GANPAT UNIVERSITY

U. V. PATEL COLLEGE OF ENGINEERING

DEPARTMENT OF ELECTRICAL ENGINEERING



SUBJECT: 2ES103 BASIC ELECTRICAL ENGINEERING (BEE)

Presented by: Prof. Anvi J. Gajjar,

Understanding Current

This flow of electrical charge is referred to as electric current. There are two types of current

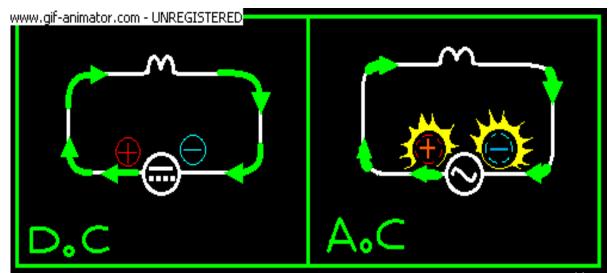


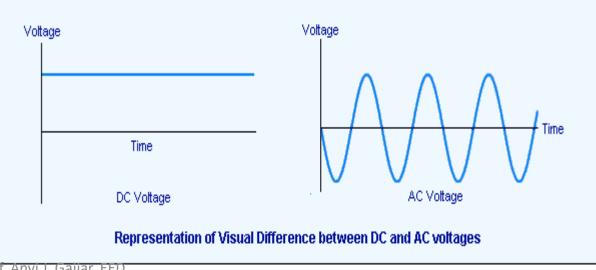
DIRECT CURRENT



ALTERNATE CURRENT

DC is current that flows in one direction with a constant voltage polarity while AC is current that changes direction periodically along with its voltage polarity.

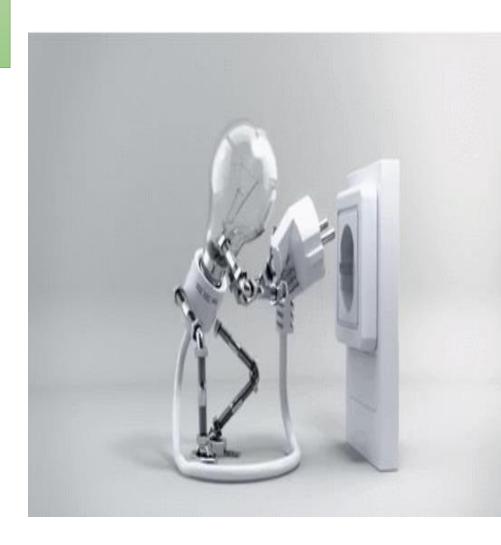




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Where is electricity used ???

- In our house....
 - T.V.
 - Refrigerator
 - Oven
 - · Mobile Phone
 - Laptop
 - Computers
 - AC
 - Fans, tube lights.....
- In industries....

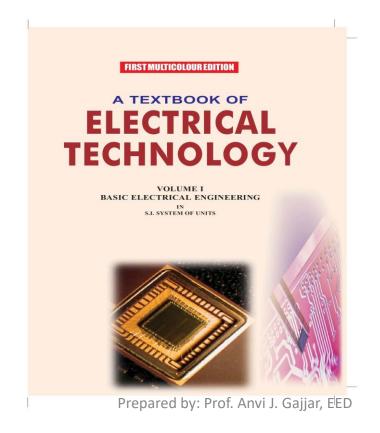


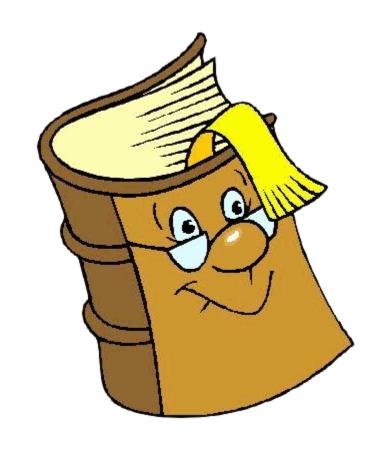
CHAPTER 5: AC CIRCUITS

- Representation of sinusoidal waveforms,
- peak and rms values,
- phasor representation,
- real power, reactive power,
- apparent power, power factor.
- Analysis of single-phase ac circuits consisting of R, L, C, RL, RC, RLC combinations (Series and parallel), resonance.
- Three phase balanced circuits, voltage and current relations in star and delta connections, measurement of power in 3-phase circuits.

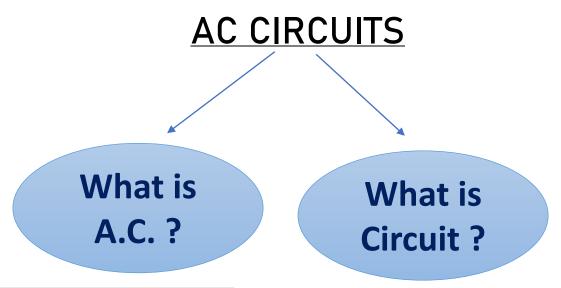
Which book to be referred???

B. L. Thereja, "Electrical Technology", S. Chand Volume-I.





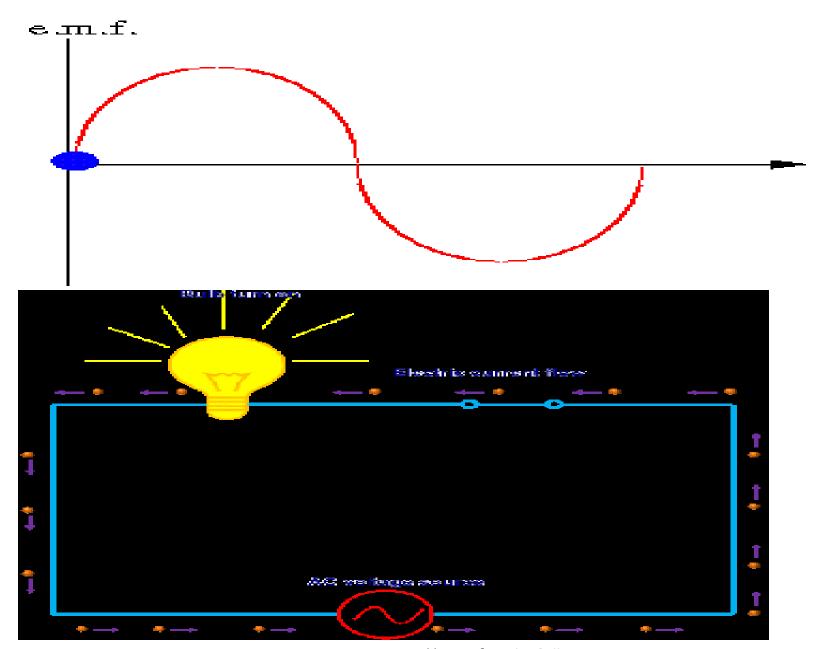
Let's us understand the title...



A.C. - Alternating Current

Alternating current is an electric current which periodically reverses direction and changes its magnitude continuously with time

This circular path, which is always required to get electricity to flow and do something useful, is called a circuit. A circuit is a path that starts and stops at the same place, which is exactly what we're doing.

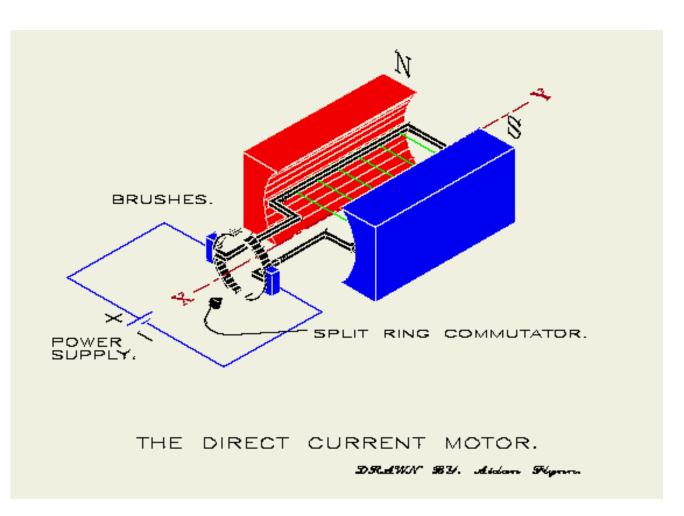


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Representation of sinusoidal waveforms

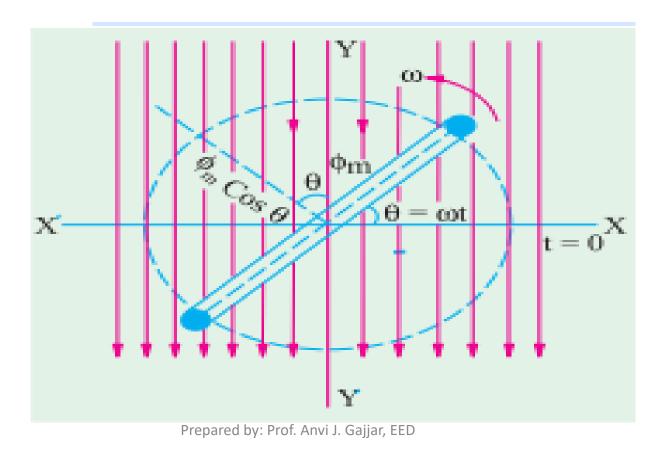
 How does A.C. generates ???

Alternating voltage may be generated by rotating a coil in a magnetic field or by rotating a magnetic field within a stationary coil.

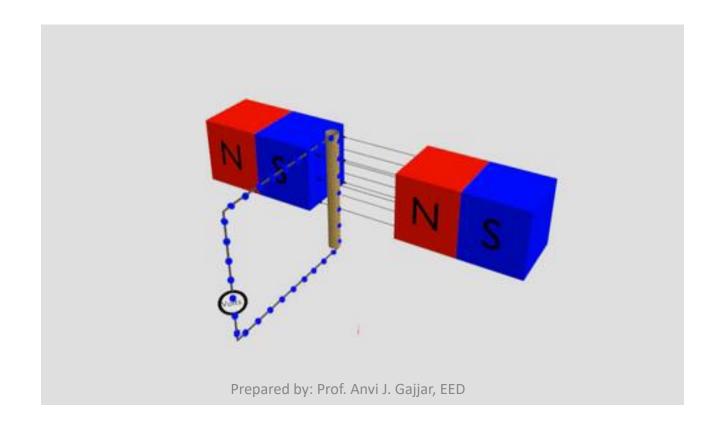


Equations of the Alternating Voltages and Currents

• Consider a rectangular coil, having N turns and rotating in a uniform magnetic field, with an angular velocity of ω radian/second, as shown in Fig.



- Let time be measured from the X-axis.
- Maximum flux \emptyset_m is linked with the coil, when its plane coincides with the X-axis.
- In time t seconds, this coil rotates through an angle $\theta = \omega t$.
- In this deflected position, the component of the flux which is perpendicular to the plane of the coil, is $\emptyset = \emptyset_m \cos \omega t$.



- Hence, flux linkages of the coil at any time are $N \emptyset = \emptyset_m N \cos \omega t$.
- According to Faraday's Laws of Electromagnetic Induction, the e.m.f. induced in the coil is given by the rate of change of flux-linkages of the coil.

•
$$e = -\frac{d}{dt} (N \emptyset)$$

= - N
$$\frac{d}{dt}$$
 (\emptyset_m N cos ω t) volt

= -
$$N\emptyset_m \omega$$
 (- $\sin \omega t$) volt

$$= \underline{\emptyset_m} \ \text{N}\omega(\sin \omega t) \ \text{volt}$$

$$= \emptyset_m \text{ N}\omega(\sin \theta) \text{ volt}$$

1

• When the coil has turned through 90° i.e. when θ = 90°, then $\sin \theta$ = 1, hence e has maximum value, say E_m .

Therefore, from Eq. (1) we get

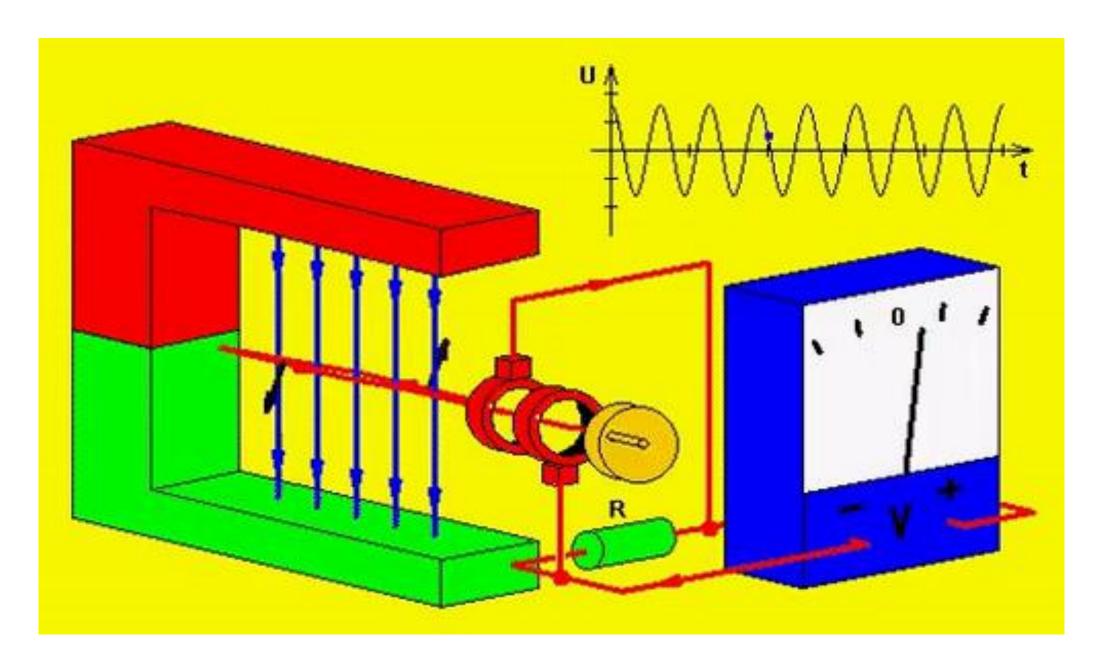
$$E_m = \omega N \emptyset_m = \omega N B_m A = 2 \pi f N B_m A \text{ volt}$$



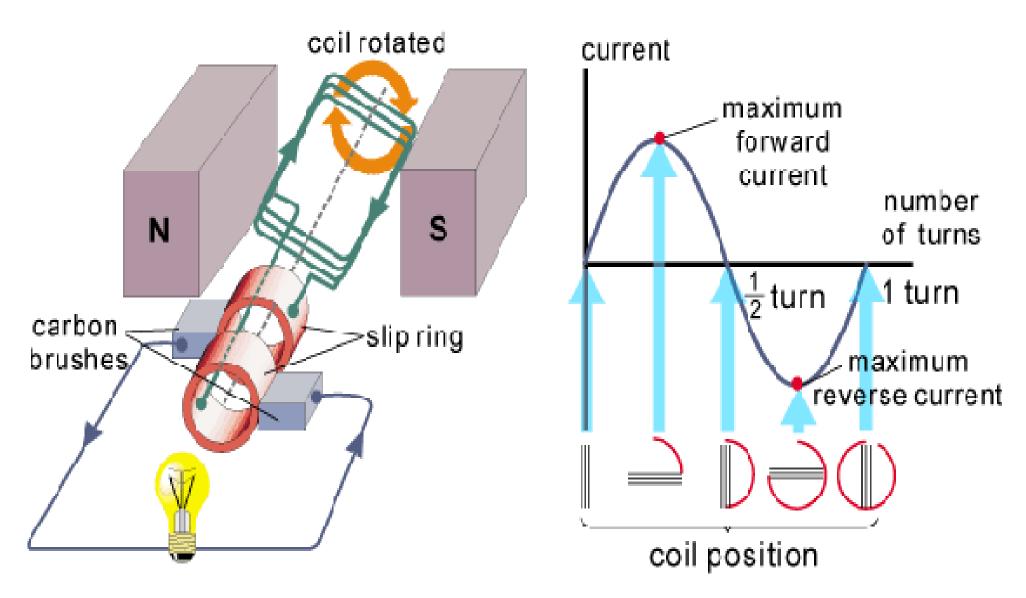
- $B_m = \text{maximum flux density in Wb/} m^2$
- A = area of the coil in m^2
- f = frequency of rotation of the coil in rev/second

Substituting this value of E_m in Eq. (1), we get

- $e = E_m \sin \theta = E_m \sin \omega t$
- Similarly, the equation of induced alternating current is $i = I_m \sin \omega t$



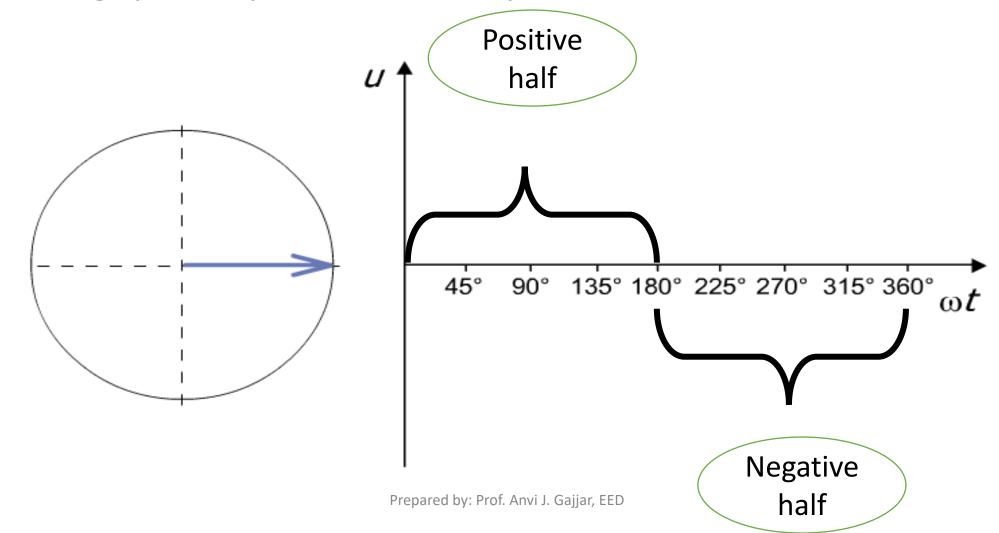
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- The term AC or to give it its full description of Alternating Current, generally refers to a time-varying waveform with the most common of all being called a **Sinusoid** better known as a **Sinusoidal Waveform**.
- Sinusoidal waveforms are more generally called by their short description as **Sine Waves**. Sine waves are by far one of the most important types of AC waveform used in electrical engineering.
- The shape obtained by plotting the instantaneous ordinate values of either voltage or current against time is called an **AC Waveform**.
- An AC waveform is constantly changing its polarity every half cycle alternating between a positive maximum value and a negative maximum value respectively with regards to time with a common example of this being the domestic mains voltage supply we use in our homes.

Let's us get familiarize with few terms...

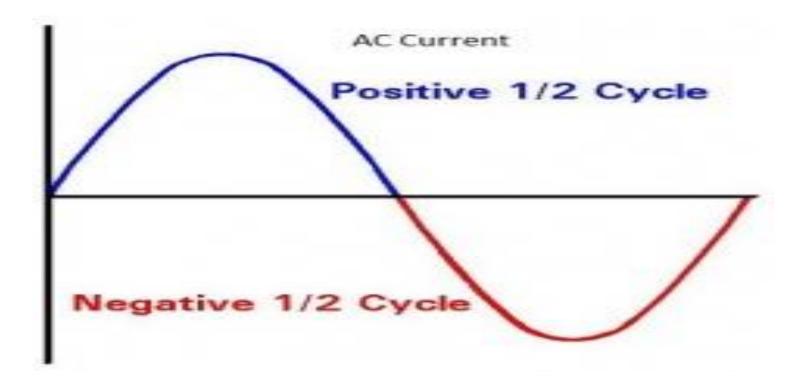
1) <u>CYCLE</u>: One complete set of positive and negative values of alternating quantity is known as cycle.



Therefore,

1 positive half + 1 negative half = 1 complete "CYCLE"

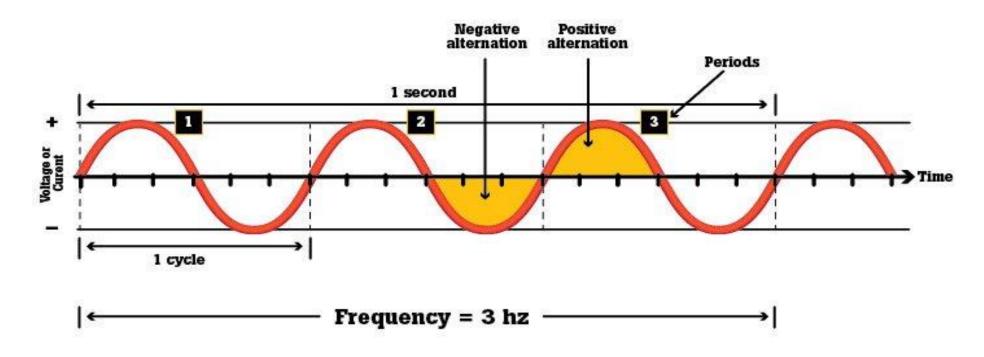
• A cycle may also be sometimes specified in terms of angular measure. In that case, one complete cycle is said to spread over 360o or 2π radians.



2) <u>FREQUENCY</u>: The number of cycles/second is called the frequency of the alternating quantity. Its unit is hertz (Hz).

Hertz (Hz) = One hertz is equal to one cycle per second.

Example: If an alternating current is said to have a frequency of 3 Hz (see diagram below), that indicates its waveform repeats 3 times in 1 second.



3) <u>TIME PERIOD</u>: The time taken by an alternating quantity to complete one cycle is called its time period T.

For example, a 50-Hz alternating current has a time period of 1/50 second.

The frequency of Alternating Current in India is 50 Hz.

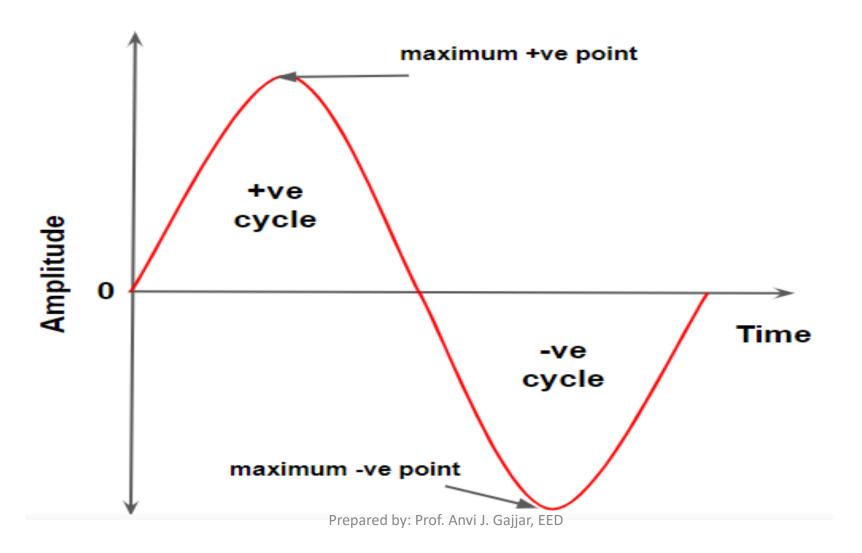
Calculate time period for Alternating current in India for 50 Hz

Time Period = 1/f

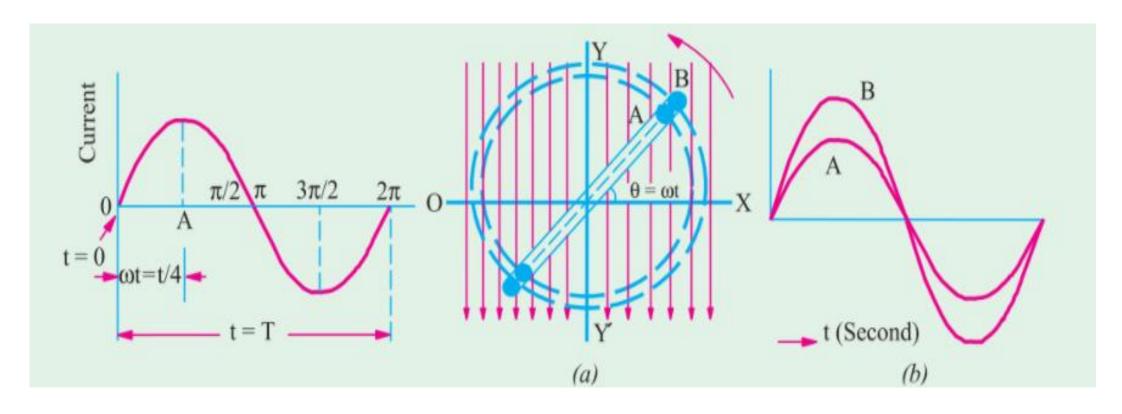
= 1/50 seconds

= 0.02 seconds

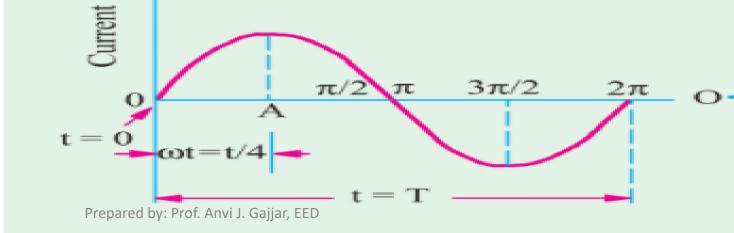
4) <u>AMPLITUDE</u>: The maximum value, positive or negative, of an alternating quantity is known as its amplitude.



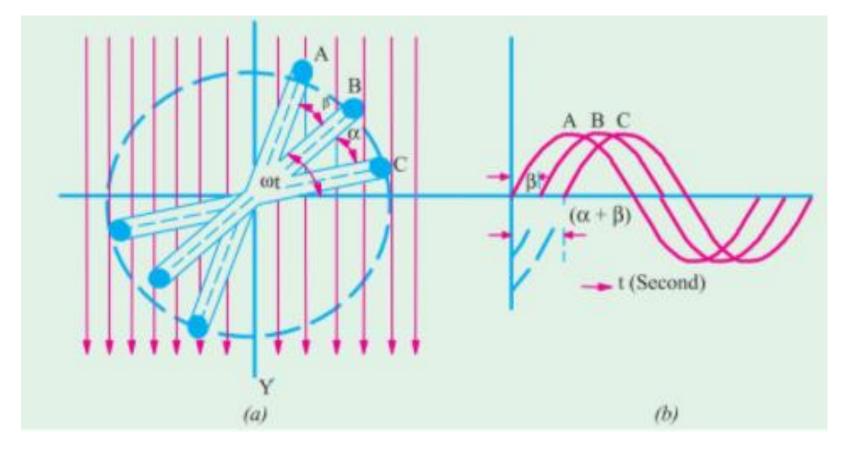
5) <u>PHASE</u>: By phase of an alternating current is meant the fraction of the time period of that alternating current which has elapsed since the current last passed through the zero position of reference.



- For example, the phase of current at point A is T/4 second, where T is time period or expressed in terms of angle, it is $\pi/2$ radians
- Similarly, the phase of the rotating coil at the instant ωt which is, therefore, called its phase angle.
- 6) Instantaneous Value: The value of an alternating quantity at a particular instant is called Instantaneous value.
- 7) Waveform: The graph of instantaneous values plotted of an alternating quantity plotted against time is called waveform.



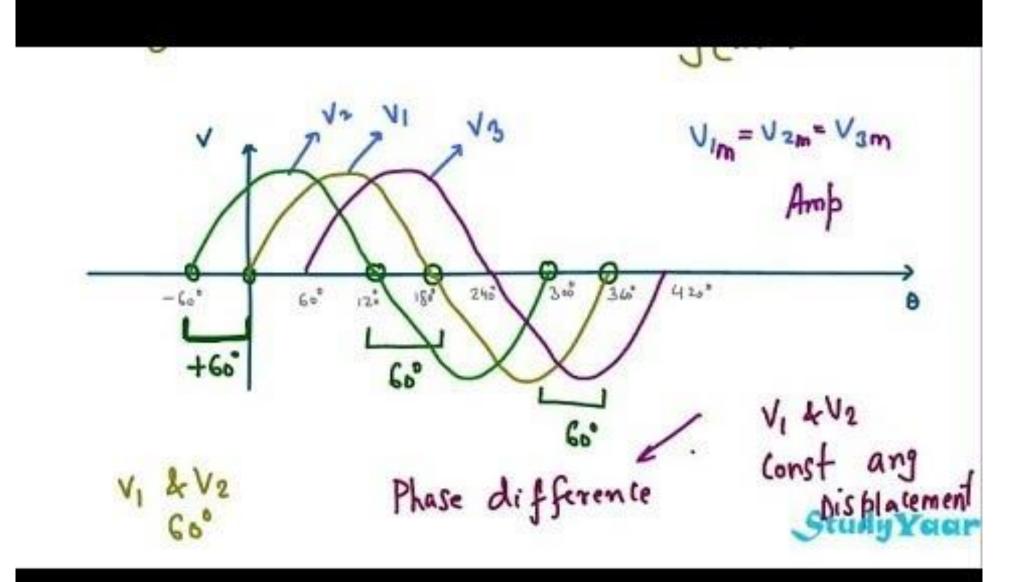
8) PHASE DIFFERENCE: Now, consider three similar single-turn coils displaced from each other by angles α and β and rotating in a uniform magnetic field with the same angular velocity [Fig. 11.13 (a)].



- In this case, the value of induced e.m.fs. in the three coils are the same, but there is one important difference.
- The e.m.fs. in these coils do not reach their maximum or zero values simultaneously but one after another.
- It is seen that curves B and C are displaced from curve A and angles β and $(\alpha + \beta)$ respectively. Hence, it means that phase difference between A and B is β and between B and C is α but between A and C is $(\alpha + \beta)$.
- The statement, however, does not give indication as to which e.m.f. reaches its maximum value first.
- This deficiency is supplied by using the terms 'lag' or 'lead'.

- A leading alternating quantity is one which reaches its maximum (or zero)
 value earlier as compared to the other quantity.
- Similarly, a lagging alternating quantity is one which reaches its maximum or zero value later than the other quantity.
- For example, in Fig. 11.13 (b), B lags behind A by β and C lags behind A by $(\alpha + \beta)$ because they reach their maximum values later.

- The three equations for the instantaneous induced e.m.fs. are
 - $e_A = E_m \sin \omega t$... reference quantity
 - $e_B = E_m \sin(\omega t \beta)$
 - $e_C = E_m \sin [\omega t (\alpha + \beta)]$



let's write the equations for the given waveforms

$$V_1 = V_m \sin(\omega t) = V_m \sin\theta$$

$$V_2 = V_m \sin(\theta + 60^\circ)$$

$$V_3 = V_m \sin(\theta - 60^\circ)$$

Let's us take an example....

- The maximum values of the alternating voltage and current are 400 V and 20 A respectively in a circuit connected to 50 Hz supply and these quantities are sinusoidal. The instantaneous values of the voltage and current are 283 V and 10 A respectively at t = 0 both increasing positively.
- (i) Write down the expression for voltage and current at time t.

Solution:

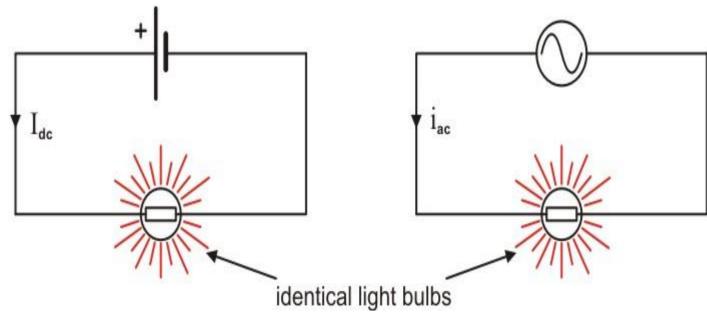
- In general, the expression for an a.c. voltage is v = Vm sin ($\omega t + \varphi$) where φ is the phase difference with respect to the point where t = 0.
- $v = 283 \ V \ and \ V_m = 400 \ V \ and \ t = 0$
- $283 = 400 \sin (\omega \times 0 + \phi)$
- $Sin \phi = 0.7070$
- Φ = 45°
- So general expression for voltage is $v = 400 \sin(wt + 45^\circ) = 400 \sin(2\pi \times 50t + 45^\circ)$

Solution:

- In general, the expression for an a.c. voltage is $i = Im \sin(\omega t + \phi)$ where ϕ is the phase difference with respect to the point where t = 0.
- $i = 10 A \ and \ I_m = 20 A \ and t = 0$
- $10 = 20 \sin (\omega \times 0 + \phi)$
- $Sin \phi = 0.5$
- Ф = 30°
- So general expression for voltage is $i = 20 \sin(wt + 30^\circ) = 20 \sin(2\pi \times 50t + 30^\circ)$

8) R.M.S Value - (Root Mean Square):

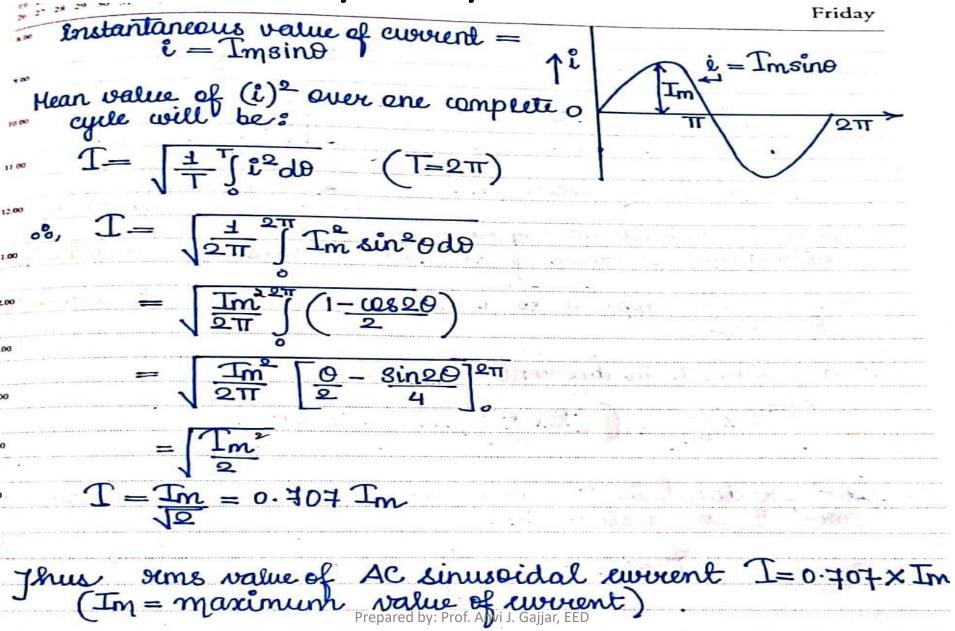
The r.m.s. value of an alternating current is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.



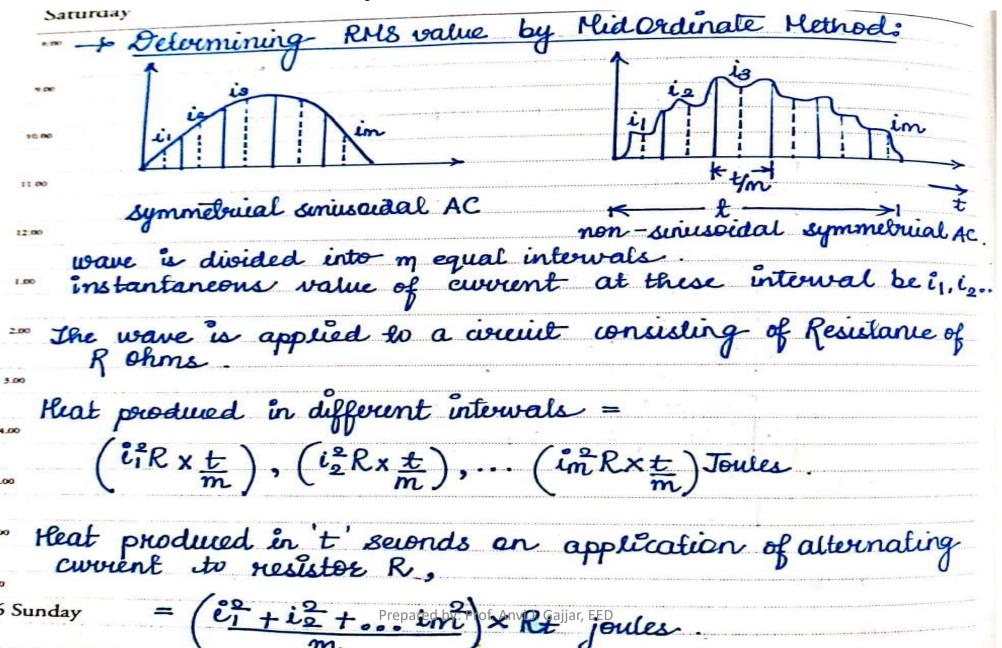
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• It is also known as the *effective* or *virtual* value of the alternating current, the former term being used more extensively. For computing the r.m.s. value of symmetrical sinusoidal alternating currents, either mid-ordinate method or analytical method may be used, although for symmetrical but non-sinusoidal waves, the mid-ordinate method would be found more convenient.

Determine RMS value by Analytical Method



Determine RMS value by Mid-Ordinate Method

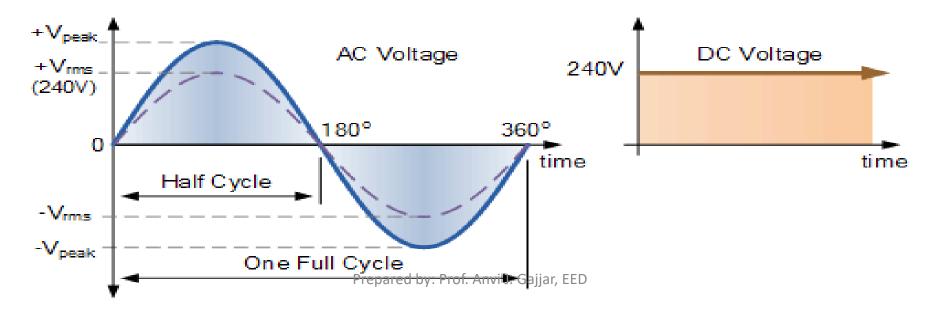


Let I be the value of direct current which when serving through the same circuit of resistance Rohms for same time 't's evends, produces same amount of heat. Heat produced by d.c = T2Rt Both heats produced are equal. + ... + im = Mean value of (i) if + intootim I = \ Hean value of (i)2

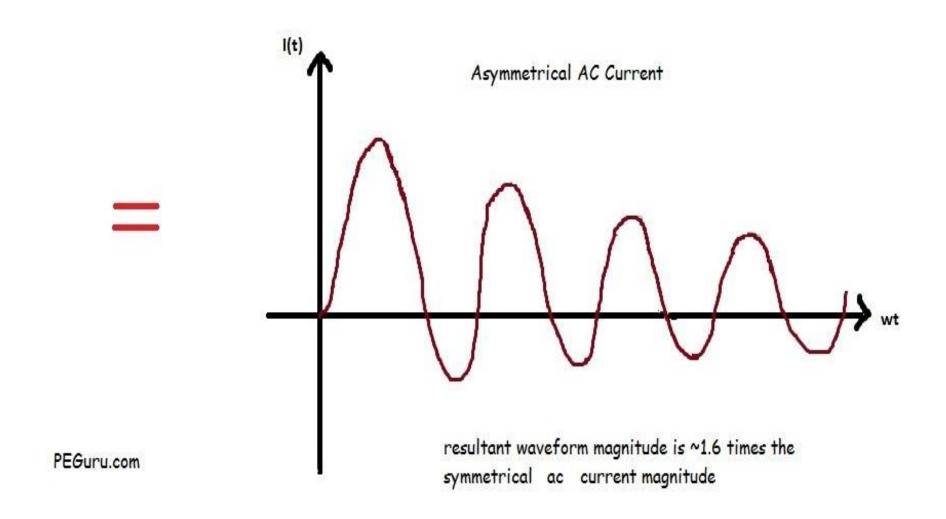
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9) AVERAGE VALUE

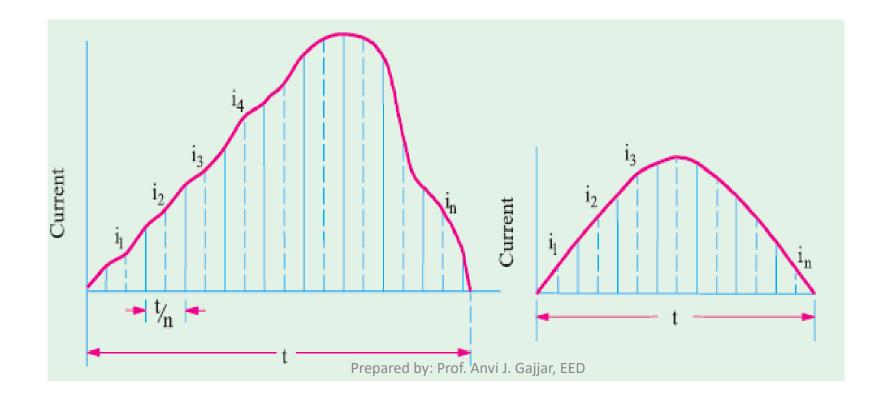
- The average value Ia of an alternating current is expressed by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.
- In the case of a symmetrical alternating current (*i.e.* one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle only.



• But in the case of an unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.



- In the case of a symmetrical alternating current (*i.e.* one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle only.
- But in the case of an unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.



(i) Mid-ordinate Method:

$$I_{av} = \frac{i_1 + i_2 + \dots + i_n}{n}$$

This method may be used both for sinusoidal and non-sinusoidal waves.

(ii) Analytical Method:

The standard equation of an alternating current is, $i = I_m \sin \theta$

$$I_{av} = \int_0^{\pi} \frac{id\theta}{(\pi - 0)} = \frac{I_m}{\pi} \int_0^{\pi} \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi} \left| -\cos\theta \left|_0^{\pi} = \frac{I_m}{\pi} \right| + 1 - (-1) \right| = \frac{2I_m}{\pi} = \frac{I_m}{\pi/2} = \frac{\text{twice the maximum current}}{\pi}$$

$$I_{av} = I_m / \frac{1}{2} \pi = 0.637 I_m$$

 $I_{mv} = I_m / \frac{1}{2} \pi = 0.637 I_m$: average value of current = 0.637 × maximum value

10) FORM FACTOR:

It is defined as the ratio, $K_f = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{0.707 I_m}{0.637 I_m} = 1.1$. (for sinusoidal alternating currents only)

In the case of sinusoidal alternating voltage also, $K_f = \frac{0.707 E_m}{0.637 E} = 1.11$

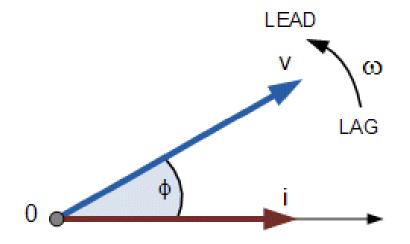
11) AMPLITUDE FACTOR (Crest Factor):

It is defined as the ratio
$$K_a = \frac{\text{maximum value}}{\text{r.m.s. value}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$$
 (for sinusoidal a.c. only)

For sinusoidal alternating voltage also,
$$K_a = \frac{E_m}{E_m/\sqrt{2}} = 1.414$$

12)Phasor Diagrams

Phasor Diagrams are a graphical way of representing the magnitude and directional relationship between two or more alternating quantities.



Sinusoidal waveforms of the same frequency can have a Phase Difference between themselves which represents the angular difference of the two sinusoidal waveforms. Also the terms "lead" and "lag" as well as "in-phase" and "out-of-phase" are commonly used to indicate the relationship of one waveform to the other with the generalized sinusoidal expression given as: $A_{(t)} = A_m \sin(\omega t \pm \Phi)$ representing the sinusoid in the time-domain form.

13) Phasors

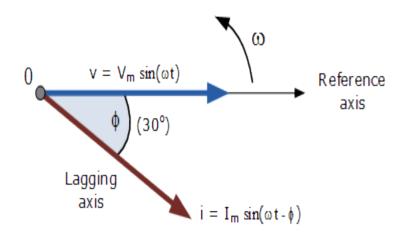
a rotating vector - called a "**Phasor**" is a scaled line whose length represents an AC quantity that has both magnitude ("peak amplitude") and direction ("phase") which is "frozen" at some point in time.

vectors are assumed to pivot at one end around a fixed zero point known as the "point of origin" while the arrowed end representing the quantity, freely rotates in an **anti-clockwise** direction at an angular velocity, (ω) of one full revolution for every cycle.

This anti-clockwise rotation of the vector is considered to be a positive rotation. Likewise, a clockwise rotation is considered to be a negative rotation.



Phasor Diagram of a Sinusoidal Waveform



- phasor diagram is drawn corresponding to time zero (t = 0) on the horizontal axis.
- lengths of the phasors are proportional to the values of the voltage, (V) and the current, (I) at the instant in time that the phasor diagram is drawn.
- current phasor lags the voltage phasor by the angle, Φ , as the two phasors rotate in an *anticlockwise* direction as stated earlier, therefore the angle, Φ is also measured in the same anticlockwise direction.

Let's us solve some numericals to understand more clearly...

- 1) An alternating current has f = 50 Hz, Peak Value = 100 A.
 - Write the equation for instantaneous value of current, and
 - Find the time taken to reach 80 A for 1st time?

Solution:

Instantaneous value of current i = I_m sin ω t

Now ω = $2\pi f$ = 2 x 3.14 x 50 = 314 radian/seconds

$$:$$
, i = 100 sin (314)t

$$\therefore$$
, 80 = 100 sin (314)t

:,
$$t = 1/314 \sin^{-1}(0.8) = 1/314(53.13^{\circ}) = 1/314(0.9268)$$

= 0.00295 seconds

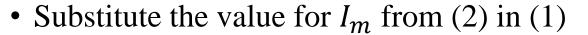
A.C. through pure Resistance alone in a circuit...

• Fig. shows A.C. Circuit consisting of a pure resistor R connected across alternating voltage

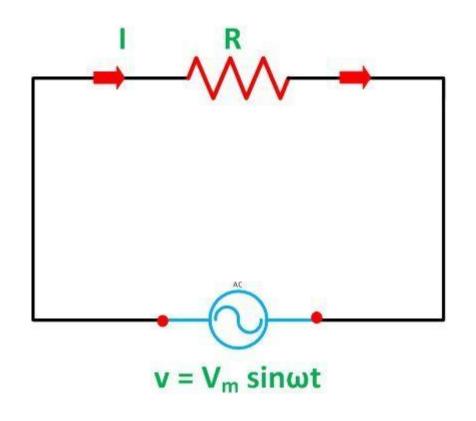
$$e = E_m sin\omega t$$

• Instantaneous current $\mathbf{i} = \frac{e}{R} = \frac{E_m sin\omega t}{R}$ (1)

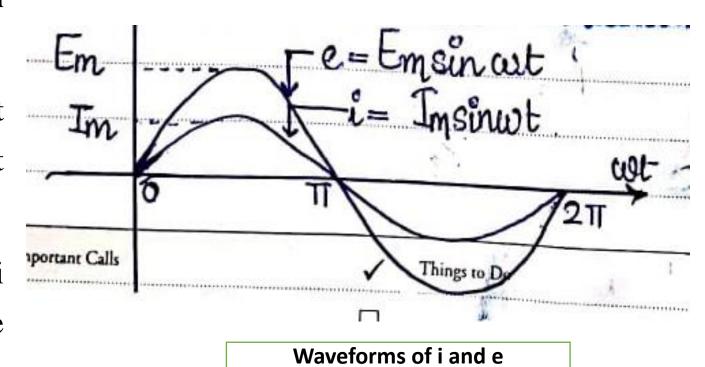
•
$$\frac{E_m}{R} = I_m$$
 (Maximum current)(2)



$$\therefore$$
, $\mathbf{i} = I_m \operatorname{sin}\omega \mathbf{t}$



- For a purely resistive circuit, both voltage and current are in phase
- They may have different peak value but they attain 0 and maximum value at same time.
- The phasor diagram shows the phasor i and v. Both are **in phase** (phase difference between v and i is 0°)



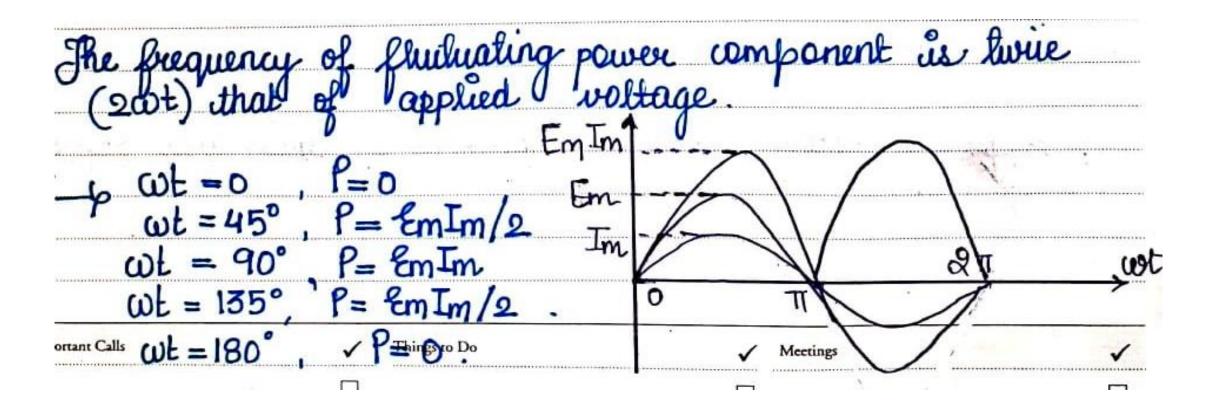
$$V_R = IR$$

$$i = I_m sin\omega t$$

Phasod Diagrams of jand e

0	_ & Instantaneous former = instantaneous vollage exavvient
	=0.41
0	=(Ensinwt)(Insinwt).
0	= 2mTm sin2cot
**********	$= \mathop{\rm Em} \operatorname{Im} \mathop{\rm sin}^2 \omega t$ $= \mathop{\rm Em} \operatorname{Im} \left(1 - \cos 2\omega t\right)$
0	2
)	- In In = Em In cas 2 cot
	= fin In = fin In cos 2 cot.
4	Instantaneous Pawer consists of two parts:
	(1) A constant part = Em Im
	(2) A flustuating part = EmIm cos 202± Prepared by Prof. Anvi J. Gajjar, EED

.....



Average value of fluctuation part of power $\frac{E_m I_m cos 2\omega t}{2}$ = 0 over a complete cycle. Hence, we conclude that "in a purely resistive A.C. circuit power is never 0".

Hence, power for the whole cycle is

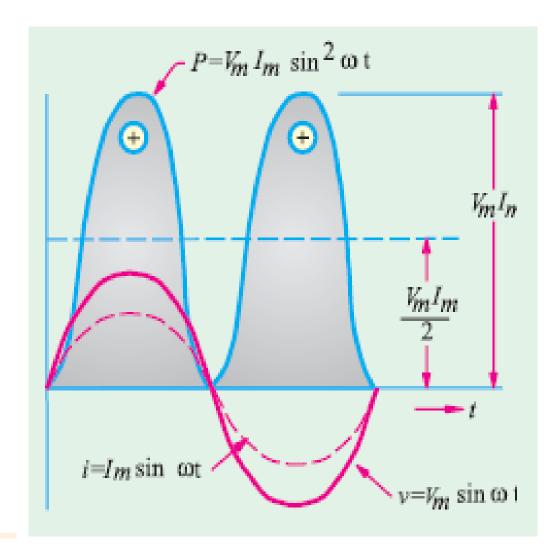
$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

or $P = V \times I$ watt

where V = r.m.s. value of applied voltage.

I = r.m.s. value of the current.

It is seen from Fig. 11.58 that no part of the power cycle becomes negative at any time. In other words, in a purely resistive circuit, power is never zero. This is so because the instantaneous values of voltage and current are always either both positive or negative and hence the product is always positive.



A.C. through purely Inductive circuit...

- The circuit containing pure inductance is shown:
- Let the alternating voltage applied to the circuit i given by the equation:

$$v = V_m Sin\omega t \dots (1)$$

• As a result, an alternating current i flows through the inductance which induces an emf in it. The equation is shown below:

$$v = V_m \sin \omega t$$

• The emf which is induced in the circuit is equal and opposite to the applied voltage. Hence, the equation becomes,

$$v = -e \dots (2)$$

• Putting the value of e in equation (2) we will get the equation as

$$v = -\left(-L \frac{di}{dt}\right)$$
 or

$$V_m Sin\omega t = L \frac{di}{dt}$$
 or

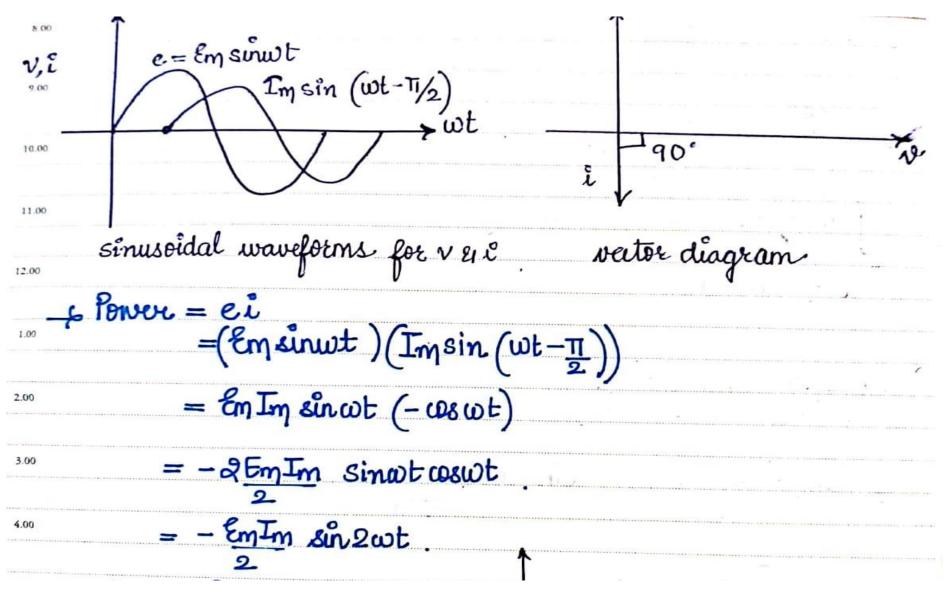
$$di = \frac{V_m}{L} \sin\omega t dt \dots \dots (3)$$

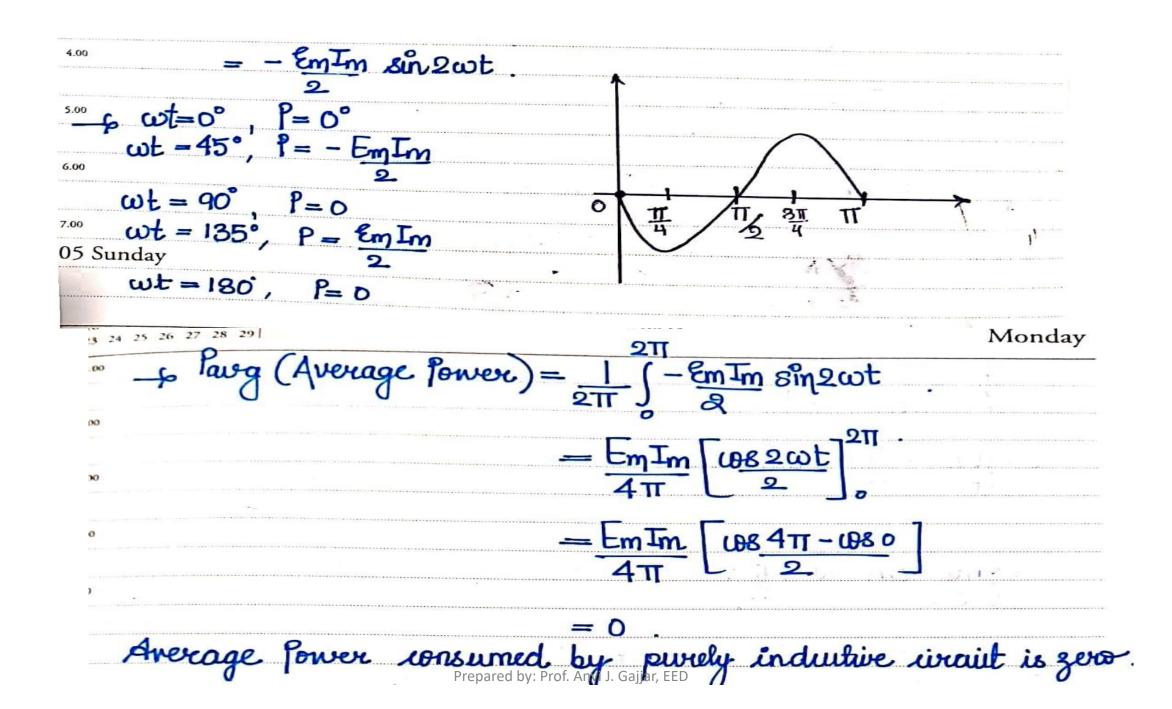
• Integrating both sides of the equation (3), we will get

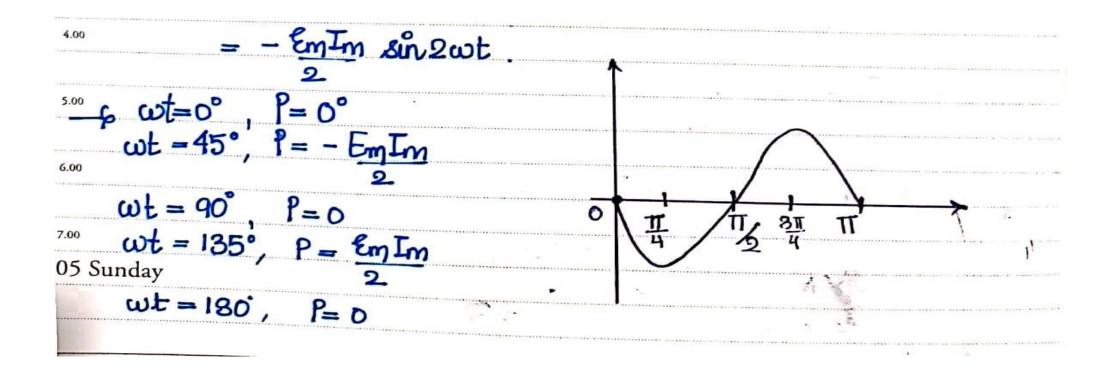
- where, $X_L = \omega$ L is the opposition offered to the flow of alternating current by a pure inductance and is called inductive reactance.
- The value of current will be maximum when $\sin (\omega t \pi/2) = 1$
- Therefore, $I_{m} = \frac{V_{m}}{X_{I}} (5)$
- Substituting this value in I_m from the equation (5) and putting it in equation (4) we will get

$$i = I_m \sin(\omega t - \frac{\pi}{2})$$
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Phasor diagram and waveforms







A.C. through purely Capacitive circuit...

- The circuit containing pure capacitance is shown:
- Let the alternating voltage applied to the circuit is given by the equation:

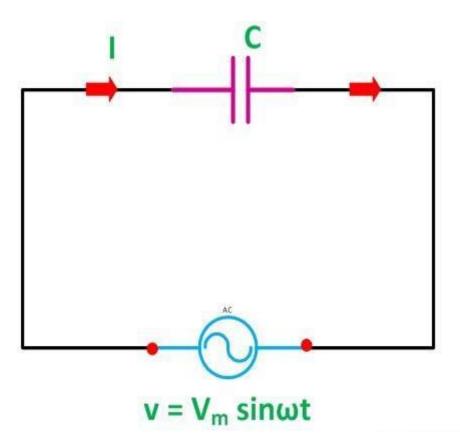
$$v = V_m Sin\omega t \dots (1)$$

• Charge of the capacitor at any instant of time is given as:

$$q = Cv(2)$$

• Current flowing through the circuit is given by the equation:

$$i = \frac{d}{q}$$
Produced by: Prof. Anvi J. Gajjar, EED



• Putting the value of q from the equation (2) in equation (3) we will get

$$i = \frac{d}{dt} (Cv) (3)$$

• Now, putting the value of v from the equation (1) in the equation (3) we will get

$$\begin{split} i &= \frac{d}{dt} \; C \; V_m Sin\omega t = C \; V_m \; \frac{d}{dt} \; sin\omega t \quad \text{or} \\ i &= \; \omega \; C \; V_m \; cos\omega t = \frac{v_m}{1/_{\omega C}} \; sin \big(\omega t + \; ^{\pi}/_2 \big) \; \text{or} \\ i &= \; \frac{V_m}{X_C} \; sin \big(\omega t + \; ^{\pi}/_2 \big) \ldots \ldots (4) \end{split}$$

- Where $Xc = 1/\omega C$ is the opposition offered to the flow of alternating current by a pure capacitor and is called **Capacitive Reactance**.
- The value of current will be maximum when $\sin(\omega t + \pi/2) = 1$. Therefore, the value of maximum current I_m will be given as:

$$I_{\mathbf{m}} = rac{\mathbf{v_{m}}}{\mathbf{X_{C}}}$$

Substituting the value of I_m in the equation (4) we will get:

$$i = I_m \sin(\omega t + \pi/2)$$

