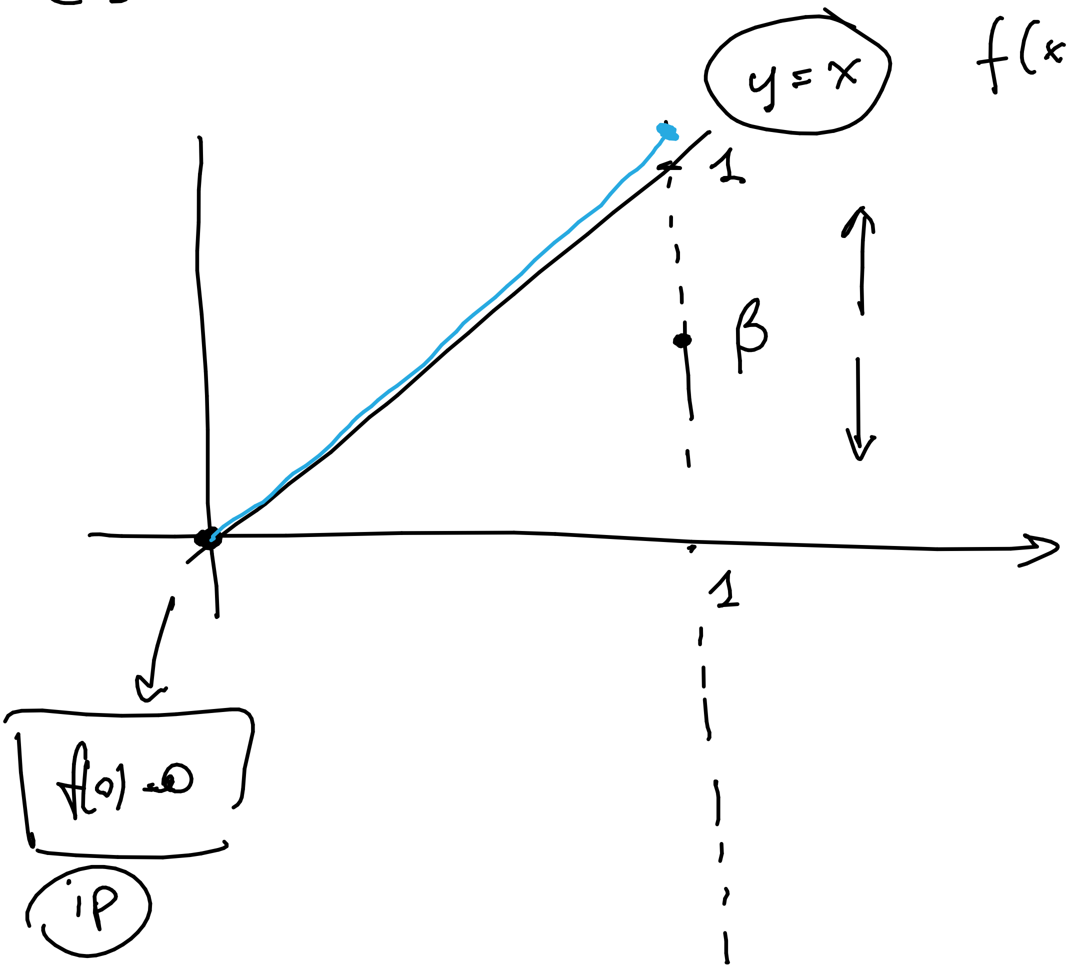
# ES 2 TUTORATO.

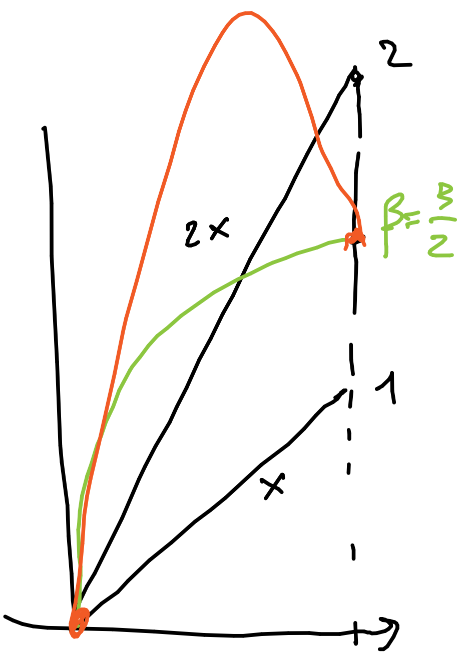
$f(x) = x \qquad f'(x) = 1$




$1 < f'(x) \le 2 \qquad x \in [0,1]$

$\boxed{\beta > 1}$

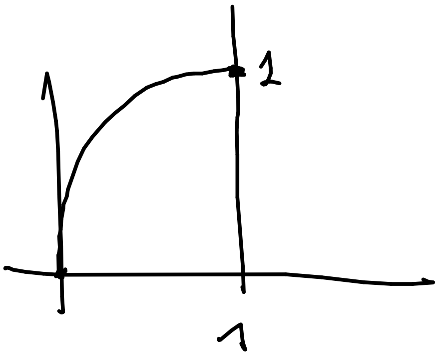
$f'(x) \le (2x)' = 2$




$$\underbrace{(x)'}_{=1} < f'(x) \le \underbrace{(2x)'}_{=2}$$

$1 < \beta = f(1) \le 2$

$f(x) = \sqrt{x}$ $\qquad f: [0,1] \to \mathbb{R}$




$\beta = 1$ NON È NEL NOSTRO INTERESSE

$f'(x) = \frac{1}{2\sqrt{x}}$ $\qquad f': (0,1] \to \mathbb{R}$

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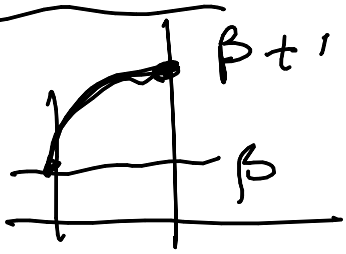
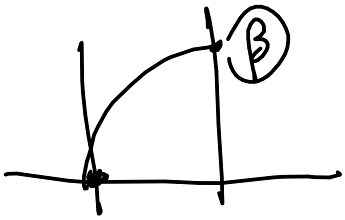
$\sqrt{x} + \beta$ trasla




$\beta\sqrt{x}$ dilata




$$\boxed{f(x) = \beta\sqrt{x}}$$

$1 < \beta \leq 2$

verifica i, ii. $\longrightarrow$ $f(0) = 0 \qquad f(1) = \beta$

NON VERIFICA iii

$f'(x) > 1$ in un intorno di $x = 0$

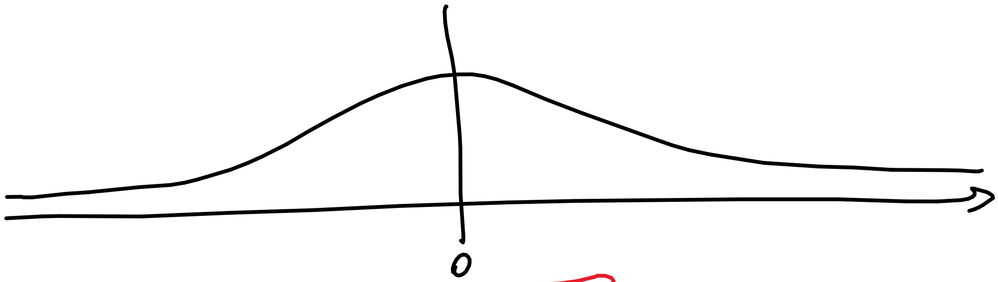
e in $x = 0$ NON È DEFINITA.

Foglio 8

Es: f)

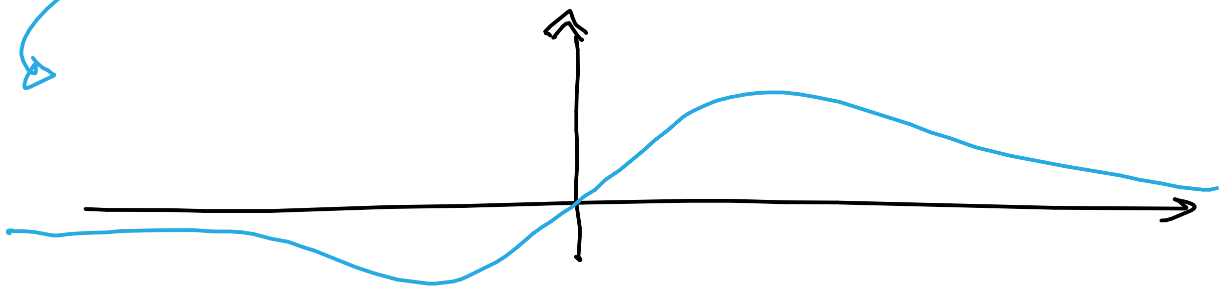
$$f(x) = (x - 2|x|)\, e^{-x^2}$$

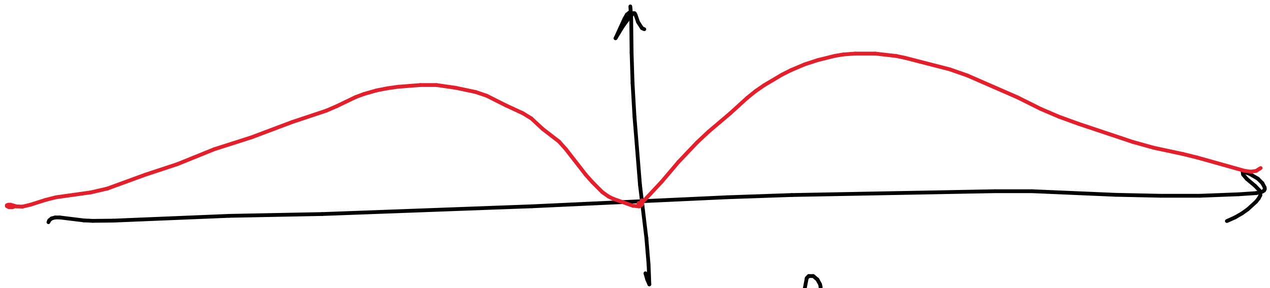
$e^{-x^2}$





$$f(x) = \boxed{x\, e^{-x^2}} - \boxed{2|x|\, e^{-x^2}}$$





$$f(x) = \begin{cases} (x - 2x)\, e^{-x^2} & x \geq 0 \\ (x + 2x)\, e^{-x^2} & x < 0 \end{cases}$$

$$f(x) = \begin{cases} -x\, e^{-x^2} & x \geq 0 \\ 3x\, e^{-x^2} & x < 0 \end{cases}$$

Per $x \geq 0$

$$f(x) = -x e^{-x^2}$$

segno: $x > 0$ sempre $\forall x \geq 0$

$e^{-x^2} > 0$ " $\forall x \geq 0$

} dominio che consideriamo

$$f(x) < 0 \quad \forall \; x \geq 0$$

$$\lim_{x \to +\infty} -x \underbrace{e^{-x^2}}_{0} = \lim_{x \to +\infty} -\frac{x}{e^{x^2}} \overset{x^2 = z}{=} \lim_{z \to +\infty} \frac{-z^{\boxed{\frac{1}{2}}}}{e^z} = 0^-$$

potenza

espon.

$z \to +\infty$ ↓ $+\infty$

$$f'(x) = -1\, e^{-x^2} - x \underbrace{(-2x)\, e^{-x^2}}_{\left(e^{-x^2}\right)'} =$$

$$= -e^{-x^2} + 2x^2 e^{-x^2} = (2x^2 - 1)\, e^{-x^2}$$

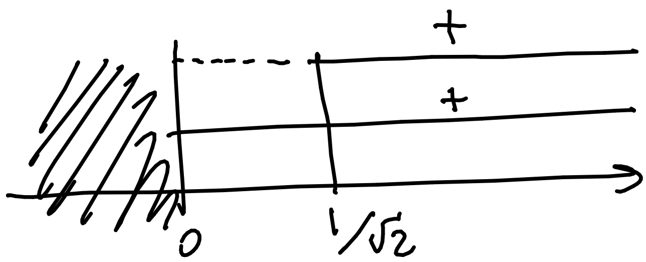
(sempre per $x \geq 0$)

Consideriamo solo $x \geq 0$

Positività di $f'(x)$:

$2x^2 - 1 > 0 \qquad 2x^2 > 1 \qquad x^2 > \frac{1}{2} \qquad x > \frac{1}{\sqrt{2}}$

$e^{-x^2} > 0 \quad \underline{\forall x \geq 0}$



Minimo in $x_M = \frac{1}{\sqrt{2}}$

valore: $f(x_M) = -\frac{1}{\sqrt{2}} \boxed{e^{-\frac{1}{2}}} < 0$

$\frac{1}{\sqrt{e}}$

$e \simeq 2.7 \qquad \frac{5}{2} < e < 3$

$\sqrt{e}$ : $\sqrt{\frac{5}{2}} < \sqrt{e} < \sqrt{3}$

$\frac{1}{\sqrt{e}}$ : $\sqrt{\frac{2}{5}} > \frac{1}{\sqrt{e}} > \frac{1}{\sqrt{3}}$ $\quad -\frac{1}{\sqrt{2}}$

$$-\frac{1}{\sqrt{5}} < f(x_M) < -\frac{1}{\sqrt{6}}$$

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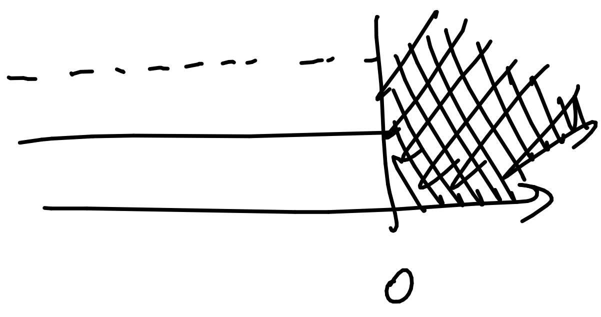
$\boxed{x < 0}$

$$f(x) = 3x\,e^{-x^2} \qquad \underline{< 0 \quad \forall x < 0}$$

Segno: $f(x) > 0$

$x > 0$ $\quad$ mai

$e^{-x^2} > 0$ $\quad$ sempre



$$\lim_{x\to-\infty} 3x\, e^{-x^2} = 0^-$$

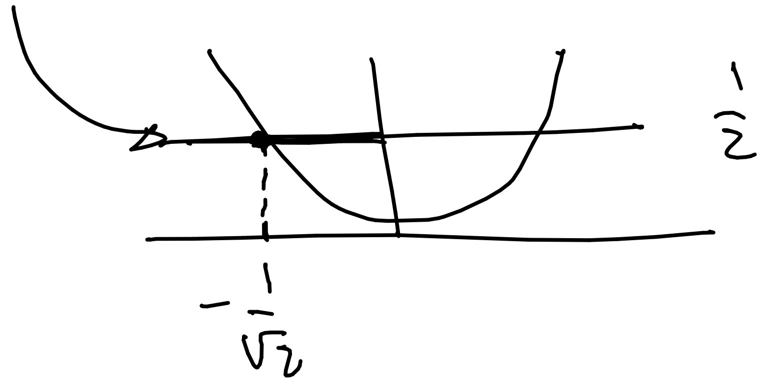
$$f'(x) = 3e^{-x^2} + 3x(-2x)\,e^{-x^2}$$

$$= (3-6x^2)\,e^{-x^2} = 3(1-2x^2)\,e^{-x^2}$$

Positività di $f'(x)$. quando $x<0$

dipende da $1-2x^2>0$

$2x^2<1$ $\quad x^2 < \frac{1}{2}$ $\quad x > -\frac{1}{\sqrt{2}}$




$f$ è crescente per $-\frac{1}{\sqrt{2}} < x < 0$

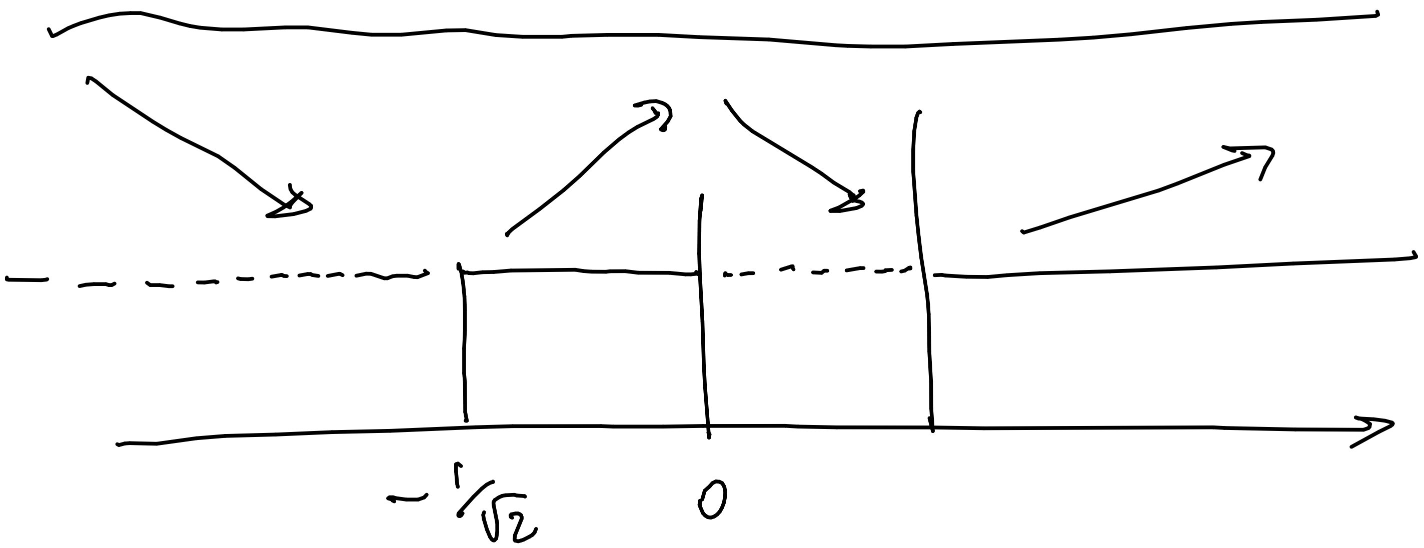
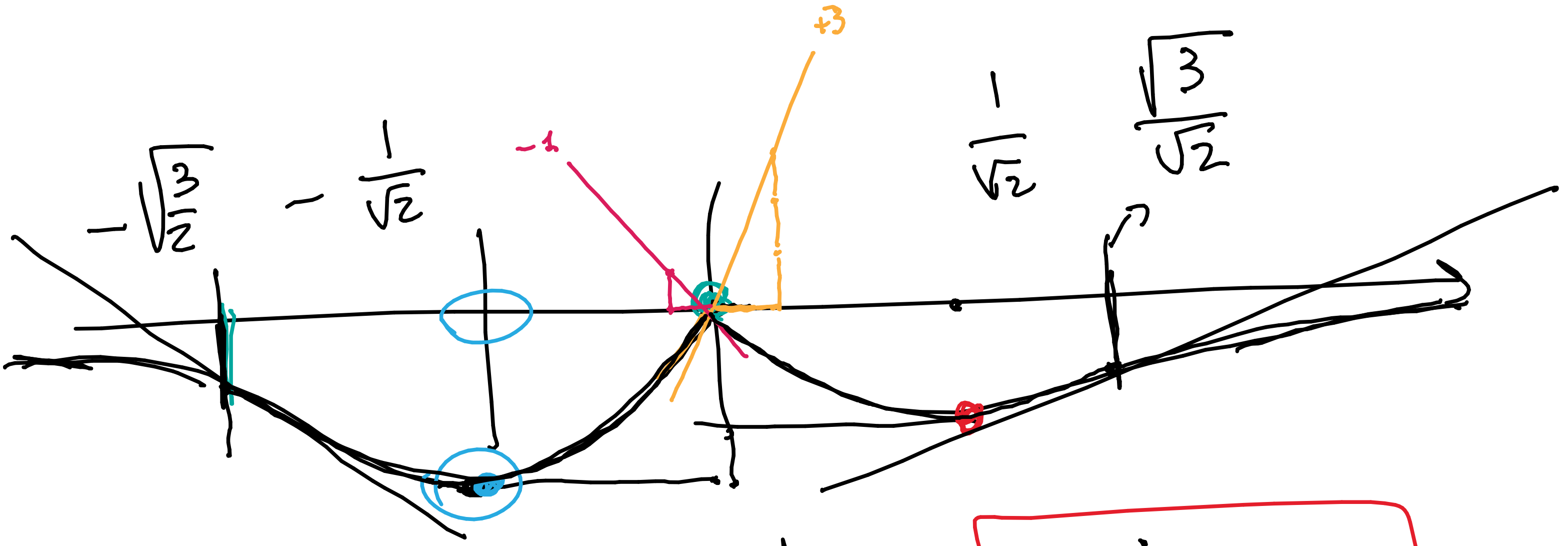


grafico :




$$f\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}\, e^{-\frac{1}{2}} \quad \leftarrow \quad \boxed{\text{minimo relativo}}$$

$$f\left(-\frac{1}{\sqrt{2}}\right) = -\frac{3}{\sqrt{2}}\, e^{-\frac{1}{2}} \quad \leftarrow \quad \boxed{\text{minimo assoluto.}}$$

$$f(0) = 0 \quad \boxed{\text{massimo assoluto}}$$

$$f'(x) = \begin{cases} 3(1-2x^2)\, e^{-x^2} & x<0 \\ (2x^2-1)\, e^{-x^2} & x>0 \end{cases} \quad \boxed{\begin{matrix} ? \\ x=0 \end{matrix}}$$

$$\lim_{x\to 0^-} 3\underbrace{(1-2x^2)}_{1}\underbrace{e^{-x^2}}_{\to 1}$$

$$\lim_{x\to 0^+} \underbrace{(2x^2-1)}_{-1}\underbrace{e^{-x^2}}_{\to 1}$$

$$\left.\begin{aligned}\lim_{x\to 0^-} f'(x) &= 3\\ \lim_{x\to 0^+} f'(x) &= -1\end{aligned}\right] \neq$$

$f$ non è derivabile in $x=0$

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È superiormente limitata $f(x) \le 0 \quad \forall x \in \mathbb{R}$

È inferiormente limitata $f(x) \ge f\left(-\frac{1}{\sqrt{2}}\right) \quad \forall x \in \mathbb{R}$

$\mathrm{Im}(f) = \left[ f\left(-\frac{1}{\sqrt{2}}\right), 0 \right]$ per il teo di valori intermedi

$\boxed{x>0}$

$$f(x) = -x\,e^{-x^2}$$

$$f'(x) = (2x^2-1)\,e^{-x^2}$$

$$f''(x) = 4x\,e^{-x^2} + \underbrace{(2x^2-1)(-2x)}\,e^{-x^2}$$

$$= e^{-x^2}\left(-4x^3+2x+4x\right)$$

$$= 2e^{-x^2}\left(-2x^3+3x\right)$$

$$= 2x\left(3-2x^2\right)e^{-x^2}$$

$$f''(x)>0 \quad \begin{cases} 3-2x^2>0 & x^2<\frac{3}{2} \\ x>0 & \text{sempre.} \end{cases} \quad x<\frac{\sqrt{3}}{\sqrt{2}}$$

Per $x < \sqrt{\frac{3}{2}}$ $f$ è convessa

$x > \sqrt{\frac{3}{2}}$ $f$ è concava

$\boxed{x<0}$

$$f'(x) = 3(1-2x^2)e^{-x^2}$$

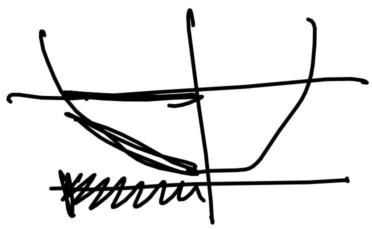
$$f''(x) = 3\left[(-4x)e^{-x^2} + (1-2x^2)(-2x)e^{-x^2}\right]$$

$$= 3e^{-x^2}(-2x)\left[2+1-2x^2\right]$$

$$= -6x\underbrace{(3-2x^2)}e^{-x^2}$$

$3-2x^2>0$ $\qquad$ $f''(x)>0$

$x^2<\frac{3}{2}$ $\qquad$ $(x<0)$



$x > -\frac{\sqrt{3}}{\sqrt{2}}$

$x > -\sqrt{\frac{3}{2}}$

$f$ è convessa $\quad x > -\sqrt{\frac{3}{2}}$

Calcolo del flesso.

$$f''(x_0) = 0$$

$$\downarrow$$

$$\underbrace{f'(x_0)} \qquad \underbrace{y - f(x_0) = f'(x_0)(x - x_0)}$$

retta tg.

nel pt di flesso $x_0$