CHAPTER 3

OPTICAL DESIGN AND CALIBRATIONS

This chapter will discuss ALI from the initial planning to a completed system including calibration and testing. First a discussion of the Acousto-Optical Tunable Filter (AOTF) will occur that covers the solution of the wave equation, diffraction efficiency, diffracted angle output, and tuning curve. Following is a calibration of the specific AOTF used in ALI. With the completed characterization of AOTF, discussion of the two primary optical layout considered for the instrument with the final design and optical specifications presented. Next, the system will be tested as a whole and finally an outline of addition components will be addressed.

# 3.1 AOTF Theory and Background

The fundamental piece of technology that allows for the building of ALI is an Acousto-Optical Tunable Filter (AOTF) which allows a signal to be passed through a band gap wavelength filter. The use of AOTF technology for space based initiatives is only recently possible due the recent advances in creating AOTFs with the ability to maintain imaging qualities over a wide acceptance angles. This section will discuss the theory behind the AOTF.

## 3.1.1 Solution to the Acoustic Equation

An AOTF is a device that through phonon-phonon interactions and Bragg diffraction allows a broadband light source to be filtered into a spectral image. Two primary types of AOTFs exist a collinear (*i.e.* the acoustic wave is aligned with the incident beam, (*Harris and Wallace*, 1969)) and a non-collinear (*i.e.* the acoustic wave and the optical beam do not propagate collinearly in the crystal, (*Chang*, 1977)) configurations and both use an optically anisotropic medium (*Saito and Yano*, 1976). An anisotropic medium is a material that is transparent and has a different index of refraction based upon the polarization of the incoming light and its propagation direction, commonly called birefringence. For images purposes a wide aperture is required for an AOTF and have been developed (*Gass and Sambles*, 1991) and are currently readily available for imaging purposes. In order to fully understand the principles behind an AOTF a stress analysis through the acousto-wave will be used to solve the wave equation.

A derivation of the wave equation for the Acousto-Optic (AO) effect will be presented starting from two of Maxwell’s equations, Amperes law and Faradays law, seen in the following

|  |  |
| --- | --- |
|  | (3.3) |
|  | (3.4) |

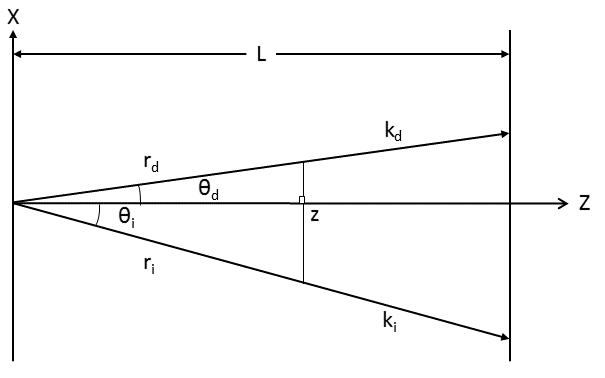
where is an electric field, is the magnetic field, is the current density, is the permeability, and is the permittivity. By taking the curl of the equation 3.3 combining it with equation 3.4 and using that the AOTF crystal is non-conductive (*i.e.* ) along with the identity and assuming the crystal has no net charge (*i.e.* ) gives the simplified form

|  |  |
| --- | --- |
|  | (3.5) |

Since the dielectric is not a constant with time it induces a susceptibility yielding the following form

|  |  |
| --- | --- |
|  | (3.6) |

where is the induced polarization due to the stress in the AO medium given by and the change is the susceptibility, is given by . In the previous definition and are the indices of refraction for the incident and diffracted electric fields and is the elasto-optic coefficient which is dependent on medium and orientation of the crystal used and is the strain wave induced by the acousto wave. A solution for this equation will be presented in the Bragg region meaning there will only be first order diffraction effects.



**Figure 3-1**: Geometry for the AOTF wave derivation assuming the acousto wave is along the x-axis and the AO interaction occurs along the z axis over a interaction length, . , , and are the position vector, wave vector, and angle of the incident electric field and similarly for the diffracted electric field. Figure recreated from *Xu and Stroud* (1992)

Assuming the incoming electric field is a plane wave the above differential equation can be solved. A standard acousto-optical geometry used in the solution and is shown in Figure 3-1. The acoustic wave is propagating in the x direction of the crystal causing a stress wave which leads to the modulation of the index of refraction within the acoustic region of the crystal denoted by . The system will be orientated such that the acousto interaction occurs along on the z axis and the electric field entering the device will be a plane wave described by and in the form

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| |  |  | | --- | --- | |  | (3.7) | | |  |  | | --- | --- | |  | (3.5a) | |

and the diffracted electric field is described as

|  |  |
| --- | --- |
|  | (3.8) |

where and are the frequencies and wave vectors for the incident and diffracted beam and c.c. is the complex conjugate of the exponential term. The modulation caused by the acoustic wave is a strain on the crystal given as

|  |  |
| --- | --- |
|  | (3.9) |

where is the frequency of the RF wave, and is the acousto wave vector.

Equations 3.6-3.9 will be used to determine the coupled wave equations. The induced polarization is given by the incident wave and the strain wave interacting in the form of . The polarization wave will in turn stimulate the diffracted electric field yielding the first half of the coupled equations in the form

|  |  |
| --- | --- |
|  | (3.10) |
|  |  |

where However, once the interaction between the incident electric field forms the diffracted field, the diffracted field in turn interacts to form a polarization wave that stimulates incident wave yielding the second coupled equation

|  |  |
| --- | --- |
|  | (3.11) |
|  |  |

Solving the coupled wave equations yields the following solutions

|  |  |
| --- | --- |
|  | (3.12) |
|  | (3.13) |

where . It should be noted that the frequency of the diffracted wavelength will be though the coupled interaction. To find the unknown coefficients the boundary conditions of the system, are used being

|  |  |
| --- | --- |
|  | (3.14) |

and solving for the coefficients yields

|  |  |
| --- | --- |
|  | (3.15) |
|  | (3.16) |

## 3.1.2 Diffraction Efficiency

The diffraction efficiency, of the AOTF is ratio of the energy of the incident electric field compared to the energy of the diffracted electric field at the end of the acoustic interaction region given by

|  |  |
| --- | --- |
|  | (3.17) |

This form yields the common sinc function shape for the spectral Point Spread Function (PSF). However, this form can be altered to better identify how to increase the diffraction efficiency of an AOTF. The diffraction efficiency will be converted into a form that uses the RF driving power assuming exact momentum matching (*i.e.* and that the interaction is occurring within a birefringent medium. The RF driving power is the power at which the piezoelectric transducer pumps the RF signal into the AO medium. The average energy flow of the acoustic power is defined by

|  |  |
| --- | --- |
|  | (3.18) |

where is mass density of the medium, is the acoustic velocity in the crystal, and is the height and length of the acoustic wave. Another variable that will be defined is known as the acousto-optic figure of merit, , which is completely determined by the medium properties defined by

|  |  |
| --- | --- |
|  | (3.19) |

Using equations 3.18 and 3.19 and simplifying equation 3.17 yields the following for the diffraction efficiency

|  |  |
| --- | --- |
|  | (3.20) |

The efficiency of the diffraction at a wavelength, , can be increased through design considerations. First, a medium should be picked that yields the largest possible AO figure of merit. Second, the active region of the AO region should be narrow and long increasing the ratio. And lastly, the driving power of RF wave should be large enough to equate the component inside of the sinusoid function in equation 3.20 to . It should be noted that increasing the RF power too high can have the possibility of deceasing the AOTF diffraction efficiency.

## 3.1.3 Diffraction Angle

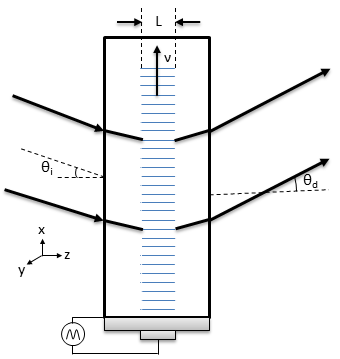
Although the wave equations are useful in determining the diffraction efficiency and the form of the electric fields. It is not convenient to determine angle of the diffracted wave or the RF acousto wave to wavelength relation known as the tuning curve (covered in the section 3.1.4). A discussion on the diffraction angle will be underwent using the interaction between the acoustic sound wave and the phonon light interaction by

|  |  |
| --- | --- |
|  | (3.21) |

known as the momentum matching criteria. For this analysis only the -1 diffraction interaction will be performed although a similar analysis can be performed for the +1 case. The wave vectors are defined as

|  |  |
| --- | --- |
|  | (3.22) |
|  | (3.23) |
|  | (3.24) |

It will be assumed that the extraordinary light undergoes the momentum matching through the device.



**Figure 3-2**: A standard non-collinear AOTF experiential set up. The crystal is assumed to infinitely long in the y direction. Figure recreated after *Guenther* (1990) number 14B-1.

A standard acousto optical experimental setup, which can be seen in Figure 3-2, will be used to determine the diffraction angle and tuning curve. Using equation 3.21, the x component of the wave vector is

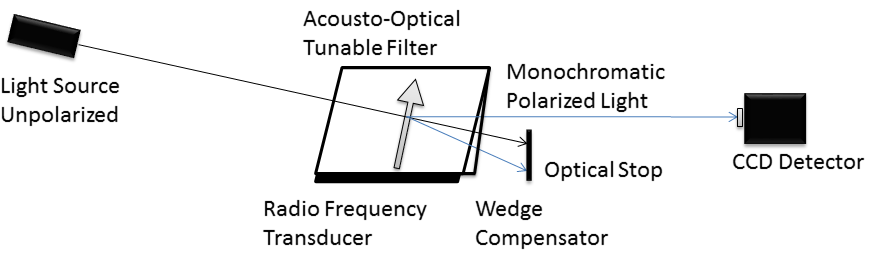
|  |  |
| --- | --- |
|  | (3.25) |

and the magnitude of the diffracted wave vector is

|  |  |
| --- | --- |
|  | (3.26) |

and combining the results from equation 3.25 and equation 3.26 the angular deviation of the diffracted source is

|  |  |
| --- | --- |
|  | (3.27) |



**Figure 3-3**: General Layout of an AOTF. A randomly polarized incoming light source hits the front surface of the birefringent crystal. The black bar below the crystal is the piezo-electric transducer that produces the RF signal and forms the acousto wave represented by the grey arrow. The momentum matching Bragg diffraction occurs and monochromatic polarized light (-1 order) exits the AOTF at a constant angle with the 0th order and +1 order being blocked by an optical stop.

The diffracted light will leave the AOTF at a different angle depending on the RF frequency which in turn alters the wavelength of light that leaves the device. In order for the device to be usable in an imaging optical system the diffracted light must always leave the device following the same path no matter what wavelength is being filter. Thus a crystal wedge or compensator is fashioned to the back of the device to compensate for this effect using a prism like effect causing the diffracted beam to always leave at the same angle. A general optical layout with the deflection in the optical path and an attached compensating wedge is shown in Figure 3-3.

## 3.1.4 Tuning Curve

The tuning curve is the AOTF relationship between the outputted diffracted light wavelength and the set acoustic sound wave frequency. The analysis will be performed using the momentum matching criteria stated in equation 3.21. Figure 3-4 shows the wave vectors in the orientation of a tellurium oxide (TeO2) crystal in a birefringent orientation where is the propagation angle of the acoustic wave with respect to the crystal orientation. The indices of ordinary and extraordinary refraction for TeO2 is given by (*Uchida*, 1971)

|  |  |
| --- | --- |
|  | (3.30) |
|  | (3.31) |

for 400 to 1000 nm.

The wave vector diagram can be used to define the incident and diffracted indices of refection in terms of the ordinary and extraordinary indices of refraction the above refractive indices defined above to be

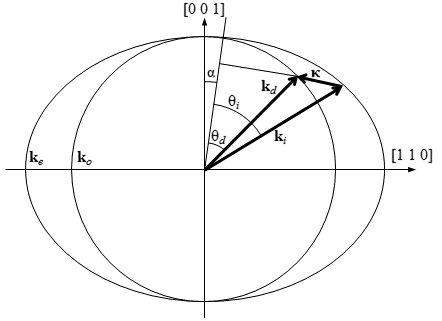
|  |  |
| --- | --- |
|  | (3.28) |
|  | (3.29) |

If the difference in the index of refraction is small, as it is for TeO2, can be approximated as (*Voloshinov et al*., 2007)

|  |  |
| --- | --- |
|  | (3.30) |

where is the difference between the extraordinary and ordinary indices of refraction (*i.e.* ). The wave vectors, seen in Figure 3-4, of the system need to follow the momentum matching criteria from equation 3.21. Separating the wave vectors into their directional components with respect to the propagation angle, , the tangential and perpendicular directions respectively are

|  |  |
| --- | --- |
|  | (3.31) |
|  | (3.32) |



**Figure 3-4**: The wave vectors generated by the AOTF experiment set up in Figure 3-1. From the above figure and are the wave vectors of the extraordinary and ordinary axis of the AOTF crystal.

The tangential and perpendicular directions of the wave vector can be used in combination with the wave vectors definitions (equations 3.22-3.24) to yield

|  |  |
| --- | --- |
|  | (3.33) |

The above can be written as

|  |  |
| --- | --- |
|  | (3.34) |

assuming difference in indices of refraction is small (equation 3.30) and the Taylor expansion approximation of the square root is used (*Voloshinov and Mosquera*, 2006). This equation has several implications to the operation of the device which affects the design possibilities in an imaging system. First, the wavelength diffracted by the AOTF is inversely related to frequency of the RF wave. Second, the wavelength of diffracted signal is dependent on the angle of incidence of the incoming wave therefore passing a signal though the AOTF at different incident angles will result in different outgoing wavelengths. Also, through the described interaction the diffracted light goes through a 90o rotation in polarization (*Voloshinov*, 1996).

# 3.1.2 AOTF Calibration

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