INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

Department of Mathematics

SI 427 (Probability I)

Tutorial Sheet-VIII

1. Let X and Y be independent and identically distributed (i.i.d.) random variables such that $P(X = i) = \frac{1}{N}$, $i = 1, 2, \ldots, N$. Find (i) $P(X \geq Y)$ (ii) $P(X = Y)$.

2. Let X and Y be i.i.d. random variables such that $P(X = i) = \frac{1}{N}$, $i = 1, 2, \ldots, N$. Find the distribution function of $\min\{X, Y\}$.

3. Let X_1, X_2 be independent Binomial $(10, \frac{1}{2})$ random variables. Find the pmf of $(X_1, X_1 + X_2)$.

4. Let X_1, X_2 are independent Poisson random variables with parameter 1. Find the pmf of $X_1 + X_2$.

5. Let X_1, X_2 as in Q4. Find the joint pmf of X_1 and $X_1 + X_2$.

6. Let X and Y be continuous random variables with joint pdf f. Find the joint pdf of $W = X - 2$ and $Z = 2Y + 1$.

7. Let X, Y be continuous random variables with joint pdf f. Find the joint pdf of X^2 and Y^2.

8. Let X, Y be continuous random variables with joint podf given by

$$f(x, y) = \begin{cases} \lambda^2 e^{-\lambda y}, & 0 \leq x \leq y \\ 0 & \text{otherwise}, \end{cases}$$

where $\lambda > 0$. Find the marginal pdfs of X and Y. Also find the joint distribution function of X, Y.

9. Let $f(x, y) = ce^{-\frac{(x^2 - xy + 4y^2)}{2}}$, $x, y \in \mathbb{R}$. Find the value of c such that f is a pdf.

10. Let X and Y be independent continuous random variables with joint pdf f. Find the pdf of $Y - X$.

11. Let X and Y be independent and identical distributed continuous random variables. Let f be the marginal pdf of X. Find the pdf of $Y - X$.

12. Let X and Y be continuous random variables with joint pdf f. Find the pdf of XY.

13. Let X and Y be i.i.d. random variables such that X is exponential with parameter λ. Find the pdf of $\frac{X}{Y}$.

14. Let (X, Y) be a normally distributed with parameters

$$\mu = (0,0),\ \Sigma \ = \ \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}.$$

Find the pdf (in terms of A) of $(U, V) = (X, Y)A$, where A is a non singular matrix and hence find A such that U and V are independent.

15. Let X be standard normal random variable with parameters 1 and 1. Is $(X, 2X)$ normally distributed? Does $(X, 2X)$ has a pdf? Justify your answers.

16. Let (X, Y) has pdf given by

$$f(x,y) \ = \ \frac{1}{2\pi} e^{-\frac{1}{2}(x^2+y^2)}, x, y \in \mathbb{R}.$$

Are X and Y independent? Justify your answer. Define (R, Θ) such that

$$X \ = \ R\cos\Theta, Y \ = \ R\sin\Theta, 0 < R < \infty, 0 \le \Theta < 2\pi.$$

Find the pdf of (R, Θ). Are R and Θ independent? Justify your answer.

17. Let $X_1, X_2, \cdots, X_{10}$ be i.i.d. uniform (0, 2) random variables. Compute
(i)$P(X^{(2)} - X^{(1)} < \frac{1}{2})$ (ii) $P(X^{(10)} - X^{(1)} \ge 1)$.