14/10

$$A+B,\ \lambda A,\ AB,\ \det A = \sum_{j=1}^{n} (-1)^{i} a_{ij} \det A_{ij} \quad \left(A \in \mathbb{R}^{n\times n}\right)$$

TEOREMA

$$\det A \neq 0 \iff \exists A^{-1} \text{ (INVERTIBILE)}$$

(NON SINGOLARE)

$$\det(A\ B) = \det A \ \det B$$

$$\shortparallel$$

$$\det(A+B) \neq \det A + \det B$$

$$\det(\lambda A) = \lambda^{n} \det A$$

$$\det(-I_n) = \det\begin{pmatrix} -1 & & 0 \\ & \ddots & \\ 0 & & -1 \end{pmatrix} = (-1)^{n}$$

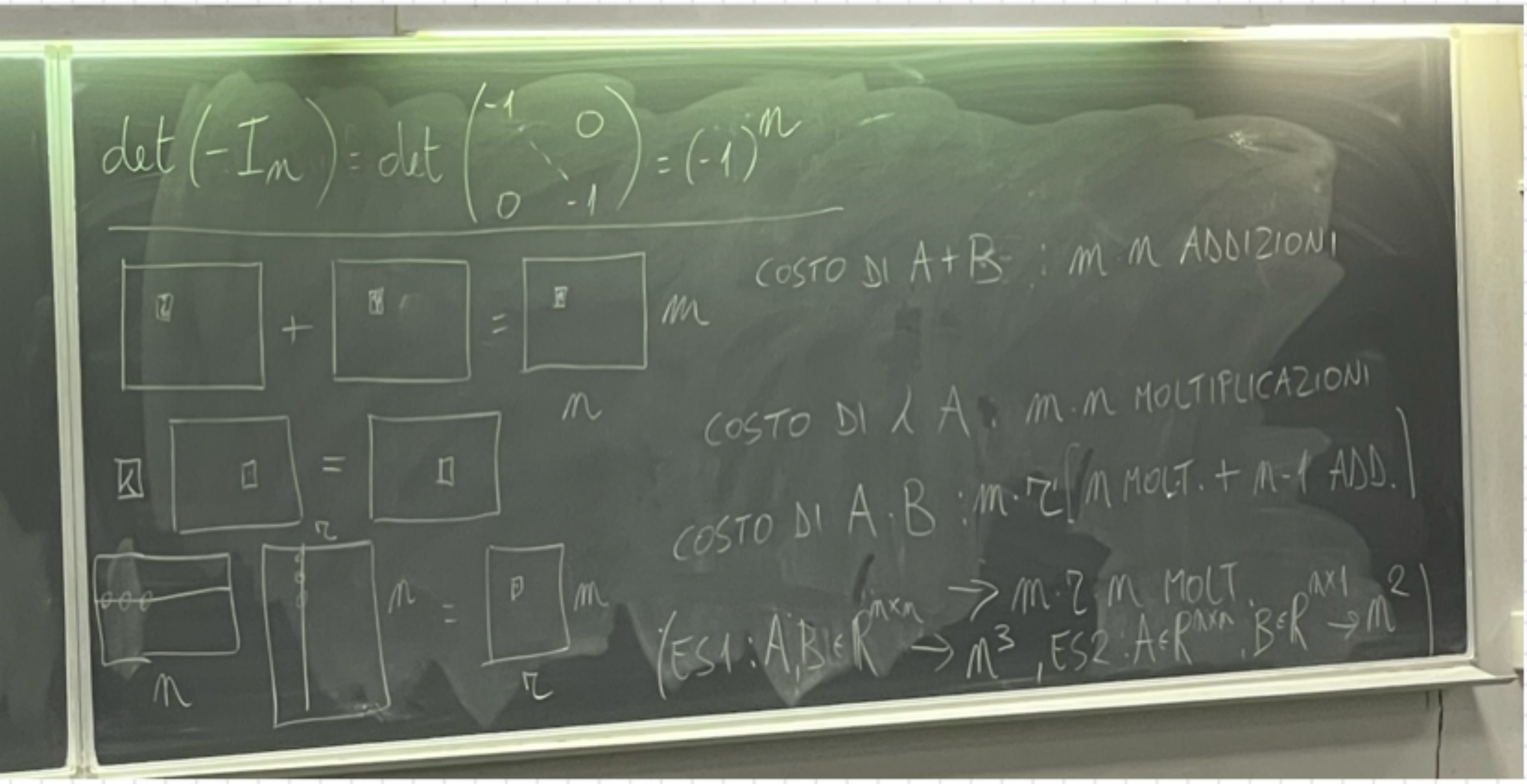
$\exists$ METODO DI CALCOLO DI $A^{-1}$ BASATO SU det

(NON EFFICIENTE)

<u>CASO $n=2$</u> :

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{\underbrace{ad - bc}_{\det\begin{pmatrix} a & b \\ c & d \end{pmatrix}}} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

SUPPONIAMO DI AVERE 2 MATRICI E DI VOLERLE SOMMARE:




$C_n$ = COSTO DI DET $n \times n$ (CON LAPLACE)

$C_1 = 0 \qquad C_2 = 2 \qquad C_n = n\, C_{n-1} + n > n \cdot C_{n-1}$

$$\Rightarrow C_n > n!$$

## SISTEMI LINEARI

ESEMPIO

$$\begin{cases} x + y = 2 \\ 1001x + 1000y = 2001 \end{cases} \rightsquigarrow A = \begin{pmatrix} 1 & -1 \\ 1001 & 1000 \end{pmatrix}$$

$\|$

$$b = \begin{pmatrix} 2 \\ 2001 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \end{pmatrix}$$

IN GENERALE

$$\begin{cases} a_{11}x_1 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + \dots + a_{mn}x_n = b_m \end{cases}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & \dots & \dots & a_{2n} \\ a_{m1} & \dots & \dots & a_{mn} \end{pmatrix} \in \mathbb{R}^{m \times n}$$

MATRICE DEI COEFFICIENTI

(MATRICE INCOMPLETA)

$$b = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix} \in \mathbb{R}^{m \times 1}$$

VETTORE TERMINI NOTI

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} \in \mathbb{R}^{m \times 1}$$

VETTORE INCOGNITE

$$A \cdot x = b$$

ESEMPIO 2

$$\begin{cases} 2x_1 - x_2 + x_4 = 5 \\ x_2 + x_3 - 2x_4 = 0 \end{cases}$$

$$b = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \qquad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$m = 2$ $\qquad$ $m = 4$

$$A = \begin{pmatrix} 2 & -1 & 0 & 1 \\ 0 & 1 & 1 & -2 \end{pmatrix} \in \mathbb{R}^{2 \times 4}$$

ESEMPIO 3

$$\begin{cases} x_1 - x_3 = 0 \\ x_2 + 2x_3 = 0 \end{cases} \qquad b = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad m = 2$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad n = 3$$

$Ax = b$ SISTEMA OMOGENEO

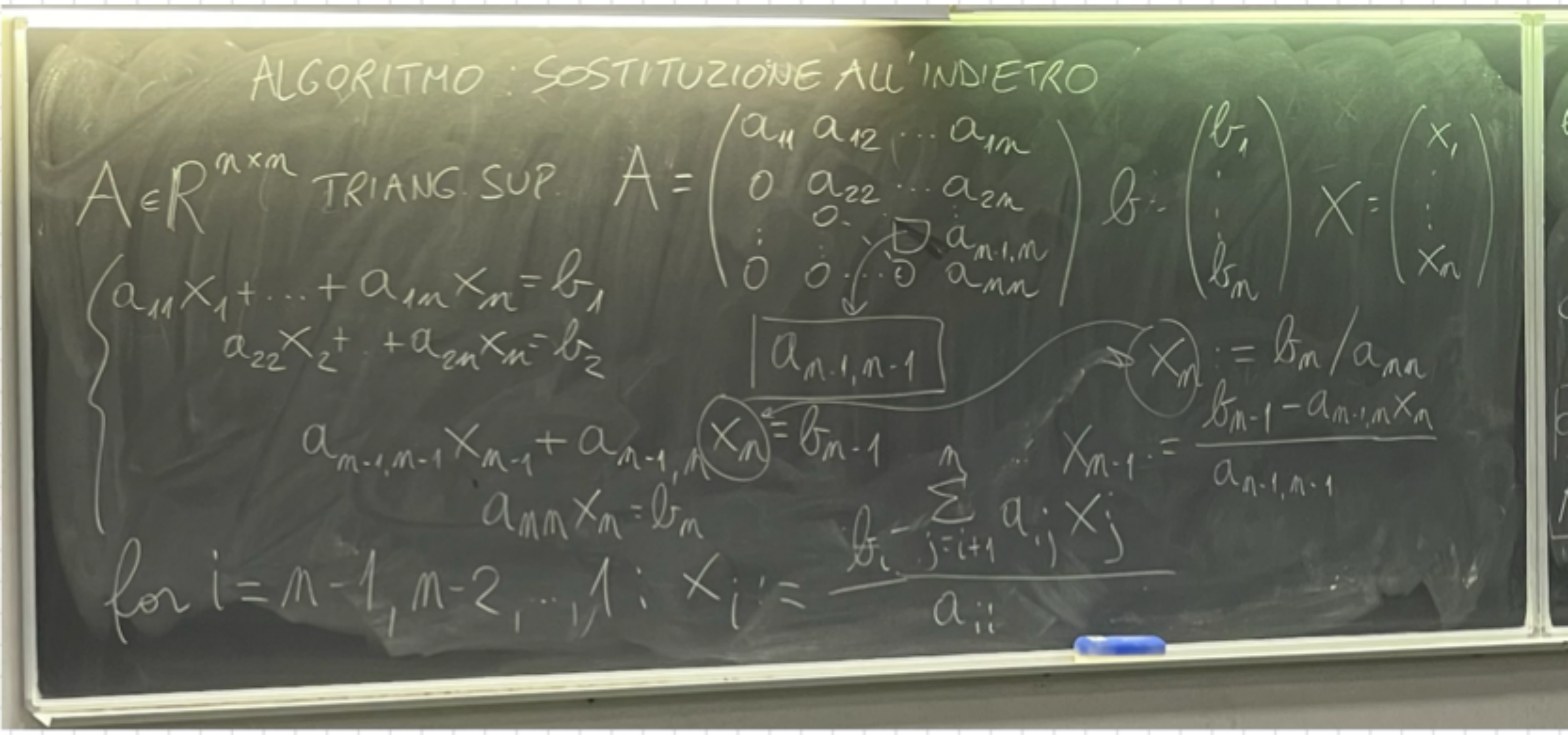
$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix} \qquad Ax = 0$$

CASO $m = n$ : $\underset{m \times n}{A} X = \underset{m \times 1}{b}$

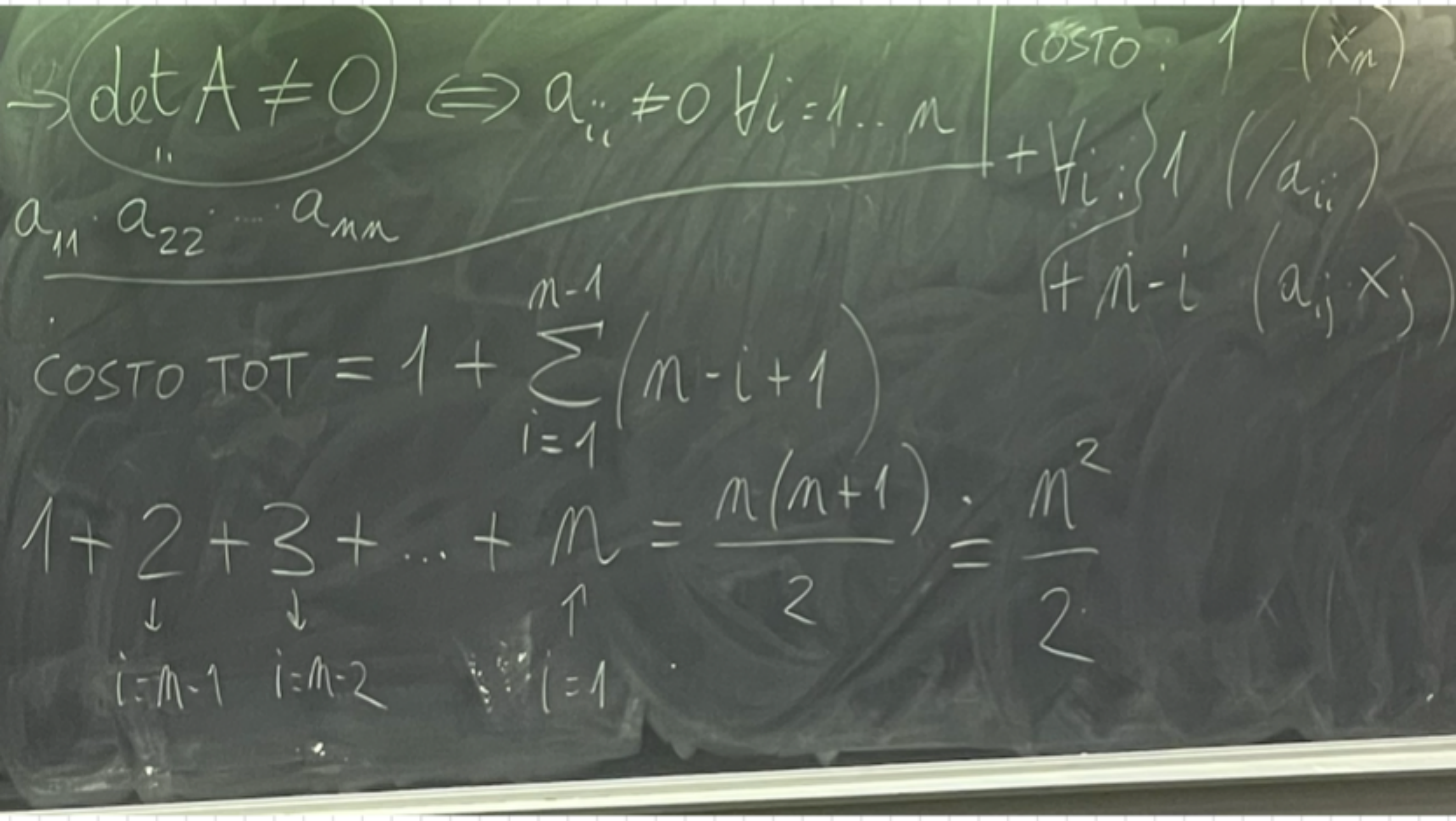
$\det A \neq 0 \Rightarrow \exists A^{-1}$

$A^{-1} A X = A^{-1} b$

ALGORITMO SOSTITUZIONE ALL'INDIETRO




$\det A \neq 0 \iff a_{ii} \neq 0 \quad \forall i = 1$



# MATRICI ELEMENTARI