DATA UNA FUNZIONE $f: \mathbb{R} \to \mathbb{R}$ CONTINUA IN $\mathbb{R}$ SI DEFINISCONO PRIMITIVE DELLA FUNZIONE L'INSIEME DELLE FUNZIONI REALI DI VARIABILE REALE AVENTI COME DERIVATA PRIMA LA FUNZIONE $f$: DETTA $G(x)$ UNA PRIMITIVA DI $f(x)$, ESSA E' TALE CHE

$G'(x) = f(x)$

RICORDIAMO CHE:

1) $\int f(x)\,dx$ E' DETTO INTEGRALE INDEFINITO

2) $\int_a^b f(x)\,dx$ E' DETTO INTEGRALE DEFINITO

ESERCIZI

1) $\int \left(x^2 + \frac{1}{x^4} - 2\right) dx$

$\int x^2 + \int x^{-4} + \int -2 \; dx$

$$\underbrace{\frac{x^3}{3} - \frac{1}{3x^3}} \underbrace{- 2x} + C$$

$\int k\,dx = kx + c$

$\int x^a\,dx = \frac{x^{a+1}}{a+1}$

2) $$\int \left( \frac{\sqrt{x}}{3} + \frac{1}{x} - e^x \right) dx$$

$$\frac{1}{3}\int \sqrt{x} + \int \frac{1}{x} - \int e^x \; dx$$

$$\frac{1}{3} \cdot \frac{(x)^{2/3}}{2/3} + \ln|x| - e^x + c$$

$$\frac{1}{3} \cdot \frac{2}{3} \cdot \sqrt{x^3} + \ln|x| - e^x + c$$

$$\frac{2}{9}\sqrt{x^3} + \ln|x| - e^x + c$$

3) $\int \frac{x^5 - 4x^2 + 1}{x^4} \, dx$

$\int \frac{x^5}{x^4} - \int \frac{4x^2}{x^4} + \frac{1}{x^4} \, dx$

$\int x - \int \frac{4}{x^2} + \int \frac{1}{x^4}$

$\frac{x^2}{2} - 4\left(\frac{x^{-2+1}}{-2+1}\right) + \left(\frac{x^{-4+1}}{-4+1}\right) =$

$= \frac{x^2}{2} + \frac{4}{x} - \frac{1}{3x^3} + C$

4) $\int \left(2\cos x - \frac{3}{5}\sin x\right) dx$

$$\int (2\cos x) - \int \frac{3}{5} \sin x =$$

$$= 2\sin x + \frac{3}{5}\cos x + c$$

5) $$\int \left( - \frac{6}{\sqrt[3]{x}} + 1 \right) dx$$

$$-6 \int \frac{1}{\sqrt[3]{x}} dx + \int dx = -6 \cdot \frac{3}{2} \cdot \sqrt[3]{x^2} + x + c$$

$$= -9\sqrt[3]{x^2} + x + c$$

6) $$\int (2x - a)^7 \, dx$$

$$\int [f(x)]^a \cdot f'(x) \, dx = \frac{[f(x)]^{a+1}}{a+1} + c$$

$$\frac{1}{2}\int 2\cdot(2x-4)^7\,dx$$

$$\frac{1}{2}\cdot\frac{(2x-4)^8}{8} = \frac{(2x-4)^8}{16} + c$$

7) $$\int x\,(x^2+2)^5\,dx$$

$$\frac{1}{2}\int 2x\,(x^2+2)^5\,dx$$

$$\frac{1}{2}\cdot\frac{(x^2+2)^6}{6} = \frac{(x^2+2)^6}{12} + c$$

8) $$\int \frac{1}{x^2-6x+9}\,dx$$

$$\Downarrow$$

$$\int \frac{1}{(x-3)^2}\,dx = \int (x-3)^{-2}\,dx$$

$$\frac{(x-3)^{-1}}{-1} + c = -\frac{1}{(x-3)} + c$$

9) $$\int \frac{10}{5x+6}\,dx$$

$$\int \frac{\varphi'(x)}{\varphi(x)} = \ln|\varphi(x)| + c$$

$$2\int \frac{5}{5x+6}\,dx = \ln|5x+6| + c$$

10) $$\int \frac{4x+1}{2x^2+x-2}\,dx$$

$$\ln|2x^2+x-2| + c$$

11) $\int (\sin x) \cdot \sqrt{\cos x}\, dx$

$$-\int -(\sin x) \cdot (\cos x)^{1/2} \, dx$$

$$-\frac{(\cos x)^{3/2}}{\frac{3}{2}} = -\frac{2}{3}\sqrt{(\cos x)^3} + c$$

12) $\int \frac{e^x}{4+2e^x} \, dx$

$$\frac{1}{2}\int \frac{2e^x}{4+2e^x} = \frac{1}{2}\ln|4+2e^x| + c$$

13) $\int \frac{1}{x \ln x}\, dx$

$\int \frac{1}{x} \cdot \frac{1}{\ln x} = \ln|\ln x| + c$

INTEGRAZIONE PER SOSTITUZIONE

1) $\int \frac{\sin x}{\underbrace{\sqrt{1-\cos x}}_{t}}\, dx$

$1-\cos x = t$

$\downarrow$

$\boxed{\sin x\, dx = dt}$

$\downarrow$

$\int \frac{1}{\sqrt{t}}\, dt \Rightarrow \int t^{-1/2}\, dt \Rightarrow$

$$\Rightarrow \frac{t^{1/2}}{1/2} + c = \frac{\sqrt{1-\cos x}}{\frac{1}{2}} =$$

$$= 2\sqrt{1-\cos x} + c$$

2) $\int \frac{e^x}{e^{2x}+1}\,dx$

$e^x = t$

$\downarrow$

$e^x\,dx = 1\,dt$

$e^{2x} = (e^x)^2 = t^2$

$$\int \frac{1}{t^2+1}\,dt =$$

$$= \arctan(t) + c \Rightarrow \arctan(e^x) + c$$

3) $\displaystyle\int \frac{1+e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$\sqrt{x} = t$

$\downarrow$

$\cancel{2}\cdot \dfrac{1}{\cancel{2}\sqrt{x}}\,dx = dt\cdot 2$

$\dfrac{dx}{\sqrt{x}} = 2\,dt$

$\int (1+e^{t})\,(2\,dt)$

$\Downarrow$

$2\int 1+e^{t}\,dt$

$2^{t} + 2e^{t} + c$

$$2\sqrt{x} + 2e^{\sqrt{x}} + c$$

4) $$\int \frac{3+e^{\sqrt{x}}}{\sqrt{x}}\, dx$$

$$\sqrt{x} = t \Rightarrow x^2 = t \Rightarrow 1 = 2t$$

$$\Rightarrow dx = 2t \cdot dt$$

$$\int \frac{3+e^{t}}{\cancel{t}}\, 2\cancel{t} \cdot dt = \int (6 + 2e^{t})\, dt$$

$$6\int dt + 2\int e^{t}\, dt = 6t + 2e^{t} + c$$

$$= 2(3t + e^{t}) + c \Rightarrow 2(3\sqrt{x} + 2e^{\sqrt{x}}) + c$$

5) $\displaystyle\int \frac{2e^x}{e^x - e^{-x}}\, dx$

$e^x = t \implies x = \ln t \implies 1 = \frac{1}{t} \implies$

$dx = \frac{1}{t}\, dt$

$\displaystyle\int \frac{2e^x}{e^x - \frac{1}{e^x}}\, dx = \int \frac{2t}{t - \frac{1}{t}} \cdot \frac{1}{t}\, dt \implies$

$\displaystyle\int \frac{2}{t - \frac{1}{t}}\, dt = \int \frac{2t}{t^2 - 1}\, dt \implies$

$$\Rightarrow \ln|t^2-1|+c \Rightarrow \ln|e^{2x}-1|+c$$

6) $$\int \frac{3e^x}{1+2e^{2x}}\,dx$$

$$e^x = t \Rightarrow e^x\,dx = 1\,dt$$

$$\int \frac{3e^x}{1+(e^x)^2} = 3\int \frac{t}{1+t^2}\,dt =$$

$$= 3\,\mathrm{ARCTAN}(t)+c = 3\,\mathrm{ARCTAN}(e^x)+c$$

7) $$\int e^{2x-1}\,dx$$

$$t = 2x-1 \Rightarrow dt = 2dx \Rightarrow dx = \frac{1}{2}\,dt$$

$$\int e^{t}\,\frac{1}{2}\,dt = \frac{1}{2}\int e^{t}\,dt =$$

$$= \frac{1}{2}\,e^{t}\,dt = \frac{1}{2}\,e^{2x-1} + c$$

8) $\int x\sqrt{5-x}\,dx$

$$t = \sqrt{5-x} \Rightarrow t^{2} = 5-x \Rightarrow$$

$$x = 5 - t^{2} \Rightarrow dx = -2t\,dt$$

$$\int (5-t^{2})\cdot t\cdot(-2t)\,dt \Rightarrow \int(-10t^{2}+2t^{4})\,dt =$$

$$= -10\,\frac{t^{3}}{3} + 2\,\frac{t^{5}}{5} + c =$$

$$= -\frac{10}{3}(5-x)\sqrt{5-x} + \frac{2}{5}(5-x)^2\sqrt{5-x} + c$$

9) $\frac{\sqrt{x}}{1+x}\,dx$

$\sqrt{x} = t \Rightarrow x = t^2 \Rightarrow dx = 2t\,dt$

$$\int \frac{t}{1+t^2} \cdot 2t\,dt = \int \frac{2t^2}{1+t^2}\,dt =$$

$$= 2\int \frac{t^2}{1+t^2}\,dt = 2\int\left(\frac{t^2+1}{1+t^2} - \frac{1}{1+t^2}\right)dt =$$

$$= 2\int \frac{t^2+1}{1+t^2}\,dt - 2\int \frac{1}{1+t^2}\,dt =$$

$= 2t - 2 \arctan t + c \Rightarrow$

$2\sqrt{x} - 2 \arctan \sqrt{x} + C$

10) $\int x \cdot \sqrt{x-1}\, dx$

$\sqrt{x-1} = t \Rightarrow x-1 = t^2 \Rightarrow$

$dx = 2t\, dt$

$\int (t^2+1) \cdot 2t^2\, dt = \int 2t^4 + 2t^2\, dt =$

$= 2\int t^4\, dt + 2\int t^2\, dt = 2\frac{t^5}{5} + 2\frac{t^3}{3} + c$

$2\left(\frac{t^5}{5} + \frac{t^3}{3}\right) + c \Rightarrow 2\left(\frac{(\sqrt{x-1})^5}{5} + \frac{(\sqrt{x-1})^3}{3}\right)$

$$= \frac{2}{5}\sqrt{(x-1)^5} + \frac{2}{3}\sqrt{(x-1)^3} + c$$