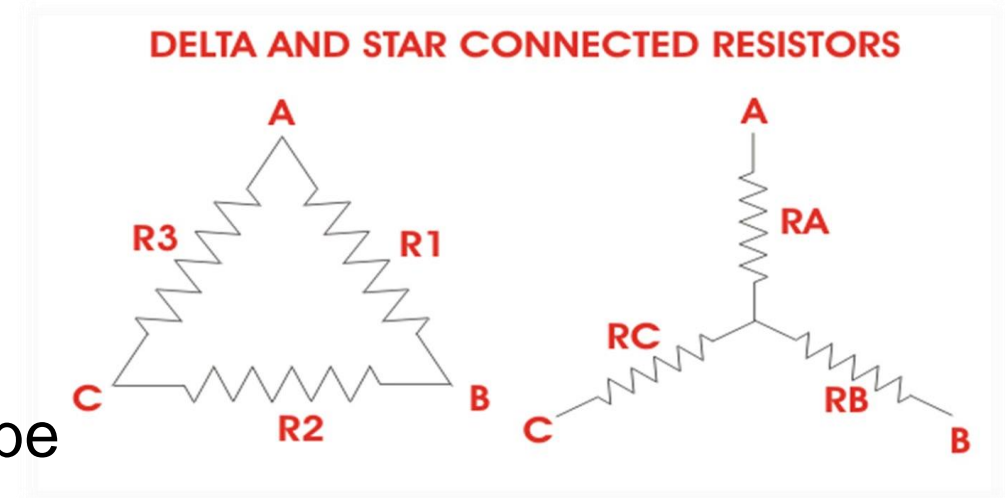
- To convert a delta to star network we need to derive a transformation formula for equating the various resistors to each other between the various terminals. Consider the circuit below.

- The resistance between the points A & B will be

$$R_A + R_B = \frac{R_1 \cdot (R_2 + R_3)}{R_1 + R_2 + R_3} = \frac{R_1 R_2 + R_1 R_3}{R_1 + R_2 + R_3}$$

- The resistance between the points B & C will be

$$R_B + R_C = \frac{R_2 \cdot (R_3 + R_1)}{R_1 + R_2 + R_3} = \frac{R_2 R_3 + R_1 R_2}{R_1 + R_2 + R_3}$$

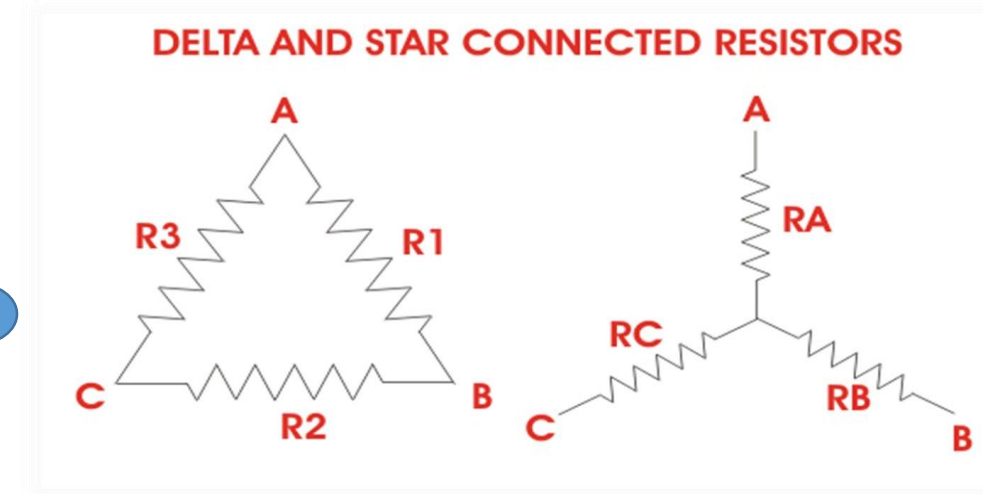


1

2

- The resistance between the points C & A will be

$$R_C + R_A = \frac{R_3 \cdot (R_1 + R_2)}{R_1 + R_2 + R_3} = \frac{R_1 R_3 + R_2 R_3}{R_1 + R_2 + R_3} \quad \text{--- 3}$$



- Adding equations 1 , 2 and 3 will be

$$2R_A + 2R_B + 2R_C = \frac{2(R_1 \cdot R_2) + 2(R_2 \cdot R_3) + 2(R_3 \cdot R_1)}{R_1 + R_2 + R_3}$$

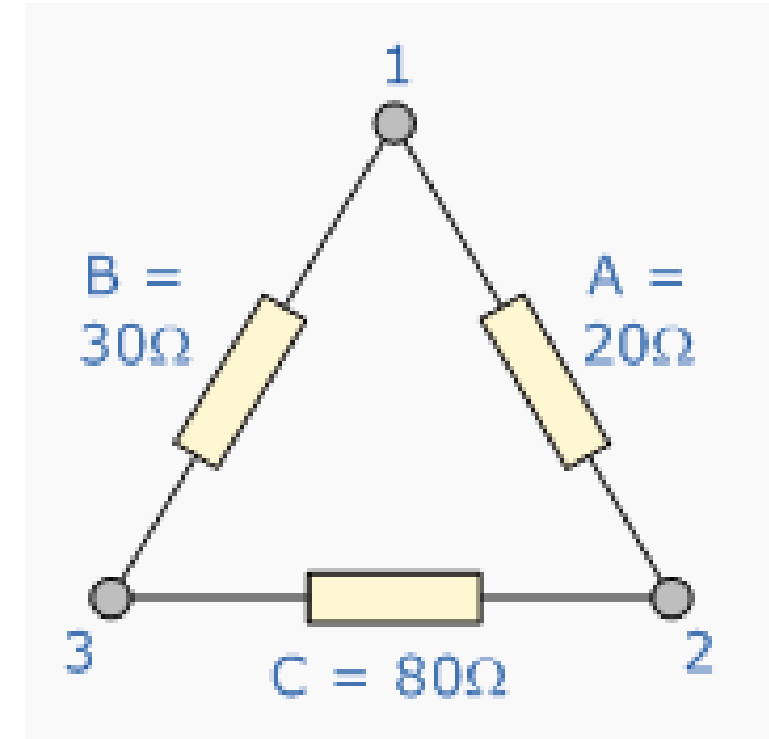
$$R_A + R_B + R_C = \frac{R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1}{R_1 + R_2 + R_3} \quad \text{--- 4}$$

- Subtracting equations 1, 2 and 3 from equation 4 we get,

$$R_A = \frac{R_3 \cdot R_1}{R_1 + R_2 + R_3} \quad \text{--- 5}$$

$$R_B = \frac{R_1 \cdot R_2}{R_1 + R_2 + R_3} \quad \text{--- 6}$$

$$R_C = \frac{R_2 \cdot R_3}{R_1 + R_2 + R_3} \quad \text{--- 7}$$



Star – Delta Transformation :

- To convert star to delta multiply equations 5,6 & 6,7 & 7,5.

$$R_A R_B + R_B R_C + R_C R_A = \frac{R_1 \cdot R_2 \cdot R_3^2 + R_2 \cdot R_3 \cdot R_1^2 + R_3 \cdot R_1 \cdot R_2^2}{(R_1 + R_2 + R_3)^2}$$
$$= \frac{R_1 \cdot R_2 \cdot R_3}{R_1 + R_2 + R_3} \quad \text{—————} \quad \textcircled{8}$$

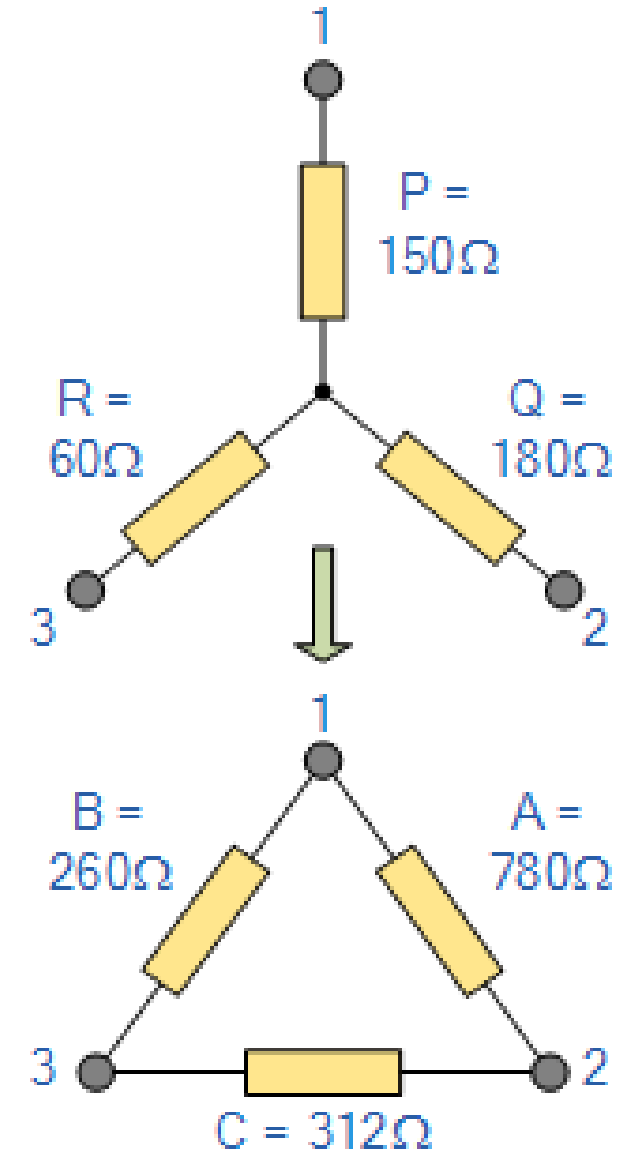
- Now dividing equation 8 by 5, 6, and 7 we get,

$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A} \quad \text{—————} \quad \textcircled{9}$$

- we get,

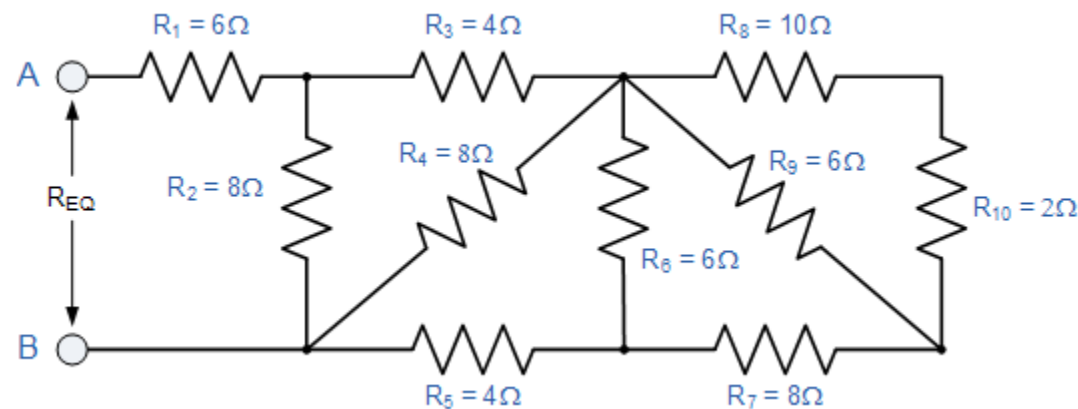
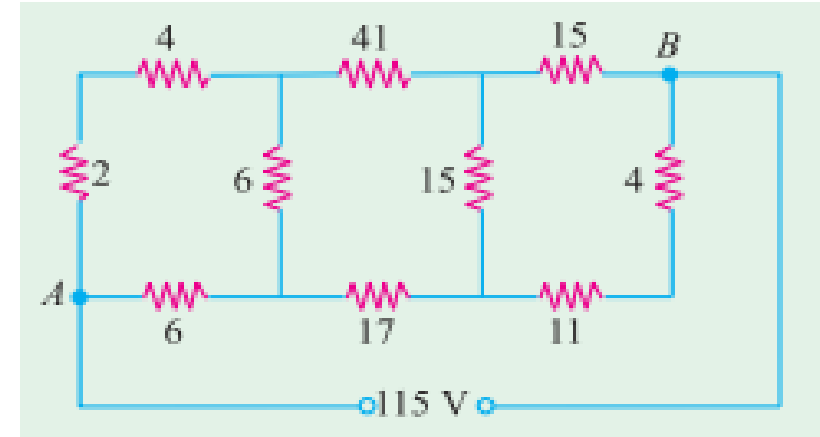
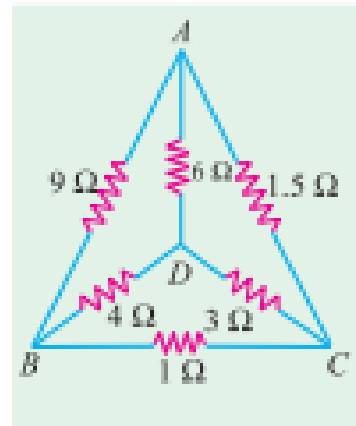
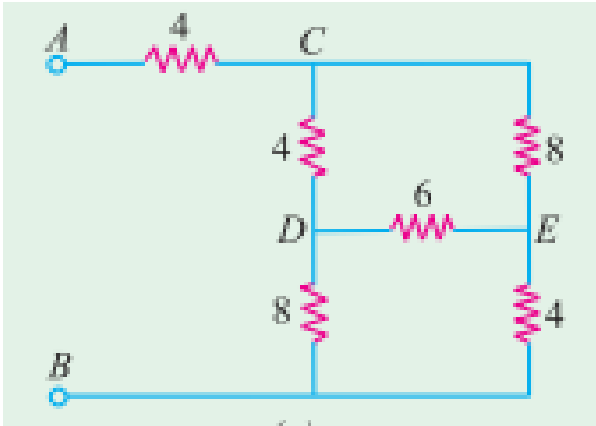
$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B} \quad \text{--- 10}$$

$$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C} \quad \text{--- 11}$$



Examples:

Determine the current passing through circuit



Superposition Theorem :

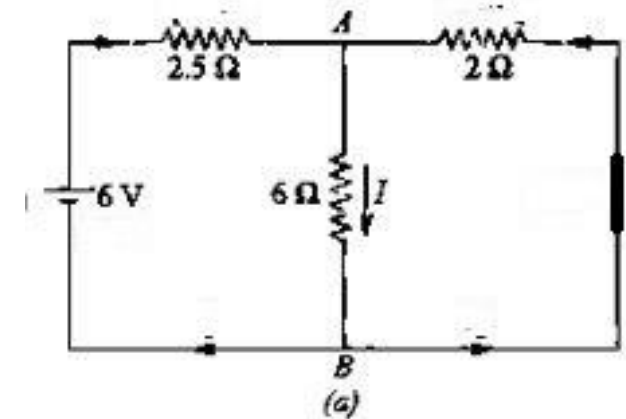
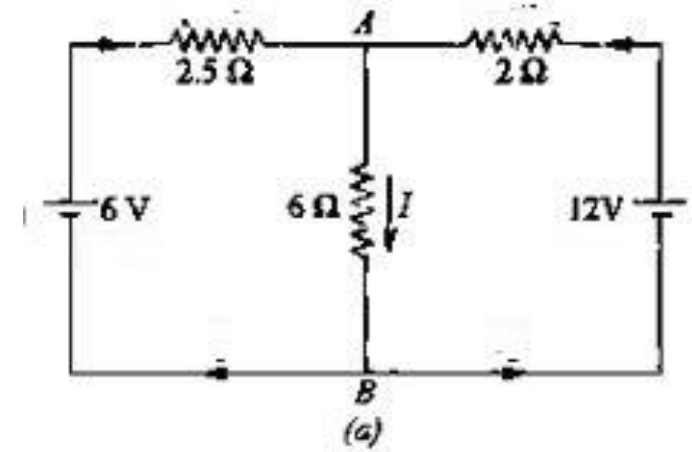
It states that “If there are several sources acting simultaneously in an electrical circuit, then the current through any branch of the circuit is summation of currents which would flow through the branch for each source keeping all other sources dead.” .

- **Step 1:** Take one voltage source at a time and replace the other one with either short or internal resistance.
- **Step 2:** Determine the particular current by only one source.

$$+6 - 2.5 I_1 - 6 (I_1 - I_2) = 0 \quad 8.5 I_1 - 6 I_2 = 6 \quad I_1 = 1.5 \text{ A}$$

$$-6 (I_2 - I_1) - 2 I_2 = 0 \quad -6 I_1 + 8 I_2 = 0 \quad I_2 = 1.125 \text{ A}$$

$$\text{Current passing through } 6 \Omega \text{ resistor } I' = I_1 - I_2 = 1.50 - 1.125 \\ = 0.375$$

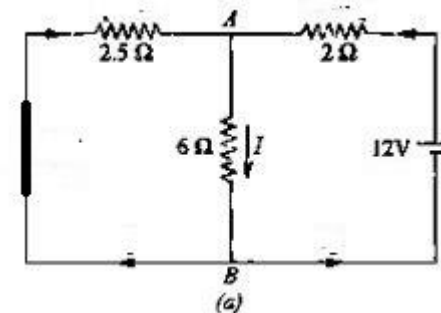


- **Step 3:** take next source and repeat steps 1 and 2.

$$\text{Total resistance} = (2 + (6 \parallel 2.5)) = \left(2 + \frac{6 * 2.5}{6 + 2.5}\right) = 3.76 \Omega$$

$$\text{Total Current } I = \left(\frac{12}{3.76}\right) = 3.2 \text{ amp}$$

$$\text{Current passing through } 6 \Omega \text{ resistor } I'' = I \left(\frac{2.5}{8.5}\right) = 3.2 * \frac{2.5}{8.5} = 0.94 \text{ A}$$



- **Step 4:** Calculate the total current passing through the circuit.

$$\text{Net Current passing through } 6 \Omega \text{ resistor } I = I' + I'' = 0.375 + 0.94 = 1.314 \text{ A}$$

Example: Using Superposition theorem, find the value of following.

(1) Current passing through $2\ \Omega$ resistor.

(2) The value of the output voltage.

- **Step 1:** Take one current source at a time and replace the other one with either short or internal resistance.
- **Step 2:** Keep only 6A Source is active.

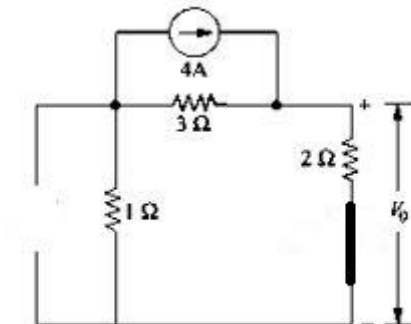
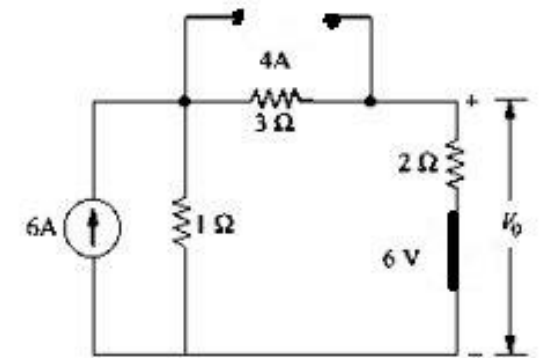
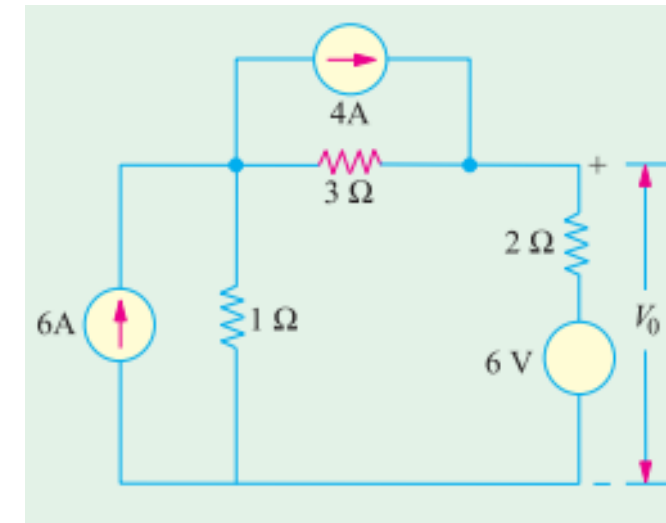
$$\text{Current passing through } 2\ \Omega \text{ resistor } I' = I \left(\frac{1}{6} \right) = 6 * \frac{1}{6} = 1\ \text{A}$$

$$\text{Voltage across } 2\ \Omega \text{ resistor } V_0' = I' * 2 = 2\ \text{V}$$

- **Step 3:** Keep only 4A Source is active.

$$\text{Current passing through } 2\ \Omega \text{ resistor } I'' = I \left(\frac{3}{6} \right) = 4 * \frac{3}{6} = 2\ \text{A}$$

$$\text{Voltage across } 2\ \Omega \text{ resistor } V_0'' = I'' * 2 = 4\ \text{V}$$



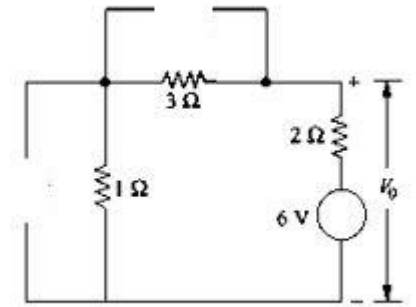
- **Step 4:** Keep only 6 V Source is active.

$$-I''' - 3I''' - 2I''' + 6 = 0$$

$$6I''' = 6$$

$$I''' = +1 \text{ Amp}$$

$$\text{Output Voltage } V_0''' = -6 + I''' * 2 = -4 \text{ V}$$



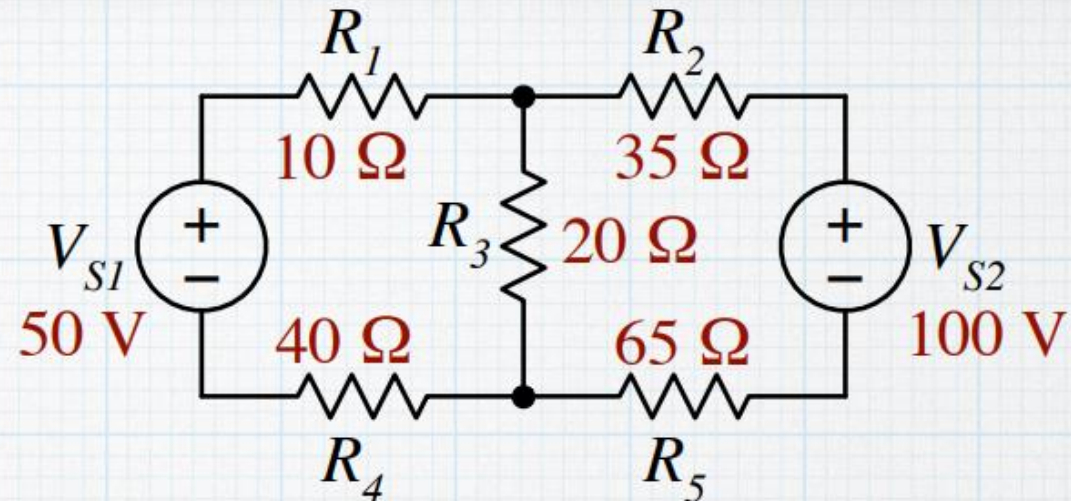
- **Step 5:** Find the total current and output voltage.

$$\text{Net Current passing through } 2 \Omega \text{ resistor } I = I' + I'' + I''' = 1 + 2 + 1 = 4 \text{ A}$$

$$\text{Output Voltage } V_0 = V' + V'' + V''' = 2 + 4 + (-4) = 2 \text{ V}$$

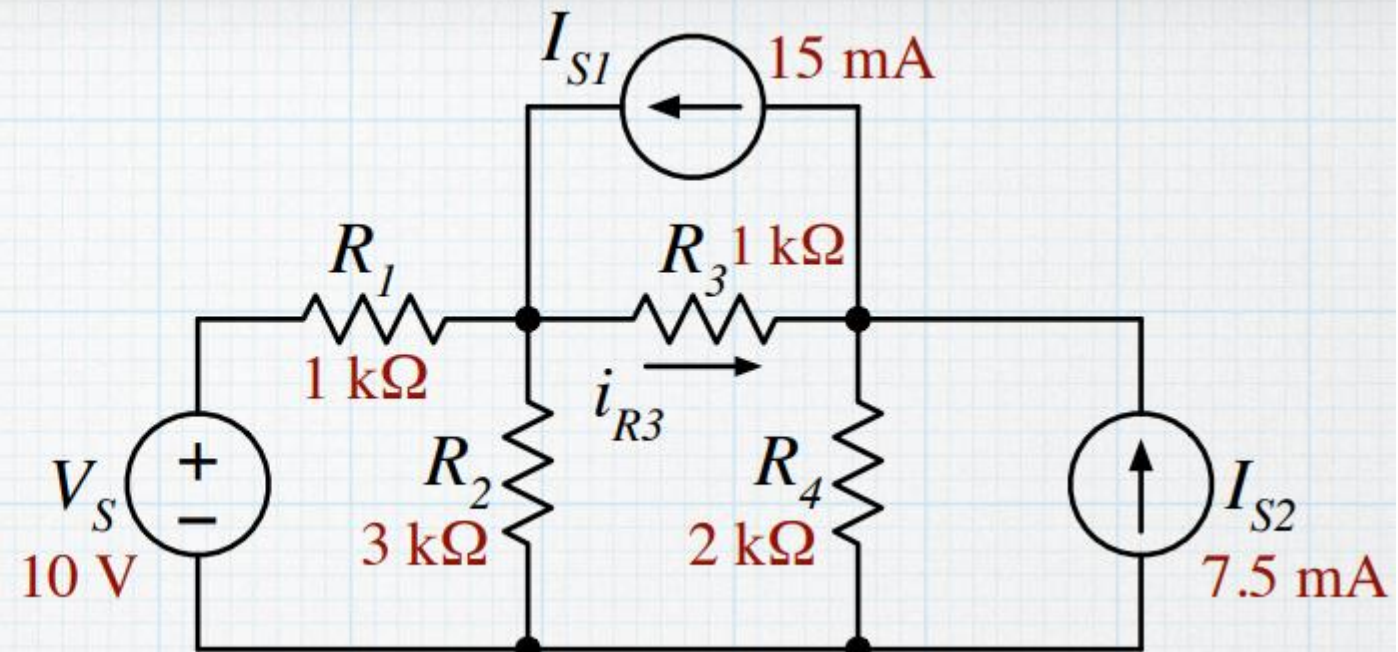
Example 3

For the circuit shown, use superposition to find the power being dissipated in R_3 .



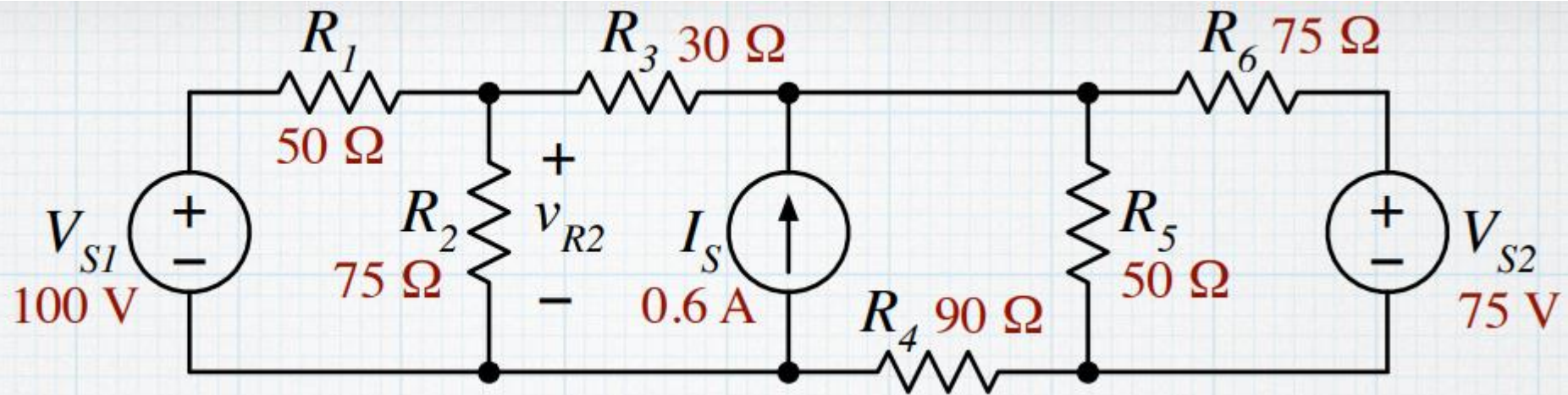
Example 4

In the circuit, find i_{R3} . With three sources, there will be three partial solutions.



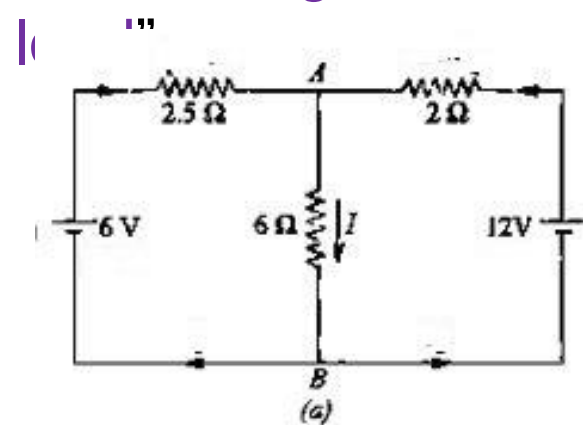
Example 5

Find v_{R2} .



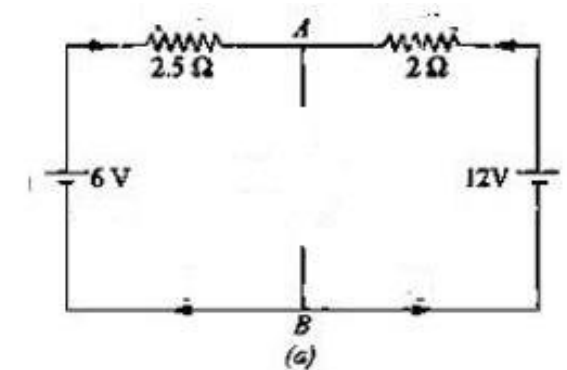
Thevenin's Theorem :

It states that “Any linear circuit containing several voltages and resistances can be replaced by just one single voltage in series with a single resistance connected across the

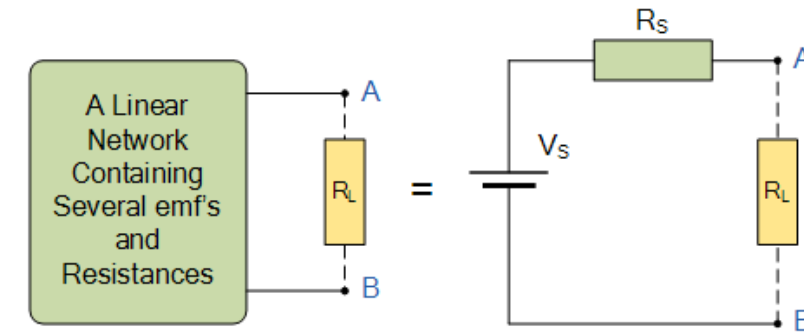


Example: Find the value of current passing through 6 ohm resistance

- **Step 1:** Temporarily remove the resistance (called load resistance R_L) whose current is required.



Thevenin's equivalent circuit



- **Step 2:** Find the open-circuit voltage V_{oc} which appears across the two terminals from where resistance has been removed. It is also called Thevenin voltage V_{th} .

$$\frac{V_A - 6}{2.5} + \frac{V_A - 12}{2} = 0$$

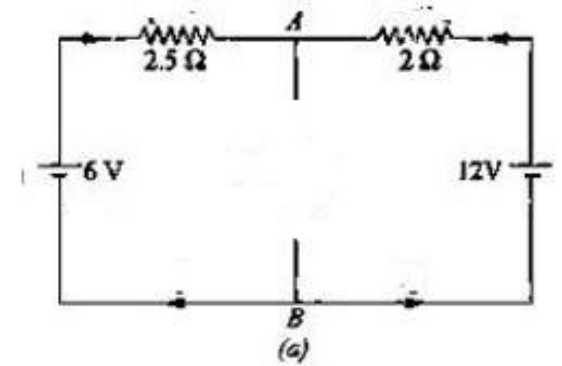
$$V_A = 9.33 \text{ V} \quad V_B = 0 \text{ V}$$

$$V_{AB} = V_A - V_B = 9.33 \text{ V}$$

$$+6 - 2.5 I - 2 I - 12 = 0$$

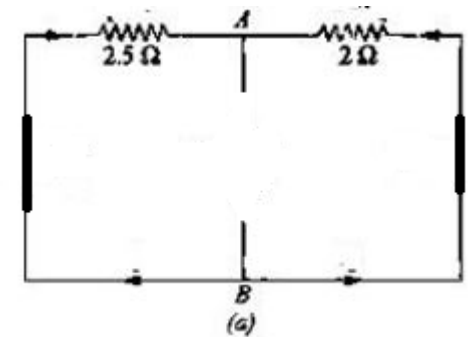
$$I = \frac{6}{-4.5} = -1.33 \text{ A}$$

$$V_{AB} = 6 - 2.5 I = 6 - 2.5 (-1.33) = 6 + 3.325 = 9.325 \text{ V}$$

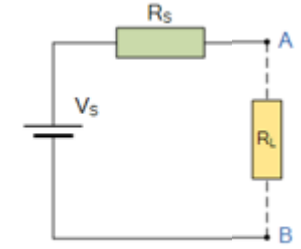


- **Step 3:** Compute the resistance of the whose network as looked into from these two terminals after all voltage sources have been removed leaving behind their internal resistances (if any) and current sources have been replaced by open-circuit i.e. infinite resistance. It is also called Thevenin resistance R_{th} or T .

$$R_{th} = (2.5 \parallel 2) = \frac{2.5 * 2}{2.5 + 2} = \frac{5}{4.5} = 1.11 \Omega$$



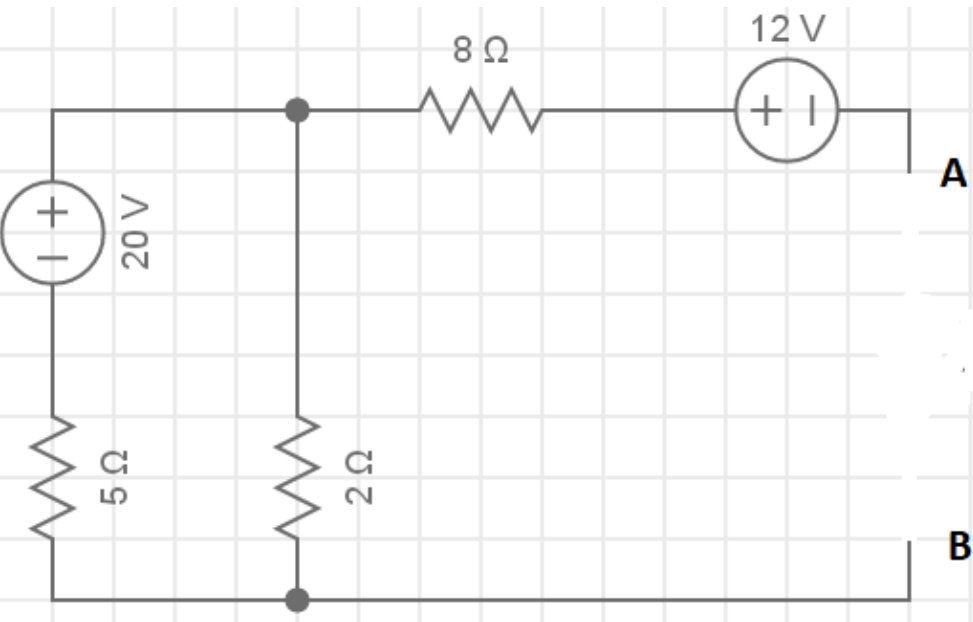
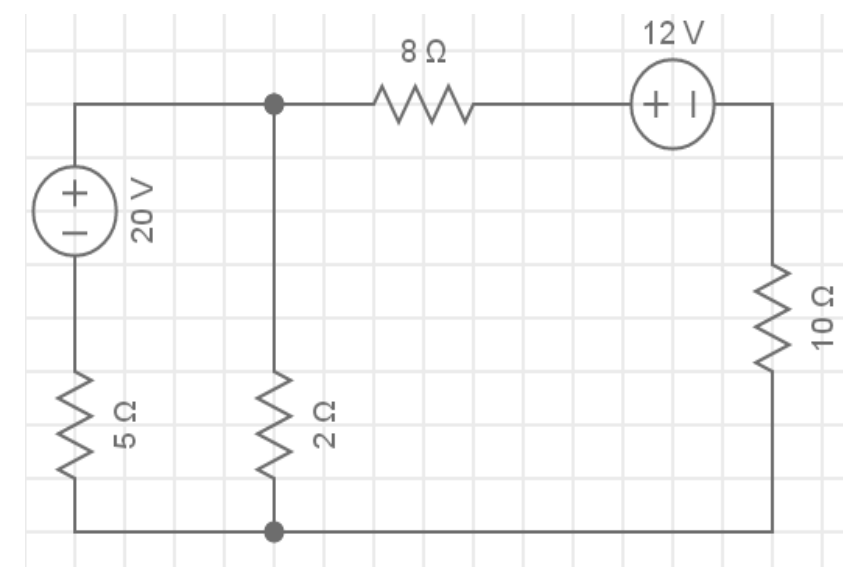
- **Step 4:** Replace the entire network by a single Thevenin source, whose voltage is V_{th} or V_{oc} and whose internal resistance is R_{th} or R_i
- **Step 5:** Connect R_L back to its terminals from where it was previously removed..
- **Step 6:** Finally, calculate the current flowing through R_L by using the equation.



$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{9.33}{1.11 + 6} = \frac{9.33}{7.11} = 1.312 \text{ A}$$

Example: Find value of current passing through 10 ohm resistance

- Step 1:** Temporarily remove the resistance (called load resistance R_L) whose current is required.



Step 2: Determination of V_{th} .

$$-5 I_1 + 20 - 2(I_1 - I_2) = 0$$

$$I_2 = 0 \text{ Amp}$$

$$I_1 = \frac{20}{7} = 2.85 \text{ Amp}$$

$$-2(I_2 - I_1) - 8 I_2 - 12 - V_{th} = 0$$

$$I_2 = 0 \text{ Amp}$$

$$-2(-I_1) - 12 - V_{th} = 0$$

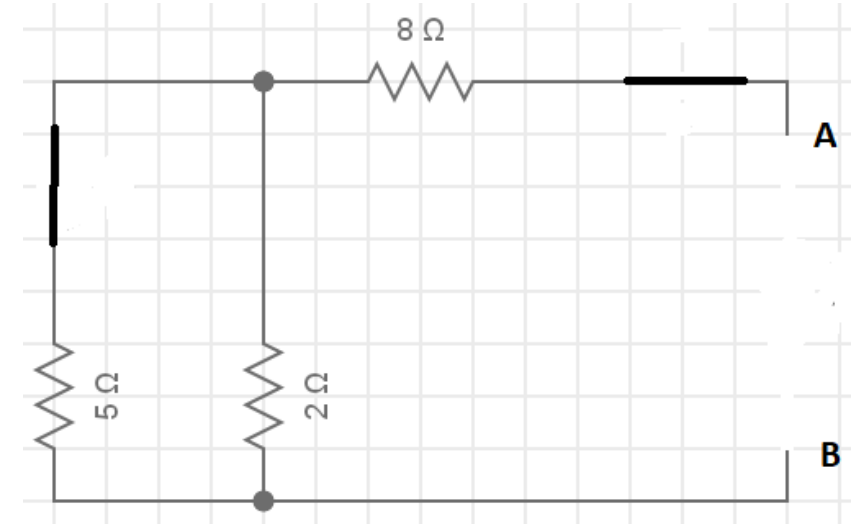
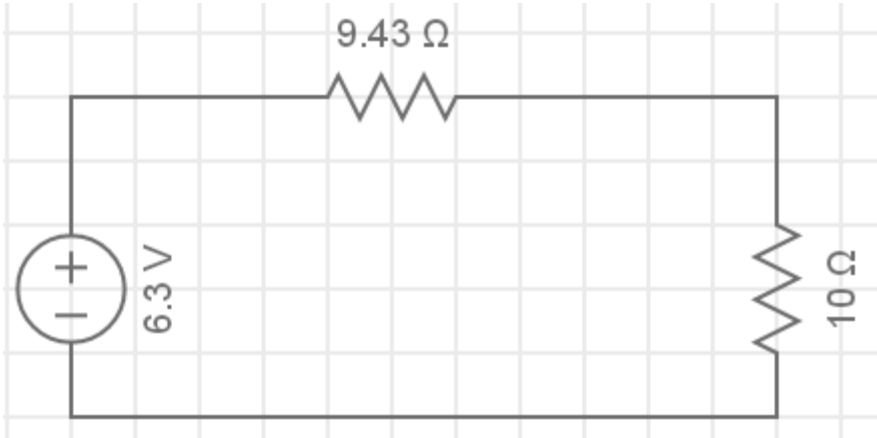
$$-2(-2.85) - 12 - V_{th} = 0$$

$$V_{th} = 5.7 - 12 = -6.3 \text{ V}$$

- **Step 3:** Determination of R_{th} .

$$R_{th} = 8 + (5 \parallel 2) = 8 + \frac{10}{7} = 8 + 1.42 = 9.42 \Omega$$

- **Step 4:** Replace the network by Thevenin's Equivalent.



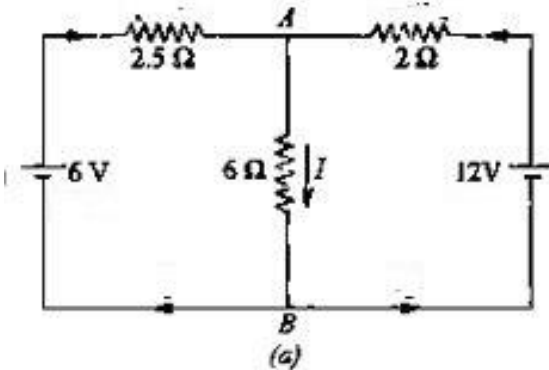
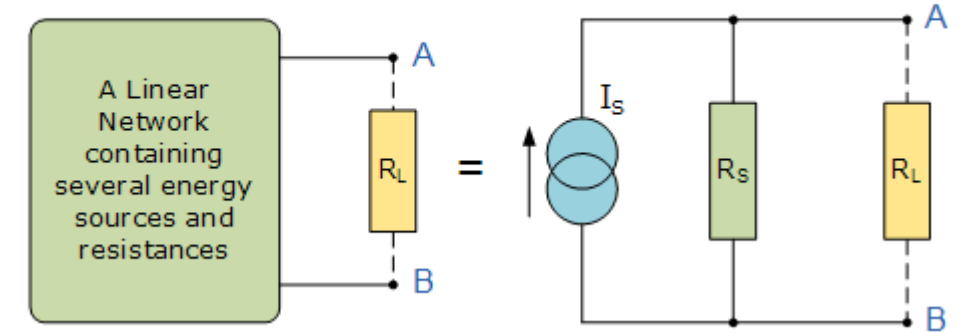
- **Step 5:** Reconnect R_L
- **Step 6:** Calculate I_L

$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{-6.3}{9.43 + 10} = \frac{-6.3}{19.43} = -0.3242 A$$

Norton's Theorem :

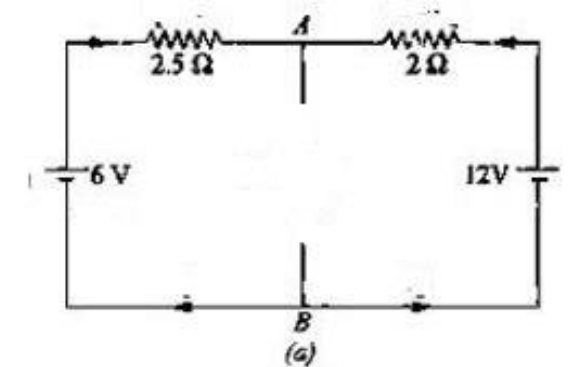
It states that “Any linear circuit containing several energy sources and resistances can be replaced by a single Constant Current generator in parallel with a Single Resistor”.

Nortons equivalent circuit



Example: Find the value of current passing through 6 ohm resistance

- **Step 1:** Temporarily remove the resistance (called load resistance R_L) whose current is required. Short the terminal A & B.

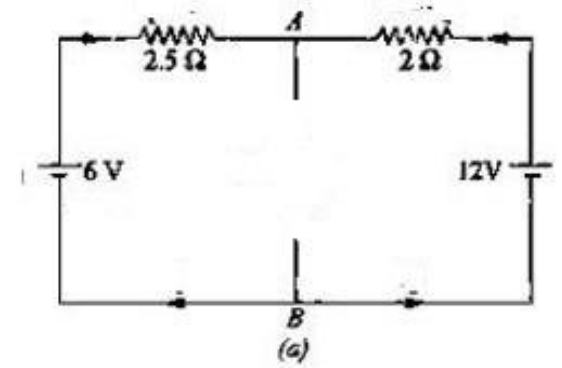


- **Step 2:** Find the Short circuit current I_{sc} which flows between the two terminals. It is also called Norton's Equivalent Current Source I_N .

$$+6 - 2.5 I_1 = 0 \Rightarrow I_1 = \frac{6}{2.5} = 2.4 \text{ A}$$

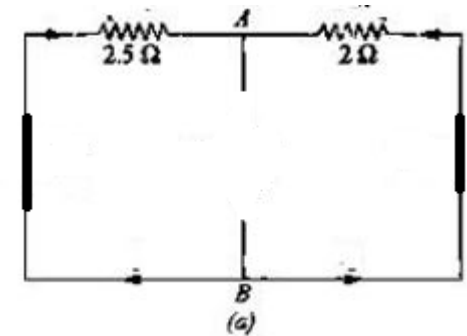
$$-2 I_2 - 12 = 0 \Rightarrow I_2 = \frac{-12}{2} = -6 \text{ A}$$

$$I_{sc} = I_1 - I_2 = 2.4 - (-6) = 8.4 \text{ A}$$

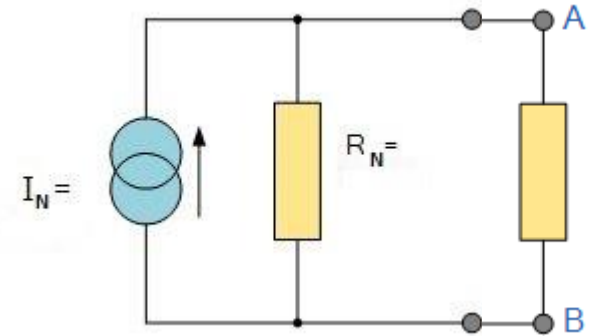


- **Step 3:** Compute the resistance of the whose network as looked into from these two terminals after all voltage sources have been removed leaving behind their internal resistances (if any) and current sources have been replaced by open-circuit i.e. infinite resistance. It is also called Norton's resistance R_N .

$$R_N = (2.5 \parallel 2) = \frac{2.5 * 2}{2.5 + 2} = \frac{5}{4.5} = 1.11 \Omega$$



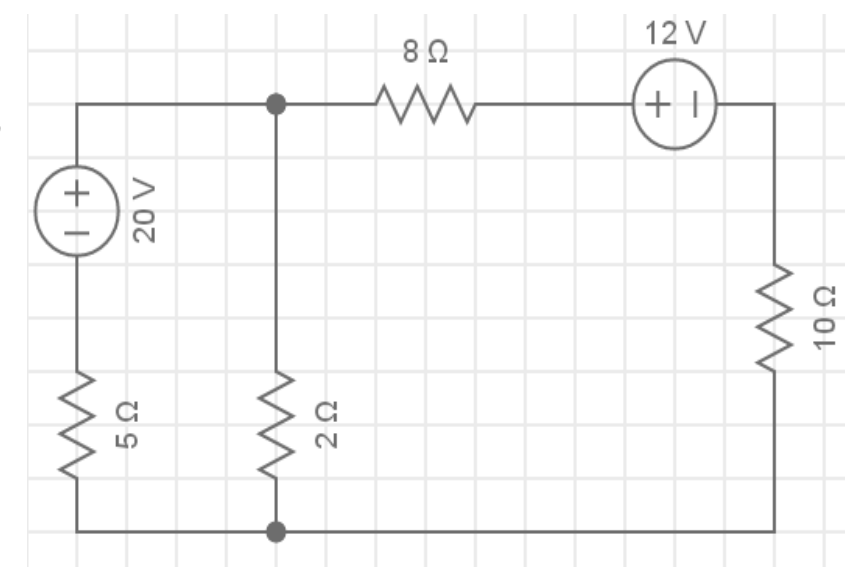
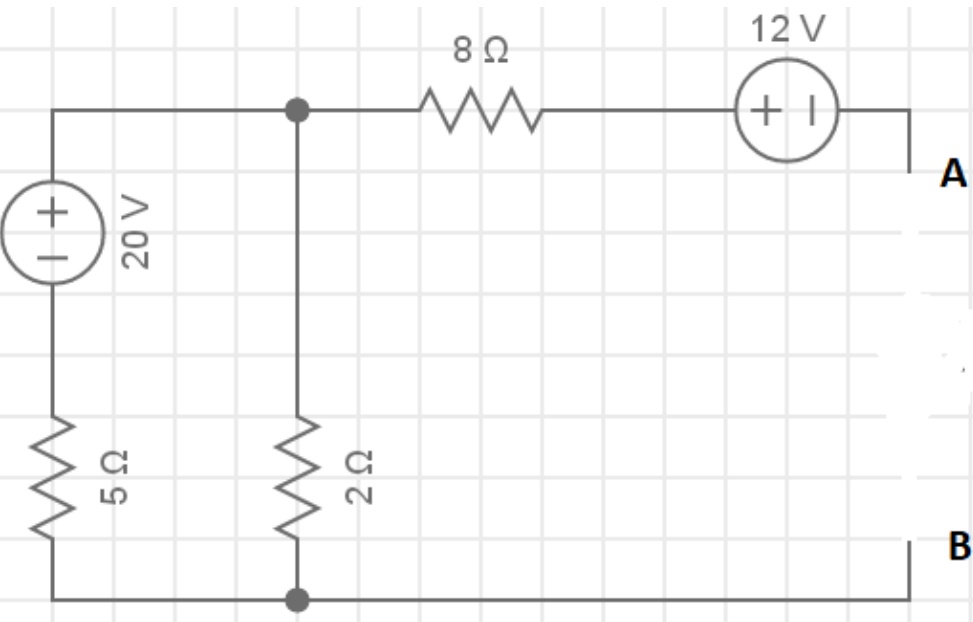
- **Step 4:** Replace the entire network by a single Norton Current Source in Parallel with Norton's Equivalent Resistance,
- **Step 5:** Connect RL back to its terminals from where was previously removed..
- **Step 6:** Finally, calculate the current flowing through RL by using the equation.



$$I_L = I_N \frac{R_N}{R_N + R_L} = 8.4 \frac{1.11}{1.11 + 6} = \frac{9.33}{7.11} = 1.312 \text{ A}$$

Example: Find value of current passing through 10 ohm resistance

- Step 1:** Temporarily remove the resistance (called load resistance R_L) whose current is required. Short the terminal A & B



Step 2: Determination of I_{sc} or I_N .

$$-5 I_1 + 20 - 2(I_1 - I_2) = 0$$

$$7 I_1 - 2 I_2 = 20$$

$$-2 (I_2 - I_1) - 8 I_2 - 12 = 0$$

$$-2 I_1 + 10 I_2 = -12$$

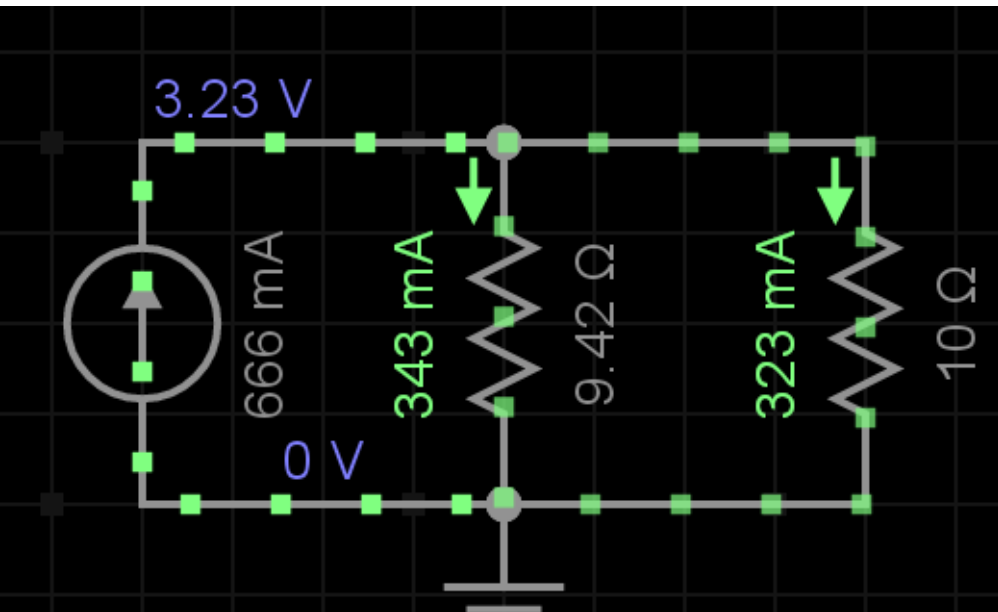
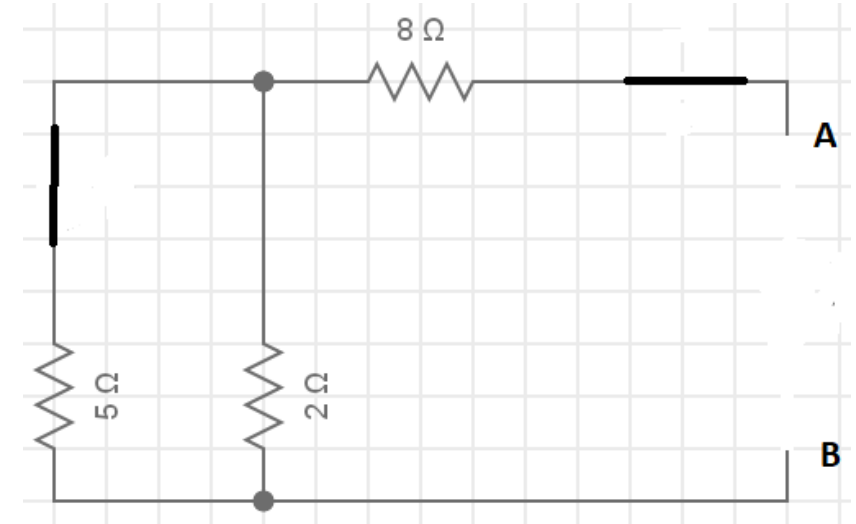
$$I_1 = 2.666 \text{ Amp}$$

$$I_N = I_2 = -0.666 \text{ Amp}$$

- **Step 3:** Determination of R_N .

$$R_N = 8 + (5 \parallel 2) = 8 + \frac{10}{7} = 8 + 1.42 = 9.42 \Omega$$

- **Step 4:** Replace the network by Norton's Equivalent.

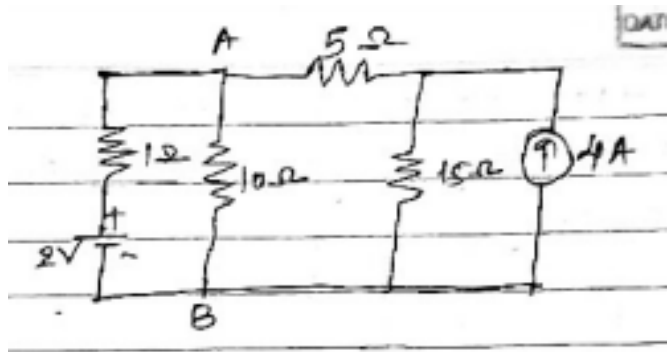


Step 5: Reconnect R_L

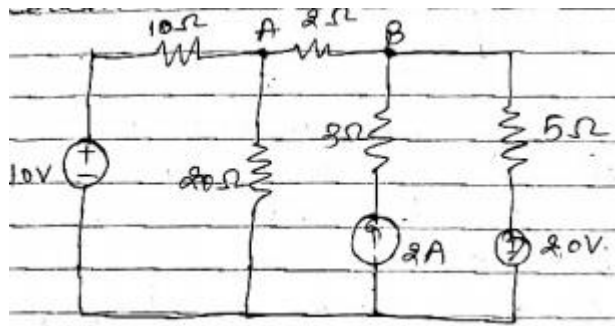
Step 6: Calculate I_L

$$I_L = I_N \frac{R_N}{R_N + R_L} = -0.66 \frac{9.42}{9.42 + 10} = \frac{9.33}{19.42} = 0.3201 A$$

Find the current passing through 10 Ω Resistor.



Find the current passing through 2 Ω Resistor.



Find the current passing through 15 Ω Resistor.

