30/09/24

Errore Algoritmico

- $\varepsilon_{a \pm b} = \frac{a}{a+b}\varepsilon_a \pm \frac{b}{a+b}\varepsilon_b + \varepsilon$

- $\varepsilon_{a \cdot b} = \varepsilon_a + \varepsilon_b + \varepsilon, \quad \varepsilon_{a/b} = \varepsilon_a - \varepsilon_b + \varepsilon$

Esempio 1

$f(x) = x^2 - 7x = x(x \boxed{-} 7)$ → CANCELLAZIONE

$a := x^2, \; p = 7x$

$y_1 = a \boxed{-} p$ → CANCELLAZIONE

$d := x - 7$

$y_2 = x \cdot d$

$$\varepsilon_{ALG1} = \varepsilon_a \frac{x}{x-7} - \varepsilon_p \frac{7}{x-7} + \varepsilon_{y_1}, \quad \varepsilon_{ALG2} = \varepsilon_d + \varepsilon_{y_2}$$

$x \simeq 7 \rightarrow \varepsilon_{Alg1}$ ALTO MA $\varepsilon_{Alg1} \simeq \varepsilon_{min} \Rightarrow \varepsilon_{Alg1}$ STABILE

$$\varepsilon_{in} \simeq C_f \cdot \varepsilon_x$$

$$C_f = \frac{x f'(x)}{f(x)}$$

DEF: ALCONTINO STABILE SE $\{_{ALOG} \approx \{_{INF} \left(o \{_{ALOG} \ll \{_{INF} \right)$

ESEMPIO 2

$$\varphi(x) = 1 - \cos x$$

$$c\varphi = \frac{x \sin x}{1 - \cos x}, \quad \lim_{x \to 0} c\varphi = 2$$

ALOG 1 $\quad c = \cos x \quad \left(\{_c \right)$

$$y_1 = 1 - c \left(\{_{y_1} \right)$$

$$\{_{ALOG1} = \{_c \frac{-c}{1-c} + \{_{y_1}$$

$$\hookrightarrow -\frac{\cos x}{1 - \cos x} \xrightarrow{x \to 0} -\frac{1}{0^+} = -\infty$$

INFINITO PER $x \to 0$

ALCONTINO ALTERNATIVO

$$\varphi(x) = (1 - \cos x) \cdot \frac{1 + \cos x}{1 + \cos x} \Rightarrow$$

$$\frac{1 - \cos x}{1 + \cos^2 x} = \frac{\sin^2 x}{1 + \cos x}$$

ALG 2 $\quad c \overset{\varepsilon_c}{=} \cos x, \quad s = \sin x \overset{\varepsilon_s}{}$

$m := s \cdot s, \quad d = 1 + c$

$y_2 = m / d$

$\varepsilon_d^{(TOT)} = \varepsilon_c \cdot \frac{c}{1+c} + \varepsilon_d$

ERRORE TOTALE ACCUMULATO

$\varepsilon_m^{(TOT)} = \varepsilon_s + \varepsilon_s + \varepsilon_m = 2\varepsilon_s + \varepsilon_m$

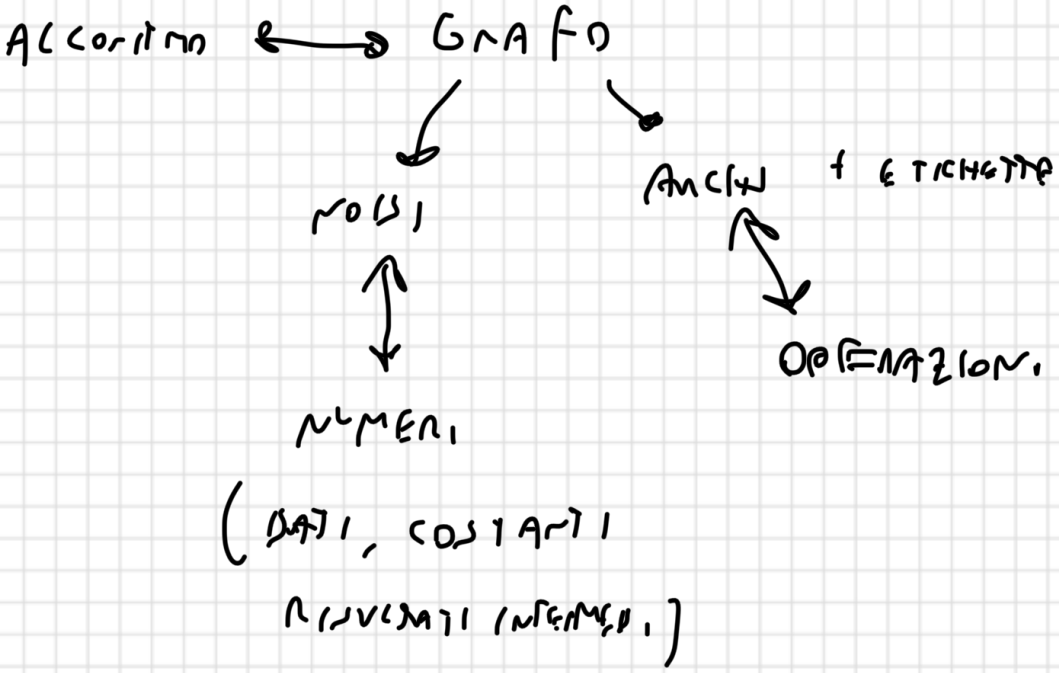
$\varepsilon_{ALG2} = \varepsilon_m^{(TOT)} - \varepsilon_d^{(TOT)} + \varepsilon_{y_2} =$

$= 2\varepsilon_s + \varepsilon_m - \left(\varepsilon_c \frac{c}{1+c} + \varepsilon_d\right) + \varepsilon_{y_2} =$

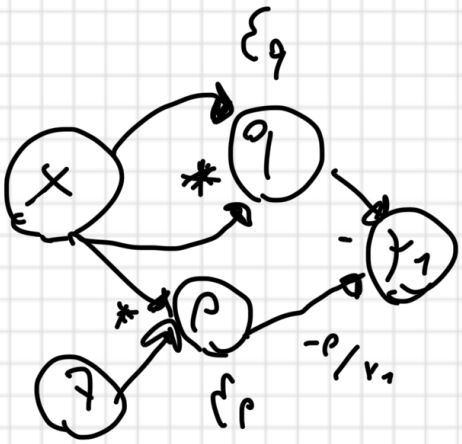
$2\varepsilon_s + \varepsilon_m - \varepsilon_c \boxed{\frac{c}{1+c}} - \varepsilon_d + \varepsilon_{y_2}$

$\lim_{x \to 0} \frac{\cos x}{1 + \cos x} = \frac{1}{2}$

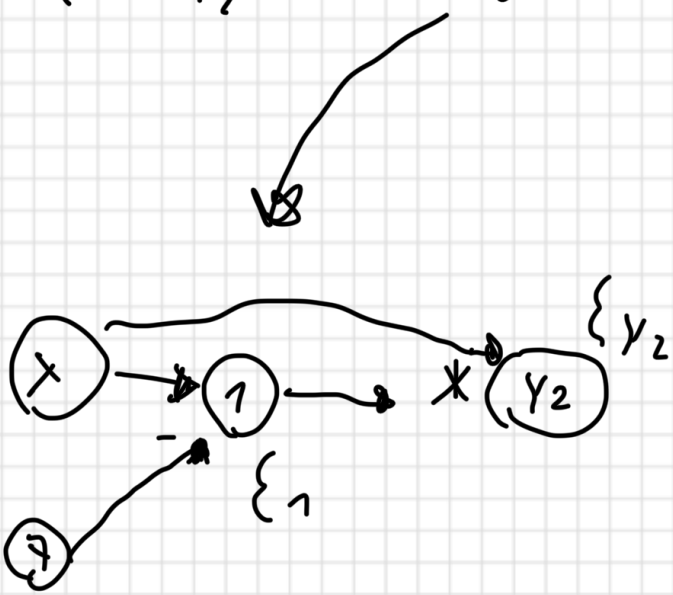
ALG 2 È STABILE PER $x \approx 0$



$f(x) = x^2 - 7x \rightsquigarrow$ ALG 1



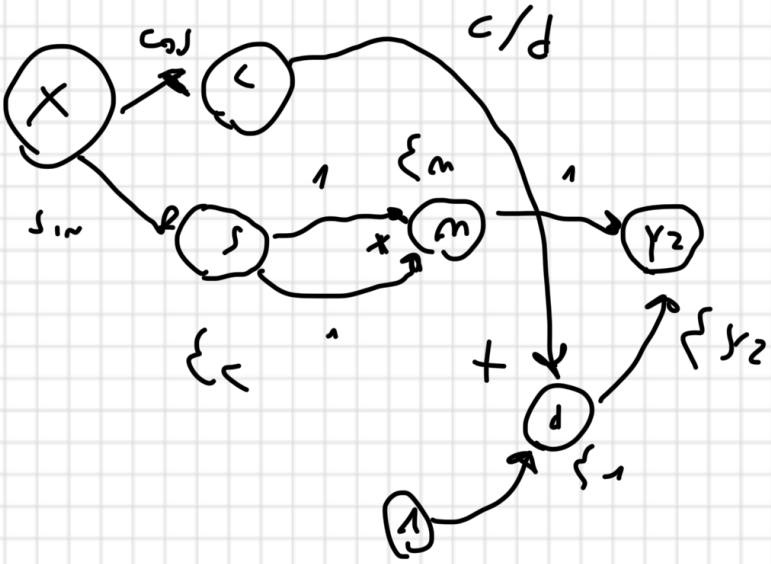
$x(x-7) \rightsquigarrow$ ALG 2



$1 + \cos x$

$\Downarrow$

Alog 2



NODI ERRORI LOCALI

(SOLO PER MOLTIPLI. INTERMEDI)

ANCHE COEFF DI

AMPLIFICAZIONE

REGOLA

$$\varepsilon_{Alog} = \sum (\text{ERRORI LOCALI}) \cdot (\text{COEFF})$$

$$\text{COEFF} = \prod (\text{ETICHETTA ANCHE USCENTI}) [1^\circ \text{ CAMMINO}] +$$

$$\cdots [2^\circ \text{ CAMMINO}] + \ldots$$

Es

$$f(x) = \frac{1}{1-\sqrt{x}} - \frac{1}{1+\sqrt{x}}$$

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