20/12

ESERCITAZIONE ESAME #1

$$g(x) = \alpha x^2 + \beta x + \gamma$$

x	-1	$-1/2$	0	$1/2$	1
y	7	4	-14	4	7

a)

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1/4 & -1/2 & 1 \\ 0 & 0 & 1 \\ 1/4 & 1/2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad y = \begin{bmatrix} 7 \\ 4 \\ -14 \\ 4 \\ 7 \end{bmatrix}$$

L'ORDINE DELLE COLONNE E' ARBITRARIO

b)

CALCOLARE SE POSSIBILE:

- A^2 = NON POSSIAMO VISTO CHE NON E' QUADRATA $\Rightarrow \underset{5\times 3}{A} \; \underset{5\times 3}{A}$
- $AA^T = 5\times 5$
- $A^TA = 3\times 3$
- $A^Ty = 3\times 1$
- $y^TA = 1\times 3$

$$\boxed{(A\cdot B)^T = B^T \cdot A^T}$$

$$(A^T y)^T = y^T \overbrace{(A^T)^T}^{A}$$

c) MODO 1:

$$c_1 \begin{pmatrix} \underline{a}_1 \end{pmatrix} + c_2 \begin{pmatrix} \underline{a}_2 \end{pmatrix} + c_3 \begin{pmatrix} \underline{y} \end{pmatrix} = \begin{pmatrix} \underline{0} \end{pmatrix} \qquad c_1, c_2, c_3 \in \mathbb{R}$$

$$\Downarrow$$

$$\boxed{c_1 = c_2 = c_3 = 0 \;\; \text{] UNICO}}$$

$$c_1 \begin{pmatrix} 1 \\ 1/4 \\ 0 \\ 1/4 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} -1 \\ -1/2 \\ 0 \\ 1/2 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 7 \\ 4 \\ -14 \\ -4 \\ 7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Longleftrightarrow \begin{cases} c_1 - c_2 + 7c_3 = 0 \\ 1/4\, c_1 - 1/2\, c_2 + 4c_3 = 0 \\ 0c_1 + 0c_2 - 14c_3 = 0 \\ 1/4\, c_1 + 1/2\, c_2 - 4c_3 = 0 \\ c_1 + c_2 + 7c_3 = 0 \end{cases}$$

$$\Rightarrow \quad c_1 = 0, \quad c_2 = 0, \quad c_3 = 0$$

<u>MODO 2:</u> GAUSS $\Rightarrow$

$$\left(\begin{array}{ccc|c} 1 & -1 & 7 & 0 \\ 1/4 & -1/2 & 4 & 0 \\ 0 & 0 & -14 & 0 \\ 1/4 & 1/2 & -4 & 0 \\ 1 & 1 & 7 & 0 \end{array}\right)$$

<u>MODO 3:</u> TROVARE SOTTOMATRICE 3×3 CON $\det \neq 0$

d) $\bar y$ È UN VETTORE DA TROVARE $\bar y \neq \underline{0}$ ORTOGONALE AD $\bar a_1$ E $\bar a_2$

$$\boxed{\underline{a}_1 \perp \bar y} \quad \text{E} \quad \boxed{\underline{a}_2 \perp \bar y}$$

$$\Updownarrow \qquad\qquad\qquad \Updownarrow$$

$$\boxed{\bar a_1^T \bar y = 0} \qquad \boxed{\bar a_2^T \bar y = 0}$$

$$\langle \underline{a}_1, \bar y \rangle = 0 \qquad \langle \underline{a}_2, \bar y \rangle = 0$$

$$\begin{array}{c} \bar a_1^T \\ \bar a_1^T \end{array} \begin{pmatrix} 1 & 1/4 & 0 & 1/4 & 1 \\ -1 & -1/2 & 0 & 1/2 & 1 \end{pmatrix} \begin{pmatrix} \bar y_1 \\ \bar y_2 \\ \bar y_3 \\ \bar y_4 \\ \bar y_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

POSSO SCEGLIERE IL VETTORE

$$\bar y = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} \bar y_1 + 1/4\, \bar y_2 + 1/4\, \bar y_4 + \bar y_5 = 0 \\ -\bar y_1 - 1/2\, \bar y_2 + 1/2\, \bar y_4 + \bar y_5 = 0 \end{cases}$$

SCEGLIAMO 1 IN QUANTO $\bar y_3$ NON COMPARE NEL SISTEMA

SIMULAZIONE #2

d VARIANTE) TROVARE $\bar{y}$ DIPENDENTE DA $\underline{a}_1$ E $\underline{a}_2$ E DIVERSO DA

$$\bar{y} = c_1 \underline{a}_1 + c_2 \underline{a}_2 \quad , \quad c_1, c_2 \in \mathbb{R}$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \neq \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

UNA SOLUZIONE POTREBBE ESSERE:

$$\bar{y} = \underline{a}_1 + \underline{a}_2$$

e SIMULAZIONE #1)

$$A^T A \underline{x} = A^T y \qquad \underline{x} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$$\begin{pmatrix} 17/8 & 0 & 5/2 \\ 0 & 5/2 & 0 \\ 5/2 & 0 & 5 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 14 \\ -4 \\ 0 \end{pmatrix} \qquad \begin{cases} 17/8\,\alpha + 5/2\,\gamma = 14 \\ 5/2\,\beta = -4 \\ 5/2\,\alpha + 5\gamma = 0 \end{cases} \Rightarrow$$

$$\boxed{\beta = -\frac{8}{5}} \qquad \begin{cases} 17/8\,\alpha + 5/2\,\gamma = 14 \\ 5/2\,\alpha + 5\gamma = 0 \end{cases} \Rightarrow \begin{cases} 17/8\,\alpha + 5/2\left(-1/2\,\alpha\right) = 14 \\ \gamma = -1/2\,\alpha \end{cases} \Rightarrow$$

$$\begin{cases} 7/8\,\alpha = 14 \\ \boxed{\gamma = -8} \end{cases} \qquad \boxed{\alpha = 16}$$

$$g(x) = \alpha x^2 + \beta x + \gamma = 16x^2 - 8/5\,x - 8$$