CS217: Artificial Intelligence and Machine Learning (associated lab: CS240)

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Decision Tree

Match	Pitch Type	Host	Batting First	Winner
M1	Spin-friendly	India	India	India
M2	Pace-friendly	Australia	Australia	Australia
M3	Balanced	India	Australia	India
M4	Spin-friendly	Australia	India	Australia
M5	Pace-friendly	India	Australia	Australia
M6	Spin-friendly	India	Australia	India
M7	Balanced	Australia	India	India
M8	Pace-friendly	Australia	India	Australia
M9	Spin-friendly	India	India	India
M10	Balanced	Australia	Australia	Australia

1. Make decision to predict if India can beat Australia in the upcoming match?
    a. Use Information Gain/Gain Ratio/Gini Index to build decision tree.

Decision Tree

Entropy == Randomness

$$X \sim \mathrm{Bern}(P)$$

$$P(x=1) = P$$

$$P(x=0) = 1 - P$$

$$H(x) = \sum_{z=1}^{k} -P(x=i)\log P(x=i)$$

$$= -\int_{-\infty}^{\infty} P(x)\log P(x)dx$$

$$= E_{x\sim P(x)}[-\log P(x)]$$

Find *P* that maximizes the entropy for a Bern(P) => MLE

$$H(\mathrm{Bern}\, n(P)) = -P\log P - (1-P)\log(1-P)$$

$$\frac{\partial H}{\partial P} = \frac{-P}{P} - \log P + \log(1-P)$$

$$H = 0$$

$$\log\left(\frac{1-P)}{P}\right) = 0$$

$$\Rightarrow 1 - P = P$$

$$\Rightarrow P = 1/2$$

$$H(\mathrm{Bern}(1/2)) = -1/2\log 1/2 - (1-1/2)\log(1-1/2)$$

$$H(\mathrm{Bern}(1/2)) = (1/2)\log 2 + (1/2)\log 2 = \log 2$$

$$\lim_{P\to 0^+} -P\log P - (1-P)\log(1-P)$$

$$\lim_{P\to 0^+} \frac{-\log P}{1/P} = \frac{-\log P}{-1/P} = \lim_{P\to 0^+} P = 0$$

$$\text{Similarily, } \lim_{P\to 1^-} H(P) = 0$$

What needs to be decided on?

- Split feature
    - based on Purity on feature
- Split point
- When to stop splitting

Purity:

- how homogeneous a node is in terms of class labels
- goal of splitting is to **create child nodes that are purer** than the parent node
- meaning they contain more instances of a single class

Different Purity measures?

1. Gini Impurity

$$Gini = 1 - \sum_{i=1}^{c} p_i^2$$

- Measures the probability of incorrectly classifying a randomly chosen element.
- Lower values indicate purer nodes.
- Used in **CART (Classification and Regression Trees)**.

2. Entropy (Information Gain)

$$Entropy = -\sum_{i=1}^{c} p_i \log_2 p_i$$

- Measures the uncertainty in a node.
- Used in **ID3, C4.5, and C5.0** algorithms.
- A split is chosen to maximize **Information Gain**:

$$IG = Entropy(parent) - \sum \frac{|child|}{|parent|} \times Entropy(child)$$

Different Purity measures?

Variance Reduction (for Regression Trees)

$$\text{Variance} = \frac{1}{N}\sum(y_i - \bar{y})^2$$

- Used for regression tasks.
- The split is chosen to minimize variance within child nodes.

Splitting Strategy- At each step, the algorithm:

- Evaluates all possible splits.
- Computes the purity measure for each split.
- Selects the split that results in the highest improvement in purity.

Information Gain to construct Decision Tree

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Information Gain to construct Decision Tree

$$Entropy(S) = -p_1 \log_2 p_1 - p_2 \log_2 p_2$$

where p_1 and p_2 are the probabilities of **India winning** and **India not winning (Australia winning)**.

From the table:

- **Total matches** = 10
- **India wins** = 5
- **Australia wins** = 5

$$p(India) = \frac{5}{10} = 0.5, \quad p(Australia) = \frac{5}{10} = 0.5$$

$$Entropy(S) = -\left(0.5 \log_2 0.5 + 0.5 \log_2 0.5\right)$$

$$= -(0.5 \times -1 + 0.5 \times -1)$$

$$= -(-0.5 - 0.5) = 1.0$$

Entropy for Spin

$$p(India) = \frac{3}{4}, \quad p(Australia) = \frac{1}{4}$$

$$Entropy(Spin) = -\left(\frac{3}{4} \log_2 \frac{3}{4} + \frac{1}{4} \log_2 \frac{1}{4}\right)$$

$$= -(0.75 \times -0.415 + 0.25 \times -2)$$

$$= -(-0.311 - 0.5) = 0.811$$

Information Gain to construct Decision Tree

Entropy for Pace

$$p(India) = 0, \quad p(Australia) = 1$$

$$Entropy(Pace) = -(0\log_2 0 + 1\log_2 1) = 0$$

$$p(India) = \frac{2}{3}, \quad p(Australia) = \frac{1}{3}$$

Entropy for Balanced

$$Entropy(Balanced) = -\left(\frac{2}{3}\log_2\frac{2}{3} + \frac{1}{3}\log_2\frac{1}{3}\right)$$

$$= -(0.667 \times -0.585 + 0.333 \times -1.585)$$

$$= -(-0.390 - 0.528) = 0.918$$

Weighted Entropy of Pitch

$$Entropy(Pitch) = \frac{4}{10} \times 0.811 + \frac{3}{10} \times 0 + \frac{3}{10} \times 0.918$$

$$= 0.3244 + 0 + 0.2754 = 0.5998$$

Information Gain

$$IG = Entropy(S) - Entropy(Pitch)$$

$$IG = 1.0 - 0.5998$$

$$IG = 0.4002$$

IG(Host)=1.0−0.971=0.029. | *IG(Batting)=1.0−0.971=0.029*

Stopping Criteria in Decision Tree

1. Pure Node

1. No significant IG

1. Minimum Samples in a node

1. Maximum tree depth

1. No features to split

Definition of a linear model

A linear model is considered **linear** because the model's predictions are a **linear function** of the parameters w.

Mathematically, a typical linear model takes the form:

$y=w^Tx+b$

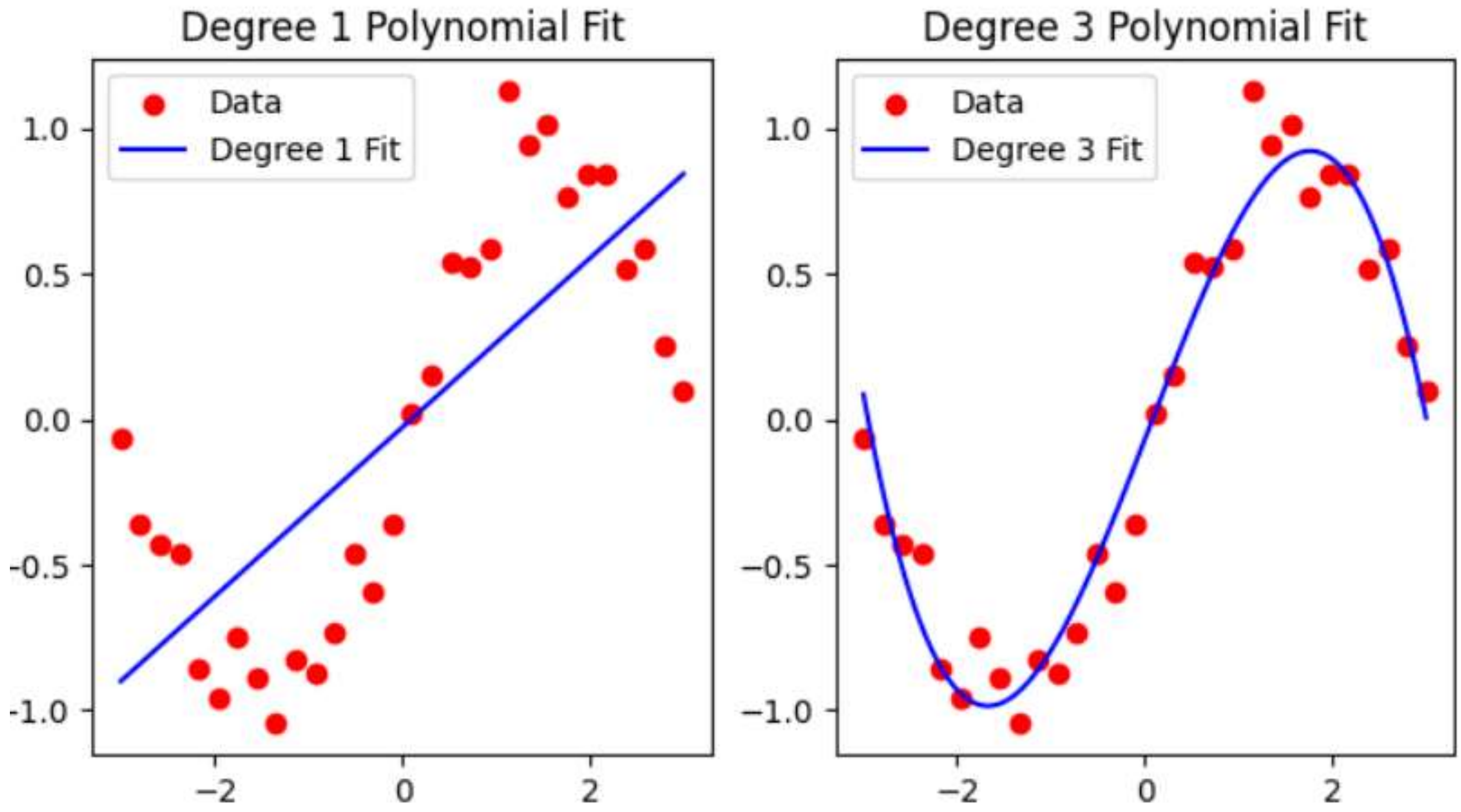
where:

- x is the input feature vector,
- w is the weight vector (parameters),
- b is the bias term,
- y is the predicted output.

Bias-Variance Tradeoff: Overfitting and Underfitting

Overfitting: The model learns not only the underlying pattern but also the noise in the training data. It performs well on training data but poorly on unseen data.

Underfitting: The model is too simple to capture the underlying pattern in the data, leading to poor performance on both training and test data.



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