INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

Department of Mathematics

SI 427 (Probability I)

Tutorial Sheet-VII

1. Let X be a continuous random variable with pdf f and $\varphi : \mathbb{R} \to \mathbb{R}$ is given by

$$\varphi(x) = \begin{cases} x+1 & \text{if} \quad x < -1 \\ 0 & \text{if} \quad -1 \leq x \leq 1 \\ x-1 & \text{if} \quad x > 1. \end{cases}$$

   Let $Y = \varphi \circ X$. Show that Y is a random variable. Does pdf of Y exists?

2. Let X, Y be a random variables. Show that $X^2 + XY + \sin(X+Y) - \ln|XY|$ is a random variable.

3. Let $F : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$F(x,y) = \begin{cases} 1 - e^{-x} - xe^{-y} & \text{if} \quad 0 \leq x \leq y \\ 1 - (1+y)e^{-y} & \text{if} \quad 0 \leq y < x \\ 0 & \text{otherwise} \end{cases}$$

   Prove or disprove that F is a distribution function.

4. Let X and Y be independent random variables respectively with uniform $(0,\ 1)$ and $N(0,1)$ distributions. Compute the joint distribution of X and Y.

5. Does there exists a non decreasing function $F : \mathbb{R} \to \mathbb{R}$ with set of discontinuities as a dense subset of $\mathbb{R}$?