24/10

SISTEMI LINEARI IMPLEMENTAZIONE

$A \in \mathbb{R}^{n \times n}$, $\det A \neq 0 \rightarrow \exists!\, x$

$A'x = b'$

$A = A^{(1)} \mapsto A^{(2)} \mapsto \dots \mapsto A^{(n)} = A'$

$$A^{(k)} = \begin{pmatrix} a_{11}^{(k)} & \dots & a_{1n}^{(k)} \\ \vdots & a_{kk}^{(k)} & \vdots \\ 0 & a_{k+1,k}^{(k+1)} & \vdots \\ & \vdots & \vdots \\ & a_{n,k}^{(k)} & \vdots \end{pmatrix}$$

PSEUDO CODICE

for $k = 1, \dots, n-1$ $\quad \left(A^{(k)} \mapsto A^{(k+1)}\right)$

pivot $= a_{kk}^{(k)}$ (SE $\neq 0$, ALTRIMENTI $R_k \leftrightarrow R_i$)

for $i = k+1, \dots, n$: ($R_i \leftarrow R_i + \lambda R_k, \dots, \lambda = -m_{ik}$)

$m_{ik} = a_{ik}^{(k)} / \text{pivot}$

for $j = k+1, \dots, n$: $a_{ij} = a_{ij} - m_{ik} \cdot a_{kj}^{(k)}$

$b_i^{(k+1)} = b_i - m_{ik} b_k^{(k)}$

COSTO = $\dfrac{n^2}{2}$

COSTO TOTALE DI GAUSS $= \displaystyle\sum_{k=1}^{n-1} (n - k + 2)(n-k)$

ESEMPIO

$$A = \begin{pmatrix} \varepsilon & 1 \\ 1 & 0 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad x = \begin{pmatrix} 1 \\ 1-\varepsilon \end{pmatrix} \qquad \varepsilon > 0$$

piccolo

$$m_{21} = 1/\varepsilon \qquad R_2 \leftarrow R_2 - \frac{1}{\varepsilon} R_1$$

$$A^{(2)} = \begin{pmatrix} \varepsilon & 1 \\ 0 & -1/\varepsilon \end{pmatrix} \qquad b^{(2)} = \begin{pmatrix} 1 \\ 1 - 1/\varepsilon \end{pmatrix}$$