16-07

SIMULAZIONI ESAME

$f(x)\quad \dfrac{1-2/x}{2+1/x} - \dfrac{1+2/x}{2-1/x} \qquad x \longrightarrow +\infty$

$f1:\ x \longrightarrow r = \dfrac{1}{x} \mapsto f1 := \dfrac{1-2r}{2+r},\quad f2 = \dfrac{1+2r2}{2-r} \mapsto y1 := f1 - f2$

$f2:\ x \mapsto m := 10x,\ d := 1-4x^2 \mapsto y2 := m/d$

$\hookrightarrow f(x) = \dfrac{10x}{1-4x^2}$

a) CALCOLO CONDIZIONAMENTO

$cf = \dfrac{x\, f'(x)}{f(x)}$

POSSIAMO SCEGLIERE UNA DELLE FUNZIONI DATE NELLA SECONDA PARTE

QUINDI SCEGLIAMO $f(x) = \dfrac{10x}{1-4x^2}$

$f'(x) = \dfrac{10\left(1-4x^2\right) - 10x(-8x)}{(1-4x^2)^2} = \dfrac{10-40x^2+80x^2}{(1-4x^2)^2}$

$\Rightarrow \dfrac{10\left(1+4x^2\right)}{(1-4x^2)^2}$ LA LASCIAMO SCOMPOSTA IN QUESTO MODO PER SEMPLIFICARE IL CALCOLO DEL LIMITE

$$c\psi = \cancel{x} \cdot \frac{\cancel{10}(1+4x^2)}{(1-4x^2)^{\cancel{2}}} \cdot \frac{\cancel{1-4x^2}}{\cancel{10}\,\cancel{x}} = \boxed{\frac{1+4x^2}{1-4x^2}}$$

$$\lim_{x \to +\infty} \frac{1+4x^2}{1-4x^2} = \frac{\infty}{-\infty} = -1$$

POSSIAMO SCEGLIERE SE HOPITAL O LA CLASSICA REGOLA DI $\frac{\infty}{\infty}$ DOVE GUARDI I COEFFICIENTI RICAVANDO ALLA X

IN QUESTO CASO ABBIAMO $\frac{4}{-4} = -1$

METODO DEL PROF: $$\lim_{x \to +\infty} \frac{4x^2\left(\frac{1}{4x^2}+1\right)}{-4x^2\left(-\frac{1}{4x^2}+1\right)} = \lim_{x \to +\infty} - \frac{\boxed{\frac{1}{4x^2}}+1}{\boxed{-\frac{1}{4x^2}}+1}$$

$\boxed{\frac{1}{4x^2}} \to 0$, $\boxed{-\frac{1}{4x^2}} \to 0$

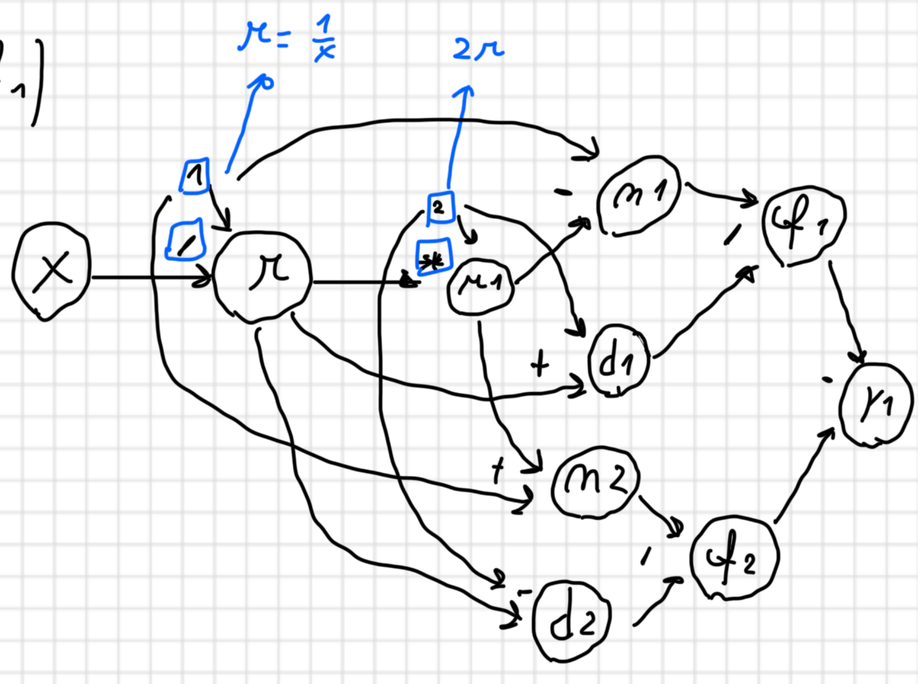
FRASE DA DIRE: IL RAPPORTO TRA GLI ERRORI TENDE A -1

SE SUCCEDE: $c\psi \xrightarrow{x \to +\infty} 0$ $\quad \varepsilon_{out} \approx c\psi\, \varepsilon_{in}$ IL PROBLEMA E' BEN CONDIZIONATO

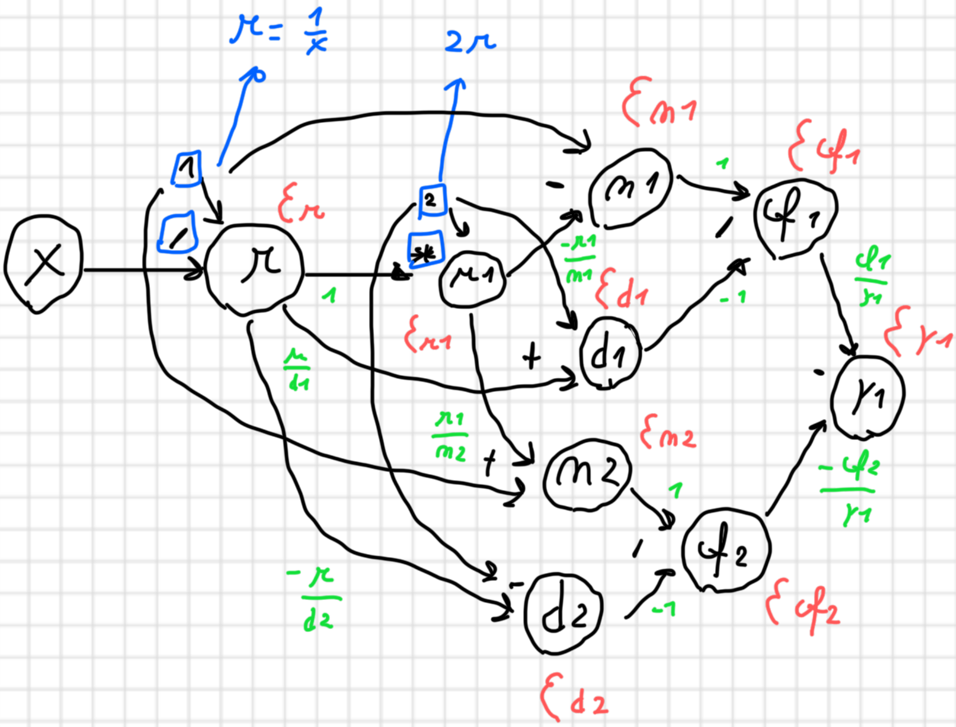
$-\infty$: MAL CONDIZIONATO

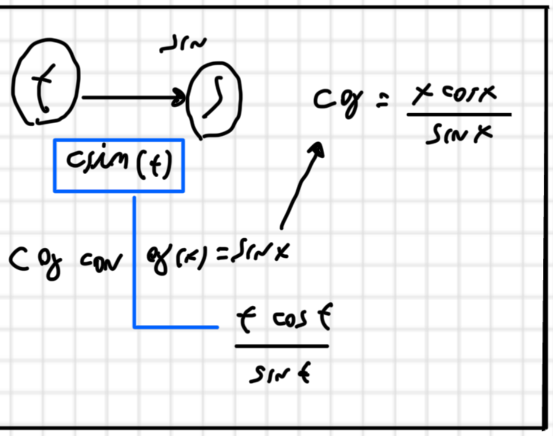
b) STUDIARE L'ERRORE DI ARROTONDAMENTO

(ψ_1)



UNA VOLTA DISEGNATO IL GRAFO DOBBIAMO ETICHETTARE NODI E ARCHI





$$\varepsilon_{a\pm b} = \frac{a}{a\pm b}\varepsilon_a \pm \frac{b}{a\pm b}\varepsilon_b$$

$$\varepsilon_{ab} = 1\varepsilon_a + 1\varepsilon_b$$

$$\varepsilon_{a/b} = 1\varepsilon_a - 1\varepsilon_b$$

REGOLE PER ETICHETTARE

UNA VOLTA FINITO DOBBIAMO CALCOLARE L'ERRORE TOTALE

SI INTENDE I CAMMINI

$$\varepsilon_{Algo} = \varepsilon_r\left\{ *2 \right\} + \varepsilon_{r1}\left\{\text{"sopra"} + \text{"sotto"}\right\}^{*} + \varepsilon_{m_1}\left\{1\cdot\frac{\varphi_1}{y_1}\right\} + \varepsilon_{\varphi_1}\left\{\frac{\varphi_1}{y_1}\right\} +$$

$$\varepsilon_{d_1}\left\{(-1)\frac{\varphi_1}{y_1}\right\} + \varepsilon_{m_2}\left\{1\cdot\left(-\frac{\varphi_2}{y_1}\right)\right\} + \varepsilon_{d_2}\left\{(-1)\left(-\frac{\varphi_2}{y_1}\right)\right\} + \varepsilon_{\varphi_2}\left\{-\frac{\varphi_2}{y_1}\right\}$$

$$* \ \varepsilon_{r1}\left\{\left(\frac{r1}{m1}\right) + 1\,\frac{\varphi1}{y1} + \frac{r1}{m2}\cdot 1\cdot\left(-\frac{\varphi_2}{y_1}\right)\right\}$$

$$*_2 \quad \varepsilon_\mu \left\{ 1\cdot\left(-\frac{\mu_1}{m_1}\right) + 1\cdot\frac{\psi_1}{m_1} + 1\cdot\frac{\mu_1}{m_2} + 1\cdot\left(-\frac{\psi_2}{m_1}\right) + \frac{\mu}{d_1}(1)\,\frac{\psi_1}{y_1} + \left(-\frac{\mu}{d_2}\right)(-1)\left(-\frac{\psi_2}{y_1}\right) \right\}$$

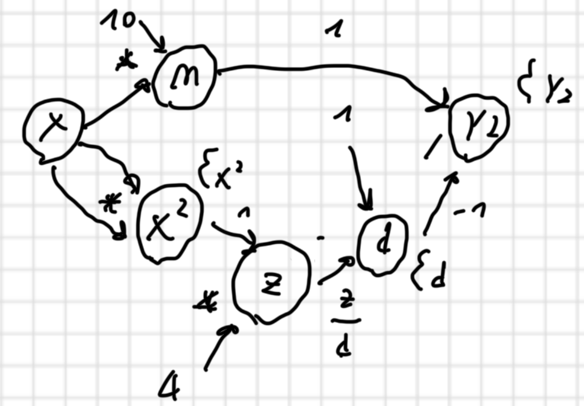
SICCOME È PRESENTE UN "−" NELLA FUNZIONE PRINCIPALE MI CONVIENE ANDARE A VEDERE NEL GRAFO L'ETICHETTA CON "−" E CALCOLARE IL LIMITE

$$\lim_{x\to+\infty} \frac{\psi_1}{y_1} = \lim_{x\to+\infty} \frac{\left[\frac{1-2/x}{2+\sqrt{x}}\right]^2}{\left[\frac{10x}{1-4x^2}\right]^{0^-}} = \pm\infty$$

$\boxed{\text{SAREBBE } -\infty \text{ MA IL PROF PREFERISCE NON SPECIFICARE}}$

SICCOME TENDE A $\pm$ L'ALGORITMO È INSTABILE

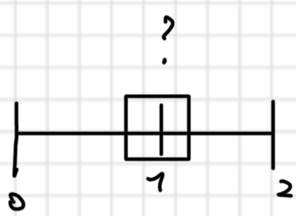
ORA BISOGNA RIFARE TUTTO COL SECONDO ALGORITMO



$$\varepsilon_{ALG2} = \varepsilon_m\{1\} + \varepsilon_{y_2} + \varepsilon_{x^2}\left\{1\left(-\frac{z}{d}\right)(-1)\right\} + \varepsilon_z\left\{\left(-\frac{z}{d}\right)\cdot(-1)\right\} + \varepsilon_d(-1)$$

$$\lim_{x\to+\infty} \frac{z}{d} : \frac{4x^2}{1-4x^2} = -1$$ STABILE

$\boxed{\text{SPLINE}}$



$$S(x) = \begin{cases} x^3 + \alpha x + \beta & \boxed{0} \le x \le 1 \\ -x^3 + \alpha x^2 & 1 \le x \le \boxed{2} \end{cases}$$

a) α, β t.c. SPLINE SU 0,1,2

SPLINE → POLINOMIO A TRATTI GRADI ≤ 3
→ S, S', S'' CONTINUE

LA PRIMA CONDIZIONE E' VERIFICATA

$$S \text{ CONTINUA} \iff S(1) = \lim_{x \to 1^-} S(x) = \lim_{x \to 1^+} S(x)$$

$$\lim_{x \to 1^-} S(x) = \lim_{x \to 1^-} \left(x^3 + \alpha x + \beta \right) = 1 + \alpha + \beta$$

$$\lim_{x \to 1^+} S(x) = \lim_{x \to 1^+} \left(-x^3 + \alpha x^2 \right) = -1 + \alpha$$

$$1 + \alpha + \beta = -1 + \alpha \Rightarrow \boxed{\beta = -2}$$

$$S'(x) = \begin{cases} 3x^2 + \alpha \\ -3x^2 + 2\alpha x \end{cases} \qquad S''(x) = \begin{cases} 6x \\ -6x + 2\alpha \end{cases}$$

$$\left.\begin{aligned} \lim_{x \to 1^-} S'(x) &= \lim_{x \to 1^-} \left(3x^2 + \alpha \right) = 3 + \alpha \\ \lim_{x \to 1^+} S'(x) &= \lim_{x \to 1^+} \left(-3x^2 + 2\alpha x \right) = -3 + 2\alpha \end{aligned}\right\} \alpha = 6$$

$$\lim_{x \to 1^-} (6x) = 6$$

$$\lim_{x \to 1^+} (-6x + 2\alpha) = -6 + 12 = 6$$

$$S(x) = \begin{cases} x^3 + 6x - 2 \\ -x^3 + 6x^2 \end{cases}$$

b) VERIFICARE SE E' NATURALE

SPLINE NATURALE $\iff S''(\underset{\underset{0}{\shortparallel}}{a}) = S''(\underset{\underset{2}{\shortparallel}}{b}) = 0$

0 E 2 PERCHE' SONO I VALORI ESTREMI

$S''(0) = 6\cdot 0 = 0$

VERIFICATO

$S''(2) = -6\cdot 2 + 2\cdot \overset{\overset{6}{\shortparallel}}{a} = 0$

$\boxed{\text{GIVENS}}$

$$x = \begin{pmatrix} 3 \\ 0 \\ -2 \\ 0 \end{pmatrix} \longmapsto \begin{pmatrix} 0 \\ m \\ 0 \\ 0 \end{pmatrix}$$

$$G(i,j,\theta) = \begin{array}{cc} \begin{pmatrix} c & -s \\ s & c \\ & \\ & \end{pmatrix} & \begin{array}{l} \leftarrow i \\ \leftarrow j \\ \\ \\ \end{array} \\ \begin{array}{cc} \uparrow & \uparrow \\ i & j \end{array} & \end{array} \qquad \begin{array}{cc} \begin{pmatrix} c & s \\ -s & c \end{pmatrix} & \begin{array}{l} \leftarrow j \\ \leftarrow i \end{array} \end{array}$$

$c = \cos\theta$

$s = \sin\theta$

$$G(i,j,\theta)\,x = \begin{array}{cc} \begin{pmatrix} * \\ 0 \end{pmatrix} & \begin{array}{l} \leftarrow i \\ \leftarrow j \end{array} \end{array}$$

$$G(2,1,\theta)\begin{pmatrix} 3 \\ 0 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ * \\ -2 \\ 0 \end{pmatrix}$$

$$c = \frac{x_i}{\sqrt{x_i^2 + x_j^2}} = 0 \qquad s = \frac{-x_j}{\sqrt{x_i^2 + x_j^2}} = -1$$

$$G(2,1,\theta) = \begin{pmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{\|x\| = \left\| \begin{pmatrix} 0 \\ \sqrt{13} \\ 0 \\ 0 \end{pmatrix} \right\| \qquad \|x\| = \sqrt{3^2+0^2+(-2)^2+0^2} = \sqrt{13}}$$

LO SI FA COME VERIFICA FINALE

LA NORMA DI X DEVE ESSERE UGUALE DEL VETTORE FINALE DI GIVENS

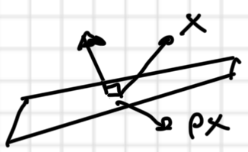
$\boxed{\text{HOUSEHOLDER}}$

$$x = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \qquad x \mapsto \begin{pmatrix} m \\ 0 \end{pmatrix}$$

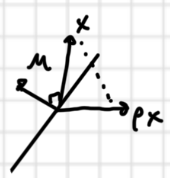
$$P = I - 2ww^T$$

- $\|w\| = 1$
- $\|x\| = |m|$
- $\mu = x - m e_1$
- $w = \dfrac{\mu}{|\mu|}$
- $P = I - 2\dfrac{\mu\mu^T}{\mu^T\mu}$

INTERPRETAZIONE GEOMETRICA: $\mathbb{R}^n$ μ, w



$\mathbb{R}^2 =$



$$m = \|x\| = \sqrt{(-3)^2+4^2} = 5$$

$$\mu = \begin{pmatrix} -3 \\ 4 \end{pmatrix} - 5\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \end{pmatrix}$$

$$\mu\mu^T = \begin{pmatrix} -8 \\ 4 \end{pmatrix}\begin{pmatrix} -8 & 4 \end{pmatrix} = \begin{pmatrix} 64 & -32 \\ -32 & 16 \end{pmatrix}$$

$$u^T u = \begin{pmatrix}-8 & 4\end{pmatrix}\begin{pmatrix}-8\\4\end{pmatrix} = 64 + 16 = 80$$

$$P = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} - \frac{2}{80}\begin{pmatrix}64 & -32\\-32 & 16\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} - \begin{pmatrix}8/5 & -4/5\\-4/5 & 2/5\end{pmatrix} = \begin{pmatrix}-3/5 & 4/5\\4/5 & 3/5\end{pmatrix}$$

$$\begin{pmatrix}-3/5 & 4/5\\4/5 & 3/5\end{pmatrix}\begin{pmatrix}-3\\4\end{pmatrix} = \begin{pmatrix}5\\0\end{pmatrix}$$ VERIFICATO