ESERCIZIO 9

1) $$\lim_{x \to -\infty} \underset{\substack{\downarrow\\+\infty}}{\boxed{x^4}} + \underset{\substack{\downarrow\\-\infty}}{\boxed{x^3}} - \underset{\substack{\downarrow\\-\infty}}{\boxed{x}} = \underbrace{+\infty}_{\substack{\downarrow\\4}} \underbrace{-\infty}_{3} \underbrace{+\infty}_{1}$$

$$\boxed{= +\infty}$$

$x = -10$

$$\left.\begin{array}{l} x^4 = 10^4 = 10000 \\ -x = 10 \end{array}\right\} 10010$$

$x = -100 \longrightarrow \longrightarrow$

$x = -100$

$x^4 = (100)^4 = 100'000'000$

$-x = +100$

$x^4 - x = \boxed{100'000'100}$

$x^3 = -(100)^3 = \boxed{-1'000'000}$

$$\lim_{x \to +\infty} \frac{3\overset{\to +\infty}{x^2}+1}{\underset{\to +\infty}{x^2}-1} = \frac{+\infty}{+\infty} = \boxed{F.\ I.}$$

velocità

Moltiplico fattori che $\to +\infty$ $\Rightarrow$ rafforzo l'infinito

Es $x^4 \cdot x^3 = x^7$ $\quad x^{4+3} = x^7$

Divido fattori che $\to +\infty$ $\Rightarrow$ indebolisce l'infinito

rallento

Es $x^4 / x^3 = x^{4-3} = x^1$

$$\lim_{x \to +\infty} \frac{\cancel{x^2}\left(3+\underbrace{\frac{1}{x^2}}_{\to 0}\right)}{\cancel{x^2}\left(1-\underbrace{\frac{1}{x^2}}_{\to 0}\right)} = \frac{3}{1} = 3$$

$$\lim_{x\to+\infty}\frac{x}{3x^2+1}=\lim_{x\to+\infty}\frac{\cancel{x}\;1}{\cancel{x^2}\left(3+\underbrace{\frac{1}{x^2}}_{\to 0}\right)}=\frac{1}{\infty}=0$$

$$\boxed{\frac{x}{x^2}=x^{1-2}=x^{-1}=\frac{1}{x}}$$

$\boxed{x=10}$

$x^2=100$

$$\frac{x}{x^2}=\frac{10}{100}=\boxed{0.1}$$

$\boxed{x=100}$

$x^2=10'000$

$$\frac{x}{x^2}=\frac{1\cancel{0}\cancel{0}}{10'0\cancel{0}\cancel{0}}=\boxed{0.01}$$

$$\lim_{x \to +\infty} \frac{x^{5} + 3x^{2} + 3x + 1}{2x^{2} - x} = \left(\sim \boxed{x^{3}} \right)$$

$$\lim_{x \to +\infty} \frac{x^{\overset{3}{\cancel{5}}} \left(1 + \overset{\to 0}{\boxed{3/x^{3}}} + \overset{\to 0}{\boxed{3/x^{4}}} + \overset{\to 0}{\boxed{1/x^{5}}} \right)}{\cancel{x^{2}} \left(2 - \underset{\to 0}{\boxed{\frac{1}{x}}} \right)} = +\infty$$

$$\lim_{x \to -\infty} \frac{x^{3} \left(\boxed{1 +} \boxed{3/x^{3}} + \boxed{3/x^{4}} + \boxed{\frac{1}{x^{5}}} \right)}{\left(\boxed{2} - \boxed{\frac{1}{x}} \right)} = -\infty$$

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x^2} = \frac{1-1}{0} = \frac{0}{0}$$

$$(a \ominus b)(a \oplus b) = a^2 \ominus b^2 \quad \leftarrow$$

$$\lim_{x \to 0} \frac{\sqrt{1+x} \ominus 1}{x^2} \frac{(\sqrt{1+x} \oplus 1)}{(\sqrt{1+x} + 1)} =$$

$|x| < \delta < 1$

$$\Bigg\| \quad \lim_{x \to 0} \frac{(\cancel{(1} + x) \ominus \cancel{1})}{x^2} \frac{1}{\sqrt{1+x} + 1}$$

$$\lim_{x\to 0^+} \frac{\cancel{x}}{\underset{\downarrow\ 0^+}{x^{\cancel{2}}}} \underset{\to \frac{1}{2}}{\boxed{\frac{1}{\underbrace{\sqrt{1+x}+1}_{\to 2>0}}}} = \frac{1}{0^+} = +\infty$$

$$\lim_{x\to 0^-} \frac{1}{\underset{\uparrow\ 0^-}{\boxed{x}}} \underset{\to \frac{1}{2}>0}{\boxed{\frac{1}{\sqrt{1+x}+1}}} = \frac{1}{0^-} = -\infty$$

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x^2} \to \nexists$$

$$\frac{\sqrt{1+x}-1}{x^2} = \frac{1}{x}\,\frac{1}{\sqrt{1+x}+1}$$

10) $$\lim_{x \to +\infty} \ln(\underbrace{2x}_{+\infty}) - \ln(\underbrace{x-1}_{+\infty}) = \infty - \infty$$

$$= \lim_{x \to +\infty} \ln \frac{2x}{x-1} =$$

$$= \lim_{x \to +\infty} \ln\left[\frac{\cancel{x}(2)}{\cancel{x}\left(1-\underset{\underset{0}{\downarrow}}{\frac{1}{x}}\right)}\right] = \ln 2$$

$$\lim_{x \to +\infty} e^{\underbrace{2+x}_{\substack{\downarrow \\ +\infty}}} - e^{\underbrace{x}_{\substack{\downarrow \\ +\infty}}} = \infty - \infty = \text{F.I.}$$

$$\lim_{x \to +\infty} e^{x}\left(\underbrace{e^{2}-1}_{>3}\right) = +\infty$$

$$\lim_{x \to +\infty} e^{x-1} - e^{x} = \infty - \infty$$

$$\lim_{x \to +\infty} e^{x}\left(\underbrace{e^{-1}-1}_{<0}\right) = -\infty$$

$$\lim_{x\to-\infty}\left(1+\overset{0\nwarrow}{\frac{1}{x}}\right)^{x^2} = 1^{\infty} = \boxed{F.I.}$$

$$\lim_{x\to-\infty} e^{x^2\ln\left(1+\frac{1}{x}\right)} =$$

$$\lim_{x\to-\infty} x^2\,\underbrace{\ln\left(1+\overset{\nearrow 0}{\frac{1}{x}}\right)}_{\ln(1+z)} = \infty\cdot 0 \quad \boxed{F.I.}$$

$$z\to 0^-$$

$$\boxed{z=\frac{1}{x}}$$

CAMBIO DI
VARIABILE

$\boxed{z = \frac{1}{x}}$ $\quad x \to -\infty \quad z \longrightarrow 0^-$ $\quad \boxed{x = \frac{1}{z}}$

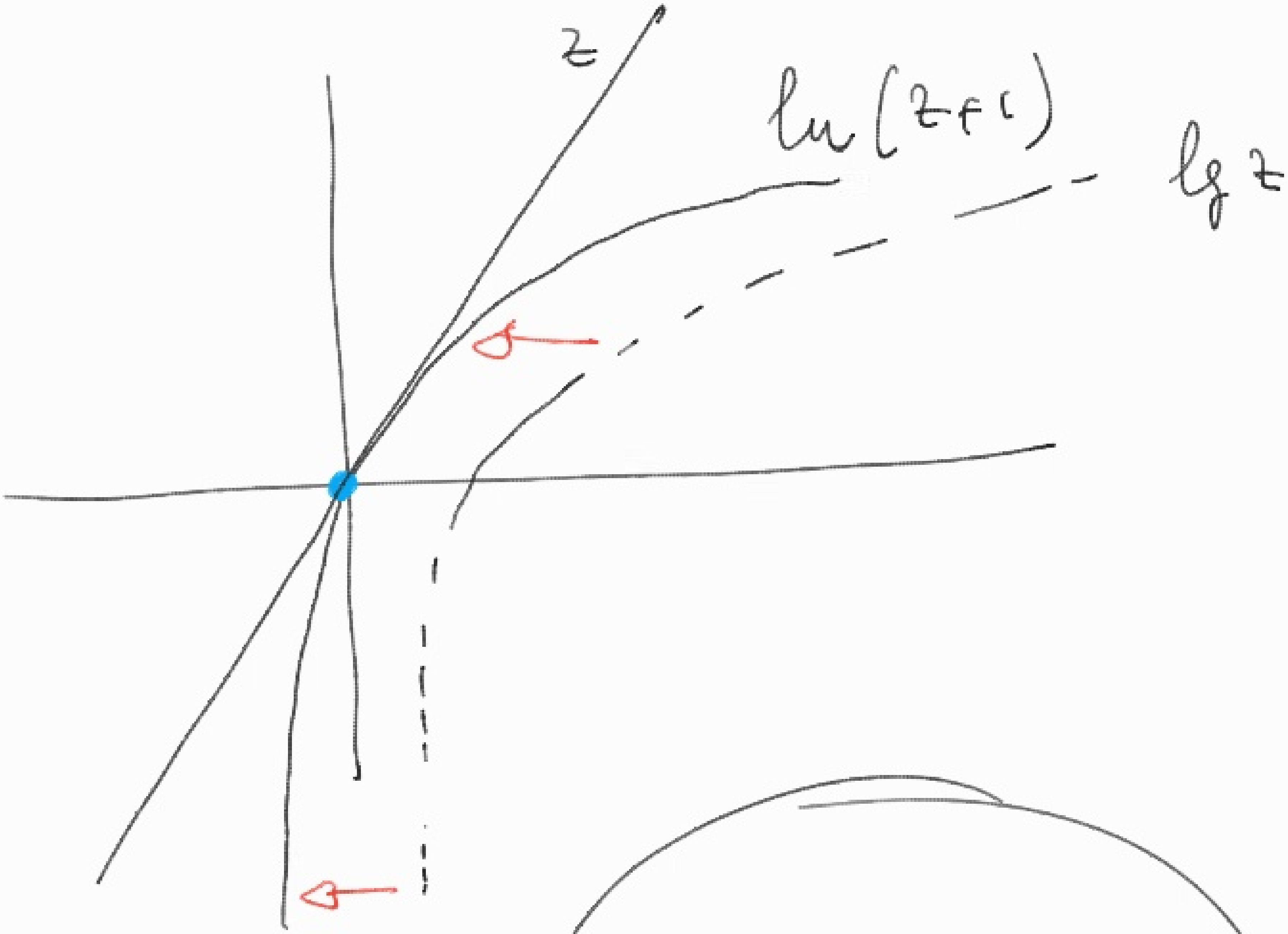
$x^2 = \frac{1}{z^2}$

$$\lim_{z \to 0^-} \boxed{\frac{1}{z^2}} \ln(1+z) =$$

$$= \lim_{z \to 0^-} \frac{1}{\underset{\downarrow\, 0^-}{\boxed{z}}} \overset{\to 1}{\boxed{\frac{\ln(1+z)}{z}}} = -\infty$$

→ <u>limite notevole</u>

$$\lim_{x \to -\infty} e^{\underbrace{x^2 \ln\left(1 + \frac{1}{x}\right)}_{-\infty}} = 0$$



$$\lim_{z\to 0}\frac{\boxed{\ln(1+z)}}{z}$$

$$\boxed{a^b} = e^{b \ln a}$$

$$e^{b \ln a} = e^{(\ln(a^b))} = a^b$$

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{x \ln x}$$

$$\lim_{x \to 0^+} \overset{0}{x} \underbrace{\ln x}_{-\infty} \qquad x = e^{-z} \quad \lim_{z \to +\infty} -e^{-z} z$$

$$\lim_{z \to +\infty} - \frac{z}{e^{z}}$$

$$\underline{\lim_{x \to +\infty} \frac{x^{\textcircled{n}}}{e^{x}} = 0} \qquad \underline{\forall n \in \mathbb{N}}$$

$$\xcancel{n \to +\infty}$$

$n = 5$

$n = 7$

$n = 10^{10^{10^{10}}}$