

# Calculus 1

## Esercizi tutorato 4

1. Risolvere:

(a)  $\tan(\pi - 2x) \geq -\sqrt{3}$

(b)  $\arcsin \sqrt{3x+2} > \pi/6$

(c)  $\arctan(\ln(x^2 + e^x)) < \pi/2$

(d)  $\arccos(3x^2 - 4) \geq \pi/4$

**Soluzioni:** (a)  $S = \cup_{k \in \mathbb{Z}} (\pi/4 + k\pi/2, 2\pi/3 + k\pi/2)$ ; (b)  $S = (-7/12, -1/3]$ ; (c)  $S = \mathbb{R}$ ; (d)  $S = [-\sqrt{(8+\sqrt{2})/6}, -1] \cup [1, \sqrt{(8+\sqrt{2})/6}]$ .

2. Determinare il dominio delle seguenti funzioni.

(a)  $f(x) = \sqrt{\cos(x^2)}$

(b)  $g(x) = \ln\left(\frac{1}{2} - \cos \frac{1}{x}\right)$

(c)  $h(x) = \sqrt{\arcsin(1 - 2 \sin x)}$

(d)  $u(x) = e^{\arcsin(\frac{1}{x})}$

**Soluzioni:** (a)  $D = [-\sqrt{\pi/2}, \sqrt{\pi/2}] \cup (\cup_{k \in \mathbb{Z}, k > 0} [-\sqrt{\pi/2 + 2k\pi}, -\sqrt{-\pi/2 + 2k\pi}] \cup [\sqrt{-\pi/2 + 2k\pi}, \sqrt{\pi/2 + 2k\pi}])$ ; (b)  $\cup_{k \in \mathbb{Z}} [(5/3\pi + 2k\pi)^{-1}, (\pi/3 + 2k\pi)^{-1}]$ ; (c)  $D = \cup_{k \in \mathbb{Z}} [2k\pi, \pi + 2k\pi]$ ; (d)  $D = (-\infty, -1] \cup [1, +\infty)$ .

3. Disegnare il grafico delle seguenti funzioni.

(a)  $f(x) = \begin{cases} 1/x & \text{se } x < 0 \\ \sqrt[3]{x} & \text{se } x \geq 0 \end{cases}$

(b)  $g(x) = \begin{cases} -x^{-2} & \text{se } x < 0 \\ \sin x & \text{se } 0 < x \leq \pi \\ \tan x & \text{se } \pi < x < \frac{3}{2}\pi \end{cases}$

(c)  $h(x) = \text{sign}(x) \sin x, x \in \mathbb{R}$

(d)  $u(x) = \text{sign}(x) \cos x, x \in \mathbb{R}$

(e)  $v(x) = \begin{cases} \log(-x) & \text{se } x < 0 \\ 2^x + 1 & \text{se } x \geq 0 \end{cases}$

4. Verificare, usando la definizione, che

$$\lim_{x \rightarrow 1} (x^2 - 2) = -1.$$