FOGLIO 5

Es 1 (4) $f(x) = \dfrac{\sqrt{|x-1|} - \sqrt{|x+1|}}{x-1}$

① $\begin{cases} |x-1| \geq 0 \\ |x+1| \geq 0 \\ x-1 \neq 0 \end{cases} \rightarrow \begin{cases} \forall x \in \mathbb{R} \\ \forall x \in \mathbb{R} \\ x \neq 1 \end{cases}$

$\text{dom} f = (-\infty, 1) \cup (1, +\infty)$

② • $\lim\limits_{x \to -\infty} \dfrac{\sqrt{|x-1|} - \sqrt{|x+1|}}{x-1} = \dfrac{\sqrt{+\infty} - \sqrt{+\infty}}{-\infty} = \dfrac{+\infty - \infty}{-\infty}$

$= \lim\limits_{x \to -\infty} \dfrac{(\overset{A}{\sqrt{|x-1|}} \overset{-}{-} \overset{B}{\sqrt{|x+1|}})(\overset{A}{\sqrt{|x-1|}} \overset{+}{+} \overset{B}{\sqrt{|x+1|}})}{(x-1)(\sqrt{|x-1|} + \sqrt{|x+1|})} =$

$= \lim\limits_{x \to -\infty} \dfrac{|x-1| - |x+1|}{(x-1)(\sqrt{|x-1|} + \sqrt{|x+1|})} = \lim\limits_{x \to -\infty} \dfrac{1 - \cancel{x} - (-1 - \cancel{x})}{(x-1)(\sqrt{1-x} + \sqrt{-1-x})}$

$|x-1| = \begin{cases} x-1 & \text{se } x \geq 1 \\ 1-x & \text{se } x < 1 \end{cases}$

$|x+1| = \begin{cases} x+1 & \text{se } x \geq -1 \\ -1-x & \text{se } x < -1 \end{cases}$

$\lim\limits_{x \to -\infty} \dfrac{2}{(x-1)(\sqrt{1-x} + \sqrt{-1-x})} = \dfrac{2}{(-\infty)(+\infty + \infty)} = \dfrac{2}{-\infty} = 0$

• $\lim\limits_{x \to +\infty} \dfrac{|x-1| - |x+1|}{(x-1)(\sqrt{|x-1|} + \sqrt{|x+1|})} = \lim\limits_{x \to +\infty} \dfrac{x-1 - (x+1)}{(x-1)(\sqrt{|x-1|} + \sqrt{|x+1|})}$

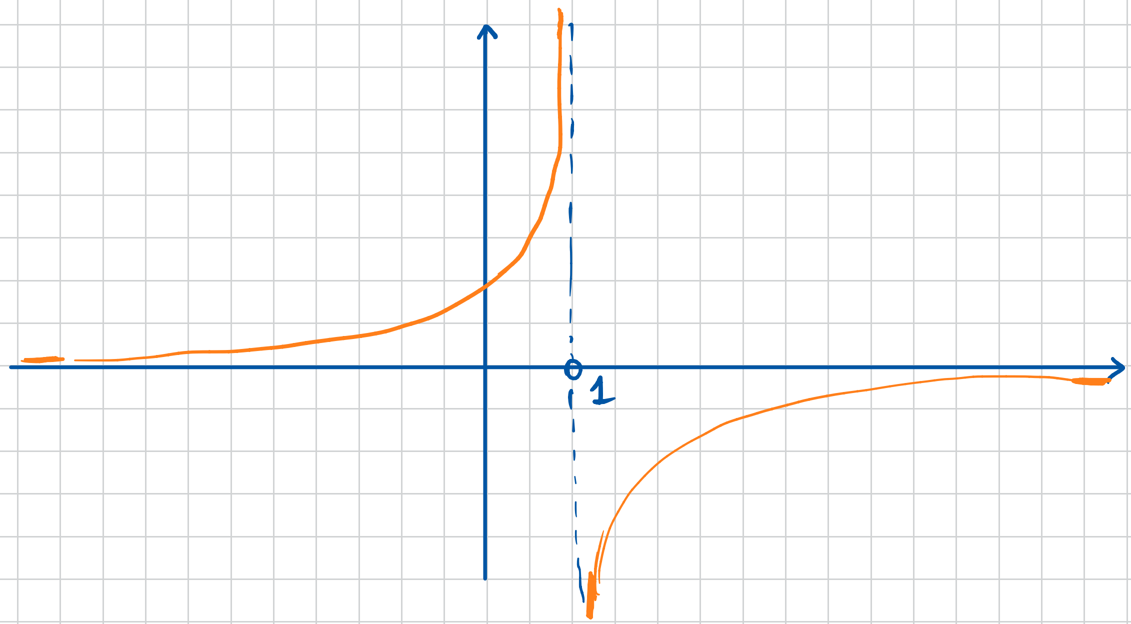
$$= \lim_{x\to+\infty} \frac{-2}{\underbrace{(x-1)}_{+\infty}\left(\underbrace{\sqrt{|x-1|}}_{+\infty}+\underbrace{\sqrt{|x+1|}}_{+\infty}\right)} = -\frac{2}{+\infty} = 0$$

$$\lim_{x\to1^+} \frac{\sqrt{|x-1|}-\sqrt{|x+1|}}{x-1} = \lim_{x\to1^+} \frac{\sqrt{x-1}-\sqrt{x+1}}{x-1} =$$

$$= \lim_{x\to1^+} \frac{\sqrt{0^+}-\sqrt{2}}{0^+} = \frac{-\sqrt{2}}{0^+} = -\infty$$

$$\lim_{x\to1^-} \frac{\sqrt{|x-1|}-\sqrt{|x+1|}}{x-1} = \lim_{x\to1^-} \frac{\sqrt{1-x}-\sqrt{2}}{x-1} = \frac{-\sqrt{2}}{0^-}$$

$$= +\infty$$



PROLUNGABILE in $x = 1$?

1 Svolgo il $\lim_{x\to1^\pm} f(x)$

2 Osservo che $\lim_{x\to1^\pm} f(x) = \pm\infty \leftarrow$ non è un numero

allora f non è prolungabile per continuità.

(2) $f(x) = \arctan\left(\frac{1}{x}\right) - \frac{\pi}{2}\cdot\frac{x}{|x|}$

dominio : $x \neq 0$ $\quad$ $\operatorname{dom} f = (-\infty, 0) \cup (0, +\infty)$

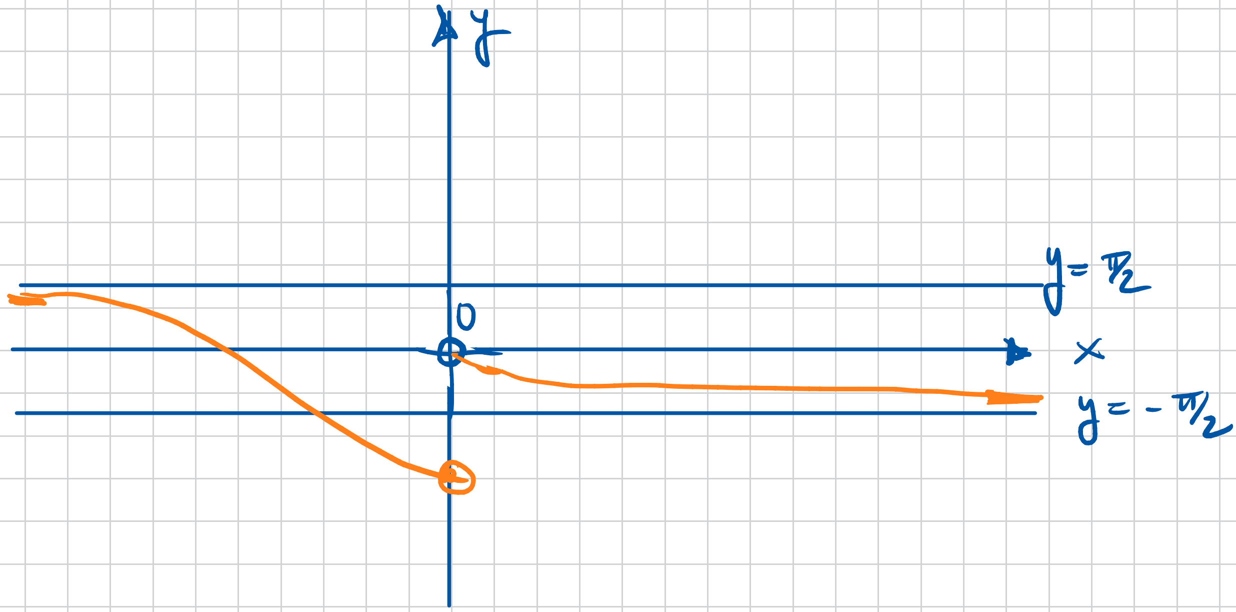
$$f(x) = \begin{cases} \arctan\frac{1}{x} - \frac{\pi}{2}\cdot\frac{\cancel{x}}{\cancel{x}} & \text{se } x > 0 \\ \arctan\frac{1}{x} - \frac{\pi}{2}\cdot\frac{x}{(-x)} & \text{se } x < 0 \end{cases}$$

$$= \begin{cases} \arctan\frac{1}{x} - \pi/2 & \text{se } x > 0 \\ \arctan\frac{1}{x} + \pi/2 & \text{se } x < 0 \end{cases}$$

$$\lim_{x \to \pm\infty} \arctan\frac{1}{x} \mp \pi/2 = 0 \mp \pi/2 \quad \begin{cases} y = -\pi/2 \\ y = +\pi/2 \end{cases}$$

$$\lim_{x \to 0^\pm} \arctan\frac{1}{x} \mp \pi/2 = \begin{cases} \pi/2 - \pi/2 & \text{se } x \to 0^+ \\ -\pi/2 - \pi/2 & \text{se } x \to 0^- \end{cases}$$

$$= \begin{cases} 0 & \text{se } x \to 0^+ \\ -\pi & \text{se } x \to 0^- \end{cases}$$



f non è prolungabile per continuità

Es 2 $\qquad f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{se } x>0 \\ x^2+a & \text{se } x\le 0 \end{cases}$

f continua in $x=0 \iff \lim_{x\to 0} f(x) = f(0)$

$f(0) = 0^2 + a = a$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \dfrac{\sin x}{x} = 1$ (limite notevole)

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2+a = a$

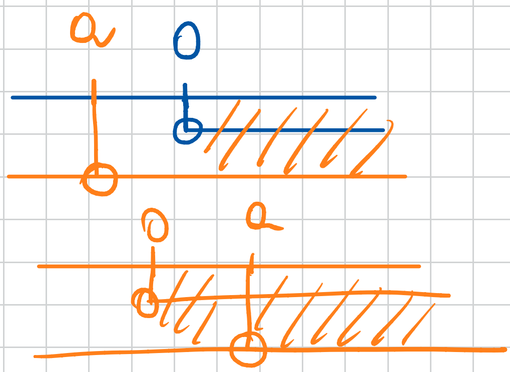
f è continua in $x=0 \iff \lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0)$

$\iff 1 = a \iff a=1$

Quindi f è continua in $x=0 \iff a=1$

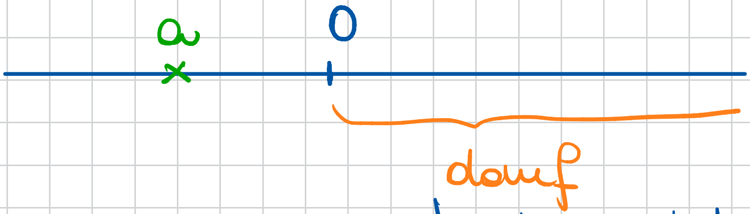
(6) $\quad f(x) = \dfrac{\ln x}{x-a}$

$\begin{cases} x>0 \\ x-a\neq 0 \end{cases} \to \begin{cases} x>0 \\ x\neq a \end{cases}$



$\text{dom}f = \begin{cases} (0,+\infty) & a\le 0 \\ (0,a)\cup(a,+\infty) & a>0 \end{cases}$

- se $a\le 0$ $\quad \text{dom}f = (0,+\infty)$ e f è continua in $(0,+\infty)$ perché quoziente di funzioni continue in dom$f$



se $a<0$ $\quad x = a$ punto isolato

se $a=0$ $\quad x = 0$ punto di accumulazione

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\ln x}{x-0} = \lim_{x \to 0^+} \frac{\ln x}{x} = -\infty$$

$\Rightarrow$ f non può essere estesa per continuità in $x=0$

<u>Se $a>0$</u>

$$\lim_{x \to a} f(x) = \lim_{x \to a^\pm} \frac{\ln x}{x-a} = \frac{\ln a}{0^\pm} = \infty$$

allora $\nexists\, a>0$ t.c. f sia prolungabile per continuità in $x=a$.