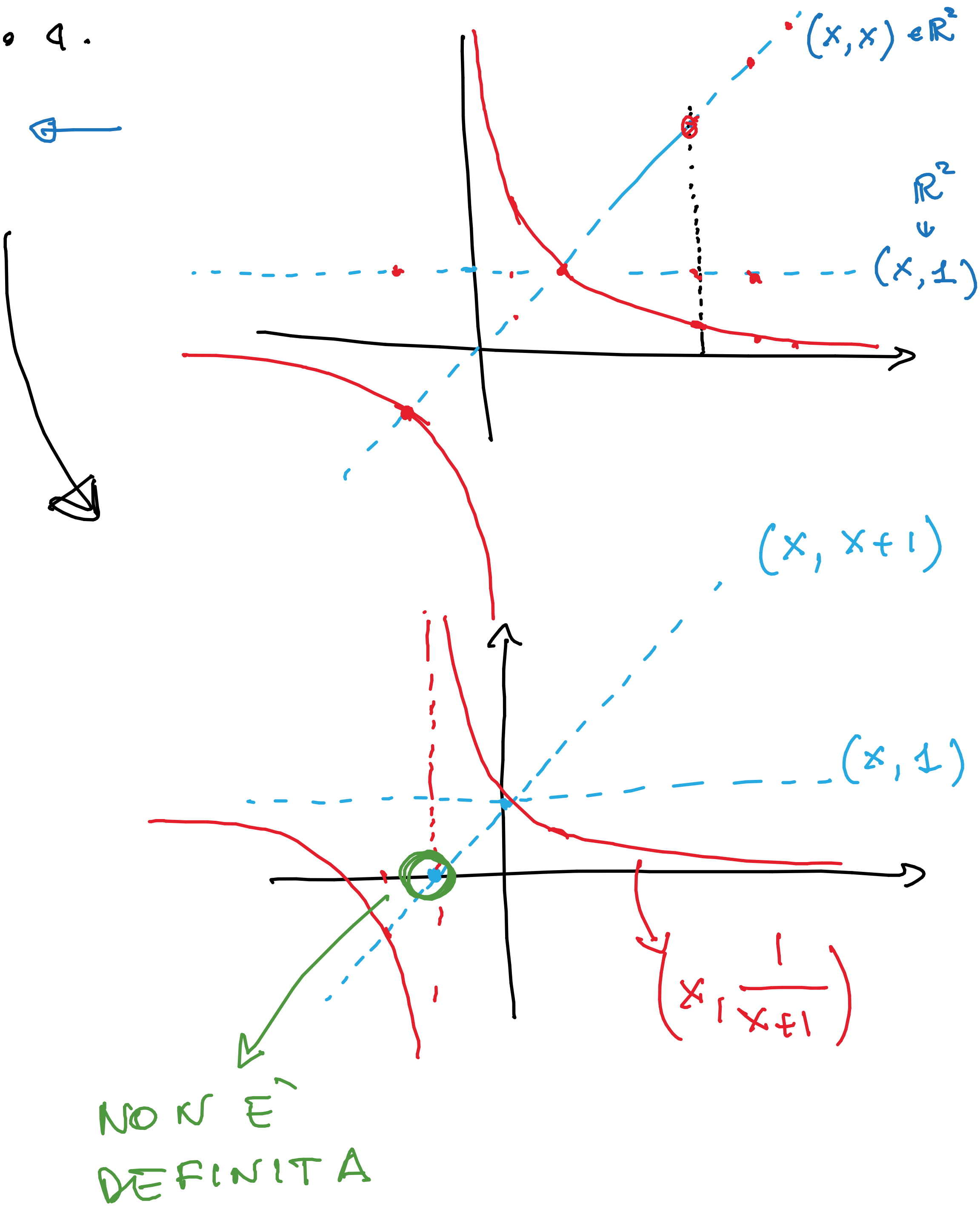
Es 1 foglio 4.

$$f(x) = \frac{1}{x+1}$$



$$\mathrm{Dom}(f) = \{x \in \mathbb{R} : x+1 \neq 0\}$$

$$= \{x \in \mathbb{R} : x \neq -1\}$$

$$= (-\infty, -1) \cup (-1, +\infty)$$

$$\lim_{x \to -\infty} \frac{1}{x+1} = 0^-$$

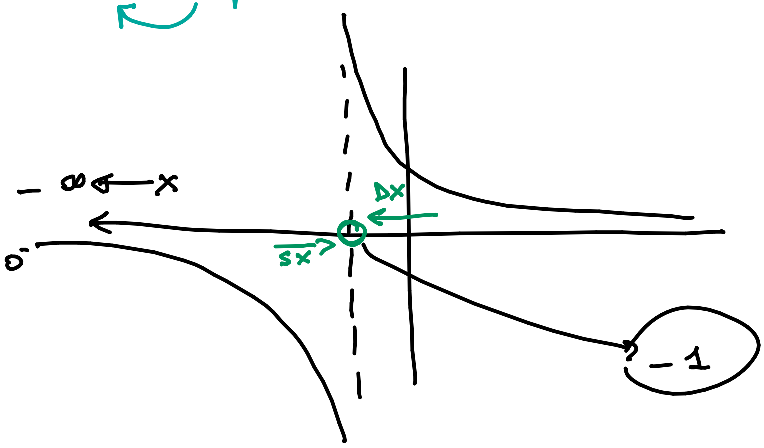
N numero arbitrariamente grande

" a piacere

"

$(\forall N > 0)$ $(\forall N < 0)$

$\frac{1}{N}$ (grande) $= \frac{1}{N}$ (piccolo)



$$\lim_{x \to -1^-} \frac{1}{x+1} = \left\{ \frac{1}{-1{,}1+1} = +\frac{1}{-0{,}1} \right\} = -\infty$$

$f(-1.1)$

$-1{,}1 \longrightarrow f(-1.1) = \frac{1}{-0.1}$

$-1{,}01 \longrightarrow f(-1{,}01) = \frac{1}{-0.01}$

$-1{,}001$ < -1

....

$-1{,}0 \ldots 01 \longrightarrow f(-1.0\ldots01) = \frac{1}{-0.0\ldots01}$

$\forall N > 0 \quad \exists \delta > 0 : 0 < |x - x_0| < \delta$

$x < x_0$

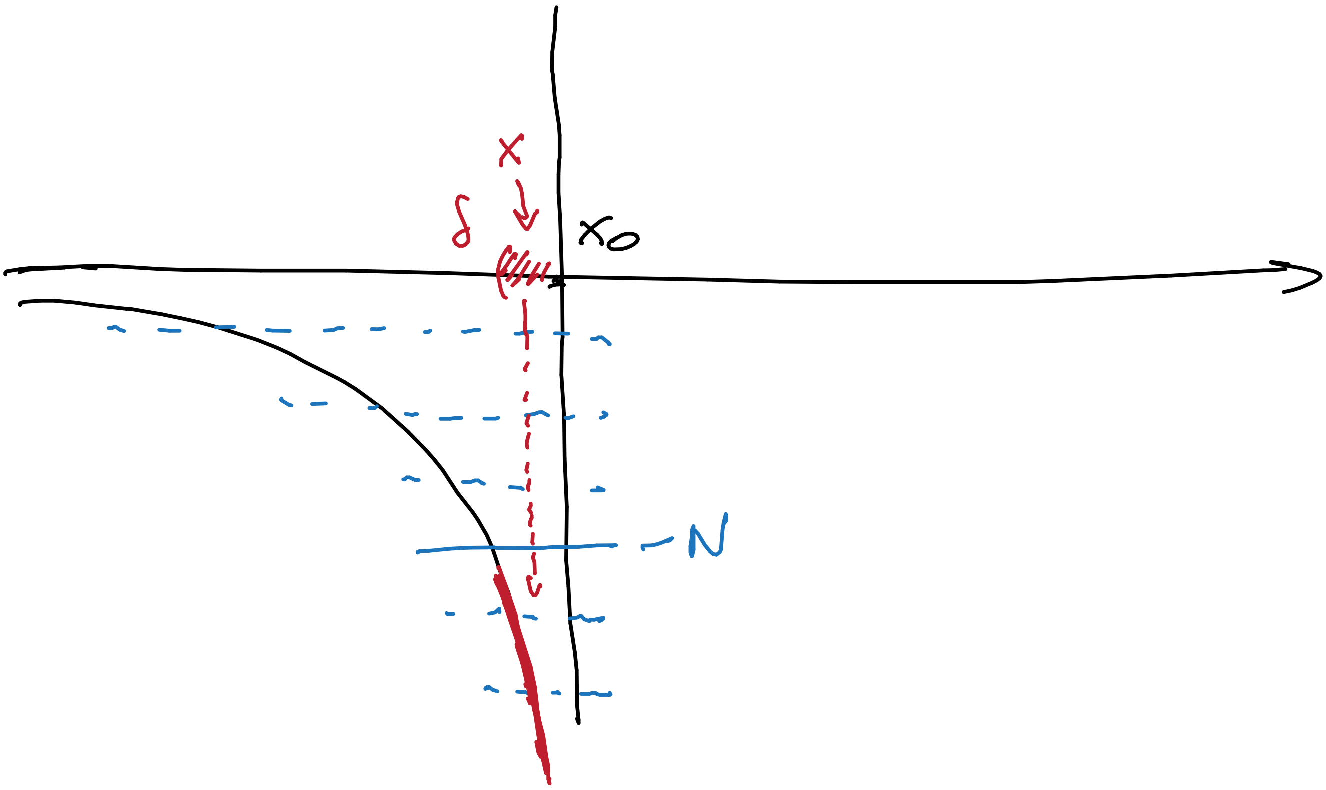
$x > x_0$

$\Rightarrow \quad f(x) < -N$

$x \in (x_0 - \delta, x_0 + \delta)$

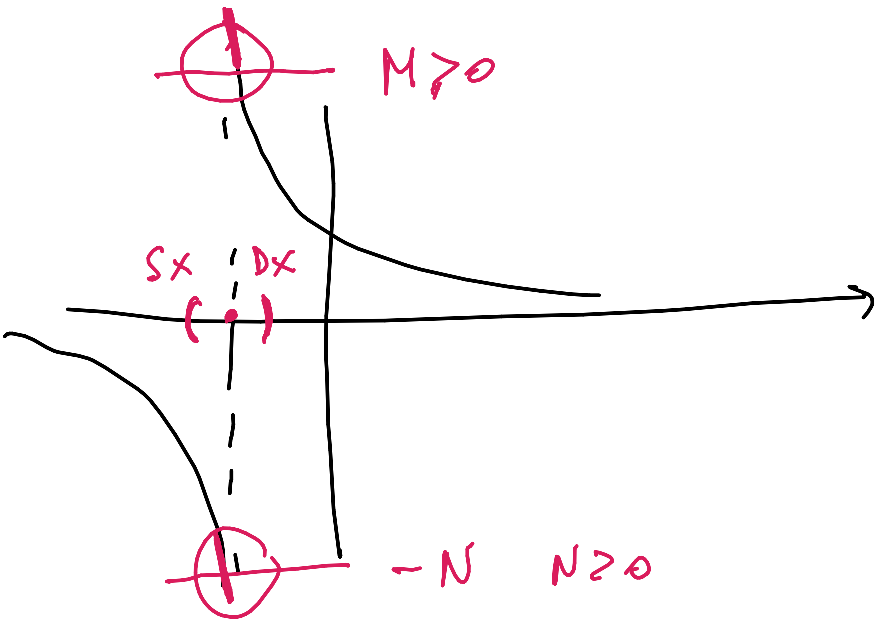
$x \neq x_0$

$$\boxed{\lim_{x \to x_0} f(x) = -\infty}$$



$$\lim_{x \to -1^+} \frac{1}{x+1} = +\infty$$

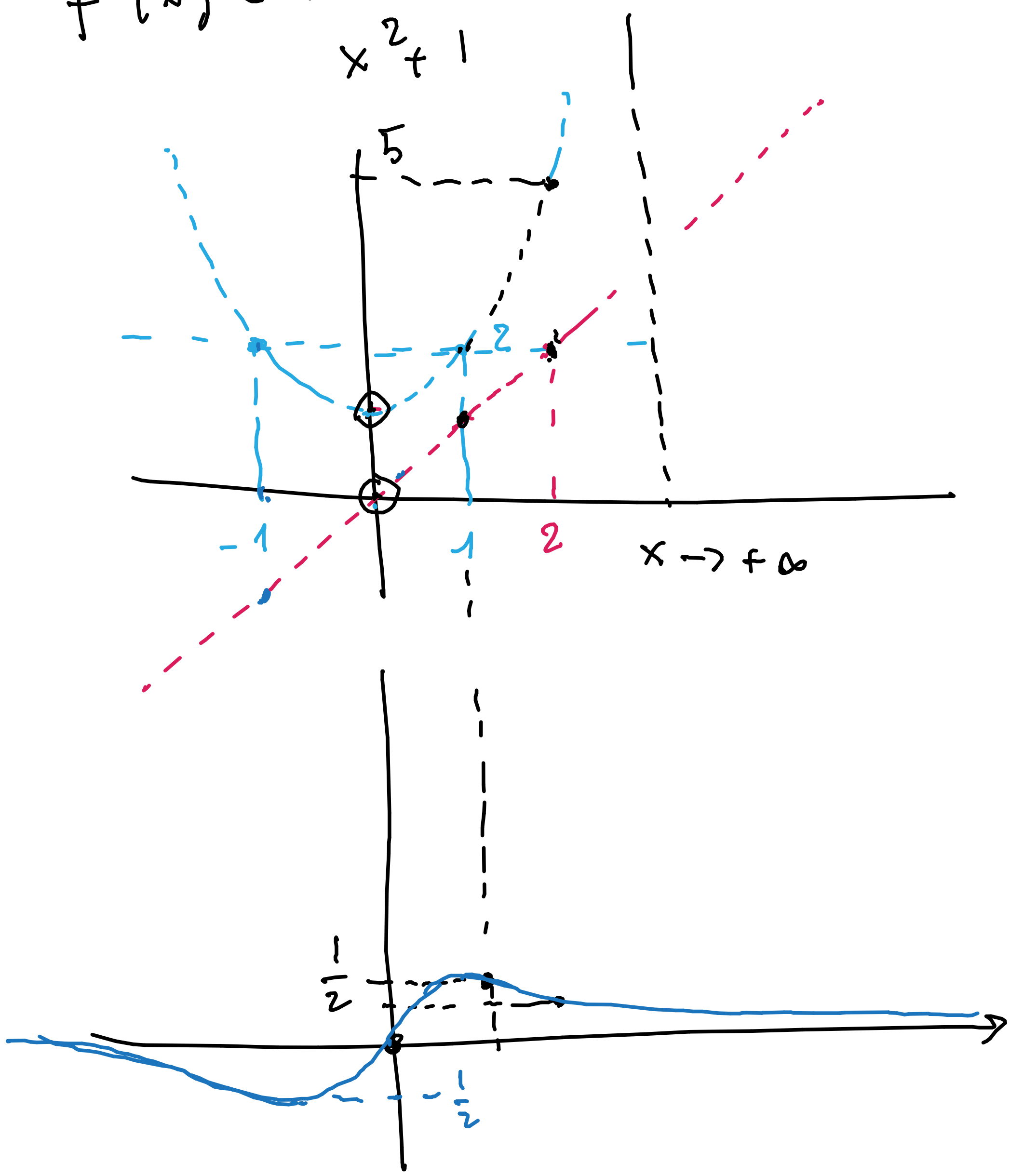
$$-1^+ \begin{cases} -0.9 \\ -0.99 \\ -0.999 \\ \vdots \\ -0.9\ldots9 \end{cases} \xrightarrow{f(x)} \quad \frac{1}{-0.9+1} = \frac{1}{0.1} \quad \vdots \quad \frac{1}{-0.9\ldots9+1} = \frac{1}{0.0\ldots1}$$



$$\lim_{x \to +\infty} \frac{1}{x+1} = 0^+$$

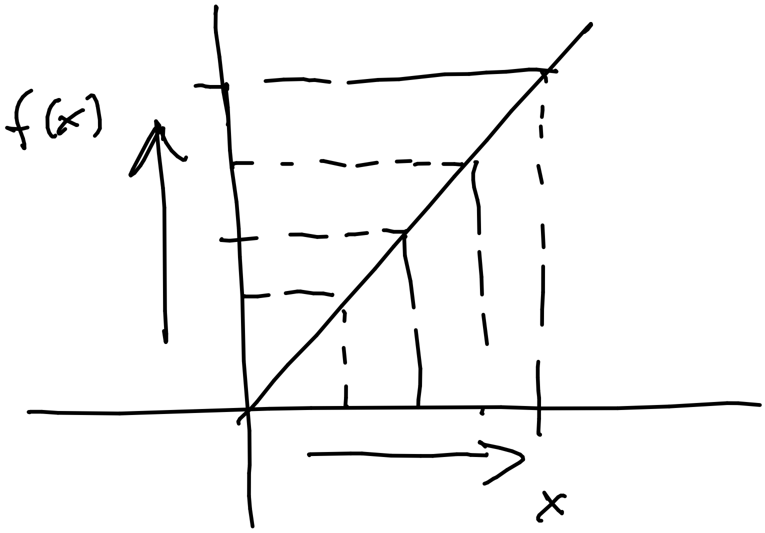
Es 1 n 2.

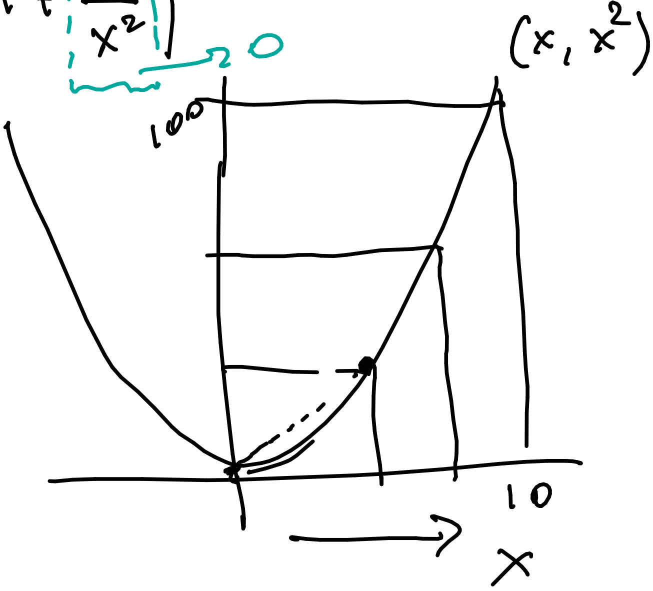
$$f(x) = \frac{x}{x^2+1}$$



$$\mathrm{Dom}(f) = \mathbb{R} = (-\infty, +\infty)$$

$$\lim_{x \to +\infty} \frac{x}{x^2+1} = \lim_{x \to +\infty} \frac{\cancel{x}\ 1}{x^{\cancel{2}}\left(1+\underbrace{\boxed{\frac{1}{x^2}}}_{\to 0}\right)} = 0^+$$

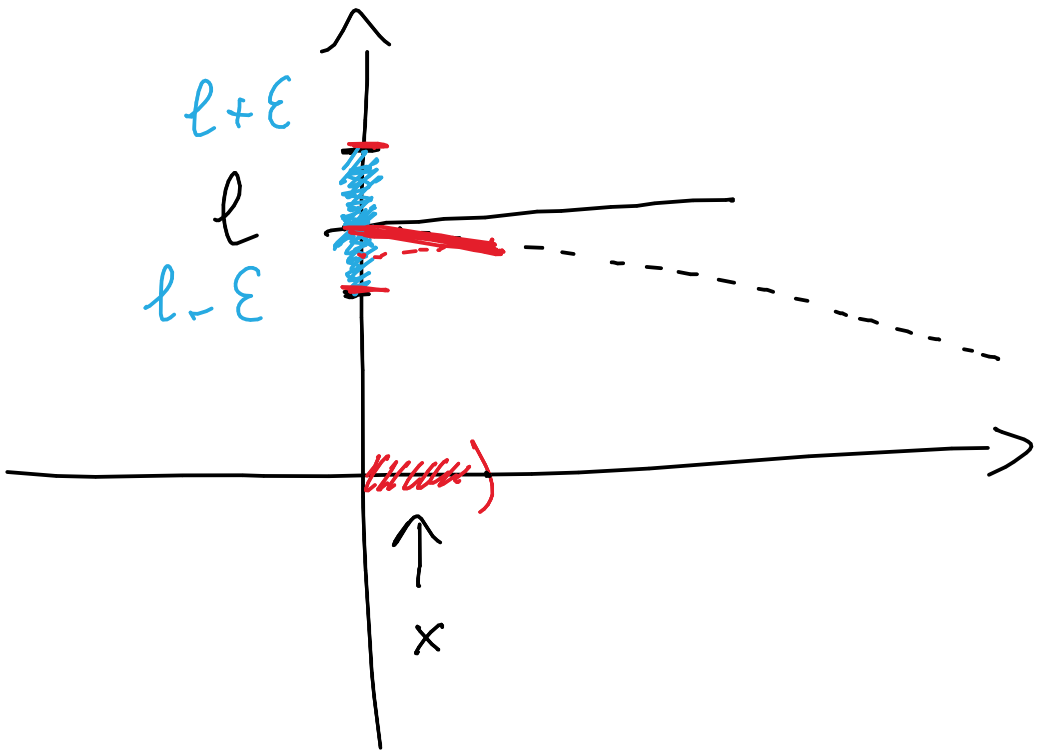




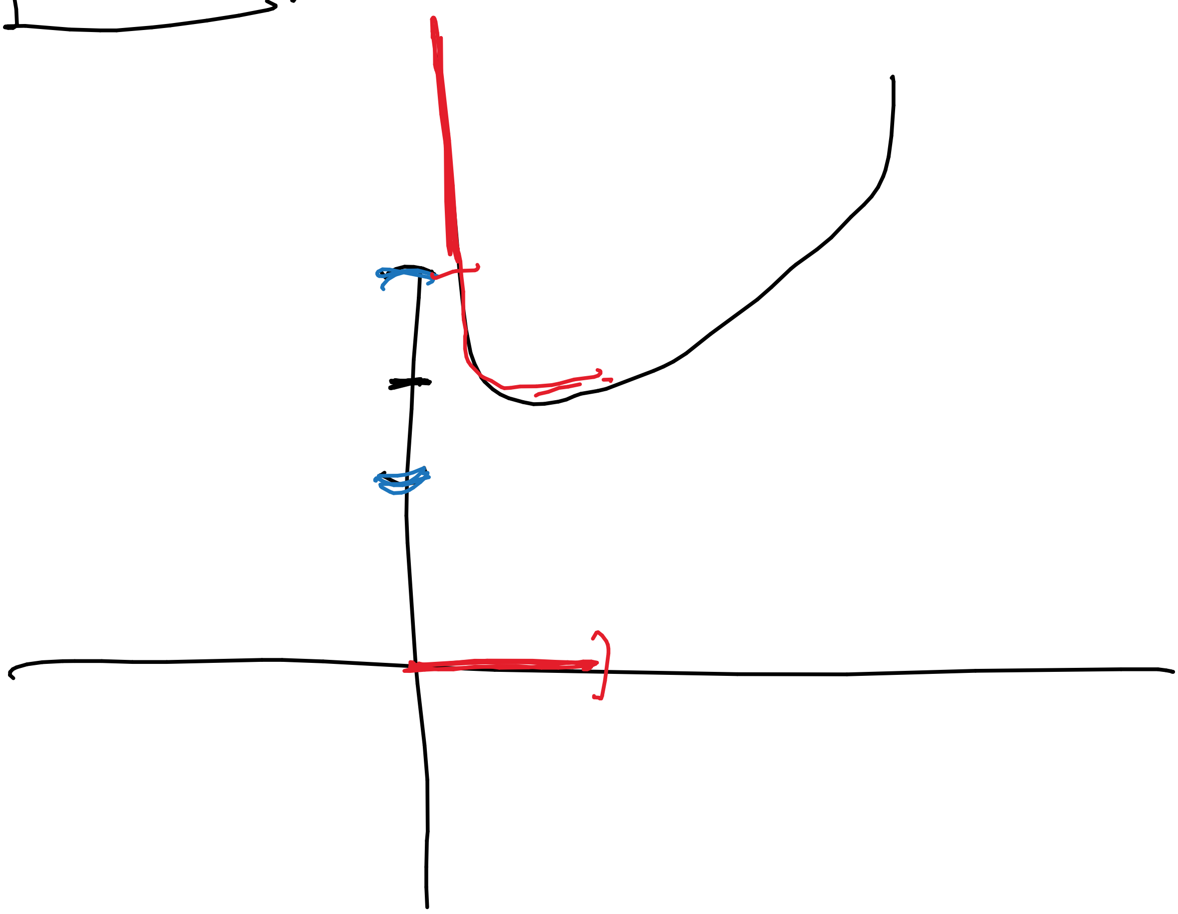
$$\lim_{x \to -\infty} \frac{x}{x^2+1} = \lim_{x \to -\infty} \frac{\cancel{x}\ 1}{x^{\cancel{2}}\left(1+\underbrace{\boxed{\frac{1}{x^2}}}_{\to 0}\right)} = 0^-$$

$$\forall \varepsilon>0 \quad \exists \delta>0 : \quad \begin{array}{l} 0<|x-x_0|<\delta \\ \text{e } \ x\in \mathrm{Dom}(f) \end{array} \Rightarrow |\underbrace{f(x)}|-\ell|<\varepsilon$$

$\mathrm{Dom}(f) = (0,+\infty)$



$\boxed{x_0=0}$



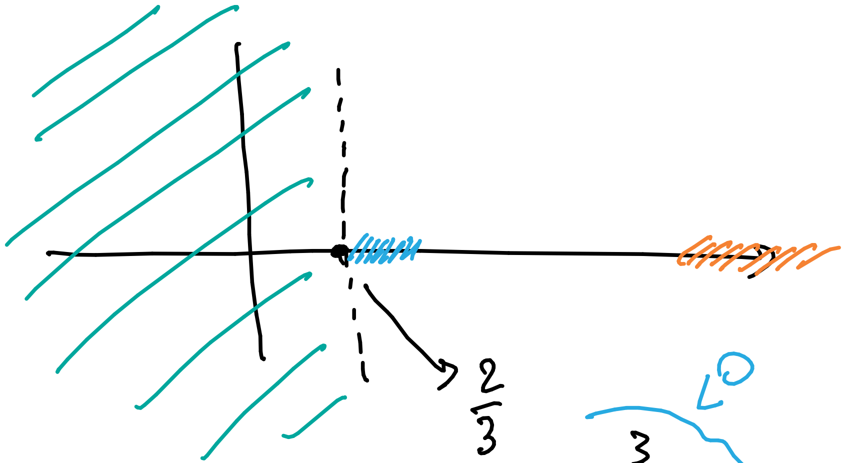
$$f(x) = \frac{x^3 - 1}{\sqrt{3x-2}}$$

$$Dom(f) = \left(\frac{2}{3}, +\infty\right)$$

$3x - 2 \geq 0$ argomento della radice

$\sqrt{3x-2} \neq 0$ argomento del denominatore

$$3x - 2 > 0 \Rightarrow x > \frac{2}{3}$$



$$\frac{8}{27} - 1 = -\frac{19}{27}$$

$$\lim_{x \to \frac{2}{3}} \frac{x^3 - 1}{\sqrt{3x-2}} = \frac{\overbrace{\left(\frac{2}{3}\right)^3 - 1}^{<0}}{\underbrace{\sqrt{3x-2}}_{0^+}} = -\infty$$

$$\lim_{x \to +\infty} \frac{x^3 - 1}{\sqrt{3x-2}} = \lim_{x \to +\infty} \frac{x^3\left(1 - \frac{1}{x^3}\right)}{\sqrt{x\left(3 - \frac{2}{x}\right)}}$$

$$= \lim_{x \to +\infty} \frac{\overbrace{x^{3-\frac{1}{2}}}^{x^{5/2}}}{\underbrace{x^{\frac{1}{2}-\frac{1}{2}}}_{=1}} \boxed{\frac{1 - \underbrace{\frac{1}{x^3}}_{\to 0}}{\sqrt{3 - \underbrace{\frac{2}{x}}_{\to 0}}}}_{\to \frac{1}{\sqrt{3}}} = +\infty$$

$$f(x) = \frac{x+1}{\sqrt{2x^2+3}}$$

$\sqrt{2x^2+3} \neq 0$ frazione $\Rightarrow$ $2x^2+3 > 0 \quad \forall x \in \mathbb{R}$

$2x^2+3 \geq 0$ radice

$\mathrm{Dom}(f) = \mathbb{R}$

$$\lim_{x\to-\infty} \frac{x+1}{\sqrt{2x^2+3}} = \lim_{x\to-\infty} \frac{x\left(1+\frac{1}{x}\right)}{\sqrt{x^2\left(2+\frac{3}{x^2}\right)}} =$$

$$\lim_{x\to-\infty} \frac{x}{\sqrt{x^2}} \frac{\left(1+\frac{1}{x}\right)}{\sqrt{2+\frac{3}{x^2}}} = \quad \left\{ \frac{x}{\sqrt{x^2}} \approx \frac{-\infty}{\sqrt{(-\infty)^2}} \right\}$$

$\frac{1}{x} \to 0$, $\frac{3}{x^2} \to 0$

PER FARLO FORMALMENTE SERVE UN CAMBIO DI VARIABILE

PRIMA FACCIAMO $x \to +\infty$

$$\lim_{x\to+\infty} \frac{x+1}{\sqrt{2x^2+3}} = \lim_{x\to+\infty} \frac{x}{\sqrt{x^2}} \frac{\left(1+\frac{1}{x}\right)}{\sqrt{2+\frac{3}{x^2}}} =$$

PER $x \to \infty$ (IN UN INTORNO DI $+\infty$)

$x \in (a, +\infty)$ per $a > 0$

$\boxed{\sqrt{x^2} = x} \quad \forall x > 0$

$$= \lim_{x\to+\infty} \frac{\cancel{x}}{\cancel{x}} \frac{1+\frac{1}{x}}{\sqrt{2+\frac{3}{x^2}}} = \frac{1}{\sqrt{2}}$$

$\frac{1}{x} \to 0$, $\frac{3}{x^2} \to 0$

Per $x \to -\infty$: cambio $z = -x$

$$\lim_{x\to-\infty} \frac{x+1}{\sqrt{2x^2+3}} = \lim_{z\to+\infty} \frac{-z+1}{\sqrt{2(-z)^2+3}} =$$

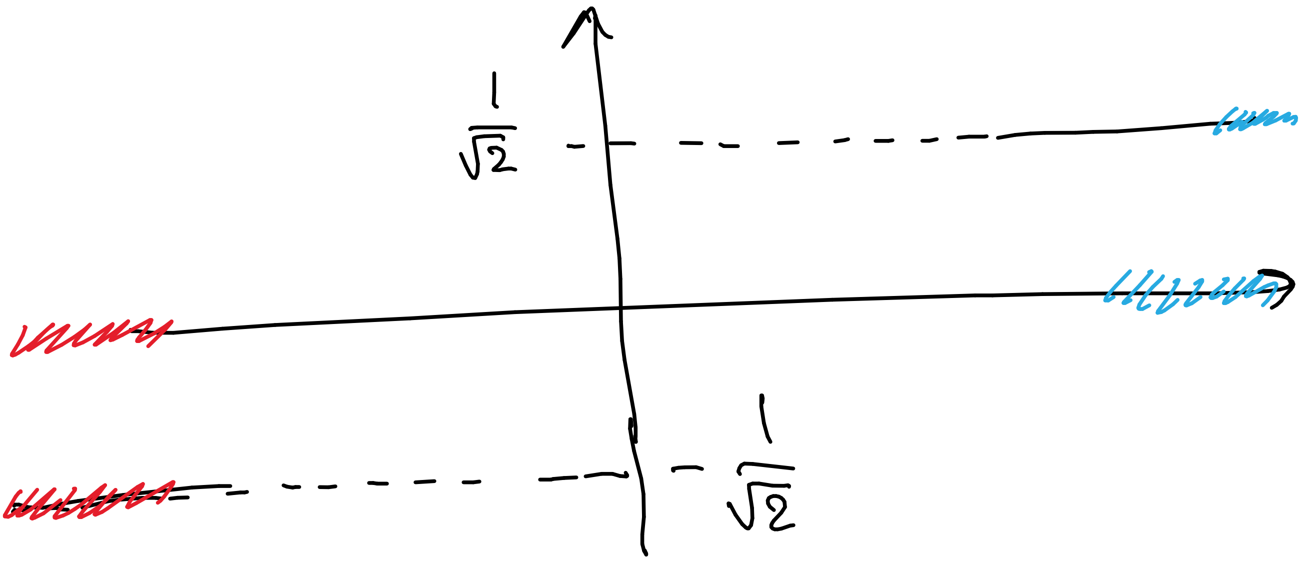
$$= \lim_{z\to+\infty} \frac{-z+1}{\sqrt{2z^2+3}} = \lim_{z\to+\infty} \frac{-z}{\sqrt{z^2}} \frac{\left(1-\frac{1}{z}\right)}{\sqrt{2-\frac{3}{z^2}}}$$

$z \geq 0$

$(\sqrt{z^2} = z)$

$$= \lim_{z\to+\infty} -\frac{\cancel{z}}{\cancel{z}} \frac{\left(1-\frac{1}{z}\right)}{\sqrt{2-\frac{3}{z^2}}} = -\frac{1}{\sqrt{2}}$$

$\frac{1}{z} \to 0$, $\frac{3}{z^2} \to 0$



Es 6.

1) $\lim_{x \to 1} \frac{1}{(x-1)^2}$ $\quad$ $Dom(f) = (-\infty, 1) \cup (1, +\infty)$

sia da dx

sia da sx

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x)$$

$$\lim_{x \to 1^+} \frac{1}{\underbrace{(x-1)^2}_{0^+}} = +\infty \qquad \lim_{x \to 1^-} \frac{1}{\underbrace{(x-1)^2}_{0^-}} = +\infty$$

sono uguali

$$\Rightarrow \lim_{x \to 1} \frac{1}{(x-1)^2} = +\infty$$

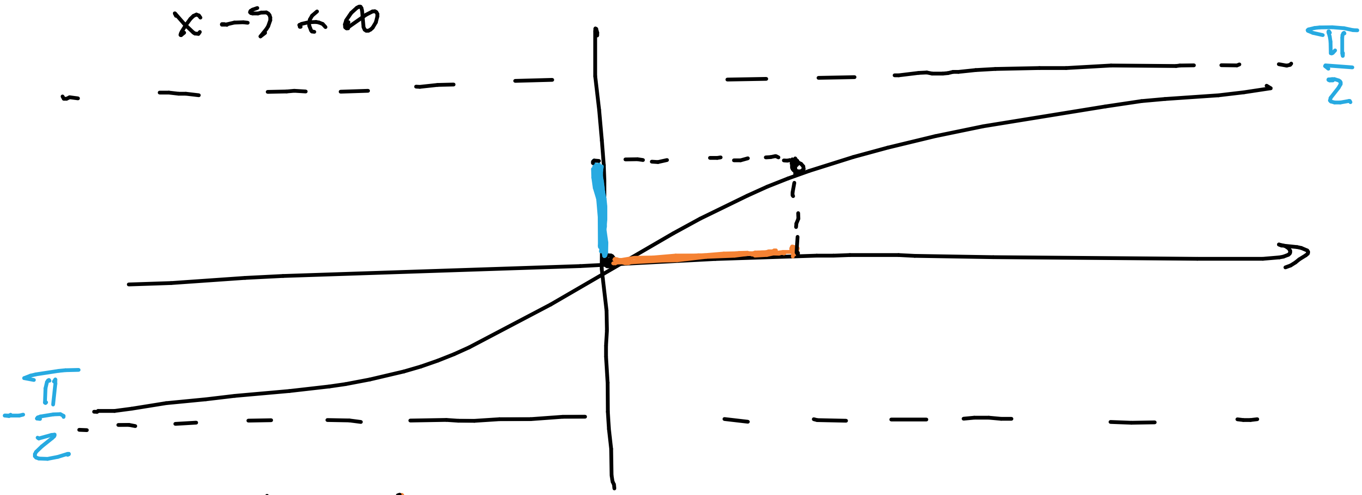
Caso $\frac{1}{(x-1)^p}$ con $p = 2, 4, 6, \dots$ (pari)

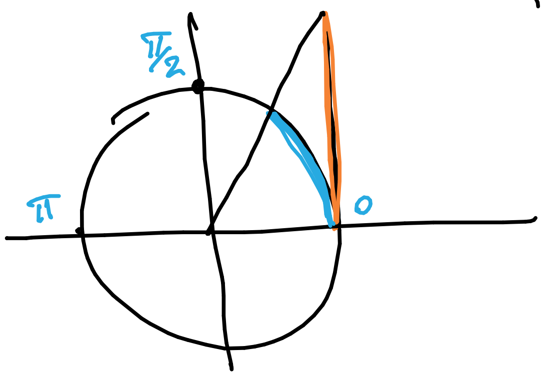
il limite a 1 esiste

Caso $\frac{1}{(x-1)^d}$ con $d = 1, 3, 5 \dots$ (dispari)

il limite a 1 non esiste

2) $$\lim_{x\to+\infty} (\arctan(x))^{1/3} = \left(\frac{\pi}{2}\right)^{\frac{1}{3}}$$





3) $$\lim_{x\to+\infty} x\,2^x = \lim_{x\to+\infty} x\,e^{x\ln 2} = +\infty$$

CONVERTIRE IN e^x : $a^x = e^{x\ln a}$

3a) $$\lim_{x\to+\infty} \frac{2^x}{x} = \lim_{x\to+\infty} \frac{e^{x\ln 2}}{x} =$$

$$\lim_{x\to+\infty} \ln 2\, \frac{e^{x\ln 2}}{x\ln 2} \overset{y=x\ln 2}{=} \lim_{y\to+\infty} \ln 2\, \frac{e^{y}}{y} = +\infty$$

Usiamo il limite notevole:

$$\boxed{\lim_{x\to+\infty} \frac{e^x}{x^N} = +\infty \quad \forall N>0}$$

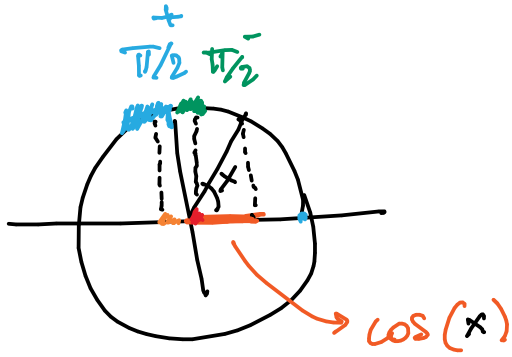
$N=1$

$$\lim_{x\to 0}\frac{3x+1}{x}$$ NON ESISTE

$$\lim_{x\to 0^+}\frac{3x+1}{x}=\frac{1}{0^+}=+\infty$$

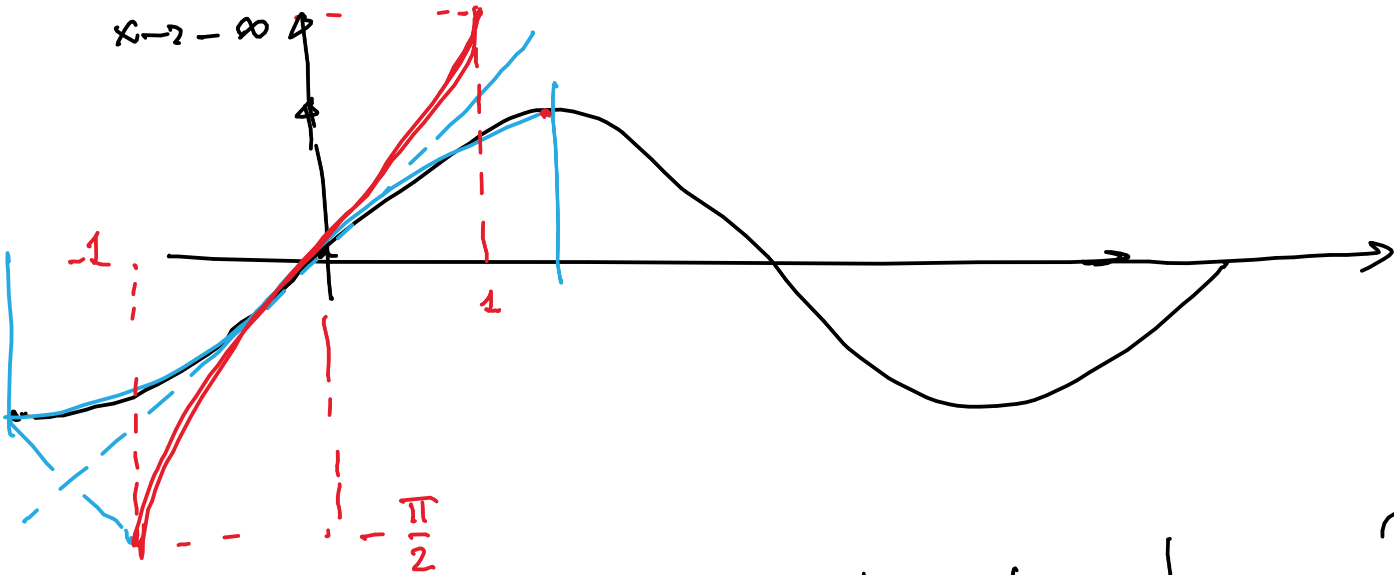
$$\lim_{x\to 0^-}\frac{3x+1}{x}=\frac{1}{0^-}=-\infty$$

7) $$\lim_{x\to\left(\frac{\pi}{2}\right)^+}\frac{1}{\cos x}=\frac{1}{0^-}=-\infty$$



8) $$\lim_{x\to\left(\frac{\pi}{2}\right)^-}\frac{1}{\cos x}=\frac{1}{0^+}=+\infty$$

9) $$\lim \arcsin\left(\frac{1}{1-x^2}\right)$$



$$\mathrm{Dom}(f)=\left\{x\in\mathbb{R} : \left|\frac{1}{1-x^2}\right|\le 1\right\}$$

$$\frac{1}{1-x^2} < 1$$

$$\frac{1}{1-x^2} > -1$$