**INDIAN INSTITUTE OF TECHNOLOGY BOMBAY**

*Department of Mathematics*

*SI 427 (Probability Theory)*

**Tutorial Sheet-X**

1. Find the characteristic function of the random variable with pmf given by
$$f(x) \;=\; 2^{-x},\; x = 1, 2, \ldots .$$

2. Find the characteristic function of the random variable $X$ with pdf given by
$$f(x) \;=\; \begin{cases} 1 - |x| & \text{if} \quad |x| \le 1 \\ 0 & \text{otherwise .} \end{cases}$$

3. Find the characteristic function of the standard normal random variable.

4. Let $X$ be a random variable such that $P\big(X \in \mathbb{Z}\big) = 1$. Show that
$$\phi_X(t) \;= \phi_X(t + 2\pi),\; t \in \mathbb{R}\,.$$

5. Find the distribution of the random variable $X$ with characteristic function
$$\phi_X(t) \;=\; \frac{1}{2}e^{-it} + \frac{1}{3} + \frac{1}{6}e^{it}\,.$$

6. Find the characteristic function of $X \sim f$, where
$$f(x) \;=\; \frac{1}{2}e^{-|x|}, x \in \mathbb{R}.$$

7. Find the characteristic function of Cauchy distribution. Hint: Use Q6.

8. Let $X$ and $Y$ be independent geometric random variables with parameters $p_1$ and $p_2$ respectively. Find $E[Y|X + Y]$.

9. Let $X$ and $Y$ be independent and identically distributed Poisson random variables with parameter $\lambda$. Find $E[Y|X + Y]$.

10. Let $N$ be a nonnegative integer valued random variable. Let $\{Y_n|n \ge 0\}$ be a sequence of discrete random variables with finite mean. Show that
$$E[Y_N|N = n] \;=\; EY_n,\; n \ge 0\,.$$

11. Let $X$ and $Y$ be independent and identically distributed exponential random variables with parameter $\lambda$. Find the conditional pdf of $X$ given $X + Y = z,\ z > 0$.

12. Let $X$ and $Y$ be independent and identically distributed continuous random variables with density $f$. Find $P\big(X^2 > Y\big)$.

13. Let $X$ be exponential (1) and $Y = I_{\{1 \leq X \leq 3\}}$. Find $f_{X|Y}$.

14. Let $X$ be uniform $(0, 5)$ and $Y = I_{\{X \leq 2\}} + I_{\{1 \leq X \leq 3\}}$. Compute $E[(X + Y)^2|Y]$ and hence compute $E(X + Y)^2$ .

15. Let $X, Y$ be independent standard normal random variables. Find $E[X + 2Y|X = Y]$. Warning : You can only use the results from the class notes, anything else should come with a justification.