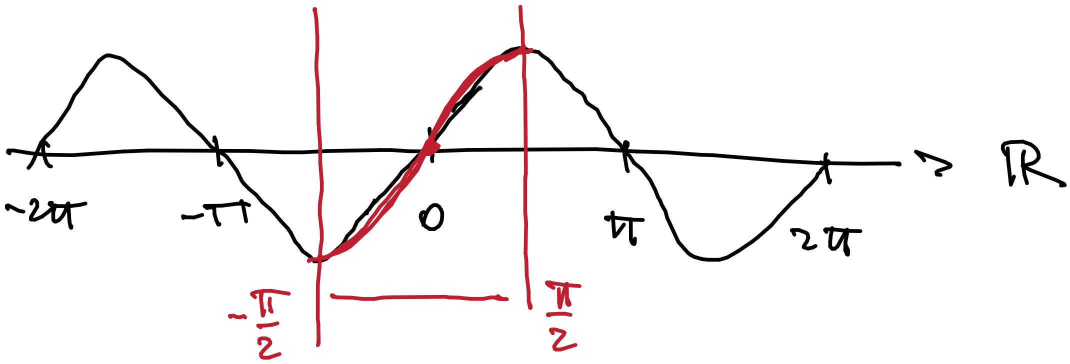
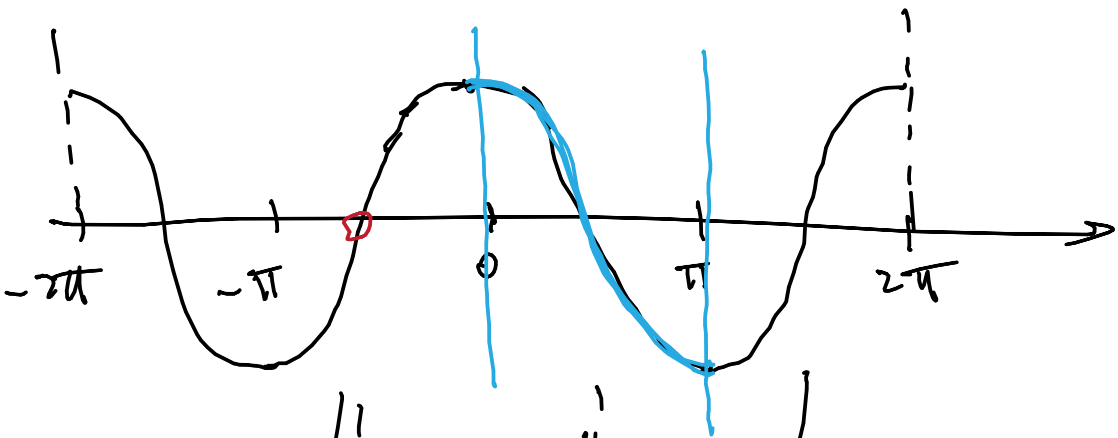
Restrizioni al dominio:

- frazione
- radice pari
- logaritmo
- tangente $\left(\frac{\sin x}{\cos x}\right)$ rientra nella frazione.
- arcsin.
- arccos.

→ sin

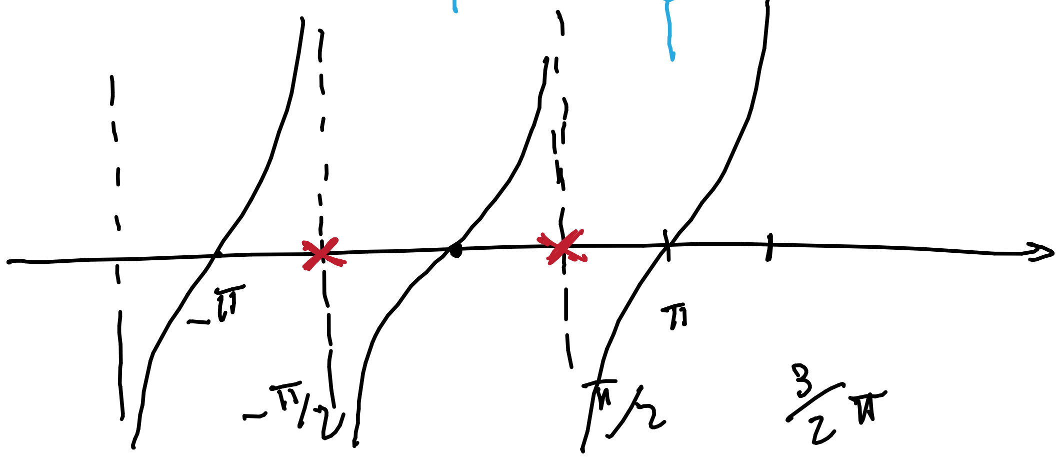


→ cos



→ tg

$\frac{\sin x}{\cos x}$



Es 1 : foglio 3.

$f(x) = x^2 + 1$

$g_a : \mathbb{R} \longrightarrow \mathbb{R}$. $g_a(x) = x + a$.

1) $(f \circ g_a)(3) = 2$

$f(x) = x^2 + 1$

$g_a(x) = x + a = y$ $\quad f(y) = y^2 + 1$

$y = g_a(x) = x + a$

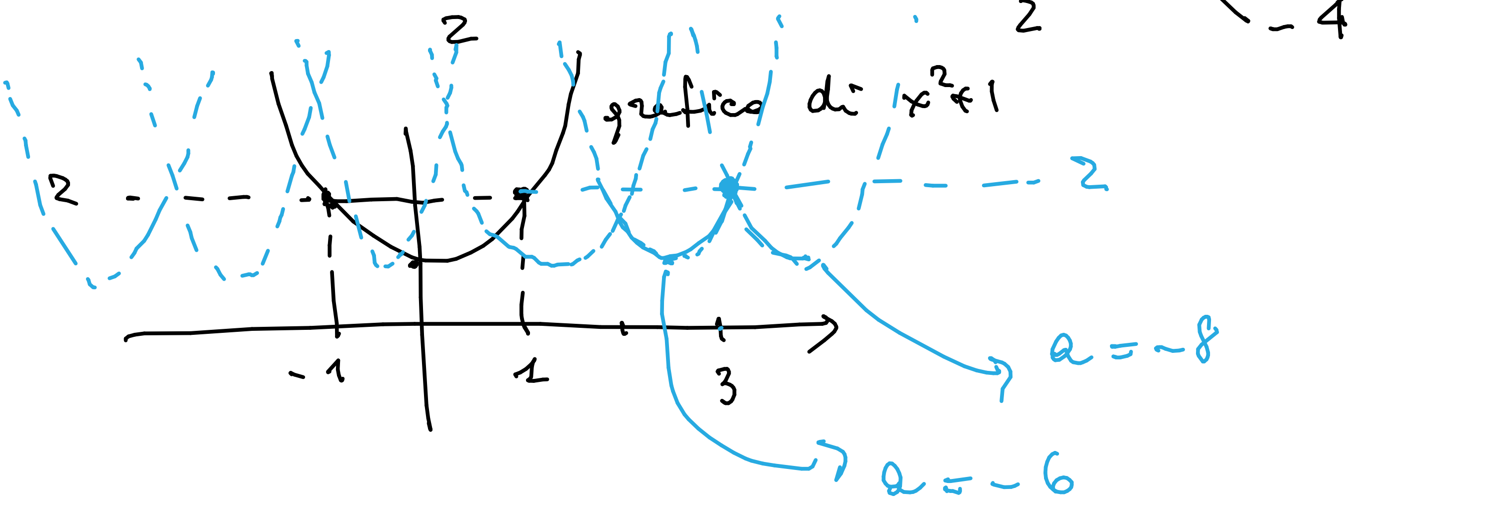
$(f \circ g_a)(x) = (x+a)^2 + 1$

$= x^2 + 2ax + a^2 + 1$

calcolare in $x=3$ $\quad = (3+a)^2 + 1$ deve essere $= 2$

$a^2 + 6a + 10 = 2 \qquad a^2 + 6a + 8 = 0$

$$a = \frac{-6 \pm \sqrt{36 - 32}}{2} = \frac{-6 \pm 2}{2} \begin{cases} -2 \\ -4 \end{cases}$$



$f(x) = x^2 + 1$

$f(\underbrace{y}) = y^2 + 1$

$\boxed{y} = h_a(x) = \boxed{ax}$

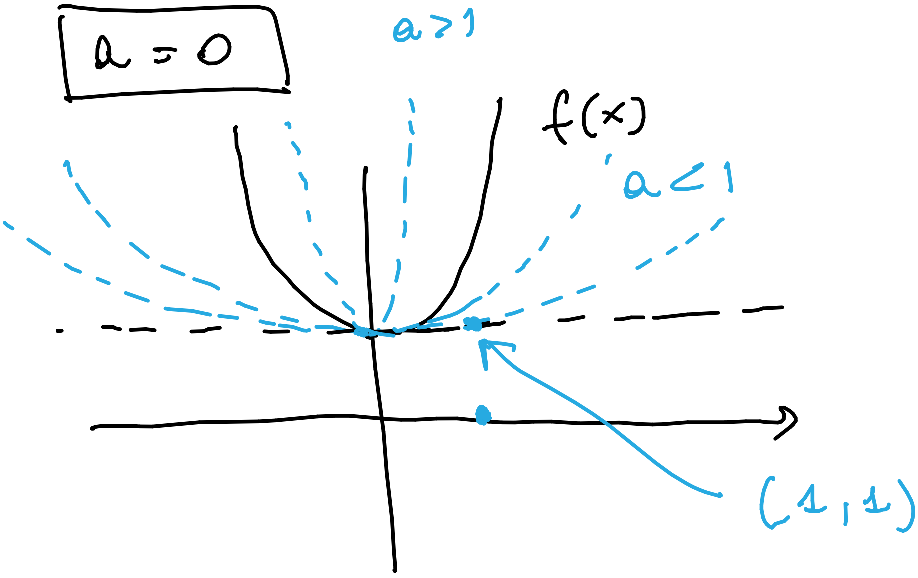
$f(x) = f(\underbrace{h_a(x)}) = (\underbrace{ax})^2 + 1$

$= a^2 x^2 + 1$

Richiesta : $f(h_a(1)) = 1$

$1 = a^2 1^2 + 1$

$\boxed{a = 0}$

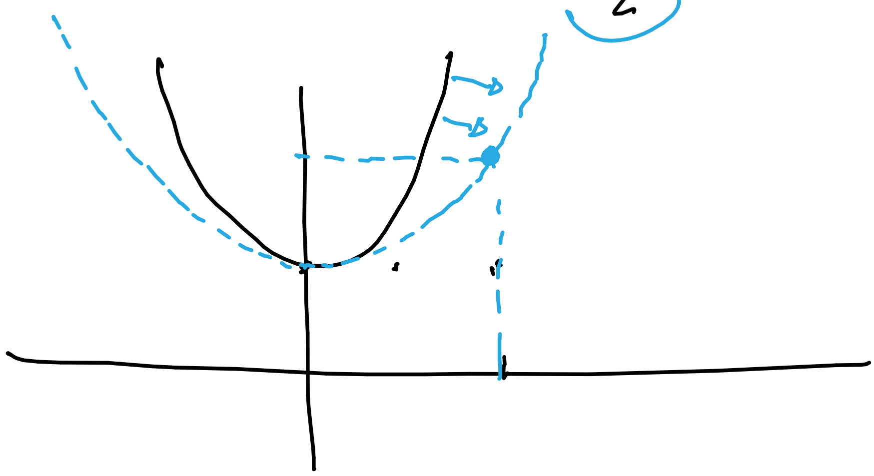


4) $f(h_a(x)) = a^2x^2 + 1$

$2 = a^2\,2^2 + 1$

$2 = 4a^2 + 1$

$a^2 = \frac{1}{4}$ $\quad a = \frac{1}{2}$ $\quad a = -\frac{1}{2}$



5) $h_a(f(x)) = a\,f(x) = a(x^2+1)$ $\quad a>0$

$= (\sqrt{a}\,x)^2 + a$ traslazione

$a<0$

precomposizione con $\sqrt{a}\,x$

$h_a(f(1)) = 1$?

$a(1^2+1) = 1$

$$\boxed{a = \frac{1}{2}}$$

Es 2 :

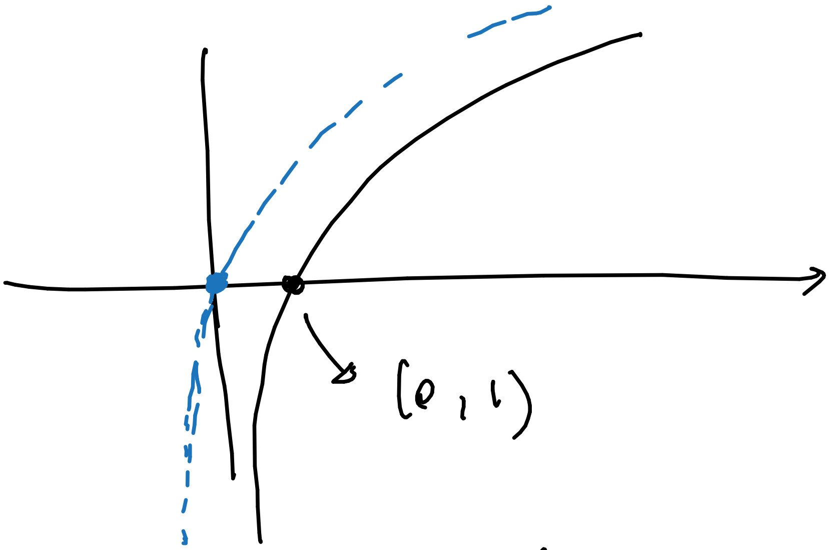
$g_a(x) = x + a$

$f(x) = \ln x$

$h(x) = \exp(x)$

Trovare a : $f \circ g_a$ passi per $(0,0)$

$\ln(g_a(x))$ passare per $(0,0)$.



Trovare la traslazione g_a : $q(x) := f(g_a(x))$ sia tale che $q(0) = 0$

$q(x) = \ln(x+1)$

$\ln(g_a(x)) = \ln(x+a)$

$x = 0$ $\quad \ln(0+a) = 0 \quad \to \quad a : \ln(a) = 0$

$\Rightarrow \boxed{a = 1} \quad \Rightarrow \quad q(x) = \ln(x+1)$

Calcolare l'inversa di

$q(x) = \ln(x+1)$

$\mathrm{Dom}(q) = (-1, +\infty)$

Argomento del logaritmo: > 0

$x + 1 > 0$

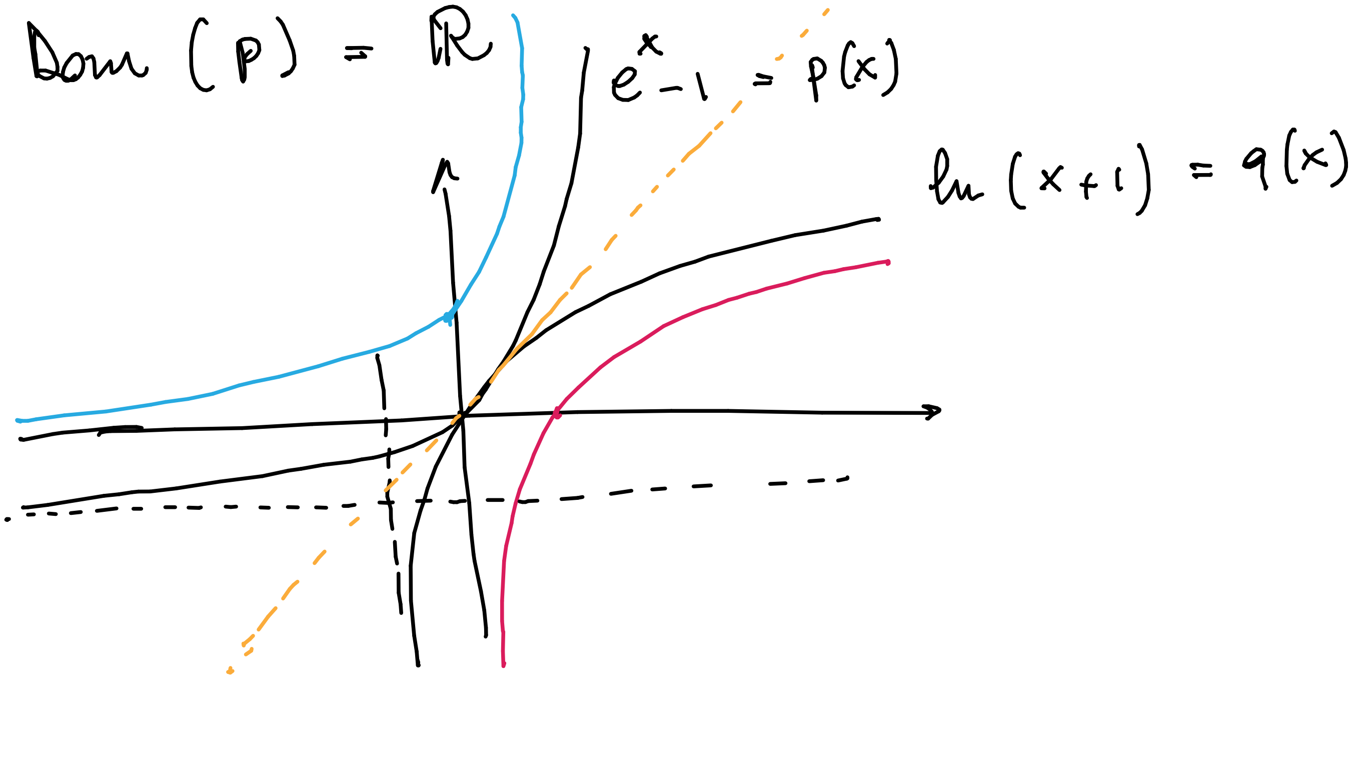
$\boxed{x > -1}$

Inversa: $y = \ln(x+1)$

$e^{y} = e^{\ln(x+1)} \Rightarrow e^{y} = x + 1$

$\boxed{x = 1 - e^{y}}$ $p(y) := e^{y} - 1$

$\mathrm{Dom}(p) = \mathbb{R}$



Es 3:

$f(x) = e^x, \ln x, \sqrt{x}, \frac{1}{x+1}$

$g(x) = \frac{1}{f(x)}$ $\qquad h(x) = f(x) \cdot f(x)$ $\qquad \mathrm{Dom}(f)$

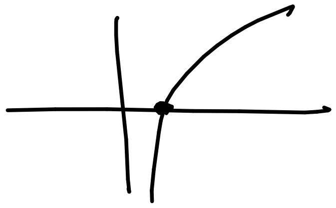
$g(x) = \frac{1}{e^x} = e^{-x}$ $\qquad \mathrm{Dom}(g) = \mathbb{R}$

$g(x) = \frac{1}{\ln x}$

Argomento ln > 0

Denominatore $\neq 0$

$\begin{cases} x > 0 \\ \ln x \neq 0 \end{cases}$



$\longrightarrow x \neq 1$

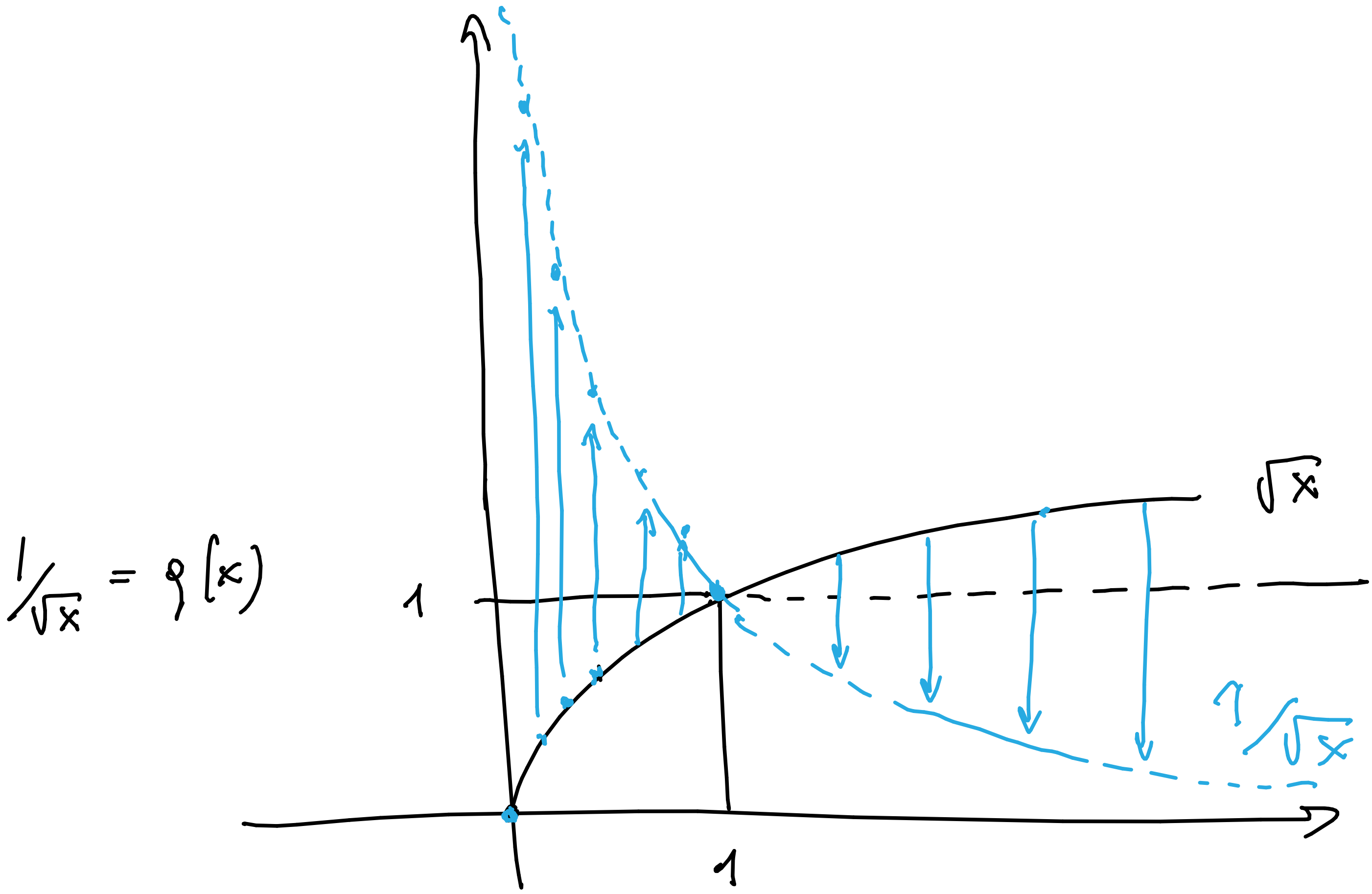
$\mathrm{Dom}(g) = (0, 1) \cup (1, +\infty)$

$g(x) = \frac{1}{\sqrt{x}}$

Arg. radice pari ≥ 0

Denominatore $\neq 0$

$\begin{cases} x \geq 0 \\ \sqrt{x} \neq 0 \end{cases}$ $\qquad \begin{cases} x \geq 0 \\ x \neq 0 \end{cases}$ $\longrightarrow \boxed{x > 0}$



$\mathrm{Dom}(\sqrt{\ }) \ni \{0\}$

$\mathrm{Dom}(1/\sqrt{\ }) \not\ni \{0\}$

$g(x) = f(x) \cdot f(x)$.

$f(x) = e^{x}$

$g(x) = (e^{x})(e^{x}) = (e^{x})^{2} = e^{2x}$

$\mathrm{Dom}(g) = \mathbb{R}$

$f(x) = \ln x$

$g(x) = (\ln x)^2$

$Dom(g) = Dom(\ln) = (0, +\infty)$

$f(x) = \sqrt{x}$

$g(x) = \sqrt{x} \cdot \sqrt{x}$ $\quad Dom(\sqrt{\ }) = [0, +\infty)$

$\boxed{Dom(g) = [0, +\infty)}$

ES 5:

$$f_a(x) = \frac{1}{1+e^{-ax}}$$

$$\mathrm{Dom}(f_a) = \{ x \in \mathbb{R} : 1+e^{-ax} \neq 0 \}$$

$1+e^{-ax} \neq 0 \qquad e^{-ax} > 0 \quad \forall a ; \forall x \in \mathbb{R}$

$1+e^{-ax} > 1$, mai $= 0$

$\mathrm{Dom}(f_a) = \mathbb{R} \quad \forall a \in \mathbb{R}$.

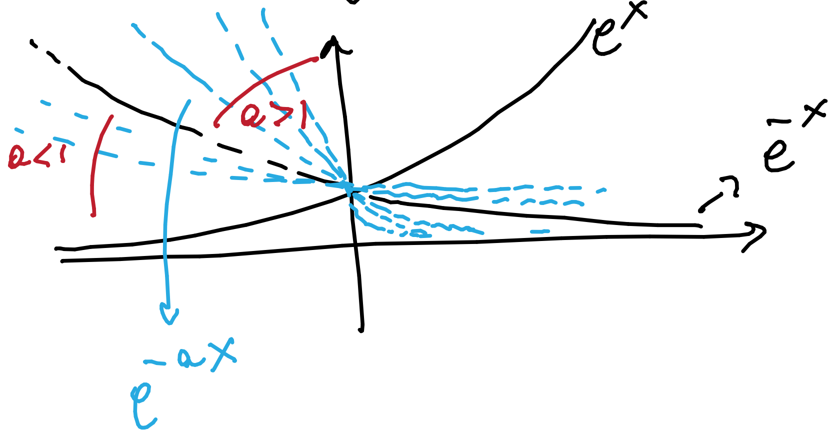
Segno di f:

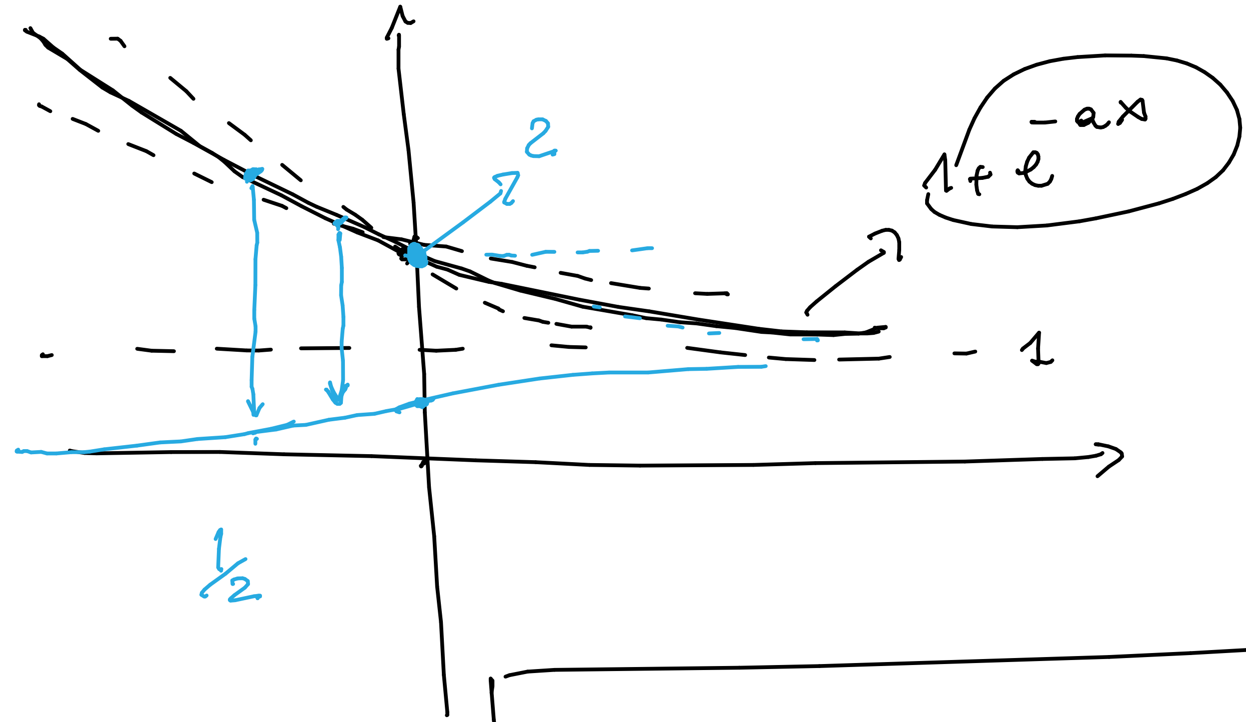
$$f_a(x) = \frac{1}{1+e^{-ax}} > 0 \quad \text{perché} \quad \begin{matrix} 1 > 0 \\ 1+e^{-ax} > 0 \end{matrix}$$

$\mathrm{Im}(f_a)$?

la funzione e^{-ax} (escludendo $a = 0$ per cui $f_0(x) = \frac{1}{2}$)

$\mathrm{Im}(f_0) = \left\{ \frac{1}{2} \right\}$





$$\frac{1}{1+e^{-ax}} :$$

$$\boxed{\begin{array}{l} \mathrm{Im}(f_a) = (0,1) \\ a \neq 0 \end{array}}$$