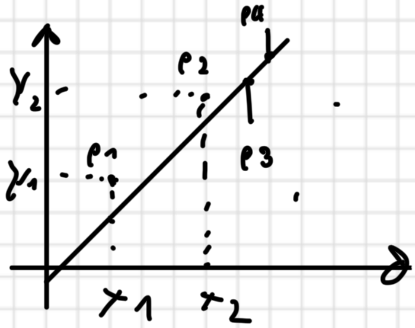
MINIMI QUADRATI / APPROSSIMAZIONE

RETTA DI REGRESSIONE



PROBLEMA

DATI PUNTI NEL PIANO

SONO $(x_i, y_i) \in \mathbb{R}^2$

$i = 1, \dots, m$ (m È IL NUMERO DEI PUNTI)

TROVARE LA RETTA CHE MEGLIO APPROSSIMA I PUNTI

$P_i = (x_i, y_i)$

$$y = \boxed{m}x + \boxed{q}$$

COEF. ANGOLARE

INTERCETTA CON L'ASSE DELLE y

L'IDEA È CHE SE FISSO (m, q) HO FISSATO UNA RETTA

QUALE RETTA? = QUALE COPPIA (m, q)?

PER "MEGLIO" SI INTENDE LA DISTANZA IN VERTICALE LUNGO LE y TRA I PUNTI DATI E IL VALORE DELLA RETTA IN QUEI PUNTI.

VETTORE y — VETTORE x

$P_1 = (x_1, y_1) \qquad y_1 - (m x_1 + q) \qquad > 0$

$P_2 = (x_2, y_2) \qquad y_2 - (m x_2 + q) \qquad > 0$

$P_3 = (x_3, y_3) \qquad y_3 - (m x_3 + q) \qquad < 0$

$P_4 = (x_4, y_4) \qquad y_4 - (m x_4 + q) \qquad > 0$

ELEVANDO TUTTI I PUNTI ALLA SECONDA OTTENGO TUTTI POSITIVI

$$\sum_{i=1}^{n} \left[y_2 - (m x_i + q) \right]^2$$

$$\begin{pmatrix} y_1 - (m x_1 + q) \\ \vdots \\ \vdots \\ y_n - (m x_n + q) \end{pmatrix} = \begin{pmatrix} \underline{y} \end{pmatrix} - \left[\underline{x} \,\middle|\, \underline{1} \right] \begin{pmatrix} m \\ q \end{pmatrix}$$

$$\underline{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{pmatrix} \qquad \underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix}$$

$$\underline{1} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \qquad \underline{v} = \begin{pmatrix} m \\ q \end{pmatrix}$$

$$\sum_{i=1}^{n} \big(\underbrace{y_i - (m x_i + q)} \big)^2 =$$

$$\underbrace{\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}}_{\underline{Y}} - \underbrace{\left(\begin{array}{c|c} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{array} \right)}_{\underline{A}} \underbrace{\begin{pmatrix} m \\ q \end{pmatrix}}_{\underline{v}} =$$

$$\underline{Y} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \qquad A = \left(\begin{array}{c|c} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{array} \right) \qquad \underline{v} = \begin{pmatrix} m \\ q \end{pmatrix}$$

$$\min_{\underline{v} = \binom{m}{q} \in \mathbb{R}^2} \{ \underbrace{\|y - A\underline{v}\|^2}_{\geq 0} \}$$

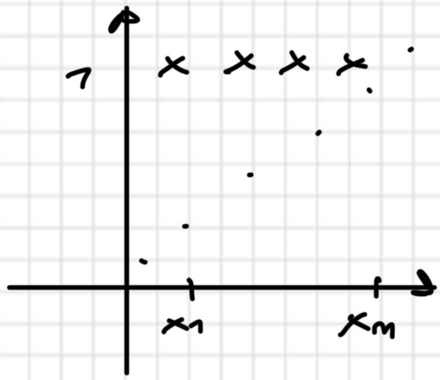
$\boxed{\text{MATRIX COOKBOOK}}$ → SITO PER FORMULE

$$\frac{d}{d\underline{v}} \|y - A\underline{v}\|^2 = 2A^T(A\underline{v} - \underline{y}) = \underline{0}$$

NOI LO FACCIAMO IN MODO GEOMETRICO E NON CON LE DERIVATE

$$A: \mathbb{R}^2 \longrightarrow \mathbb{R}^n$$

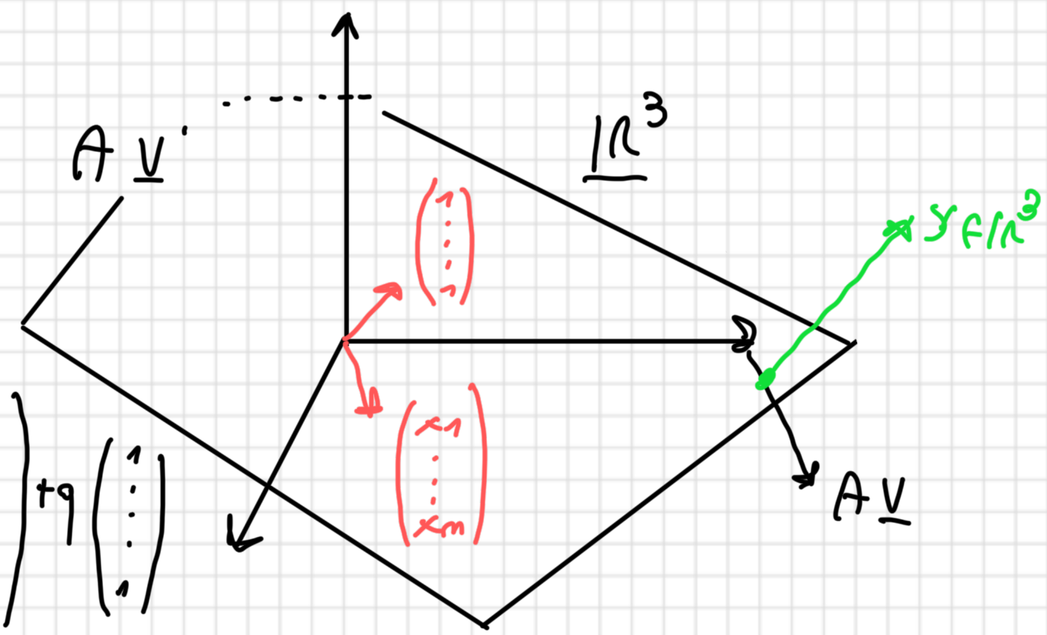
$$\binom{m}{q} = \underline{v} \longrightarrow A\underline{v}$$



$$m \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} + q \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} m x_1 + q \\ m x_2 + q \\ \vdots \\ m x_n + q \end{pmatrix}$$

CERCHIAMO LA DISTANZA IN NORMA 2 PIU' PICCOLA TRA TUTTI I $\underline{v}$

$A : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$



$$m \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} + q \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$\begin{cases} \underline{y} - A\underline{v} \perp \underline{x} \\ \underline{y} - A\underline{v} \perp \underline{1} \end{cases}$$

$$A = \left(\underline{x} \;\middle|\; \underline{1} \right) \qquad \underline{x}^T \left(\underline{y} - A\underline{v} \right) = 0$$

$$\begin{pmatrix} \underline{x}^T \\ \hline \underline{1}^T \end{pmatrix} = A^T \qquad \left(\underline{1}^T \right) \left(\underline{y} - A\underline{v} \right) = 0$$

COSA SUCCEDE ALL'ESAME

DATI I PUNTI

x	-2	-1	0	1	2
y	0	4	-5	2	3

$P_1(-2, 0)$
$P_2(-1, 4)$
$P_3(0, -5)$
$P_4(1, 2)$
$P_5(2, 3)$

$$\boxed{y = ax + b}$$

$$\left.\begin{matrix} P_1(-2,0) \\ P_2(-1,4) \\ P_3(0,-5) \\ P_4(1,2) \\ P_5(2,3) \end{matrix}\right\} \longrightarrow A^T(\underline{y} - A\underline{v}) = 0$$

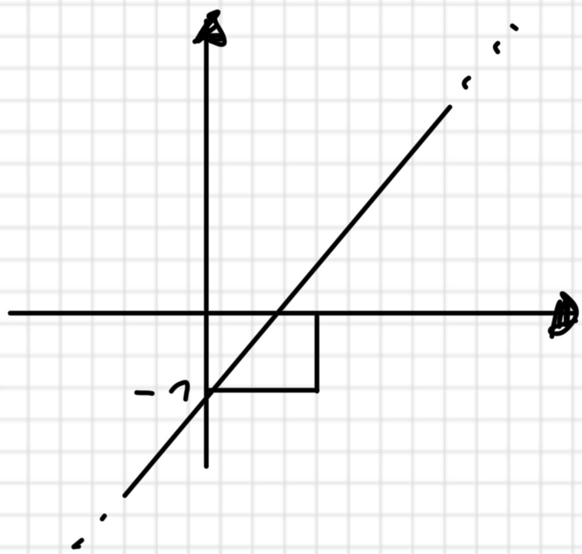
$$A = \begin{pmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} \qquad \underline{y} = \begin{pmatrix} 0 \\ 4 \\ -5 \\ 2 \\ 3 \end{pmatrix} \qquad A^T\underline{y} = A^T A\,\underline{v}$$

$m = *$
$q = *$

$$y = mx + q$$

$$m = \frac{1}{2} \longrightarrow y = \frac{1}{2}x - 1$$

$q = -1$



AL POSTO DELLA RETTA POTREMMO AVERE $y = ax^2 + bx + c$

$$\sum_{i=1}^{n} \left[y_i - \left(a x^2 + b x_i + c \right) \right]^2 \longrightarrow \text{MINIMO AL VARIARE DI } (a, b, c)$$

$$\left[\begin{pmatrix} y_1 \\ \vdots \\ \vdots \\ y_m \end{pmatrix} - \left(a \begin{pmatrix} x_1^2 \\ x_2^2 \\ \vdots \\ x_m^2 \end{pmatrix} + b \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} + c \begin{pmatrix} 1 \\ \vdots \\ \vdots \\ 1 \end{pmatrix} \right)\right]$$

$$A = \begin{bmatrix} x_1^2 & x_1 & 1 \\ \vdots & \vdots & \vdots \\ x_m^2 & x_m & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -2 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix} \quad 3\left\{\begin{pmatrix} m \\ A^T \end{pmatrix}\right._m \begin{pmatrix} 1 \\ \underline{y} \end{pmatrix} = 3\begin{pmatrix} m \end{pmatrix}_m \begin{pmatrix} 3 \\ \\ \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$A^T A \, \underline{v} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix} = A^T \underline{y}$$

$$y = a\cos(x) + b\sin(x) + c$$

$$y = a\cos(Mx) + b\sin(Mx) + c$$