

14/19

$$A+B, \lambda A, AB, \det A = \sum_{j=1}^m (-1)^{i+j} \det A_{ij} \quad (A \in \mathbb{R}^{n \times n})$$

TEOREMA

$$\det A \neq 0 \Leftrightarrow \exists A^{-1} \quad (\text{INVERSE})$$

(NON SINGOLARE)

$$\det(A+B) = \det A \quad \det B$$

$$\det(A+B) \neq \det A + \det B$$

$$\det(\lambda A) = \lambda^n \det A$$

$$\det(-I_m) = \det \begin{pmatrix} -1 & & 0 \\ & \ddots & \\ 0 & & -1 \end{pmatrix} = (-1)^m$$

\Rightarrow METODO DI CALCOLO DI A^{-1} BASATO SU \det
(NON EFFETTIVAMENTE)

$$\frac{\det m=2}{\text{CASO}} : \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{1}{ad \cdot bc} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$\rightarrow \det \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

SUPponiamo di averci 2 matrici E ci
volgono somm:

$$\det(-I_m) = \det \begin{pmatrix} -1 & & \\ & \ddots & 0 \\ 0 & & -1 \end{pmatrix} = (-1)^m$$

$$\boxed{\square} + \boxed{\square} = \boxed{\square}$$

COSTO DI $A+B$: $m \cdot m$ ADDIZIONI

$$\boxed{\square} \cdot \boxed{\square} = \boxed{\square}$$

COSTO DI λA : $m \cdot m$ MOLTIPLICAZIONI

$$\begin{array}{c} \boxed{\square} \\ \times \\ \boxed{\square} \end{array} = \boxed{\square}$$

COSTO DI $A \cdot B$: $m \cdot n^2 (m \text{ MOLT.} + m-1 \text{ ADD.})$

(ES1: $A, B \in \mathbb{R}^{n \times n} \rightarrow m \cdot n^2 \cdot m \text{ MOLT.}$, ES2: $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times 1} \rightarrow m^2$)

$C_m = \text{COSTO DI } \det m \times m$ (con CARATCE)

$$C_1 = 0 \quad C_2 = 2 \quad C_m = m C_{m-1} + m > m \cdot C_{m-1}$$

$$\Rightarrow C_m > m!$$

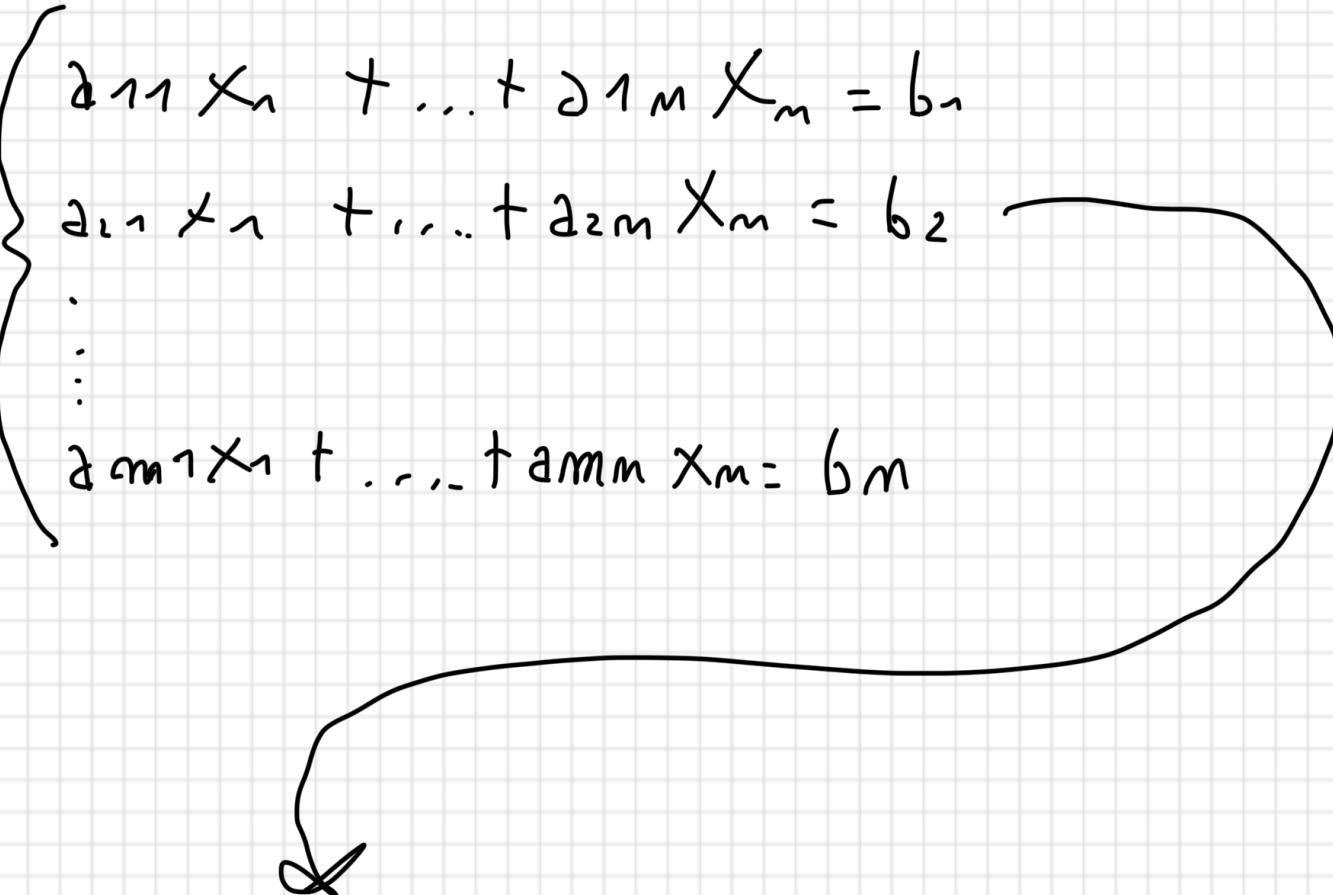
SISTEMI LINEARI

ESEMPIO

$$\begin{cases} x+y=2 \\ 100x+1000y=200 \end{cases} \rightsquigarrow A = \begin{pmatrix} 1 & 1 \\ 100 & 1000 \end{pmatrix}$$

$$b = \begin{pmatrix} 2 \\ 2001 \end{pmatrix} x = \begin{pmatrix} x \\ y \end{pmatrix}$$

Linear Equations

$$\left\{ \begin{array}{l} a_{11}x_1 + \dots + a_{1m}x_m = b_1 \\ a_{21}x_1 + \dots + a_{2m}x_m = b_2 \\ \vdots \\ a_{m1}x_1 + \dots + a_{mm}x_m = b_m \end{array} \right.$$


$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & \dots & a_{2m} \\ a_{m1} & \dots & a_{mm} \end{pmatrix} \in \mathbb{R}^{m \times m}$$

MATRICE DEF (coefficient)

(MATRICELLE (variable))

$$b = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix} \in \mathbb{R}^{m \times 1}$$

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} \in \mathbb{R}^{m \times 1}$$

VEKTORLICHE FORMEL NOTI

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$$A \cdot x = b$$

ESEMPIO 2

$$\begin{cases} 2x_1 - x_2 + x_4 = 5 \\ x_2 + x_3 - 2x_4 = 0 \end{cases}$$

$$b = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$$m = 2$$

$$m = 4$$

$$A = \begin{pmatrix} 2 & -1 & 0 & 1 \\ 0 & 1 & 1 & -2 \end{pmatrix} \in \mathbb{R}^{2 \times 4}$$

ESEMPIO 3

$$\begin{cases} x_1 + x_3 = 0 \\ x_2 + 2x_3 = 0 \end{cases}$$

$$b = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad m = 2$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad m = 3$$

$Ax = b$ SISTEMA OMOCHEO

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$Ax = 0$$

CASO $m = m$: $AX = b$

$m \times m$  $m \times 1$

$\det A \neq 0 \Rightarrow \exists A^{-1}$

$$A^{-1} Ax = A^{-1} b$$

ALGORITMO SOSTITUZIONE ALL'INDIETRO

Acc i rroccerap

ALGORITMO: SOSTITUZIONE ALL'INDIETRO

$$A \in \mathbb{R}^{n \times n} \text{ TRIANG. SUP. } A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ 0 & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \ddots & a_{m-1,m} \\ 0 & 0 & \dots & a_{mm} \end{pmatrix} \quad b = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix} \quad X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\left\{ \begin{array}{l} a_{11}x_1 + \dots + a_{1m}x_m = b_1 \\ a_{22}x_2 + \dots + a_{2m}x_m = b_2 \\ \vdots \\ a_{m-1,m-1}x_{m-1} + a_{m-1,m}x_m = b_{m-1} \\ a_{mm}x_m = b_m \end{array} \right.$$

$$X_m = b_m / a_{mm}$$

$$X_{m-1} = \frac{b_{m-1} - a_{m-1,m}X_m}{a_{m-1,m-1}}$$

$$\text{for } i = m-1, m-2, \dots, 1 : X_i = \frac{b_i - \sum_{j=i+1}^m a_{ij}X_j}{a_{ii}}$$

$$\det A \neq 0 \iff a_{ii} \neq 0 \quad \forall i=1$$

$$\rightarrow \det A \neq 0 \Leftrightarrow a_{ii} \neq 0 \quad \forall i=1..n$$

COSTO: 1 - (x_m)
 $+ \forall i: 1 / a_{ii}$
 $+ n-i (a_{ij} x_j)$

$$\frac{a_{11} \cdot a_{22} \cdots a_{nn}}{.}$$

$$\text{COSTO TOT} = 1 + \sum_{i=1}^{n-1} (n-i+1)$$

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} = \frac{n^2}{2}$$

$\downarrow \quad \downarrow \quad \uparrow \quad | \quad |$
 $i=n-1 \quad i=n-2 \quad \dots \quad i=1$

MATRICI ELEMENTARI

ELEMENTARI

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$$I_n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- I_n \leftrightarrow SCAMBIO RIGHE $i \leftrightarrow j$ in I_n : $E_{13} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
- $E_i(\lambda) \leftrightarrow$ MOLTIPLICO PER λ RIGA i -ESIMA in I_n : $E_2(3) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
- $E_{ij}(\lambda) \leftrightarrow$ SOMMO ALLA RIGA i -ESIMA $\lambda * \text{RIGA } j$ -ESIMA in I_n : $E_{21}(4) = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$