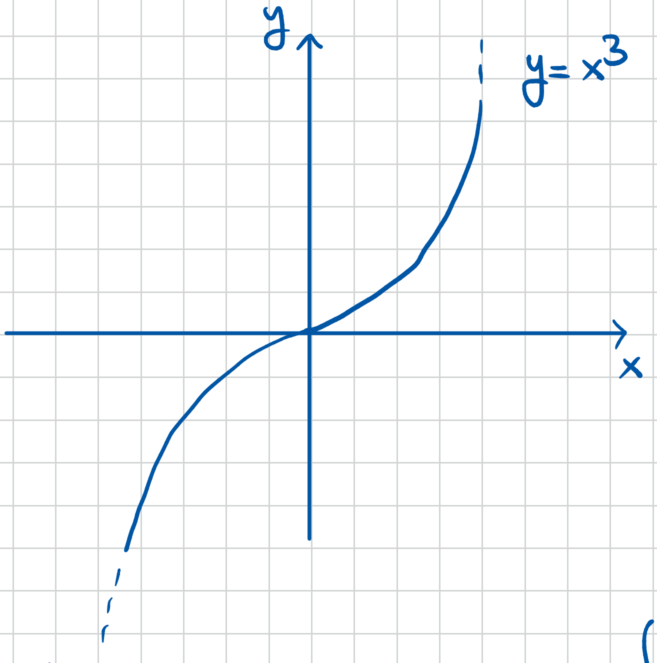
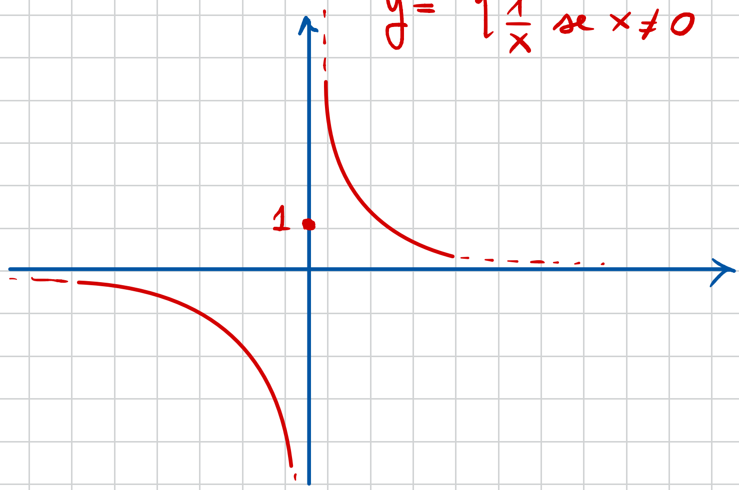
681 b) $f(x) = x^3 \qquad \mathrm{dom} f = \mathbb{R}$

$$g(x) = \begin{cases} 1 & \text{se } x = 0 \\ \frac{1}{x} & \text{se } x \neq 0 \end{cases} \qquad \mathrm{dom}\, g = \mathbb{R}$$





$$(f \circ g)(x) = f(g(x)) = \begin{cases} f(1) & \text{se } x = 0 \\ f\left(\frac{1}{x}\right) & \text{se } x \neq 0 \end{cases} = \begin{cases} 1 & \text{se } x = 0 \\ \frac{1}{x^3} & \text{se } x \neq 0 \end{cases}$$

$$(g \circ f)(x) = g(f(x)) = g(x^3) = \begin{cases} 1 & \text{se } x^3 = 0 \\ \frac{1}{x^3} & \text{se } x^3 \neq 0 \end{cases}$$

$$= \begin{cases} 1 & \text{se } x = 0 \\ \frac{1}{x^3} & \text{se } x \neq 0 \end{cases}$$

$$\mathrm{dom}(f \circ g) = \mathrm{dom}(g \circ f) = \mathbb{R}$$

$f(x) = x^2 \qquad \mathrm{dom} f = \mathbb{R} \qquad\qquad \mathrm{dom}(f \circ g) = [0, +\infty)$

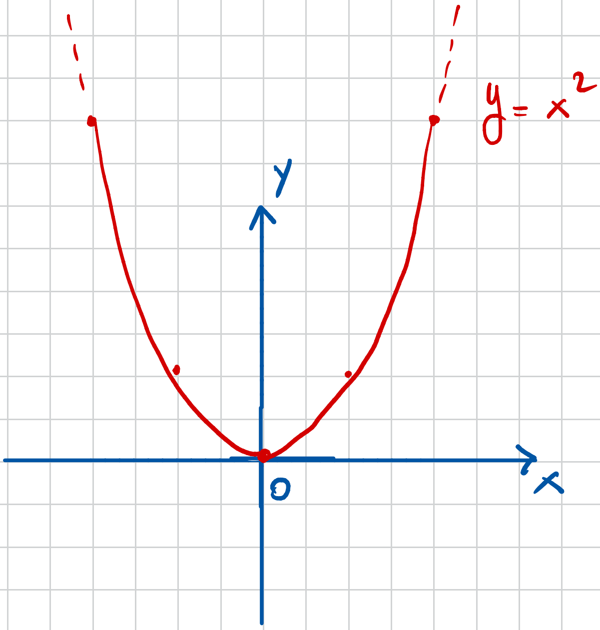
$g(x) = \sqrt{x} \qquad \mathrm{dom}\, g = [0, +\infty)$

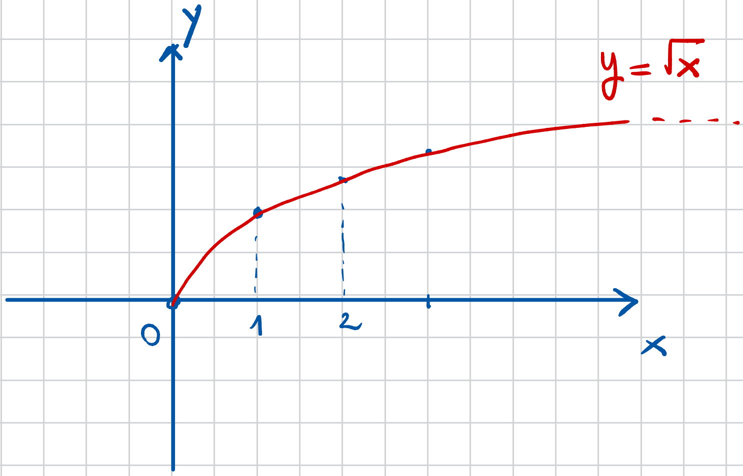
poiché $x \geq 0$

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = |x| = x$$

$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x| \neq x$

$\text{dom}(g \circ f) = \mathbb{R}$



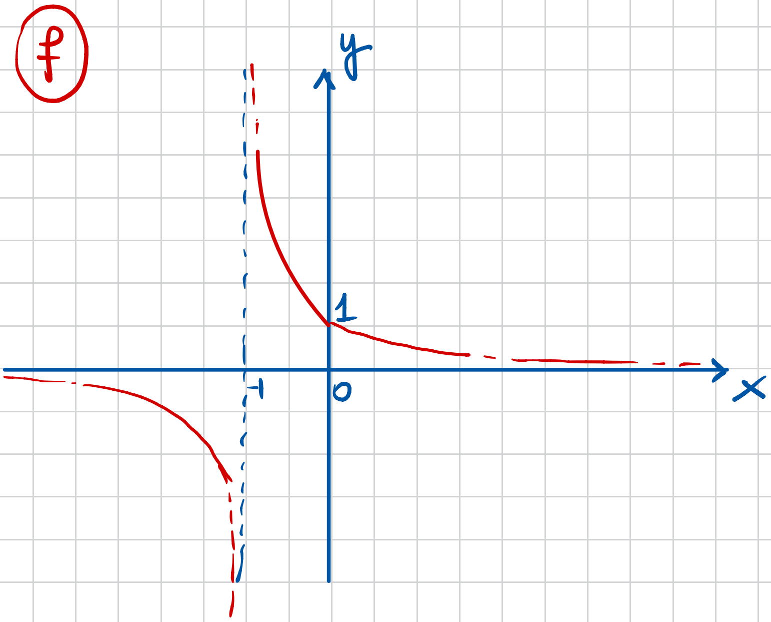


$f(x) = \dfrac{1}{x+1}$ $\quad$ $\text{dom} f = \mathbb{R} \setminus \{-1\} = (-\infty, -1) \cup (-1, +\infty)$

$g(x) = \dfrac{1}{x^2+1}$ $\quad$ $\text{dom}\, g = \mathbb{R}$ $\quad$ perché $x^2 + 1 = 0$ è impossibile

siccome $\Delta = 0^2 - 4 \cdot 1 \cdot 1 = -4 < 0$

f



$y = \dfrac{ax+b}{cx+d}$ $\quad$ IPERBOLE RIFERITA AGLI ASINTOTI

$y = \dfrac{a}{c}$ $\quad$ ASINTOTO ORIZZ

$x = -\dfrac{d}{c}$ $\quad$ ASINTOTO VERTICALE

$a = 0$

$b = c = d = 1$

$y = 0$ e $x = -1$

$$\begin{cases} y = 0 \\ y = \dfrac{1}{x+1} \end{cases} \rightarrow \quad \frac{1}{x+1} = 0 \qquad \text{IMPOSSIBILE}$$

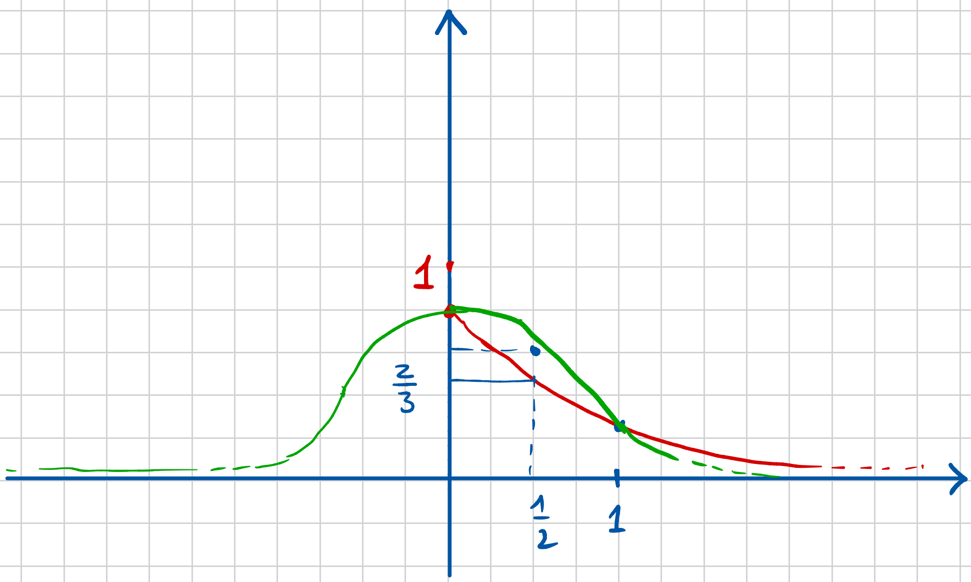
$$\begin{cases} x = 0 \\ y = \dfrac{1}{x+1} \end{cases} \rightarrow \quad \begin{cases} x = 0 \\ y = 1 \end{cases} \qquad (0,1)$$

g

$$y = \frac{1}{x} \qquad x \longmapsto x+1 \qquad \text{traslazione verso sx}$$

$$y = \frac{1}{x+1}$$

$$y = \frac{1}{x^2+1} \qquad x \longmapsto x^2$$



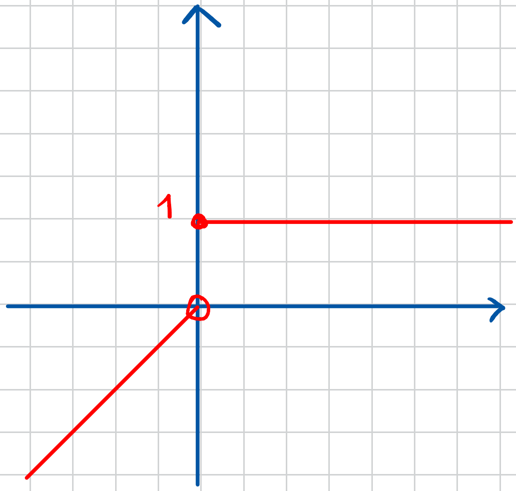
$$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x^2+1}\right) = \frac{1}{\dfrac{1}{x^2+1} + 1} = \frac{1}{\dfrac{1 + x^2+1}{x^2+1}} =$$

$$= \frac{x^2+1}{x^2+2} \qquad \text{dom}\,(f \circ g) = \mathbb{R}$$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x+1}\right) = \frac{1}{\left(\dfrac{1}{x+1}\right)^2 + 1} = \frac{1}{\dfrac{1}{(x+1)^2} + 1}$$

$$= \frac{1}{\frac{1+(x+1)^2}{(x+1)^2}} = \frac{(x+1)^2}{(x+1)^2+1}$$

$\mathrm{dom}\, g \circ f = \mathbb{R} \setminus \{-1\}$



682 $f(x) = \begin{cases} x & \text{se } x<0 \\ 1 & \text{se } x \geq 0 \end{cases}$

STR. CRESCENTE (DECRESCENTE) $\forall x_1, x_2 \in \mathrm{dom} f$ t.c. $x_1 < x_2$ si ha che $f(x_1) < f(x_2)$ $(f(x_1) > f(x_2))$

f non è str. crescente o str decrescente

ma è MONOTONA CRESCENTE

- NON INIETTIVA $f(1) = f(2) = 1$
- NON SURIETTIVA $2 \notin \mathrm{Im} f \Rightarrow \mathrm{Im} f \subsetneq \mathbb{R}$
- NON E' PARI $f(2) = f(-2)$? NO

  $f(2) = 1 \quad f(-2) = -2$
- NON E' DISPARI $f(-2) = -f(2)$? NO

  perché sarebbe $-2 = -1$ FALSO

- NON E' BIIETTIVA $\Rightarrow$ NON INVERTIBILE

$f(x) = x$

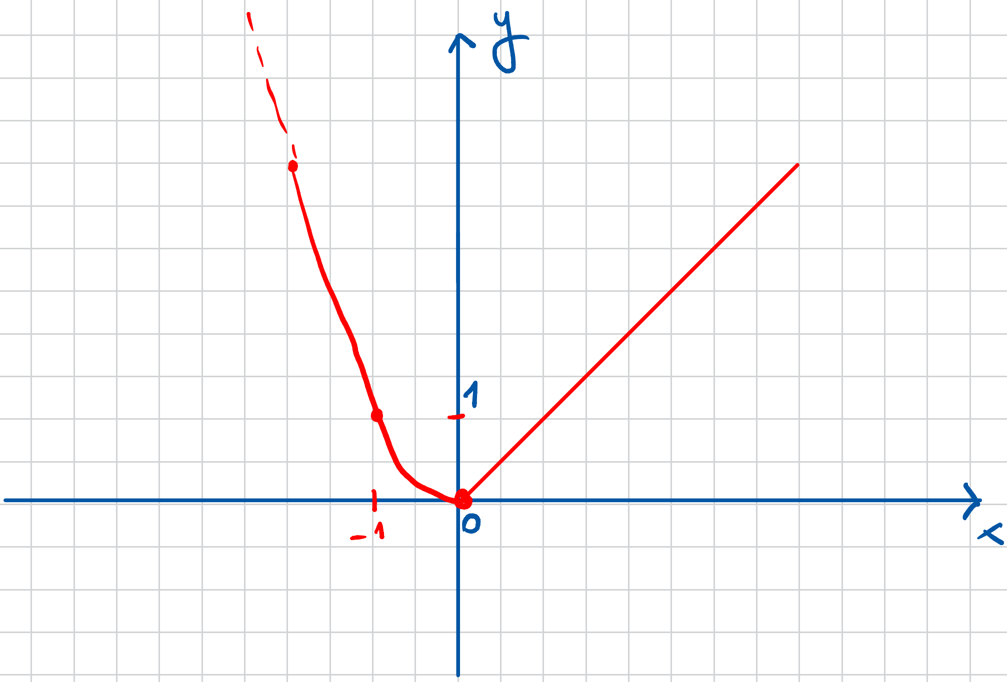
$f(\underbrace{f(x)}_{x}) = f(x) = x \qquad (f \circ f)(x) = x \qquad \rightarrow \qquad f \circ f = id$

$f^{-1} = f$

$f \circ f = f^2$

Se $f^2 = id \Rightarrow f$ è detta IDEMPOTENTE

$$f(x) = \begin{cases} x^2 & \text{se } x < 0 \\ x & \text{se } x \geq 0 \end{cases}$$



- NON È stra monotona né monotona
- NON INIETTIVA $\quad f(-1) = 1 = f(1)$
- NON SURIETTIVA $\quad -1 \notin Im f$
- NON DISPARI $\quad f(-x) = -f(x)$

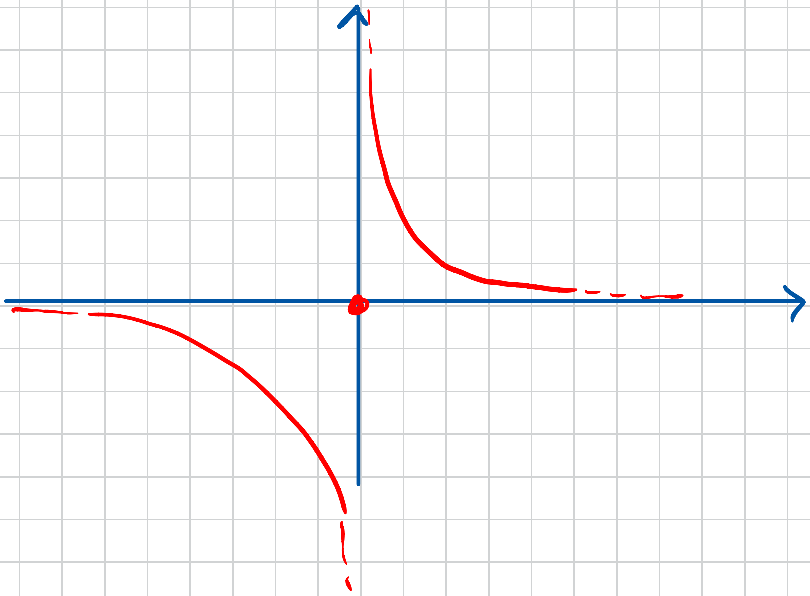
se $x = 1 \qquad f(-1) = 1 \neq -1 = f(1)$

NON È PARI $\quad f(-x) = f(x)$

$f(-2) = 4 \qquad f(2) = 2$

- NON INVERTIBILE

$$f(x) = \begin{cases} 0 & \text{se } x=0 \\ \frac{1}{x} & \text{se } x \neq 0 \end{cases}$$



INIETTIVA se $f(x_1)=f(x_2)$ allora $x_1=x_2$?

- Supponiamo $x_1 \neq 0 \wedge x_2 \neq 0$

  $f(x_1)=f(x_2) \rightarrow \frac{1}{x_1} = \frac{1}{x_2} \rightarrow x_1 = x_2$ ✓

- Se $x_1 = 0 \wedge x_2 \neq 0$ (o viceversa)

  $f(x_1) = f(x_2) \rightarrow 0 = \frac{1}{x_2}$ IMPOSSIBILE

- Se $x_1 = x_2 = 0$ ho finito

SURIETTIVA $\forall y \in \mathbb{R} \quad \exists x \in \operatorname{dom} f$ t.c $f(x) = y$

$f(x) = y$

$\rightarrow y = 0 \rightarrow f(x) = 0 \rightarrow x = 0$

$\rightarrow y \neq 0 \rightarrow y = f(x) = \frac{1}{x} \rightarrow x = \frac{1}{y}$

SURIETTIVA + INIETTIVA = BIIETTIVA $\Rightarrow$ INVERTIBILE

L'inversa è $f^{-1}(x) = \begin{cases} 0 & \text{se } x = 0 \\ \frac{1}{x} & \text{se } x \neq 0 \end{cases}$

$$f \circ f^{-1}(x) = f(f^{-1}(x)) = \begin{cases} 0 & \text{se } f^{-1}(x) = 0 \\ \frac{1}{f^{-1}(x)} & \text{se } f^{-1}(x) \neq 0 \end{cases}$$

$$= \begin{cases} 0 & \text{se } x = 0 \\ \frac{1}{\frac{1}{x}} & \text{se } x \neq 0 \end{cases} = \begin{cases} 0 & \text{se } x = 0 \\ x & \text{se } x \neq 0 \end{cases} = x$$

l'inversa è $y = x$

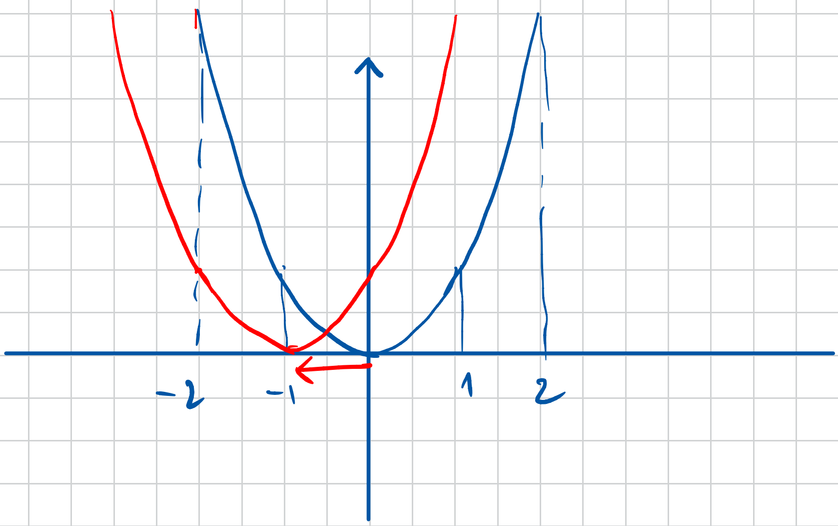
$f(x) = x^2$

$f_1(x) = x^2 + 2x + 1 = (x+1)^2$

$f \mapsto f_1$

TRASLAZIONE VERSO SX

$x \longmapsto x+1$



$$f_2(x) = x^2+4x+1 = \underbrace{x^2+4x+4}_{\text{QUADRATO}}-3 = (x+2)^2-3$$

$$x^2 \longmapsto (x+2)^2 \longmapsto (x+2)^2-3$$

TRASLAZIONE della parabola di 2 a sx

TRASLAZIONE della parabola di 3 in basso

$g_1(x) = x^2$ $\quad$ $g_2(x) = x-3$ $\quad$ $g_3(x) = x+2$

$$f_2(x) = g_2(g_1(g_3(x)))$$

$$f_3(x) = 2x^2+x-1 = 2\left(x^2+\frac{1}{2}x-\frac{1}{2}\right) =$$

$$= 2\left(x^2+\frac{1}{2}x+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-\frac{1}{2}\right) =$$

$$= 2\left(\left(x+\frac{1}{4}\right)^2-\frac{1}{16}-\frac{1}{2}\right) =$$

$$= 2\left(\left(x+\frac{1}{4}\right)^2+\frac{-1-8}{16}\right) = 2\left(\left(x+\frac{1}{4}\right)^2-\frac{9}{16}\right)$$

$$x \xmapsto{g_1} x^2 \xmapsto[\text{A SX}]{\text{TRASLAZ}} \left(x+\frac{1}{4}\right)^2 \xmapsto[\text{IN GIU'}]{\text{TRASLAZ.}} \left(x+\frac{1}{4}\right)^2-\frac{9}{16}$$

$$\downarrow \text{DILATAZ.}$$

$$2\left(\left(x+\frac{1}{4}\right)^2-\frac{9}{16}\right)$$

$$f_4(x) = -x^2+2$$

$$x \xmapsto{g_1} x^2 \longmapsto -x^2 \longmapsto -x^2+2$$

DILATAZIONE TRASLAZIONE

<u>Es 4</u>

5) $f(x) = \dfrac{3}{2+x^2}$

$$g_1(x) = x^2$$
$$g_2(x) = x+2$$
$$g_3(x) = \frac{3}{x}$$

$$f(x) = g_3(g_2(g_1(x)))$$

$$g_3(g_2(g_1(x))) = g_3(g_2(x^2)) = g_3(x^2+2) = \frac{3}{x^2+2}$$