Esercizio 1

1) $Dom = (\Lambda, \{-1\})$

2) $Dom = \mathbb{R}$

3) $\dfrac{x^3 - 1}{\sqrt{3x - 2}}$

$3x > 2$

$x > \dfrac{2}{3}$

$Dom = \left(\dfrac{2}{3}, +\infty\right)$

4) $f(x) = \dfrac{x + 1}{\sqrt{2x^2 + 3}}$

$2x^2 + 3 \neq 0 \Rightarrow x^2 > \dfrac{-3}{2}$ $\checkmark$

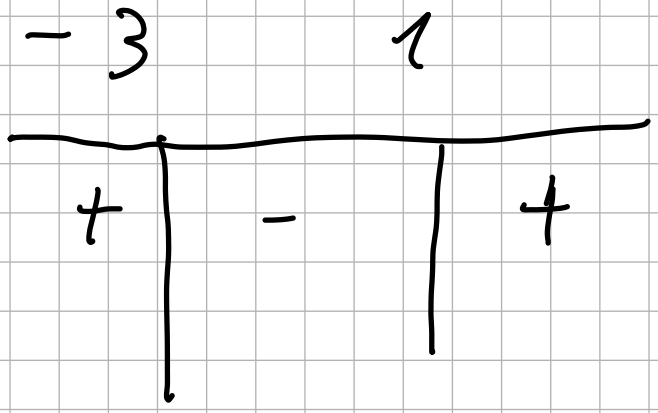
$Dom = \mathbb{R}$

Esercizio 2

1) $\sqrt{x^2+2x-3}$

$x^2+2x-3 \geq 0$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{16}}{2} \begin{cases} -3 \\ 1 \end{cases}$$



Dom: $(-\infty, -3] \cup [1, +\infty)$

2) $\sqrt[3]{x+1}$ $\quad x \geq -1$

Dom $[-1, +\infty)$

3) $f(x) = \sqrt{\log(x) + 1}$

$$\begin{cases} \log(x) + 1 \geq 0 \\ x > 0 \end{cases} \Rightarrow \begin{cases} \log(x) \geq -1 \\ x > 0 \end{cases}$$

$$\Rightarrow \begin{cases} e^{\log} \geq e^{-1} \\ x > 0 \end{cases} \Rightarrow \begin{cases} x \geq \frac{1}{e} \\ x > 0 \end{cases}$$

$$Dom = \left[\frac{1}{e}, +\infty \right)$$

4) $f(x) = \sqrt{e^x - 5}$

$$e^x \geq 5 \Rightarrow x \geq \log 5$$

Esercizio 6

1) $\lim_{x \to 1} \frac{1}{(x-1)^2} = \frac{1}{0} = +\infty$

2) $\lim_{x \to +\infty} \sqrt[3]{\arctan x} = \sqrt[3]{\frac{\pi}{2}}$

3) $\lim_{x \to +\infty} x \cdot 2^x = +\infty$

4) $\lim_{x \to 0} \frac{3x+1}{0} = \nexists$

$\lim_{x \to 0^-} = -\infty$

$\lim_{x \to 0^+} = +\infty$

5) $\lim_{x \to 0} \frac{1}{1+x^2} = 1$

6) $\lim_{x \to +\infty} \frac{1}{1+x^2} = \frac{1}{+\infty} = 0$

7) $\lim_{x \to \frac{\pi}{2}^+} \frac{1}{\cos x} = -\infty$

8) $\lim_{x \to \frac{\pi}{2}^-} \frac{1}{\cos x} = \frac{1}{0^+} = +\infty$

9) $\lim_{x \to -\infty} \arcsin \frac{1}{1-x^2} = 0$

10) $\lim_{x \to -\infty} \ln(1+e^{-x}) = +\infty$

11) $\lim_{x \to \frac{\pi}{2}^-} (\tan(x)) = +\infty$

12) $\lim_{x \to -\infty} \tan(x) = \nexists$

13) $\lim_{x \to 0^+} \frac{\log(x)}{\sqrt{3x}} = -\infty$

14) $\lim_{x \to +\infty} \sqrt{x+7} \log x = +\infty$

15) $\lim_{x \to +\infty} e^{\frac{1}{x-1}} = 1$

16) $\lim_{x \to +\infty} \sqrt{x+3} \log x = +\infty$

ESERCIZIO 7

1) $\lim_{x \to 0^+} \frac{1}{x} : \frac{1}{0^+} = +\infty$

$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

$$\lim_{x \to 2} = ?$$

$$\lim_{x \to -\infty} \frac{1}{x} = \frac{1}{-\infty} = 0^-$$

$$\lim_{x \to +\infty} \frac{1}{x} = \frac{1}{+\infty} = 0^+$$

2) $\varphi(x) = \frac{1}{x^2}$

$$\lim_{x \to 0^+} \frac{1}{x^2} = \frac{1}{0^+} = +\infty$$

$$\lim_{x \to 0^-} = +\infty$$

$$\lim_{x \to 0} = +\infty$$

$$\lim_{x \to x_0} : \underline{\lim_{x \to \cdot\infty}} = 0$$

3) $f(x) = e^{\frac{1}{x}}$

$$\lim_{x \to 0^+} = +\infty$$

$$\lim_{x \to 0} = -\infty$$

$$\lim_{x \to 0} = \nexists$$

$$\lim_{x \to +\infty} : 1$$

$$\lim_{x \to -\infty} : 1$$

4) $f(x) = e^{(x-2)}$

$\lim_{x \to 0^+} = e^{(0-2)} = e^{(-2)} = \frac{1}{e^2}$

$\lim_{x \to 0^-} = \frac{1}{e^2}$

$\lim_{x \to 0} = \frac{1}{e^2}$

$\lim_{x \to +\infty} = +\infty$

$\lim_{x \to -\infty} = 0$

Esercizio 8

1) $f(x) = x + 1$

$Dom = \mathbb{R}$

$\lim_{x \to -\infty} x + 1 : -\infty$

$\lim_{x \to +\infty} = +\infty$

2) $f(x) : x^2 - 1$

$Dom = \mathbb{R}$

$\lim_{x \to -\infty} x + 1 : +\infty$

$\lim_{x \to +\infty} x + 1 = +\infty$

3) $x^7 + 4x^4 - x^3 + 7$

$Dom = \mathbb{R}$

$$\lim_{x \to +\infty} = +\infty$$

$$\lim_{x \to -\infty} = -\infty$$

4) $f(x) = x^6 + 3x^5 + 3x^2 - 7$

$Dom = \mathbb{R}$

$$\lim_{x \to +\infty} = +\infty$$

$$\lim_{x \to -\infty} = +\infty$$

5) $f(x) = \frac{1}{x-1}$

$Dom = \mathbb{R} \setminus \{1\}$

$\lim_{x \to 1^+} = \frac{1}{0^+} = +\infty$

$\lim_{x \to 1^-} = \frac{1}{0^-} = -\infty$

6) $f(x) = \frac{x-1}{x+1}$

$Dom = \mathbb{R} \setminus \{-1\}$

$\lim_{x \to -1^+} = +\infty$

$$\lim_{x \to 2^-} = -\infty$$

7) $f(x) = \dfrac{x^2 - 1}{x - 2}$

$Dom = \mathbb{R} - \{2\}$

$$\lim_{x \to 2^+} = +\infty$$

$$\lim_{x \to 2^-} = -\infty$$

8) $f(x) = \dfrac{x + 1}{x^2 - 2}$

$Dom = \mathbb{R}$

$$\lim_{x \to +\infty} = \frac{+\infty}{+\infty} = 0$$

$$\lim_{x \to -\infty} = \frac{-\infty}{+\infty} = 0$$

ESERCIZIO 3

1) $f(x) = \log(x+1)$

$x + 1 > 0 \Rightarrow x > -1$

Dom: $(-1, +\infty)$

2) $f(x) = x + \log(x^2 + 2)$

$x^2 + 2 > 0 \Rightarrow x^2 > -2$ V

Dom = $\mathbb{R}$

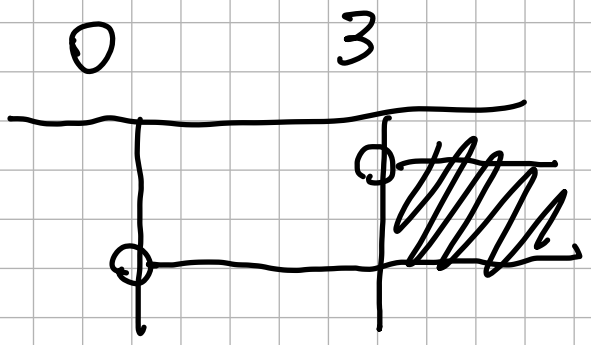
3) $f(x) = 5 + \log|x+1|$

$x+1 > 0 \Rightarrow x > -1$

$Dom: \mathbb{R} - \{-1\}$

4) $f(x) = \log(x) - (x+1) \cdot \log(x-3)$

$$\begin{cases} x-3>0 \\ x>0 \end{cases} \Rightarrow \begin{cases} x>3 \\ x>0 \end{cases}$$



$Dom: (3, +\infty)$

ESERCIZIO 4

1) $\varphi(x) = \tan(x) + \arctan(x)$

$\tan(x)$ HA DOMINIO $\mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi \right\}$

$\arctan(x)$ HA DOMINIO $\mathbb{R}$

$$\text{DOM}(\varphi) = \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi \right\}$$

2) $\varphi(x) = \tan(3x - 2)$

$$3x - 2 \neq \frac{\pi}{2} + k\pi \Rightarrow 3x \neq \frac{\pi}{2} + k\pi + 2$$

$$\Rightarrow x \neq \frac{\pi}{6} + k\frac{\pi}{3} + \frac{2}{3} \Rightarrow x \neq \frac{\pi + 4}{6} + k\pi$$

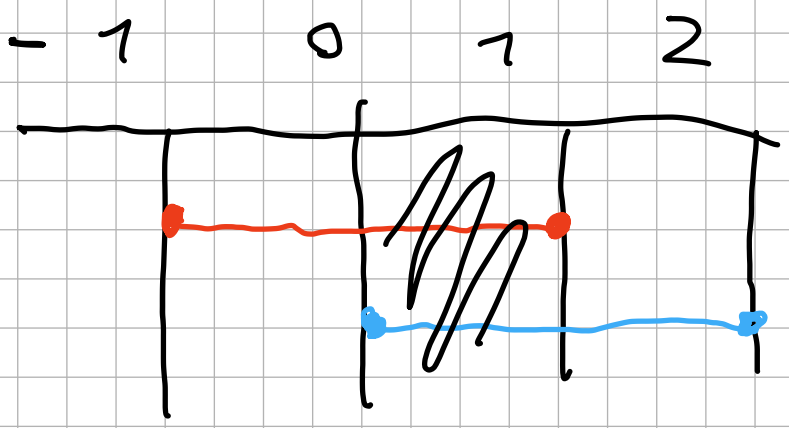
$$\text{DOM}(\varphi) = \mathbb{R} \setminus \left\{ \frac{\pi + 4}{6} + k\pi \right\}$$

3) $f(x) = \arcsin(x^2) + \arccos(x-1)$

$$\begin{cases} -1 \le x^2 \le 1 & a \\ -1 \le x-1 \le 1 & b \end{cases}$$

a $\begin{cases} x^2 \le 1 \\ x^2 \ge -1 \ \forall \end{cases}$

b $\begin{cases} x \ge 0 \\ x \le 2 \end{cases}$



Dom: $[0, 1]$

4) $\varphi(x) = \dfrac{\arcsin(x^2+1)}{\arccos(x-1)}$

$$\begin{cases} -1 \le x^2+1 \le 1 \\ -1 \le x-1 \le 1 \\ \arccos(x-1) \neq 0 \end{cases} \Rightarrow \begin{cases} -2 \le x^2 \le 0 \\ 0 \le x \le 2 \\ x-1 \neq 1 \end{cases}$$

$\text{Dom}(\varphi) = \emptyset$

5) $\varphi(x) = \arctan(x^2+1) \cdot \arccos(x^2-1)$

$$\begin{cases} x^2-1 \ge -1 \\ x^2-1 \le 1 \end{cases} \Rightarrow \begin{cases} x^2 \ge 0 \\ x^2 \le 2 \end{cases}$$

Dom: $[-\sqrt{2}, \sqrt{2}]$

ESERCIZIO 9

1) $\lim_{x \to -\infty} x^4 + x^3 - x$

$+\infty - \infty + \infty = +\infty$ PERCHÉ x^4 HA IL GRADO MAGGIORE

2) $\lim_{x \to -\infty} (x^4 - x^3 - x) \cdot e^{-x^2}$

$(+\infty \quad +\infty \quad +\infty) \cdot -\infty = 0$

3) $\lim_{x \to +\infty} \frac{3x^2 + 1}{x^2 - 1}$

$\frac{+\infty}{+\infty} = 3$

4) $\lim_{x \to -\infty} \frac{x}{3x^2 + 1}$

$\frac{-\infty}{+\infty} = \frac{x^2 \cdot \frac{1}{x}}{x^2 \cdot \left(3 + \frac{1}{x^2}\right)} = \frac{0}{3} = 0$

5) $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x^2} = \frac{0}{0} = ?$

6) $\lim_{x \to -\infty} \frac{x^7 - 15x^3}{3x^7 - 2x + 1} = \frac{-\infty + \infty}{-\infty + \infty} =$

$= \frac{-\infty}{-\infty} = \frac{1}{3}$

7) $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 9} = \frac{0}{0}$

$$\frac{x^2 + 2x - 3x - 6}{(x-3) \cdot (x+3)} = \frac{5}{6}$$

8) $\lim_{x \to +\infty} \frac{x^5 + 3x^2 + 3x + 1}{2x^2 - x}$

$$\frac{\infty}{\infty - \infty} = \frac{1}{2}$$