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MATRICI

- STRUTTURA ALGEBRICA
- PROBLEMA MATEMATICO : SISTEMA LINEARE → TEORIA / → METODI NUMERICI (+ COND PROBLEMA)

$$\begin{cases} x + y = 2 \\ 1001x + 1001y = 2001 \end{cases}$$

- INTERPRETAZIONE GEOMETRICA

MATRICE (TABELLA DI NUMERI) → $M = \overbrace{\begin{pmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & & \end{pmatrix}}^{n = \text{colonne}} \Big\} m = \text{righe}$

$m \times n$

LA MATRICE POSSO DEFINIRLA COME:

$$M = (a_{ij})_{\substack{i=1 \dots m \\ j=1 \dots n}} \quad \text{con } a_{ij} = \frac{1}{i+j-1}$$

$$M = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \dots \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \dots \\ \frac{1}{3} & \dots & & \end{pmatrix}$$

MATRICE DI HILBERT

ESEMPIO:

$$\begin{pmatrix} 1/2 & 0 & -2 \\ 45 & -7/11 & 1 \end{pmatrix} \in \mathbb{R}^{2\times 3}$$

$$\begin{pmatrix} 1 & 0 \\ -\sqrt{2} & 1 \end{pmatrix} \in \mathbb{R}^{2\times 2}$$

CASI PARTICOLARI DIMENSIONE

$m = n \longrightarrow$ **MATRICE QUADRATA**

$m = 1 \longrightarrow R_i = \begin{pmatrix} a_{i1} & a_{i2} & \dots & a_{in} \end{pmatrix} \in \mathbb{R}^{1\times n}$

$n = 1 \longrightarrow C_j = \begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{pmatrix} \in \mathbb{R}^{m\times 1}$

(COLONNA)

$\downarrow$

VETTORE

$$\begin{cases} x + y = 2 \\ 1001x + 1000y = 2001 \end{cases} \Longrightarrow A = \begin{pmatrix} 1 & 1 \\ 1001 & 1000 \end{pmatrix}$$

$$b = \begin{pmatrix} 2 \\ 2001 \end{pmatrix} \qquad x = \begin{pmatrix} x \\ y \end{pmatrix}$$

MATRICE TRIANGOLARE SUPERIORE

SE HA TANTI 0 E' PARTICOLARE SE HA SOLO 0 IN QUESTA DIAGONALE:

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ 0 & a_{22} & \dots & a_{2m} \\ 0 & 0 & \ddots & \vdots \\ \vdots & & \ddots & \\ 0 & \dots & \dots & a_{mm} \end{pmatrix}$$

MATRICE TRIANGOLARE INFERIORE

$$\begin{pmatrix} * & 0 & 0 \\ * & * & 0 \\ * & * & * \end{pmatrix}$$

$*$ = NUMERI

MATRICE DIAGONALE

$$\begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & \\ \vdots & & \ddots & \\ 0 & \dots & 0 & a_{mm} \end{pmatrix}$$

MATRICE IDENTICA

(O IDENTITA')

$$I_n = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \in \mathbb{R}^{n \times n}$$

$$X = \begin{pmatrix} 2 & 3 & 1 \\ -1 & 0 & 5 \end{pmatrix} \in \mathbb{R}^{2 \times 3} \Longrightarrow X^t = \begin{pmatrix} 2 & -1 \\ 3 & 0 \\ 1 & 5 \end{pmatrix} \in \mathbb{R}^{3 \times 2}$$

(TRASPOSTA)

$$X = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \in \mathbb{R}^{3 \times 3} \longrightarrow X^t = \begin{pmatrix} 1 & & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \in \mathbb{R}^{3 \times 3}$$

X SIMMETRICA SE $X = X^t$ $\left(\Rightarrow X \in \mathbb{R}^{n \times n} \right)$

ESEMPIO:

$$\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow D = D^t$$

$$X = \begin{pmatrix} 7 & 5 & 4 \\ 5 & 1 & 0 \\ 4 & 0 & -2 \end{pmatrix}$$ SIMMETRICA

SOMMA (1)

OPERAZIONI → MOLTIPLICAZIONE PER "SCALARE" ⟷ NUMERO REALE (2)

→ PRODOTTO (O MOLTIPLICAZIONE) (3)

(1) $A, B \in \mathbb{R}^{m \times n} \longmapsto A + B \in \mathbb{R}^{m \times n}, \; A+B = (a_{ij} + b_{ij})$

$A = (a_{ij})$, $B = (b_{ij})$

$i = 1 \dots m$

$j = 1 \dots n$

ESEMPIO

$$X = \begin{pmatrix} 2 & 3 \\ -1 & 0 \\ 6 & 4 \end{pmatrix}, \quad Y = \begin{pmatrix} 5 & -3 \\ 1 & 2 \\ -4 & -5 \end{pmatrix} \Rightarrow X+Y = \begin{pmatrix} 7 & 0 \\ 0 & 2 \\ 2 & 1 \end{pmatrix}$$

$\in \mathbb{R}^{3 \times 2}$ $\qquad \in \mathbb{R}^{3 \times 2}$ $\qquad \in \mathbb{R}^{3 \times 2}$

(2) $X \in \mathbb{R}^{m \times n}, \; \lambda \in \mathbb{R} \longmapsto \lambda X \in \mathbb{R}^{m \times n}, \; \lambda X = (\lambda x_{ij})$

$i = 1 \dots m$

$j = 1 \dots n$

ESEMPIO

$$X = \begin{pmatrix} 2 & 3 \\ -1 & 0 \\ 6 & 4 \end{pmatrix} \longmapsto 3X = \begin{pmatrix} 6 & 9 \\ -3 & 0 \\ 18 & 12 \end{pmatrix}$$

CHIUSURA

A, B DIAGONALI $\begin{pmatrix} \ddots & 0 \\ 0 & \ddots \end{pmatrix}$

$A+B$, λA DIAGONALI

A, B TRIANGOLI SUP $\begin{pmatrix} \ddots & m \\ 0 & \ddots \end{pmatrix}$

$\Rightarrow A+B$, λA TR. SUP.

A, B SIMMETRICHE $\Rightarrow$

$A+B$ SIMMETRICA

PROPRIETÀ

$(A+B)^t = A^t + B^t$, $(\lambda A)^t =$
$= \lambda (A^t)$

$(A+B)+C = A+(B+C)$
(ASSOCIATIVA)

$A+B = B+A$ (COMMUTATIVA)

$A+0 = A$ DOVE $0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

MATRICE NULLA

$\forall A \; \exists$ "$-A$" (OPPOSTO) T.C.
$A + (-A) = 0 \rightarrow -A = (-a_{ij})$

$\alpha(\beta x) = (\alpha\beta x)$, $\lambda(A+B) =$
$= \lambda A + \lambda B = (\alpha+\beta)x = \alpha x + \beta x$

③ PRODOTTO RIGA × COLONNA

$$(a_1 \dots a_m) \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$$

$$= a_1 b_1 + a_2 b_2 + a_3 b_3 + \dots + a_m b_m$$

ESEMPIO

$$(2\ 3\ 0) \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix} = -4 + 12 + 0 = 8$$

$$A = (a_{ij}) \in \mathbb{R}^{m \times \underline{m}}, \quad B = (b_{ij}) \in \mathbb{R}^{\underline{m} \times r}$$

$$\underbrace{R_i^A}_{\text{RIGA}} = (a_{i1} \dots a_{im}) \qquad \underbrace{C_j^B}_{\text{COLONNA}} = \begin{pmatrix} b_{ij} \\ b_{mj} \end{pmatrix}$$

$$\Rightarrow R_i^A \cdot C_j^B \rightsquigarrow AB = (a) = \mathbb{R}^{m \cdot r}$$

ESEMPIO

$$A = \begin{pmatrix} 2 & 3 & 0 \\ -1 & 7 & 5 \end{pmatrix} \in \mathbb{R}^{2\times 3}, \quad B = \begin{pmatrix} -2 & 1 \\ 4 & 0 \\ 5 & -3 \end{pmatrix} \in \mathbb{R}^{3\times 2}$$

$$\Rightarrow AB = \begin{pmatrix} \boxed{8} & 2 \\ \boxed{55} & \boxed{-16} \end{pmatrix} \in \mathbb{R}^{2\times 2}$$

$$R_1^A \cdot C_1^B = (2\ 3\ 0)\begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix} = -4+12+0 = 8$$

$$R_1^A C_2^B = (2\ 3\ 0)\begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix} = 2+0+0 = 2$$

$$R_2^A C_1^B = (-1\ 7\ 5)\begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix} = 2+28+25 = 55$$

$$R_2^A C_2^B = (-1\ 7\ 5)\begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix} = -1+0+(-15) = -16$$

ESEMPIO

$$\underset{3\times 2}{B} \cdot \underset{2\times 3}{A} \in \mathbb{R}^{3\times 3} \qquad B = \begin{pmatrix} -2 & 1 \\ 4 & 0 \\ 5 & -3 \end{pmatrix} \quad A = \begin{pmatrix} 2 & 3 & 0 \\ -1 & 7 & 5 \end{pmatrix}$$

$$\Downarrow$$

$$BA = \begin{pmatrix} -5 & 1 & 5 \\ 8 & 12 & 0 \\ 13 & -6 & -15 \end{pmatrix}$$

$$v = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \quad v^t \cdot v = 6$$

$$v \cdot v^t = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \\ 2 & -2 & 4 \end{pmatrix}$$

$$v^t = \begin{pmatrix} 1 & -1 & 2 \end{pmatrix}$$

$$v = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & -1 \end{pmatrix}, \quad D = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix}$$

u_1 u_2 w_3

CALCOLARE

$$(v \cdot D) v^t$$

$$\alpha \cdot u_1 \cdot u_1^t + \beta u_2 u_2^t + \gamma u_3 u_3$$