INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

Department of Mathematics

SI 427 (Probability I)

Tutorial Sheet-IX

1. Let X be a non negative integer valued random variable such that mean of X is finite. Show that
$$EX = \sum_{n=1}^{\infty} P\big(X \geq n\big) .$$

2. Let X be a geometrically distributed [1] random variable with parameter p and M be a positive integer. Find $E\min\{X, M\}$.

3. Let X be Poisson with parameter λ. Find the mean of $\frac{1}{1+X}$.

4. Give an example of discrete random variable infinite mean.

5. Give an example of a discrete random variable with finite mean but with infinite variance.

6. Let X be a discrete random variable with finite second moment. Show that it has finite mean.

7. Let X be a Binomial random variable with parameters $n = 4,\ p = \frac{1}{2}$. Find $E\sin\frac{\pi X}{2}$.

8. Let X be a discrete random variable with pmf f and $\varphi : \mathbb{R} \to \mathbb{R}$ be a function. Show that
$$E[\varphi \circ X] = \sum_{x \in D} \varphi(x) f(x),$$
provided the rhs series converges absolutely, where $D = \big\{x \in \mathbb{R} | P\big(X = x\big) > 0\big\}$.

[1] A random variable X is said be geometric with parameter $0 < p < 1$ if its pmf is given by $f(x) = p(1-p)^x,\ x = 0, 1, \ldots$

9. Let X be standard normal random variable. Find EX^2.

10. Let X be exponentially distributed with parameter λ. Find the 4th moment.

11. Let X be a nonnegative continuous random variable with finite mean. Show that
$$EX = \int_0^\infty P\big(X > t\big)\,dt\,.$$

12. Let $\varphi : \mathbb{R} \to \mathbb{R}$ be a continuous bounded function and X be a random variable with pdf f. Show that
$$E[\varphi(X)I_{\{X\leq 0\}}] = \int_{-\infty}^0 \varphi(x)f(x)dx.$$

13. Let X and Y be independent and identically distributed exponential random variables with parameter λ. Find the mean of $\max\{X, Y\}$.

14. Let X be Binomial (n, p) with $EX = 1$ and $EX^2 = 2$, find the pmf of X.

15. Let X, Y has joint pdf
$$f(x,y) = \begin{cases} 2 & \text{if} \quad 0 \leq x \leq y \leq 1 \\ 0 & \text{if} \quad \text{otherwise.} \end{cases}$$
Find the covariance matrix of (X, Y).

16. Let X be geometric $(\frac{1}{2})$ and $\varphi(x) = e^{ax}, x \in \mathbb{R}, a \geq 0$. Does $E\varphi(X)$? Justify your answer.

17. Let X be Poisson (λ). Does there exists a $\varphi : \mathbb{R} \to [0, \infty)$ such that $E\varphi(X)$ doesn't exists.

18. Let X and Y be independent, X be Binomial $(10, \frac{1}{2})$ and Y is Uniform $(0,\ 1)$ Find $E[XY]$.

19. Let X, Y has joint pdf given by
$$f(x,y) = \begin{cases} 1 & \text{if} \quad 0 \leq x \leq y \leq 2 \\ 0 & \text{if} \quad \text{otherwise.} \end{cases}$$
Find $E[X^2Y]$.

20. Let $\varphi(x) = \lfloor x \rfloor$ and X is exponential (1). Find $E[\varphi(X)]$.

21. Let $\varphi(x) = x, x \in (2n, 2n+1],\ = 0, x \in (2n+1, 2n+2], n = 0, 1, 2, 3, 4$ and $\varphi(x) = 1$ otherwise. Find $E\varphi(X)$, where X is Uniform $(0, 10)$.

22. Let X and Y be random variables such that $E[XY] = EXEY$. Is it true that X and Y are independent.

23. Let X be such that EX^2 exists, then show that EX exists.

24. Let X be a non negative random variable such that EX^2 exists. Then show that
$$EX \leq \sqrt{EX^2}.$$