ESERCIZIO 1

$f(x) = x^2 + 1$ $\quad$ $g(x) = x + a$

1) $(f \circ g) \Rightarrow f(x+a) \Rightarrow (x+a)^2 + 1$

$(f \circ g)(3) = 2 \Rightarrow (3+a)^2 + 1 = 2 \Rightarrow$

$(3+a)^2 = 1 \Rightarrow \sqrt[2]{(3+a)^2} = \sqrt[2]{1} \Rightarrow 3+a = \pm 1$

$\Rightarrow a = -3 \pm 1 \begin{cases} -4 \\ -2 \end{cases}$

2) $(g \circ f) \Rightarrow g(x^2+1) \Rightarrow x^2 + 1 + a$

$(g_a \circ f)(3) = 2 \Rightarrow 9 + 1 + a = 2 \Rightarrow$

$a = -8$

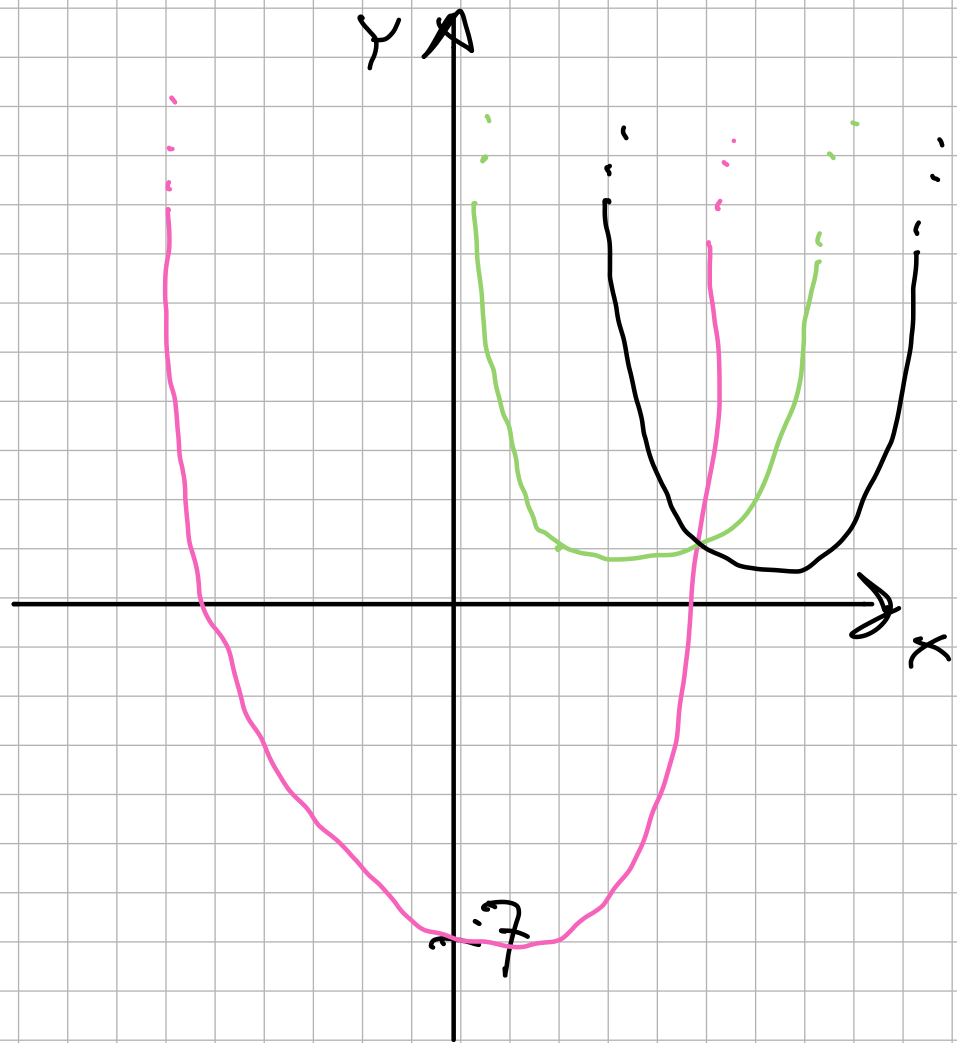
ORA DISEGNO $f \circ g$ e $g \circ f$

PER QUANTO RIGUARDA $f \circ g$ DEVO

SOSTITUENDO LA a IN $(x+a)^2+1$ OTTENENDO DUE FUNZIONI:

- $(x-2)^2+1$
- $(x-4)^2+1$

MENTRE PER $g \circ f$ HO SOLO: x^2+1-8



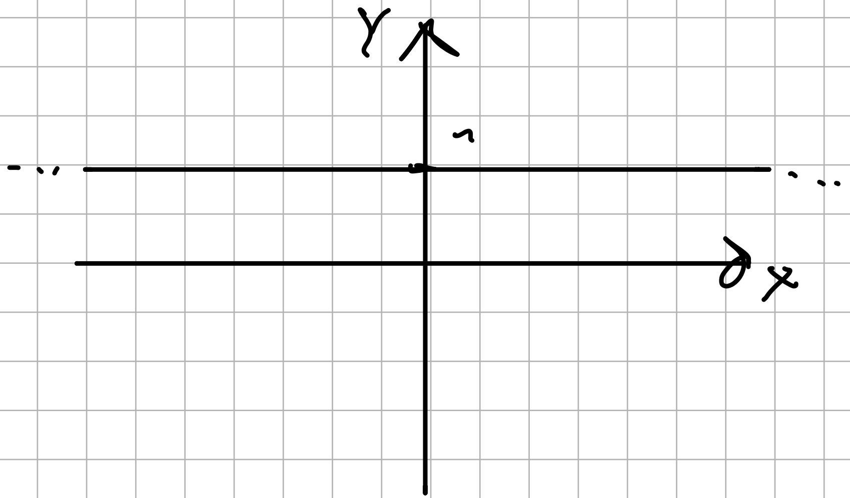
* GRAFICO QUALITATIVO

3) $H(x) = ax$

$(f \circ H) \Rightarrow f(ax) \Rightarrow (ax)^2 + 1$

$(f \circ H)(1) = 1 \Rightarrow (a \cdot 1)^2 + 1 = 1 \Rightarrow$

$a^2 = 0 \Rightarrow a = 0$



4) $(f \circ H)(2) = 2 \Rightarrow (a \cdot 2)^2 + 1 = 2 \Rightarrow$

$a^2 4 = 1 \Rightarrow a^2 = \frac{1}{4} \Rightarrow a = \pm \frac{1}{2} \begin{cases} -1/2 \\ 1/2 \end{cases}$

5) $(H \circ f) = H(x^2+1) = a(x^2+1)$

$(H \circ f)(1) = 1 \Rightarrow a(2) = 1 \Rightarrow a2 = 1 \Rightarrow$

$\Rightarrow a = \frac{1}{2}$

ESERCIZIO 2

1) $(f \circ g_a) = f(x+a) = \log(x+a)$

$(f \circ g_a)(0) = 0 \Rightarrow \log(0+a) = 0 \Rightarrow$

$\boxed{\log(a) = 0} \Rightarrow a = 1$

IL LOGARITMO DI 1 È 0 QUINDI a DEVE VALERE 1 PER OTTENERE LO 0

INVERSA: $x = \log(y+a) \Rightarrow e^x = e^{\log(y+a)} \Rightarrow$

$y = e^x - a$

2) $(g_a \circ h) = g_a(\exp(x)) \Rightarrow \exp(x) + a$

$(g_a \circ h)(0) = 0 \Rightarrow \exp(0) + a = 0 \Rightarrow$

$\Rightarrow 1 + a = 0 \Rightarrow a = -1$

INVERSA: $EXP(x) + a = y \Rightarrow x = EXP(y) + a \Rightarrow$

$x - a = EXP(y) \Rightarrow \log^{x-a} = \log^{EXP(y)} \Rightarrow y = \log(x-a)$

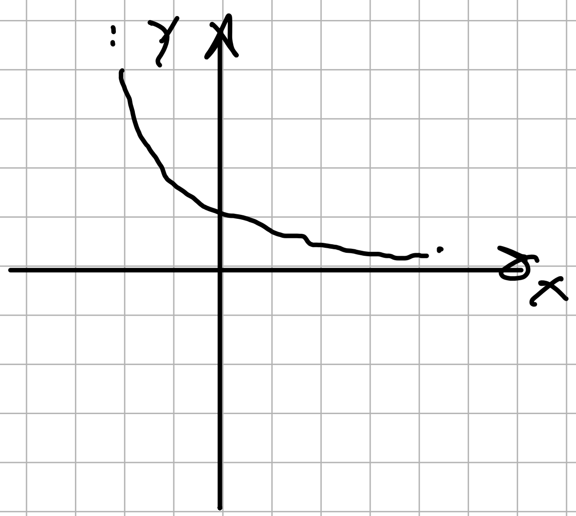
ESERCIZIO 3

1) $f(x) = EXP(x)$

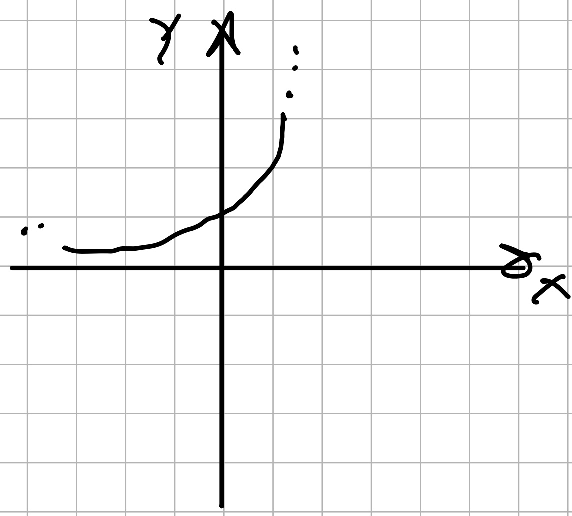
$DOM(f) = \mathbb{R}$

$DOM(g) =$ $\mathbb{R}$ PERCHÉ $EXP(x)$ È SEMPRE $\neq 0$

$DOM(h) = \mathbb{R}$



g



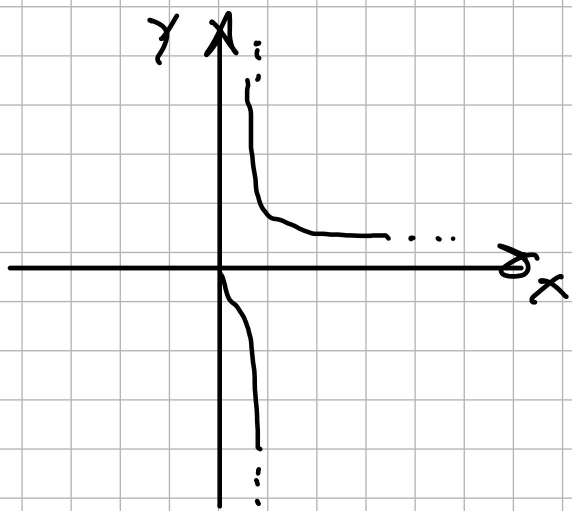
h

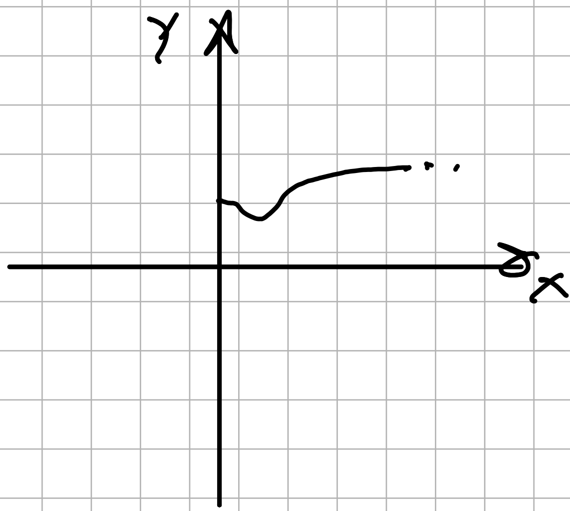
2) $f(x) = \log(x)$

$DOM(f) = (0, +\infty)$ PERCHÉ IL LOG NON È DEFINITO PER I NUMERI NEGATIVI E 0

$Dom(g) = \mathbb{R}^+ \setminus \{1\}$

$Dom(f) = \mathbb{R}^+$



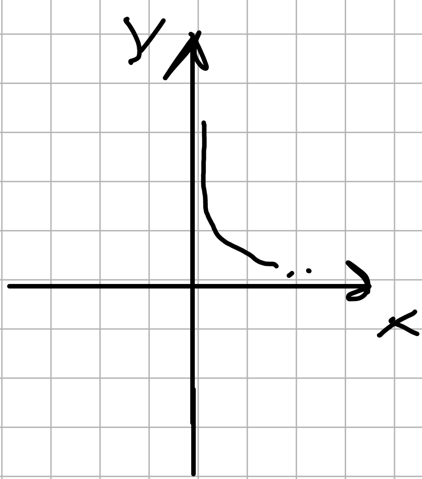


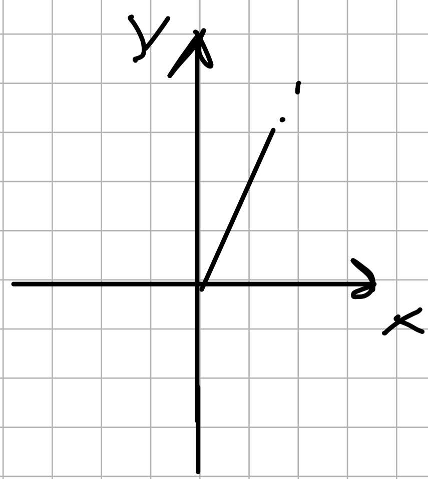
3) $\varphi(x) = \sqrt{x}$

$Dom(\varphi) = [0, +\infty)$

$Dom(g) = (0, +\infty)$

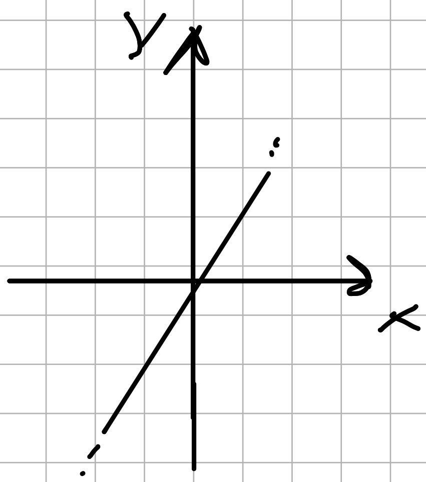
$Dom(f) = [0, +\infty)$

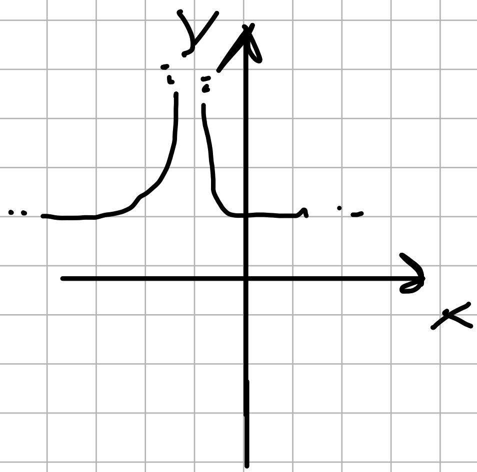




4) $f(x) = \frac{1}{x+1}$

$Dom(f) = Dom(g) = Dom(h) = \mathbb{R} \setminus \{-1\}$





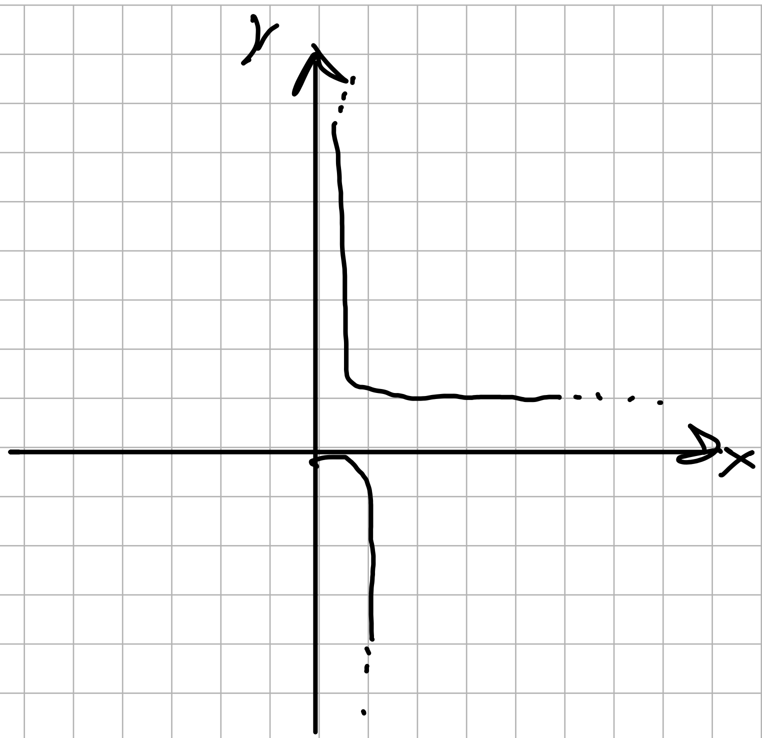
ESERCIZIO 4

$$f(x) = \frac{1}{\log(x-2)}$$

VISTO CHE LOG NON ACCETTA VALORI NEGATIVI E LO 0 DOBBIAMO ASSICURARCI CHE $x - 2$ SIA > 0.

QUINDI RISOLVIAMO: $x-2>0 \Rightarrow x>2$

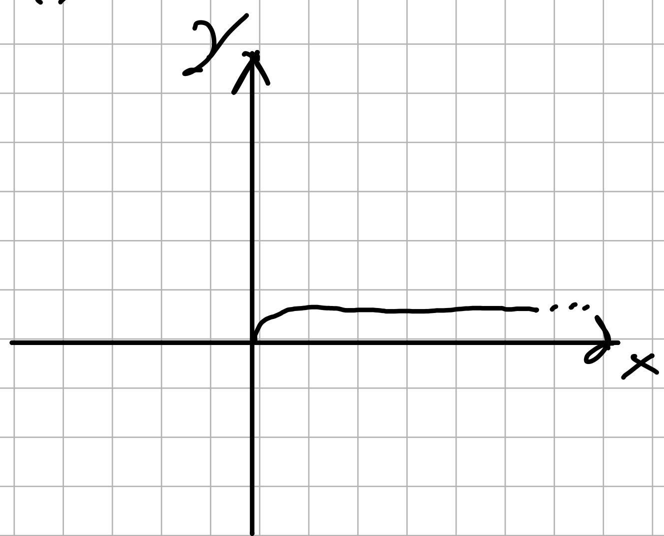
IL $Dom(f)$ E' QUINDI $(2, +\infty)$



ESERCIZIO 5

$$f(x) = \frac{1}{1+e^{-2x}}$$

AVENDO UN ESPONENZIALE AL DENOMINATORE AVRÒ SEMPRE VALORI POSITIVI PERTANTO IL DOM(f) È $\mathbb{R}$



ESERCIZIO 6

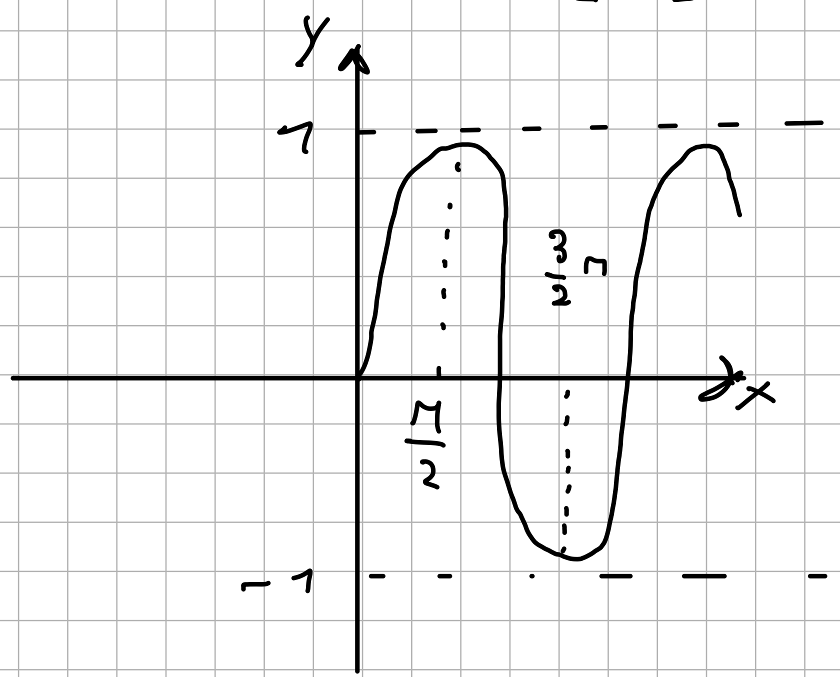
1) $f(x) = \sin(x)$

Dom = $\mathbb{R}$ ESSENDO UNA FUNZIONE PERIODICA NON E' INIETTIVA.

NON E' NEANCHE SURIETTIVA IN QUANTO SIAMO LIMITATI TRA 1 E -1.

PER TUTTO IL DOMINIO NON E' MONOTONA MAI:

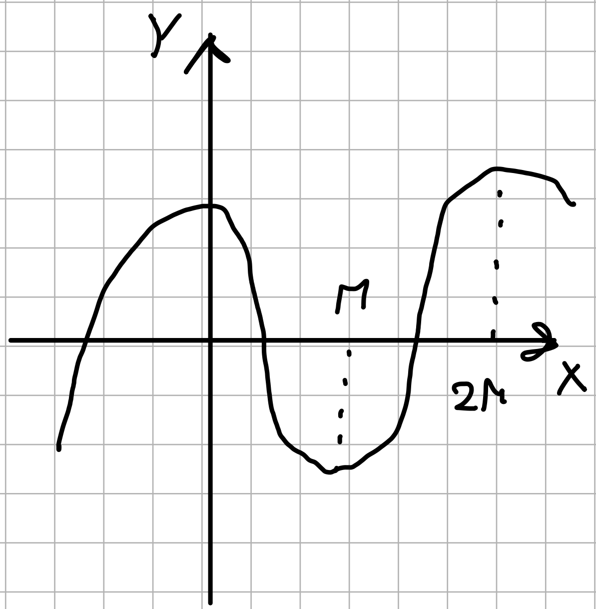
- $\sin(x)$ E' CRESCENTE IN $[0, \frac{\pi}{2}]$
- $\sin(x)$ E' DECRESCENTE IN $[\frac{\pi}{2}, \frac{3}{2}\pi]$



2) $f(x) = \cos(x)$

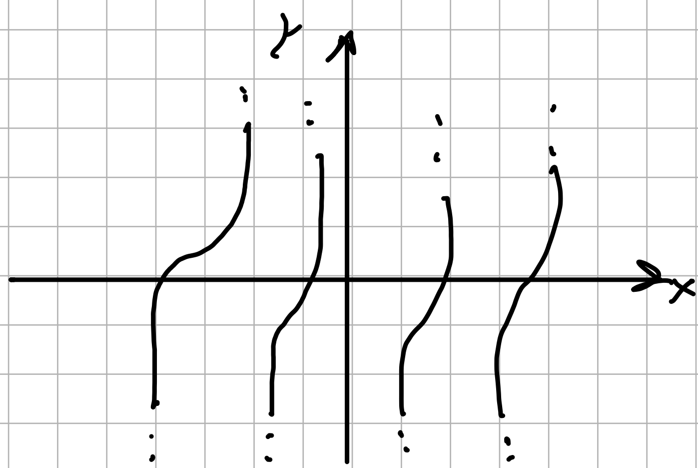
DOM = $\mathbb{R}$ SPIEGAZIONE ANALOGA A QUELLA DEL SIN(X).

E' DECRESCENTE IN $[0, \pi]$ E CRESCENTE IN $[\pi, 2\pi]$



3) $f(x) = \tan(x)$

Dom: $\mathbb{R} \setminus \{ \frac{\pi}{2} + k\pi \}$



NON E' INIETTIVA PERCHE' $TAN(0) = TAN(\pi) = 0$

MA E' SURIETTIVA.

E' CRESCENTE SOLO PER $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + k\pi$