13-9-2024

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 5 \\ 0 & -2 & -5 \end{bmatrix}$$

a) $A - \lambda I \Rightarrow \lambda I = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \Rightarrow A - \lambda I \begin{bmatrix} -1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 5 \\ 0 & -2 & -5-\lambda \end{bmatrix}$

$$\det(A-\lambda I) \Rightarrow -1-\lambda \begin{vmatrix} 2-\lambda & 5 \\ -2 & -5-\lambda \end{vmatrix} \Rightarrow -1-\lambda \left[(2-\lambda)(-5-\lambda) - (5)(-2) \right] \Rightarrow$$

$$(-1-\lambda)\left[-10 - 2\lambda + 5\lambda + \lambda^2 + 10\right] \Rightarrow (-1-\lambda)\left[\lambda^2 + 3\lambda\right]$$

$$\lambda^2 + 3\lambda = \lambda(\lambda+3)$$

b) $-\lambda - 1 = 0 \Rightarrow \lambda = -1 \qquad \lambda = 0 \qquad \lambda = -3$

$\lambda_1 = -1 \qquad \lambda_2 = 0 \qquad \lambda_3 = -3$

c)

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 5 \\ 0 & -2 & -5 \end{bmatrix} \; A3 \to A3 + A2 \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{RANGO } 2$$

CASO $\lambda = -1$

$$A + I \Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 5 \\ 0 & -2 & -4 \end{bmatrix}$$

PRENDO LA SOTTOMATRICE 2x2 E CALCOLO IL DET

$$\left[(3)(-4)\right] - \left[(5)(-2)\right] \Rightarrow -12 + 10 = -2 \quad \text{RANGO } 2$$

CASO $\lambda = 0$

STESSO RANGO DI A IN QUANTO $A - (0) \cdot I = A - 0 = A$

CASO $\lambda = -3$

$$A+3I = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 5 \\ 0 & -2 & -5 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 5 \\ 0 & -2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 5 \\ 0 & -2 & -2 \end{bmatrix} R3 \to R3 + \frac{2}{5} R2 \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 5 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{RANGO } 2$$

d) $(A - \lambda I)x = 0$

$\lambda = -1$

$$\begin{cases} 0 = 0 \\ 3x_2 + 5x_3 \\ -2x_2 - 4x_3 \end{cases} \Rightarrow \begin{cases} 0 = 0 \\ \frac{3x_2}{3} = \frac{-5x_3}{3} \\ \frac{10}{3}x_3 - 4x_3 \end{cases} \Rightarrow \begin{cases} 0 = 0 \\ x_2 = -5/3\, x_3 \\ \frac{10-12}{3} x_3 = 0 \end{cases} \Rightarrow \begin{cases} 0 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{cases}$$

$$\left\{ \underline{x_1} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \middle| x_1 \in \mathbb{R} \right\} = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\rangle$$

$\lambda = 0$

$$\begin{cases} -x_1 = 0 \\ 2x_2 + 5x_3 = 0 \\ -2x_2 - 5x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ \frac{2x_2}{2} = \frac{-5x_3}{2} \\ 5x_3 - 5x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = -\frac{5}{2} x_3 \\ 0 = 0 \end{cases}$$

$$\left\{ \underline{x_3} \begin{pmatrix} 0 \\ -5/2 \\ 1 \end{pmatrix} \middle| x_3 \in \mathbb{R} \right\} = \left\langle \begin{pmatrix} 0 \\ -5/2 \\ 1 \end{pmatrix} \right\rangle$$

CASO $\lambda = -3$

$$\begin{cases} 2x_1 = 0 \\ 5x_2 + 5x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ 5x_2 = -5x_3 \\ // \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = -1 x_3 \\ 0 = 0 \end{cases}$$

$$\left\{ x_3 \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \;\middle|\; x_3 \in \mathbb{R} \right\} = \left\langle \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \right\rangle$$

e) $V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -5/2 & -1 \\ 0 & 1 & 1 \end{pmatrix}$ $\qquad D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -3 \end{pmatrix}$

$$\det(V) = 1 \begin{vmatrix} -5/2 & -1 \\ 1 & 1 \end{vmatrix} = -\frac{5}{2} + 1 = \frac{-5+2}{2} = \frac{-3}{2} \neq 0$$

$A_{11} = -\frac{3}{2}$ $\qquad A_{12} = 0$ $\qquad A_{13} = 0$ $\qquad \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$

$A_{21} = 0$ $\qquad A_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$ $\qquad A_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = -1$

$A_{31} = 0$ $\qquad A_{32} = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = 1$ $\qquad A_{33} = \begin{vmatrix} 1 & 0 \\ 0 & -5/2 \end{vmatrix} = -5/2$

$$\mathrm{cof}(V)^T = \begin{pmatrix} -3/2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & -5/2 \end{pmatrix}$$

$$V^{-1} = \frac{1}{\det(V)} \cdot \mathrm{cof}(V)^T = \begin{bmatrix} \frac{-\frac{3}{2}}{\frac{-3}{2}} & 0 & 0 \\ 0 & \frac{1}{\frac{-3}{2}} & \frac{1}{\frac{-3}{2}} \\ 0 & \frac{-1}{\frac{-3}{2}} & \frac{-\frac{5}{2}}{\frac{-3}{2}} \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{2}{3} & -\frac{2}{3} \\ 0 & \frac{2}{3} & \frac{5}{3} \end{bmatrix}$$

$$V\Delta \begin{pmatrix} 1 & 0 & 0 \\ 0 & -5/2 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -3 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & -3 \end{pmatrix}$$

$$V\Delta V^{-1} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2/3 & 2/3 \\ 0 & 2/3 & 5/3 \end{pmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 5 \\ 0 & -2 & -5 \end{bmatrix}$$ VERIFICATO

f) $\rho = 1$ $\rho = -2$

$$\mu_1 = \frac{1}{\lambda_1 - \rho} \qquad \mu_1 = \frac{1}{\lambda_1 - \rho} = \frac{1}{-1-1} = -\frac{1}{2} \simeq -0,5$$

$$\mu_2 = \frac{1}{\lambda_2 - \rho} = \frac{1}{-\rho} = -1$$

$$\mu_3 = \frac{1}{\lambda_3 - \rho} = \frac{1}{-3-1} = -\frac{1}{4} \simeq 0,25$$

$$V = \left|\frac{\mu_2}{\mu_1}\right|^k = \left|\frac{0,5}{-1}\right|^k = (0,5)^k$$

HO MONOTONICAMENTE PER MASSIMO MODULO, μ^1 MISURA IL PIÙ GRANDE MENTRE μ_2 IL PIÙ PICCOLO (QUELLO CHE SI AVVICINA DI PIÙ A μ_1)

$$\mu_2 = \frac{1}{\mu - \rho} \qquad \mu_1 = \frac{1}{\lambda_1 - \rho} = \frac{1}{-1+2} = -1$$

$$\mu_2 = \frac{1}{\lambda_2 - \rho} = \frac{1}{2} \simeq 0,5$$

$$\mu_3 = \frac{1}{\lambda_3 - \rho} = \frac{1}{-3+2} = -1$$

$$V = \left|\frac{0,5}{1}\right|^k = (0,5)^k$$

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MINIMI QUADRATI

$a\sqrt{x} + \beta$

x	0	1	4	9
y	-1	0	1	2

a)

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \qquad Y = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

b)

A^2 = NON ESSENDO MATRICE QUADRATA NON LO POSSO CALCOLARE

$$AA^T = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 7 \\ 1 & 4 & 7 & 10 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix}$$

$$A^TY = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$$

$$Y^TA = \begin{bmatrix} -1 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 2 \end{bmatrix}$$

c)

$$M = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

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DEVO DIMOSTRARE CHE $c_1 a_1 + c_2 a_2 + c_3 y = 0 \Rightarrow c_1 = c_2 = c_3 = 0$

$$\begin{cases} c_2 - c_3 = 0 \\ c_1 + c_2 = 0 \\ 2c_1 + c_2 + c_3 = 0 \\ 3c_1 + c_2 + 2c_3 = 0 \end{cases} \Rightarrow \begin{cases} -c_1 - c_3 = 0 \\ c_2 = -c_1 \\ c_3 = -2c_1 - c_2 \\ // \end{cases} \Rightarrow \begin{cases} c_1 = -c_3 \\ c_2 = c_3 \\ c_3 = 2c_3 - c_3 \\ // \end{cases} \Rightarrow \begin{cases} // \\ // \\ -2c_3 + c_3 + c_3 = 0 \\ // \end{cases}$$

AVENDO c_3 COME VARIABILE LIBERA GLI ASSEGNO IL VALORE 0 OTTENENDO $c_1 = c_2 = c_3 = 0$ E QUINDI HO DIMOSTRATO CHE È INDIPENDENTE.

d) DIMOSTRARE CHE y SIA DIPENDENTE

BASTA SOMMARE $a_1 + a_2$ PER TROVARMI CON y DIPENDENTE.

$$\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

e) $A^T A \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = A^T y$

$$\begin{cases} 14\alpha + 6\beta = 8 \\ 6\alpha + 4\beta = 2 \end{cases} \Rightarrow \begin{cases} // \\ 4\beta = 2 - 6\alpha \end{cases} \Rightarrow \begin{cases} 14\alpha + 6\left(\frac{1}{2} - \frac{3}{2}\alpha\right) = 0 \\ \beta = \frac{1}{2} - \frac{3}{2}\alpha \end{cases} \Rightarrow$$

$$\begin{cases} 14\alpha + 3 - 9\alpha = 0 \\ // \end{cases} \Rightarrow \begin{cases} 14\alpha - 9\alpha = -3 \\ // \end{cases} \Rightarrow \begin{cases} 5\alpha = -3 \\ // \end{cases} \Rightarrow \begin{cases} \alpha = -\frac{3}{5} \\ \beta = \frac{1}{2} + \frac{9}{10} \end{cases}$$

$$\frac{5+9}{10} \Rightarrow \begin{matrix} \alpha = -\frac{3}{5} \\ \beta = \frac{7}{5} \end{matrix}$$

$$-\frac{3}{5}\sqrt{x} + \frac{7}{5} \Rightarrow -0{,}6\sqrt{x} + 1{,}4$$

F) LA NUOVA FUNZIONE $O_f(x)$ APPROSSIMA AI MINIMI QUADRATI I DATI RIPORTATI, MINIMIZZANDO LA SOMMA DEGLI SCARTI CHE ASSUME E I DATI y

SPLINE 19-07-24

$$S(x) = \begin{cases} 2x^3 + 3x^2 & \text{se } -1 \le x \le 0 \\ -x^3 + 2x^2 & \text{se } 0 \le x \le 1 \end{cases}$$

a) VERIFICARE CHE SIA SPLINE

PER ESSERE UNA SPLINE DEVE ESSERE:

- POLINOMIO A TRATTI GRADO ≤ 3
- S, S', S'' CONTINUE

PRIMA CONDIZIONE SODDISFATTA

$$\left.\begin{array}{l} \lim_{x \to 0^-} S(x) = 0 \\ \lim_{x \to 0^+} S(x) = 0 \end{array}\right\} \text{LA FUNZIONE E' CONTINUA PER } x = 0$$

$$S(x)' = \begin{cases} 6x^2 + 6x & [-1, 0] \\ -3x^2 + 2a x & [0, 1] \end{cases}$$

$$\left.\begin{array}{l} \lim_{x \to 0^-} S(x) = 0 \\ \lim_{x \to 0^+} S(x) = 0 \end{array}\right\} S'(x) \text{ CONTINUA IN } x = 0$$

$$S(x)'' = \begin{cases} 12x + 6 & [-1, 0] \\ -6x + 2a & [0, 1] \end{cases}$$

$$\left.\begin{array}{l} \lim_{x \to 0^-} = 6 \\ \lim_{x \to 0^+} = 2a \end{array}\right\} \begin{array}{l} 6 = 2a \Rightarrow -2a = -6 \Rightarrow a = 3 \\ S''(x) \text{ PER ESSERE CONTINUA DEVE AVERE } a = 3 \end{array}$$

b) INTERPOLANTE

$$f(x) = 1 - \cos\left(\frac{\pi}{2}x\right)$$

PERIODICA: $S''(x_0) = S''(x_m)$

E $S'(x_0) = S'(x_m)$

DOVE x_0 SAREBBE IL PRIMO PUNTO E x_m NEL NOSTRO CASO $x_0 = -1$ E $x_m = 1$

$$f(+1) = 1 - \cos\left(-\frac{\pi}{2}\right) = 1 - 0 = 1$$

$$f(0) = 1 - 1 = 0$$

$$f(1) = 1 - 0 = 1$$

PER ESSERE INTERPOLANTE DEVO OTTENERE GLI STESSI VALORI DI $f(x)$

$$S(-1) = 2(-1)^3 + 3(-1)^2 = 1$$

$S(0) = 0$ VERIFICATA SIA PER LA PRIMA RIGA E PER LA SECONDA

$$S(1) = 1$$

ABBIAMO DIMOSTRATO CHE $S(x)$ È INTERPOLANTE ALLA FUNZIONE $1-\cos\left(\frac{\pi}{2}x\right)$

DIAGONALIZZAZIONE

$$A = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & -1 \end{bmatrix}$$

a) $A - \lambda I \Rightarrow \lambda I : \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} =$

$$= \begin{bmatrix} -1-\lambda & 0 & 2 \\ 0 & -\lambda & 0 \\ 2 & 0 & -1-\lambda \end{bmatrix}$$

$$\det(A-\lambda I) = -\lambda \cdot \begin{bmatrix} -1-\lambda & 2 \\ 2 & -1-\lambda \end{bmatrix} \Rightarrow -\lambda \cdot \left[(-1-\lambda)(-1-\lambda) - (4)\right] \Rightarrow$$

$$-\lambda\left[1+\lambda+\lambda+\lambda^2-4\right] \Rightarrow -\lambda\left[2\lambda+\lambda^2-3\right]$$

5)

$\lambda_1 = 0$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{4-4(1)(-3)}}{2} = \frac{-2 \pm \sqrt{16}}{2} =$$

$$\frac{-2 \pm 4}{2} \begin{cases} \frac{-2-4}{2} = -3 \\ 1 \end{cases}$$

$\lambda_1 = 0 \qquad \lambda_2 = -3 \qquad \lambda_3 = 1$

6) $\begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & -1 \end{bmatrix} R3 \to 2R3 \begin{bmatrix} -1 & 0 & 2 \\ 2 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} R2 \to 2R2+2R1 \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ RANGO 2

CASO $\lambda_1 = 0 \Rightarrow$ STESSO RANGO DI A

CASO $\lambda_2 = -3 \Rightarrow A-(-3)I \Rightarrow A+3I \Rightarrow \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} =$

$= \begin{bmatrix} 2 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 2 \end{bmatrix} R3 \to 2R3-R1 \begin{bmatrix} 2 & 0 & 2 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ RANGO 2

CASO $\lambda_3 = 1$

$$A - I \Rightarrow \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 0 & 2 \\ 0 & -1 & 0 \\ 2 & 0 & -2 \end{bmatrix}$$

$\begin{bmatrix} -2 & 0 & 2 \\ 0 & -1 & 0 \\ 2 & 0 & -2 \end{bmatrix} R3 \to 2R3+R1 \begin{bmatrix} -2 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ RANGO 2

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CASO $\lambda=0$

d) $\begin{cases} -x_1 + 2x_3 = 0 \\ 3x_3 = 0 \\ 0 = 0 \end{cases} \Rightarrow \begin{cases} -x_1 = -2x_3 \\ x_3 = 0 \\ 0 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_3 = 0 \\ 0 = 0 \end{cases}$

$\left\{ \underline{x}_3 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \middle| \underline{x}_3 \in \mathbb{R} \right\} = \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\rangle$

CASO $\lambda=-3$

$\begin{cases} 2x_1 + 2x_3 = 0 \\ 3x_2 = 0 \\ 0 = 0 \end{cases} \Rightarrow \begin{cases} 2x_1 = -2x_3 \\ x_2 = 0 \\ 0 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = -1x_3 \\ x_2 = 0 \\ 0 = 0 \end{cases}$ $\left\{ \underline{x}_3 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \middle| \underline{x}_3 \in \mathbb{R} \right\} =$

$= \left\langle \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\rangle$

CASO $\lambda=1$

$\begin{cases} -2x_1 + 2x_3 = 0 \\ -x_2 = 0 \\ 0 = 0 \end{cases} \Rightarrow \begin{cases} -2x_1 = -2x_3 \\ x_2 = 0 \\ 0 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 1x_3 \\ x_2 = 0 \\ 0 = 0 \end{cases}$

$\left\{ \underline{x}_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \underline{x}_3 \in \mathbb{R} \right\} = \left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle$

e) $V = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ $D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\det(V) = -1 \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = 2 \neq 0$ INVERTIBLE

$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$

$A_{11} = 0$ $A_{12} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = -1$ $A_{13} = 1$

$A_{21} = 2$ $A_{22} = 0$ $A_{23} = 0$

$A_{31} = 0$ $A_{32} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 1$ $A_{33} = \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = 1$

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$$V^{-1} = \frac{1}{\det(V)} \cdot \mathrm{cof}(V)^T = \frac{1}{2} \cdot \begin{pmatrix} 0 & 2 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -1/2 & 0 & 1/2 \\ 1/2 & 0 & 1/2 \end{pmatrix}$$

$$VD = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 3 & 1 \\ 0 & 0 & 0 \\ 0 & -3 & 1 \end{pmatrix}$$

$$VDV^{-1} = \begin{pmatrix} 0 & 3 & 1 \\ 0 & 0 & 0 \\ 0 & -3 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ -1/2 & 0 & 1/2 \\ 1/2 & 0 & 1/2 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & -1 \end{pmatrix}$$ VERIFICATA

f) $p = 1$ $\qquad$ $p = 2$

$$\mu_1 = \frac{1}{1} = 1 \qquad \mu_2 = \frac{1}{-2} = -\frac{1}{2} \qquad \mu_3 = \frac{1}{2}$$

$$V = \left(\frac{0{,}5}{1}\right)^k = (0{,}5)^k$$

$$\mu_1 = \frac{1}{2} \qquad \mu_2 = -1 \qquad \mu_3 = \frac{1}{3}$$

$$V = \left(\frac{0{,}3}{1}\right)^k = (0{,}3)^k$$

MINIMI QUADRATI

x	$-\sqrt{2}$	-1	0	1	$\sqrt{2}$
y	1	$1/2$	0	$1/2$	1

$$p(x) = \alpha x^2 + \beta x + \gamma$$

1)

$$A = \begin{bmatrix} 2 & -\sqrt{2} & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & \sqrt{2} & 1 \end{bmatrix} \qquad y = \begin{bmatrix} 1 \\ 1/2 \\ 0 \\ 1/2 \\ 1 \end{bmatrix}$$

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A

$$A^TA = \begin{bmatrix} 2 & 1 & 0 & 1 & 2 \\ -\sqrt{2} & -1 & 0 & 1 & \sqrt{2} \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -\sqrt{2} & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & \sqrt{2} & 1 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 6 \\ 0 & 6 & 0 \\ 6 & 0 & 5 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 2 & -\sqrt{2} & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & \sqrt{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 0 & 1 & 2 \\ -\sqrt{2} & -1 & 0 & 1 & \sqrt{2} \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 3+\sqrt{2} & 1 & 3-\sqrt{2} & 3 \\ 3+\sqrt{2} & 3 & 1 & 1 & 3-\sqrt{2} \\ 1 & 1 & 1 & 1 & 1 \\ 3-\sqrt{2} & 1 & 1 & 3 & 3+\sqrt{2} \\ 3 & 3-\sqrt{2} & 1 & 3+\sqrt{2} & 7 \end{bmatrix}$$

$$YA^T = \begin{bmatrix} 1 & 1/2 & 0 & 1/2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 0 & 1 & 2 \\ -\sqrt{2} & -1 & 0 & 1 & \sqrt{2} \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} =$$

NON POSSIAMO SVOLGERE IL CALCOLO IN QUANTO IL NUMERO DI COLONNE DI YA^T NON CORRISPONDE ALLE RIGHE DI A^T

$$A^TY = \begin{bmatrix} 2 & 1 & 0 & 1 & 2 \\ -\sqrt{2} & -1 & 0 & 1 & \sqrt{2} \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1/2 \\ 0 \\ 1/2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix}$$

c) $$M = \begin{bmatrix} 2 & -\sqrt{2} & 1 \\ 1 & -1 & 1/2 \\ 0 & 0 & 0 \\ 1 & 1 & 1/2 \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

$$c_1 x_1 + c_2 x_2 + c_3 Y = 0$$

$$\begin{cases} 2c_1 - \sqrt{2}c_2 + c_3 = 0 \\ c_1 - c_2 + \frac{1}{2}c_3 = 0 \\ 0 = 0 \\ c_1 + c_2 + \frac{1}{2}c_3 = 0 \\ 2c_1 + \sqrt{2}c_2 + c_3 = 0 \end{cases} \Rightarrow \begin{cases} '' \\ -\sqrt{2}c_2 = -2c_1 - c_3 \\ '' \\ '' \\ '' \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 2c_1 - \sqrt{2}\left(\frac{2}{\sqrt{2}}c_1 - \frac{1}{\sqrt{2}}c_3\right) \\ c_2 = \frac{2}{\sqrt{2}}c_1 - \frac{1}{\sqrt{2}}c_3 \\ // \\ // \\ // \end{cases} \Rightarrow \begin{cases} 2c_1 - 2c_1 + c_3 = 0 \\ c_2 = \frac{2}{\sqrt{2}}c_1 \\ 0 = 0 \\ c_1 + \frac{2}{\sqrt{2}}c_1 = 0 \\ // \end{cases} \Rightarrow$$

$$\begin{cases} // \\ // \\ \frac{\sqrt{2}+2}{\sqrt{2}}c_1 = 0 \end{cases} \Rightarrow \begin{matrix} c_1 = 0 \\ c_2 = 0 \\ c_3 = 0 \end{matrix}$$ VERIFICATO

4) VERIFICA ORTOGONALITÀ

$$\begin{matrix} a_1^T \\ a_2^T \end{matrix} \begin{pmatrix} 2 & 1 & 0 & 1 & 2 \\ -\sqrt{2} & -1 & 0 & 1 & \sqrt{2} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

SICCOME ABBIAMO UNA VARIABILE LIBERA POSSIAMO FARE IL VERSORE

$$\tilde{y} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

e) $A^T A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^T y$

$$\begin{cases} 10x + 6z = 5 \\ 6y = 0 \\ 6x + 5z = 3 \end{cases} \Rightarrow \begin{cases} \frac{10x}{10} = \frac{5}{10} - \frac{6z}{10} \\ y = 0 \\ // \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2} - \frac{3}{5}z \\ y = 0 \\ // \end{cases} \Rightarrow \begin{cases} // \\ // \\ 6\left(\frac{1}{2} - \frac{3}{5}z\right) + 5z = 3 \end{cases}$$

$$\Rightarrow \begin{cases} x = \frac{1}{2} \\ y = 0 \\ z = 0 \end{cases}$$ QUINDI $Q_f(x) = \frac{1}{2}x^2$

f) LA PARABOLA $Q_f(x)$ MINIMIZZA LA SOMMA DEI QUADRATI DEGLI SCARTI TRA I

22-7-2021

$$S(x) = \begin{cases} -x^3 & \text{SE } x \in [-1, 0] \\ x^3 & \text{SE } x \in [0, 1] \end{cases}$$

a) VERIFICARE CHE SIA SPLINE E SE È NATURALE

PER ESSERE UNA SPLINE DEVE ESSERE:

- POLINOMIO A TRATTI GRADI ≤ 3
- S, S', S'' CONTINUE

LA PRIMA CONDIZIONE È VERIFICATA

$$\left.\begin{array}{l} \lim_{x \to 0^-} S(x) = 0 \\ \lim_{x \to 0^+} S(x) = 0 \end{array}\right\} \text{ CONTINUA IN } x = 0$$

$$S'(x) = \begin{cases} -3x^2 & [-1, 0] \\ 3x^2 & [0, 1] \end{cases}$$

$$\left.\begin{array}{l} \lim_{x \to 0^-} = 0 \\ \lim_{x \to 0^+} = 0 \end{array}\right\} S'(x) \text{ CONTINUA IN } x = 0$$

$$S''(x) = \begin{cases} -6x & [-1, 0] \\ 6x & [0, 1] \end{cases}$$

$$\left.\begin{array}{l} \lim_{x \to 0^-} = 0 \\ \lim_{x \to 0^+} = 0 \end{array}\right\} S''(x) \text{ CONTINUA IN } x = 0$$

SPLINE NATURALE $\Leftrightarrow S''(a) = S''(b) = 0$

$S(-1) = -3$

$S(1) = 1$

NON E' NATURALE

CONDIZIONAMENTO

$$A = \begin{pmatrix} 1 & 0 & -31 \\ 0 & -1 & 0 \\ -1 & 0 & 30 \end{pmatrix} \qquad x = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \qquad \delta b = \begin{pmatrix} 10^{-2} \\ -10^{-2} \\ -10^{-2} \end{pmatrix}$$

$$b = A \cdot x \Rightarrow \begin{pmatrix} 1 & 0 & -31 \\ 0 & -1 & 0 \\ -1 & 0 & 30 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 30 \\ -1 \\ -29 \end{pmatrix}$$

d) VERIFICARE A^{-1}

$$\det(A) = -1 \begin{vmatrix} 1 & -31 \\ -1 & 30 \end{vmatrix} = 1 \neq 0 \quad \text{INVERTIBILE}$$

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

$A_{11} = -30 \qquad A_{12} = 0 \qquad A_{13} = -1$

$A_{21} = 0 \qquad A_{22} = \begin{vmatrix} 1 & -31 \\ -1 & 30 \end{vmatrix} = -1 \qquad A_{23} = 0$

$A_{31} = -31 \qquad A_{32} = 0 \qquad A_{33} = -1$

$$A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{cof}(A)^T = \begin{pmatrix} -30 & 0 & -31 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{pmatrix}$$

b) $\mu_1(A)$ E $\mu_\infty(A)$

$\|A\|_1 = \max(2, 1, 61) = 61$

$\|A^{-1}\| = \max(31, 1, 32) = 32$

$\mu_1 = 61 \cdot 32 = 1952$

$\|A\|_\infty = \max(32, 1, 31) = 32$

$\|A^{-1}\|_\infty = \max(61, 1, 2) = 61$

$\mu_\infty = 61 \cdot 32 = 1952$

d) $\|\cdot\|_1$ $\|\cdot\|_2$

$\|x\|_1 = 3$ $\|x\|_2 = \sqrt{3}$

$\|b\|_1 = 60$ $\|b\|_2 = \sqrt{900 + 1 + 841} = \sqrt{1742}$

$\|\delta b\|_1 = 10^{-2} + 10^{-2} + 10^{-2} = \frac{3}{10^2}$

$\|\delta b\|_2 = \sqrt{(10^{-2})^2 + (-10^{-2})^2 + (-10^{-2})^2} = \sqrt{10^{-4} + 10^{-4} + 10^{-4}} = \sqrt{\frac{3}{10^4}} = \frac{\sqrt{3}}{100}$

e) ERRORE $\|\tilde{x} - x\|_1$

$$\boxed{\varepsilon_b = \frac{\|\delta b\|_1}{\|b\|_1}} \simeq \frac{\frac{3}{10^2}}{60} = \frac{3}{100} \cdot \frac{1}{60} = \frac{3}{6000} = \frac{1}{2000} = 0{,}0005$$

$$\boxed{\varepsilon_x = \mu_1 \cdot \varepsilon_b = 1952 \cdot 0{,}0005 = 9{,}76}$$

SPLINE

$$S(x) = \begin{cases} x^3 + 9x^2 + 2 & \text{SI } x \in [-2,-1] \\ -x^3 + 3x^2 - 6x & \text{SI } x \in [-1,0] \end{cases}$$

a)

$$\left.\begin{array}{l} \lim_{x \to -1^-} S(x) = -1 + 9 + 2 = 10 \\ \lim_{x \to -1^+} = S(x) = 1 + 3 + 6 = 10 \end{array}\right\} \text{CONTINUA}$$

$$S'(x) = \begin{cases} 3x^2 + 18x & [-2,-1] \\ -3x^2 + 6x - 6 & [-1,0] \end{cases}$$

$$\left.\begin{array}{l} \lim_{x \to -1^-} S'(x) = 3 - 18 = -15 \\ \lim_{x \to -1^+} S'(x) = 3 - 6 - 6 = -15 \end{array}\right\} \text{CONTINUA}$$

$$S''(x) = \begin{cases} 6x + 18 & [-2,-1] \\ -6x + 6 & [-1,0] \end{cases}$$

$$\left.\begin{array}{l} \lim_{x \to -1^-} = -6 + 18 = 12 \\ \lim_{x \to -1^+} = 6 + 6 = 12 \end{array}\right\} \text{CONTINUA}$$

b) NATURAL

$$S''(a) = S''(b) = 0$$

$S''(-2) = 12$ NOT NATURAL

a) $f(x) = 5x^2 - 5x$ VERIFICARE INTERPOLANTE

$f(-2) = 0$ $\quad$ $f(-2) = 20$ $\quad$ NON INTERPOLANTE

$f(-1) =$ $\quad$ $f(-1) = 10$

$f(0) =$ $\quad$ $f(0) = 0$

SVD

SIA $A = X \Lambda X^T$

$$X = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \qquad \Lambda = \begin{pmatrix} -2 & 0 \\ 0 & 3 \end{pmatrix}$$

a) DIMOSTRARE CHE X E' ORTOGONALE

PER ESSERE ORTOGONALE DEVO FARE IL PRODOTTO TRA LE DUE COLONNE ED ESSERE $= 0$. IN QUESTO CASO ABBIAMO $\frac{1}{2} - \frac{1}{2} = 0$

ORA FACCIAMO LA $\|\cdot\|_2$ DELLE COLONNE E DEVE OTTENERE 1

$$\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = 1$$

$$\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = 1$$

b) VERIFICARE CHE A E' SIMMETRICA

COME FACCIAMO UTILIZZO DELLA FORMULA $X \Lambda X^T$ SIGNIFICA CHE LA MATRICE A E' SIMMETRICA PER IL TEOREMA SPETTRALE

c) DETERMINARE LE MATRICI U, Σ, V

$\lambda_1 = -2 \qquad \lambda = 3$

$\sigma_1 = |\lambda_1| \quad \sigma_2 = |\lambda_2| \longrightarrow \sigma_1 = 2 \quad \sigma_2 = 3$

$\Sigma = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \longrightarrow$ OVVERO PRENDO IL MODULO DEI λ E LI METTO IN ORDINE DECRESCENTE

$U = \left(-\underline{x_1} \mid \underline{x_2} \right) \longrightarrow U = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \longrightarrow$ SE NEL MODULO CAMBIA SEGNO, CAMBIA IL SEGNO DI TUTTA LA COLONNA

$V^T = \begin{pmatrix} \underline{x_1} \\ \hline \underline{x_2} \end{pmatrix} \longrightarrow V^T = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \longrightarrow V = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

$V^T = |U^T|$

MINIMI QUADRATI

$g(x) = \alpha x^2 + \beta x + \gamma$

x	-2	-1	0	1	2
y	8	5	-7	-1	1

a)

$A = \begin{bmatrix} 4 & -2 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix} \qquad y = \begin{bmatrix} 8 \\ 5 \\ -7 \\ -1 \\ 1 \end{bmatrix}$

b)

$A^T A = \begin{bmatrix} 4 & 1 & 0 & 1 & 4 \\ -2 & -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 34 & 0 & 10 \\ 0 & 10 & 0 \\ 10 & 0 & 5 \end{bmatrix}$

$A^T y = \begin{bmatrix} 4 & 1 & 0 & 1 & 4 \\ -2 & -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 5 \\ -7 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 40 \\ -20 \\ 6 \end{bmatrix}$

c) $c_1 a_1 + c_2 a_2 + c_3 y = 0$

$$\begin{cases} 4c_1 - 2c_2 + 8c_3 = 0 \\ c_1 - c_2 + 5c_3 = 0 \\ -7c_3 = 0 \\ c_1 + c_2 - c_3 = 0 \\ 4c_1 + 2c_2 + c_3 = 0 \end{cases} \Rightarrow \begin{cases} // \\ -c_2 = -c_1 \\ c_3 = 0 \\ // \\ // \end{cases} \Rightarrow \begin{cases} 4c_1 - 2c_1 = 0 \\ c_2 = c_1 \\ c_3 = 0 \\ // \\ // \end{cases} \Rightarrow \begin{cases} c_1 = 0 \\ c_2 = 0 \\ c_3 = 0 \end{cases}$$

d) $$\begin{matrix} a_1^T \\ a_2^T \end{matrix} \begin{pmatrix} 4 & 1 & 0 & 1 & 4 \\ -2 & -1 & 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{y}_3 \\ \tilde{y}_4 \\ \tilde{y}_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

PER VERIFICARE CHE SIA GIUSTO MOLTIPLICHIAMO IL SISTEMA PER QUESTO VETTORE

SICCOME $\tilde{y}_3$ È VARIABILE LIBERA POSSO SCEGLIERE $\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

e) DIPENDENZA

$$a_1 + a_2 \Rightarrow \begin{pmatrix} 4 \\ 1 \\ 0 \\ 1 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 2 \\ 6 \end{pmatrix}$$

f) $A^T A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^T y$

$$\begin{cases} 34x + 10z = 40 \\ 10y = -20 \\ 10x + 5z = 6 \end{cases} \Rightarrow \begin{cases} // \\ y = -2 \\ \frac{5z}{5} = \frac{6}{5} - \frac{10x}{5} \end{cases} \Rightarrow \begin{cases} 34x + 10\left(\frac{6}{5} - 2x\right) = 40 \\ y = -2 \\ z = \frac{6}{5} - 2x \end{cases} \Rightarrow$$

$$\begin{cases} 34x + 12 - 20x = 40 \\ // \\ // \end{cases} \Rightarrow \begin{cases} 14x = 28 \\ // \\ // \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = -2 \\ z = \frac{6}{5} - 4 \end{cases}$$

$$\frac{\theta - 20}{5} \Rightarrow \begin{cases} x = 2 \\ y = -2 \\ z = -\frac{14}{5} \end{cases}$$

6) LA FUNZIONE $g(x)$ APPROSSIMA AI MINIMI QUADRATI I DATI PROVVISTI, MINIMIZZANDO LA SOMMA DEGLI SCARTI CHE ASSUME I DATI P

ESERCIZIO 1 GRAFI

$$f(x) = \frac{1}{1-3x} - \frac{2}{2+3x}$$

piccoli valori di x

7) $$f'(x) = \frac{0(1-3x) - 1(-3)}{(1-3x)^2} - \frac{0(2+3x) - 2(3)}{(2+3x)^2} = \frac{3}{(1-3x)^2} + \frac{6}{(2+3x)^2}$$

$$c_f = \frac{x \cdot \left(\frac{3}{(1-3x)^2} + \frac{6}{(2+3x)^2} \right)}{\frac{1}{1-3x} - \frac{2}{2+3x}} = \frac{0}{0}$$

DE HOPITAL

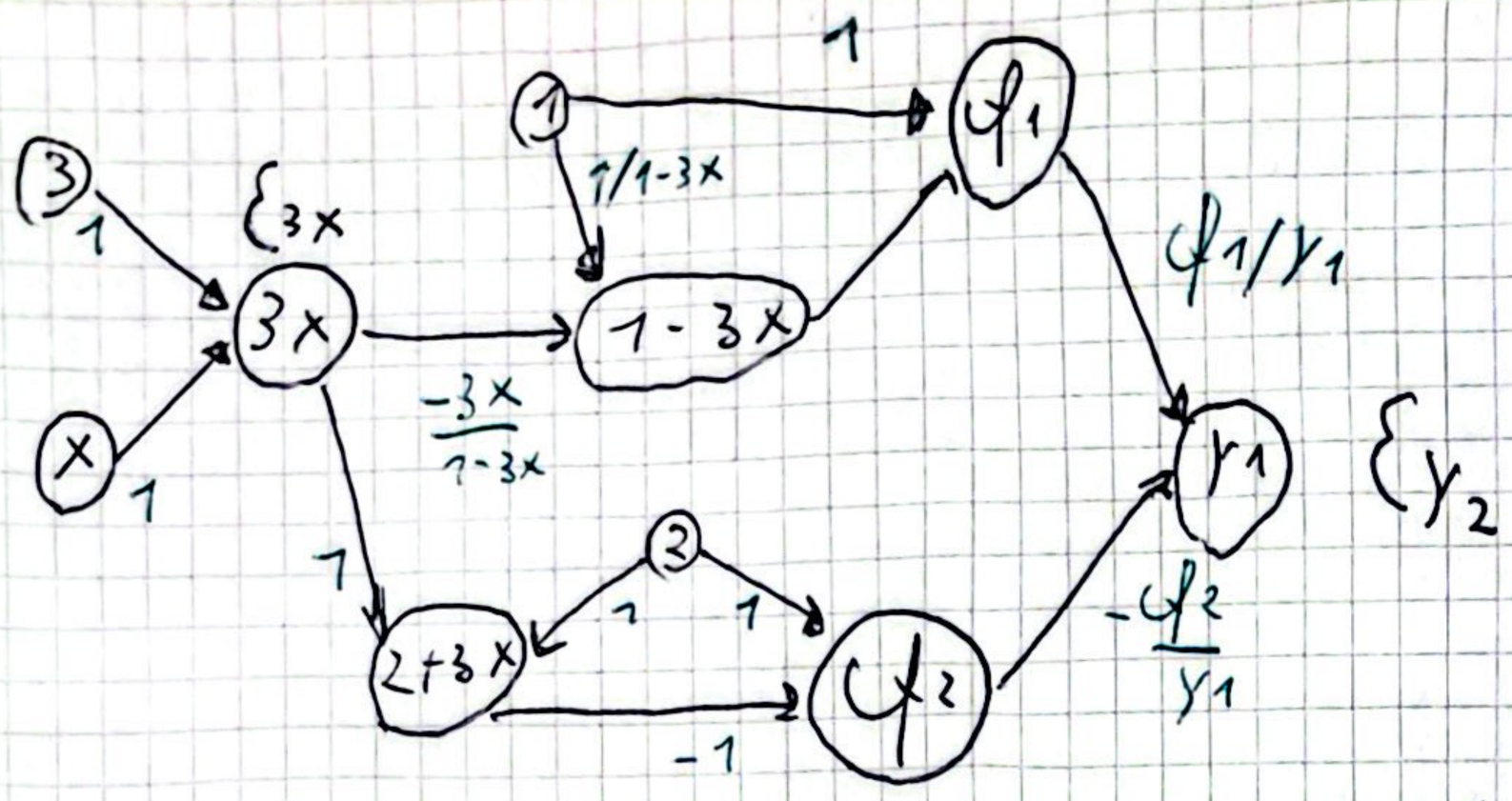
$$\frac{3(1-3x)^2 - 3x(18x-6)}{(1-3x)^4} + \frac{6(2+3x)^2 - 6x(12+18x)}{(2+3x)^4} =$$

$$= \frac{3(1-3x)^2 - 54x^2 + 18x}{(1-3x)^4} + \frac{6(2+3x)^2 - 72x - 108x}{(2+3x)^4} =$$

$$= \frac{3 + \frac{3}{2}}{3 + \frac{3}{2}} = 1$$

L'ALGORITMO È BEN CONDIZIONATO IN QUANTO L'ERRORE IN INPUT È UGUALE ALL'ERRORE IN OUTPUT

b1) $f_1 : \frac{1}{1-3x}$ $\quad f_2 : \frac{2}{2+3x}$ $\longmapsto Y_1 = f_1 - f_2$



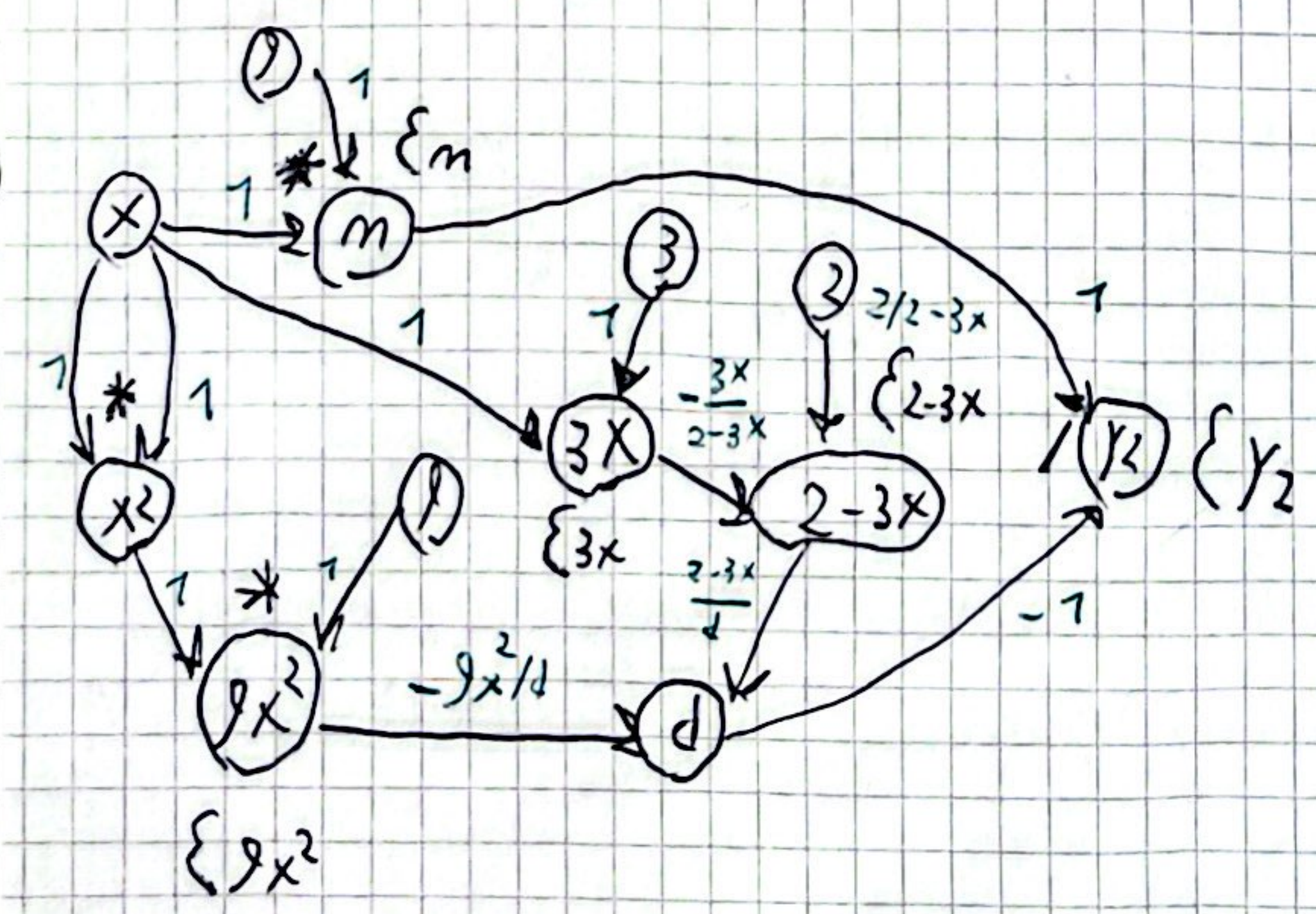
$$\xi_{Alg1} = \xi_{3x}\left\{-\frac{3x}{1-3x}\cdot(-1)\cdot\frac{\varphi_1}{y_1}\right\} + \xi_{2+3x}\left\{-1\cdot\left(-\frac{\varphi_2}{y_1}\right)\right\} +$$

$$\xi_{1-3x}\left\{-1\cdot\frac{\varphi_1}{y_1}\right\} + \xi_{\varphi_1}\left\{\frac{\varphi_1}{y_1}\right\} + \xi_{\varphi_2}\left\{-\frac{\varphi_2}{y_1}\right\} + \xi_{y_2}$$

LA CALCOLAZIONE DATA DA $\varphi_1 - \varphi_2$ POTREBBE CAUSARE PROBLEMI PER PICCOLI VALORI.

$$\lim_{x\to 0}\frac{\varphi_1}{y_1} = \frac{\frac{1}{1-3x}}{\frac{1}{1-3x}-\frac{2}{2+3x}} = \frac{1}{0} = \pm\infty \quad \text{ALGORITMO INSTABILE}$$

b2) $n := \ln x \qquad d := 2-3x-\ln x^2 \mapsto y_2 := n/d$



$$\xi_{Alg2} = \xi_{x^2}\left\{1\cdot\left(-\frac{\ln x^2}{d}\right)(-1)\right\} + \xi_n\{1\} + \xi_{3x}\left\{-\frac{3x}{2-3x}\cdot\frac{2-3x}{d}\cdot(-1)\right\}$$

$$+ \xi_{2-3x}\left\{\frac{2-3x}{d}\cdot(-1)\right\} + \xi_{\ln x^2}\left\{-\frac{\ln x^2}{d}\cdot(-1)\right\} + \xi_d\{-1\} + \xi_{y_2}$$

$$\lim_{x \to 0} \frac{-8x^2}{1} = 0$$

$$\lim_{x \to 0} \frac{-3x}{2-3x} = \frac{0}{2} = 0$$

$$\lim_{x \to 0} \frac{2-3x}{1} = 1$$

L'ALGORITMO È STABILE IN QUANTO I SUOI NASCENTI CALCOLO PONTA PROBLEMI

26 - 01 - 2016

$$f(x) = \frac{4}{x+\frac{2}{x}} - \frac{2}{x+\frac{1}{x}}$$

$$= \frac{4}{\frac{x^2+2}{x}} - \frac{2}{\frac{x^2+1}{x}} = 4 \cdot \frac{x}{x^2+2} - 2 \cdot \frac{x}{x^2+1} = \frac{4x}{x^2+2} - \frac{2x}{x^2+1} =$$

$$= \frac{4x(x^2+1) - 2x(x^2+2)}{(x^2+2)(x^2+1)} = \frac{(4x^3+4x)-(2x^3+4x)}{x^4+x^2+2x^2+2} = \frac{2x^3}{x^4+3x^2+2}$$

$$C = \frac{x \cdot f'(x)}{f(x)}$$

DERIVATA $\left(\frac{f}{g}\right) = \frac{f' \cdot g - f \cdot g'}{g^2} =$

$$= \frac{2(3x^2) \cdot x^4 + 3x^2 + 2 - 2x^3 \cdot 4x^3 + 3(2x) + 0}{(x^4+3x^2+2)^2} = \frac{6x^2(x^4+3x^2+2) - 2x^3(4x^3+6x)}{(x^4+8x^2+2)^2}$$

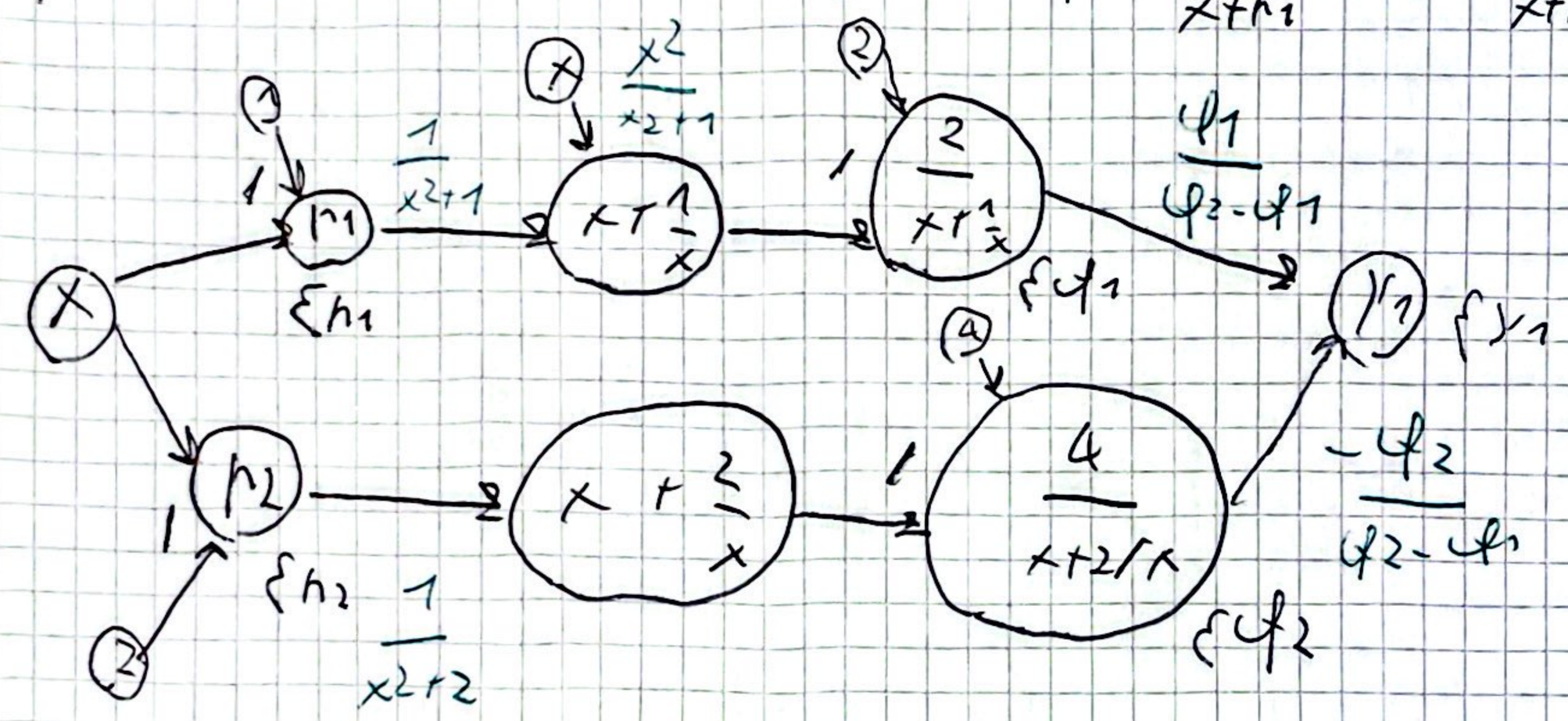
$$= \frac{(6x^6+18x^4+12x^2)-(8x^6+12x^4)}{(x^4+8x^2+2)^2} = \frac{-2x^6+6x^4+12x^2}{(x^4+3x^2+2)^2} =$$

$$= x \cdot \left(\frac{-2x^6 + 6x^4 + 12x^2}{(x^4+3x^2+2)^{\cancel{2}}} \right) \cdot \frac{x^4 + \cancel{3}x^2 + 2}{2x^3} = \frac{-2x^7 + 6x^5 + 12x^3}{2x^3(x^4 + 3x^2 + 2)} =$$

$$= \frac{-\cancel{2x^3}(x^4 - 3x^2 - 6)}{\cancel{2x^3}(x^4 + 3x^2 + 2)} = \frac{-x^4 + 3x^2 + 6}{x^4 + 3x^2 + 2}$$

$$\lim_{x \to 0} (c) = \lim_{x \to 0} \frac{6}{2} = 3$$ BEN CONDIZIONATO

b) ∂_1: $h_1 = 1/x$ $h_2 = 2/x$ $\varphi_1 = \frac{2}{x+h_1}$ $\varphi_2 = \frac{4}{x+h_2}$ $y = \varphi_2 - \varphi_1$



$$\{\partial_1 = \{h_1\left\{\frac{1}{x^2+1}(-1) \cdot \{\varphi_1\right\} + \{h_2\left\{\frac{1}{x^2+2}(-1) \cdot \{\varphi_2\right\} + \{\varphi_1\left\{\frac{\varphi_1}{\varphi_2-\varphi_1}\right\}$$

$$+ \{\varphi_2\left\{-\frac{\varphi_2}{\varphi_2-\varphi_1}\right\} + \{y_1$$

ESAME GENNAIO 2025

DIAGONALIZZAZIONE

1.

$$A = \begin{bmatrix} 0 & -4 & -3 \\ -4 & 0 & 0 \\ -3 & 0 & 0 \end{bmatrix}$$

a) $A - \lambda I$

$$\lambda I = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & -4 & -3 \\ -4 & 0 & 0 \\ -3 & 0 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} -\lambda & -4 & -3 \\ -4 & -\lambda & 0 \\ -3 & 0 & -\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = +4 \begin{vmatrix} -4 & 0 \\ -3 & -\lambda \end{vmatrix} - \lambda \begin{vmatrix} -\lambda & -3 \\ -3 & -\lambda \end{vmatrix} =$$

$\boxed{\text{SVOLGERE TUTTI I CALCOLI}}$

2

$$4 \cdot [4\lambda] - \lambda[\lambda^2 - (9)] = 16\lambda - \lambda^3 + 9\lambda \Rightarrow \lambda(16 - \lambda^2 + 9)$$

$$-\lambda^2 + 25 = 0 \Rightarrow \lambda^2 - 25 = 0 \Rightarrow \lambda^2 = 25 \Rightarrow \lambda = \pm 5$$

b)

$\lambda_1 = 0 \quad \lambda_2 = -5 \quad \lambda_3 = 5$

c) RANGO DI A E λ_1

$$\begin{bmatrix} 0 & -4 & -3 \\ -4 & 0 & 0 \\ -3 & 0 & 0 \end{bmatrix} \overset{R3 \to R2 - \frac{4}{3}R3}{\Rightarrow} \begin{bmatrix} 0 & -4 & -3 \\ -4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & -3 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3/4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A - \lambda_3 I \Rightarrow \begin{bmatrix} 0 & -4 & -3 \\ -4 & 0 & 0 \\ -3 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -5 & -4 & -3 \\ -4 & -5 & 0 \\ -3 & 0 & -5 \end{bmatrix} \quad \text{RANGO } 3$$

PERCHÉ $\begin{vmatrix} -4 & -5 \\ -3 & 0 \end{vmatrix} \neq 0$

$A+\lambda_2 I \Rightarrow \begin{bmatrix} 5 & -4 & -3 \\ -4 & 5 & 0 \\ -3 & 0 & 5 \end{bmatrix}$ RANGO 3 PERQUE' $\begin{vmatrix} -4 & 5 \\ -3 & 0 \end{vmatrix} \neq 0$

d) $(A-\lambda I)x = 0$

CASO $\lambda = 0$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3/4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} x_1 = 0 \\ x_2 + 3/4\, x_3 = 0 \\ 0 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = -\frac{3}{4} x_3 \\ 0 = 0 \end{cases} \Rightarrow \left\{ \underline{x_3} \begin{pmatrix} 0 \\ -3/4 \\ 1 \end{pmatrix} \,\middle|\, \underline{x_3} \in \mathbb{R} \right\} = \left\langle \begin{pmatrix} 0 \\ -3/4 \\ 1 \end{pmatrix} \right\rangle$$

CASO $\lambda = -5$

$$\begin{cases} 5x_1 - 4x_2 - 3x_3 = 0 \\ -4x_1 + 5x_2 = 0 \\ -3x_1 + 5x_3 = 0 \end{cases} \Rightarrow \begin{cases} 5x_1 - \frac{16}{5} x_1 = 3x_3 \\ x_2 = \frac{4}{5} x_1 \\ '' \end{cases} \Rightarrow \begin{cases} \frac{25-16}{5} x_1 = 3x_3 \\ '' \\ '' \end{cases} \Rightarrow$$

$$\begin{cases} \frac{9}{5} x_1 = 5x_3 \\ '' \\ '' \end{cases} \Rightarrow \begin{cases} x_1 = 5/3\, x_3 \\ x_2 = 4/3\, x_3 \\ -3\left(\frac{5}{3} x_3\right) + 5x_3 = 0 \end{cases} \Rightarrow \begin{cases} '' \\ '' \\ 0 = 0 \end{cases} \Rightarrow \left\{ \underline{x_3} \begin{pmatrix} 5/3 \\ 4/3 \\ 1 \end{pmatrix} \,\middle|\, \underline{x_3} \in \mathbb{R} \right\} = \left\langle \begin{pmatrix} 5/3 \\ 4/3 \\ 1 \end{pmatrix} \right\rangle$$

CASO $\lambda = 5$

$$\begin{cases} -5x_1 - 4x_2 - 3x_3 = 0 \\ -4x_1 - 5x_2 = 0 \\ -3x_1 - 5x_3 = 0 \end{cases} \Rightarrow \begin{cases} '' \\ -5x_2 = 4x_1 \\ '' \end{cases} \Rightarrow \begin{cases} -5x_1 + 16/5\, x_1 = 3x_3 \\ x_2 = -4/5\, x_1 \\ '' \end{cases} \Rightarrow$$

$$\begin{cases} \frac{-25+16}{5} x_1 = 3x_3 \\ '' \\ '' \end{cases} \Rightarrow \begin{cases} x_1 = -5/3\, x_3 \\ x_2 = -\frac{4}{5}\left(-\frac{5}{3}\right) x_3 \\ '' \end{cases} \Rightarrow \begin{cases} '' \\ x_2 = \frac{4}{3} x_3 \\ '' \end{cases} \Rightarrow$$

Scanned with CamScanner

$$\begin{cases} '' \\ '' \\ -3\left(-\frac{5}{3}\times 3\right) - 5\times 3 \end{cases} \Rightarrow \begin{cases} '' \\ '' \\ 0=0 \end{cases} \Rightarrow \left\langle \begin{pmatrix} -5/3 \\ 4/3 \\ 1 \end{pmatrix} \right\rangle$$

e)

$A = VDV^{-1}$

$\det(V) \neq 0$

$$V = \begin{pmatrix} 0 & 5/3 & -5/3 \\ -3/4 & 4/3 & 4/3 \\ 1 & 1 & 1 \end{pmatrix} \qquad D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 5 \end{pmatrix} \qquad \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

PER ESSERE DIAGONALIZZABILE DEVO VERIFICARE $A = VDV^{-1}$

$$A_{11} = \begin{vmatrix} 4/3 & 4/3 \\ 1 & 1 \end{vmatrix} = 0 \qquad A_{12} = \begin{vmatrix} -3/4 & 4/3 \\ 1 & 1 \end{vmatrix} = +\frac{25}{12} \qquad A_{13} = \begin{vmatrix} -3/4 & 4/3 \\ 1 & 1 \end{vmatrix} = \ldots$$

$$A_{21} = \begin{vmatrix} 5/3 & -5/3 \\ 1 & 1 \end{vmatrix} = -\frac{10}{3} \qquad A_{22} = \begin{vmatrix} 0 & -5/3 \\ 1 & 1 \end{vmatrix} = \frac{5}{3} \qquad A_{23} = \begin{vmatrix} 0 & 5/3 \\ 1 & 1 \end{vmatrix} = \frac{5}{3}$$

$$A_{31} = \begin{vmatrix} 5/3 & -5/3 \\ 4/3 & 4/3 \end{vmatrix} = \frac{40}{9} \qquad A_{32} = \begin{vmatrix} 0 & -5/3 \\ -3/4 & 4/3 \end{vmatrix} = +\frac{15}{12} = \frac{5}{4} \qquad A_{33} = \begin{vmatrix} 0 & 5/3 \\ -3/4 & 4/3 \end{vmatrix} = \frac{15}{12} = \frac{5}{4}$$

$$V^{-1} = \frac{1}{\det(V)} \cdot \mathrm{cof}(V)^T = \begin{bmatrix} 0 & -6/25 & 32/125 \\ 3/10 & 6/25 & 9/25 \\ -3/10 & 6/25 & 9/25 \end{bmatrix}$$

$A = VDV^{-1}$ = VERIFICATO

$$\Rightarrow \begin{cases} '' \\ '' \\ 0 = 0 \end{cases} \Rightarrow \left\langle \begin{pmatrix} -5/3 \\ 4/3 \\ 1 \end{pmatrix} \right\rangle$$

$\det(V) \neq 0$

$$\begin{matrix} -5/3 \\ 4/3 \\ 1 \end{matrix} \Bigg) \qquad D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 5 \end{pmatrix} \qquad \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

ZABILE DEVO VERIFICARE $A = VDV^{-1}$

$$3 \Big| = 0 \qquad A12 = \begin{vmatrix} -3/4 & 4/3 \\ 1 & 1 \end{vmatrix} = + \frac{25}{12} \qquad A13 = \begin{vmatrix} -3/4 & 4/3 \\ 1 & 1 \end{vmatrix} = -\frac{25}{12}$$

$$= -\frac{10}{3} \qquad A22 = \begin{vmatrix} 0 & -5/3 \\ 1 & 1 \end{vmatrix} = \frac{5}{3} \qquad A23 = \begin{vmatrix} 0 & 5/3 \\ 1 & 1 \end{vmatrix} = \frac{5}{3}$$

$$= \frac{40}{9} \qquad A32 = \begin{vmatrix} 0 & -5/3 \\ -3/4 & 4/3 \end{vmatrix} = + \frac{15}{12} = \frac{5}{4} \qquad A33 = \begin{vmatrix} 0 & 5/3 \\ -3/4 & 4/3 \end{vmatrix} = \frac{15}{12} = \frac{5}{4}$$

$$= \begin{bmatrix} 0 & -6/25 & 32/125 \\ 3/10 & 6/25 & 9/25 \\ -3/10 & 6/25 & 9/25 \end{bmatrix}$$

$125 \cdot 0$

MINMI QUADRATI

$g(x): ax + b$

x	-2	-1	0	2	3
y	6	1	1	1	-1

a) $A = \begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$ $\quad y = \begin{bmatrix} 6 \\ 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$

$\boxed{\text{SOGGONNE = NICAS} \quad \text{POSSO CALCOLARE}}$

RIGHE COLONNE

b) $y^T y = \underset{1 \times 5}{\begin{bmatrix} 6 & 1 & 1 & 1 & -1 \end{bmatrix}} \cdot \underset{5 \times 1}{\begin{bmatrix} 6 \\ 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}} = \begin{bmatrix} 40 \end{bmatrix}$

$y y^T = \underset{5 \times 1}{\begin{bmatrix} 6 \\ 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}} \cdot \underset{1 \times 5}{\begin{bmatrix} 6 & 1 & 1 & 1 & -1 \end{bmatrix}} = \begin{bmatrix} 36 & 6 & 6 & 6 & -6 \\ 6 & 1 & 1 & 1 & -1 \\ 6 & 1 & 1 & 1 & -1 \\ 6 & 1 & 1 & 1 & -1 \\ -6 & -1 & -1 & -1 & 1 \end{bmatrix}$

$A^T y A \Rightarrow$ FACCIAMO PRIMA $A^T y \Rightarrow \begin{bmatrix} -2 & -1 & 0 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -14 \\ 8 \end{bmatrix}$

$A^T y A = \underset{2 \times 1}{\begin{bmatrix} -14 \\ 8 \end{bmatrix}} \cdot \underset{5 \times 2}{\begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}} =$ NON POSSO FARLO PERCHÉ COLONNE $\neq$ RIGHE

$A^T A = \underset{2 \times 5}{\begin{bmatrix} -2 & -1 & 0 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}} \cdot \underset{5 \times 2}{\begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}} = \begin{bmatrix} 18 & 2 \\ 2 & 5 \end{bmatrix}$

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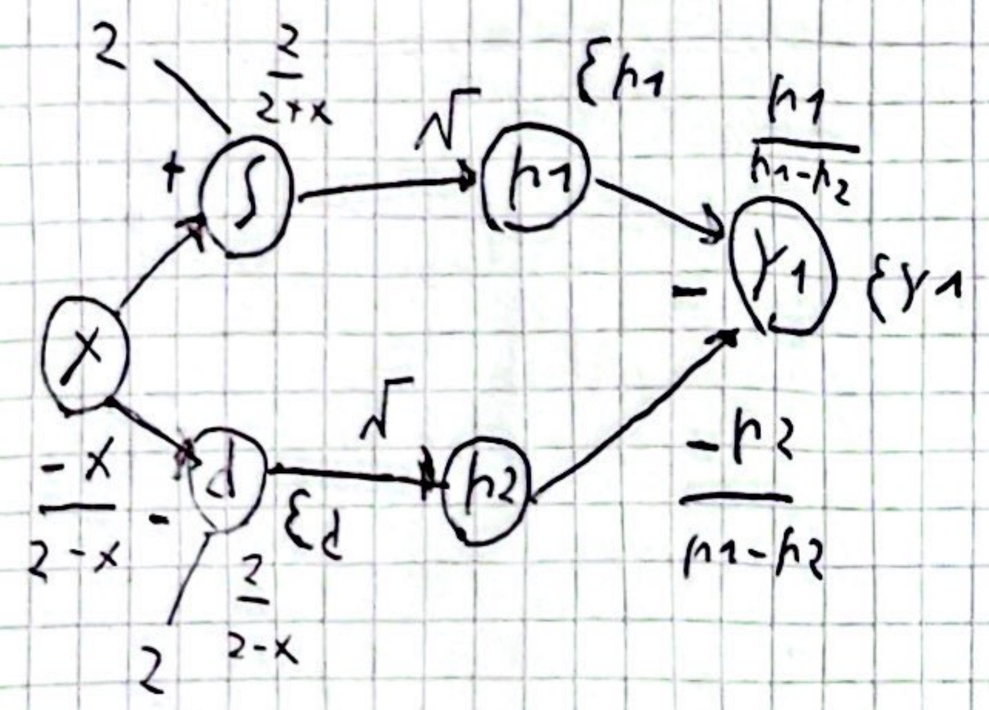
$$A^T y = \begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 & 1 & 1 & 1 & -1 \end{bmatrix} = \text{NON posso farlo}$$

CONDIZIONAMENTO (ES 1)

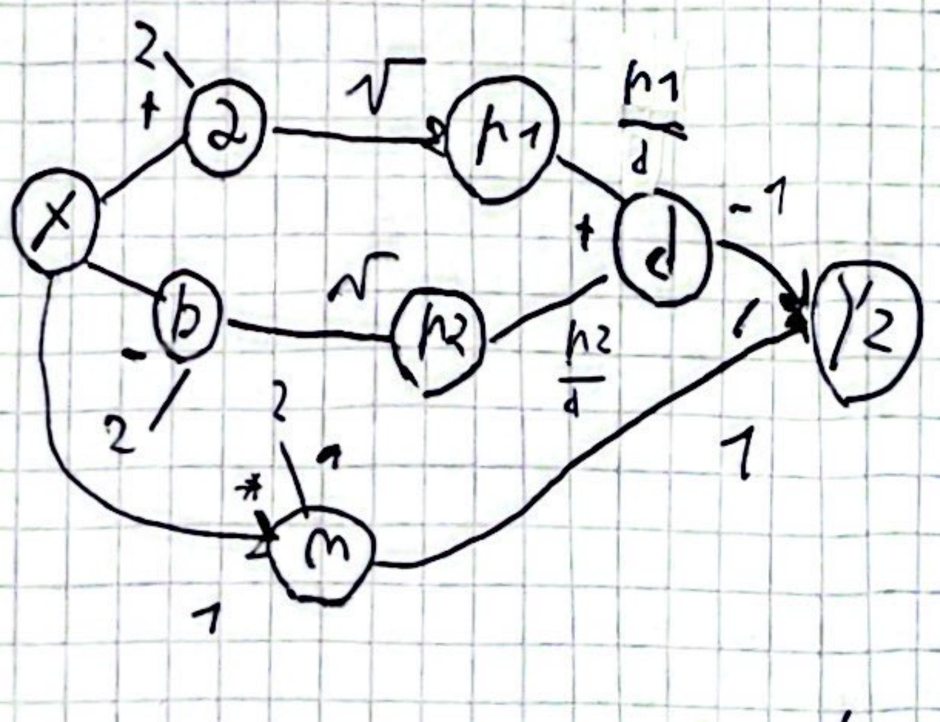
$a_1: x \to s := 2+x \quad d := 2-x \mapsto r_1 := \sqrt{s} \quad r_2 = \sqrt{d} \mapsto y_1 := r_1 - r_2$



$$\varepsilon_{Alg1} = \varepsilon_{y_1} + \varepsilon_{r_2}\left\{\frac{-r_2}{r_1-r_2}\right\} + \varepsilon_d\left\{-\frac{r_2}{r_1-r_2}\right\} + \varepsilon_{r_1}\left\{\frac{r_1}{r_1-r_2}\right\} + \varepsilon_s\left\{\frac{r_1}{r_1-r_2}\right\}$$

$$\lim_{x \to 0} = \frac{-\sqrt{2}}{\sqrt{2}-\sqrt{2}} = \pm\infty \quad \text{INSTABILE}$$

$a_2: x \mapsto r_1 := \sqrt{2+x} \quad r_2 = \sqrt{2-x} \quad m = 2x \quad d = r_1 + r_2 \quad y_2 = m/d$

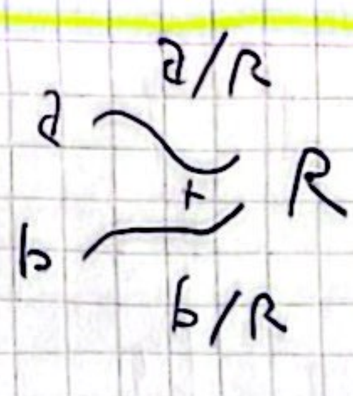


$$\varepsilon_{Alg2} = \varepsilon_{y_2} + \varepsilon_d\{-1\} + \varepsilon_{r_1}\left\{\frac{r_1}{d}\cdot(-1)\right\} + \varepsilon_{r_2}\left\{\frac{r_2}{d}\cdot(-1)\right\} +$$

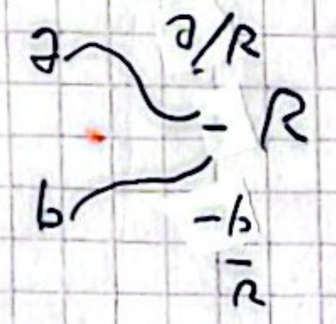
$$+ \varepsilon_m\{1\}$$

ICONA:

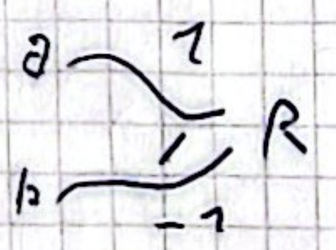
$a + b$



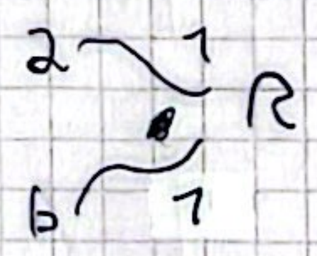
$A - b$



a / b



$a \cdot b$



MINIMI QUADRATI 16-9-21

$$\alpha(x) = a + \beta x + \gamma \frac{1}{x}$$

x	-2	-1	1	2
y	1	0	0	-1

a) $A = \begin{bmatrix} 1 & -2 & -1/2 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 2 & 1/2 \end{bmatrix}$ $\quad Y = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$

b)

A^2 = NON POSSIAMO SVOLGERLO IN QUANTO NON È QUADRATA

$$AA^T = \begin{bmatrix} 1 & -2 & -1/2 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 2 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & -1 & 1 & 2 \\ -1/2 & -1 & 1 & 1/2 \end{bmatrix} = \begin{bmatrix} 21/4 & 7/2 & -3/2 & -13/4 \\ 7/2 & 3 & -1 & -3/2 \\ -3/2 & -1 & 3 & 7/2 \\ -\frac{13}{4} & -\frac{3}{2} & \frac{7}{2} & \frac{21}{4} \end{bmatrix}$$

$$y^T y = \overset{1\times 4}{\begin{bmatrix} 1 & 0 & 0 & -1 \end{bmatrix}} \cdot \overset{4\times 1}{\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}} = \begin{bmatrix} 2 \end{bmatrix}$$

$$A^T y = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & -1 & 1 & 2 \\ -1/2 & -1 & 1 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \\ -1 \end{bmatrix}$$

c)

$$c_1 y_1 + c_2 y_2 + c_3 y = 0$$

$$M = \begin{bmatrix} 1 & -2 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} c_1 - 2c_2 + c_3 = 0 \\ c_1 - c_2 = 0 \\ c_1 + c_2 = 0 \\ c_1 + 2c_2 - c_3 = 0 \end{cases} \Rightarrow \begin{cases} c_2 = c_3 \\ c_1 = c_2 \\ c_2 + c_3 = 0 \\ '' \end{cases} \Rightarrow \begin{cases} c_1 = c_2 = c_3 = 0 \\ \\ \\ \end{cases}$$

d) ortogonali:

$$\begin{array}{c} \partial_1^T \\ \partial_2^T \end{array} \begin{pmatrix} 1 & 1 & 1 & 1 \\ -2 & -1 & 1 & 2 \end{pmatrix} \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{y}_3 \\ \tilde{y}_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} \tilde{y}_1 + \tilde{y}_2 + \tilde{y}_3 + \tilde{y}_4 = 0 \\ -2\tilde{y}_1 - \tilde{y}_2 + \tilde{y}_3 + 2\tilde{y}_4 = 0 \end{cases} \Rightarrow \begin{cases} '' \\ -\tilde{y}_2 = 2\tilde{y}_1 - \tilde{y}_3 - 2\tilde{y}_4 \end{cases} \Rightarrow$$

$$\begin{cases} \tilde{y}_1 - 2\tilde{y}_1 + \tilde{y}_3 + 2\tilde{y}_4 + \tilde{y}_3 + \tilde{y}_4 = 0 \\ \tilde{y}_2 = -2\tilde{y}_1 + \tilde{y}_3 + 2\tilde{y}_4 \end{cases} \Rightarrow \begin{cases} -\tilde{y}_1 + 2\tilde{y}_3 + 3\tilde{y}_4 = 0 \\ '' \end{cases} \Rightarrow$$

$$\tilde{y}_1 = 2\tilde{y}_3 + 3\tilde{y}_4$$
$$\tilde{y}_2 = -4\tilde{y}_3 - 6\tilde{y}_4 + \tilde{y}_3 + 2\tilde{y}_4$$

$$\Rightarrow \begin{cases} \tilde{y}_1 = 2\tilde{y}_3 + 3\tilde{y}_4 \\ \tilde{y}_2 = -3\tilde{y}_3 - 4\tilde{y}_4 \end{cases} \Rightarrow \begin{pmatrix} 5 \\ -7 \\ 1 \\ 1 \end{pmatrix}$$

PER VERIFICARE CHE QUESTO VETTORE SIA ORTOGONALE AD a_1 E a_2 BASTA VERIFICARE CHE:

$$\boxed{\begin{array}{c} a_1^T \\ a_2^T \end{array} \begin{pmatrix} 1 & 1 & 1 & 1 \\ -2 & -1 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -7 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}}$$

e) $A^T A \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = A^T y$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & -1 & 1 & 2 \\ -1/2 & -1 & 1 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & -1/2 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 2 & 1/2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 10 & 4 \\ 0 & 4 & 5/2 \end{bmatrix}$$

$$\begin{cases} 4\alpha = 0 \\ 10\beta + 4\gamma = -4 \\ 4\beta + 5/2\,\gamma = -1 \end{cases} \Rightarrow \begin{cases} \alpha = 0 \\ 4\gamma = -4 - 10\beta \\ '' \end{cases} \Rightarrow \begin{cases} \alpha = 0 \\ \gamma = -1 - \frac{5}{2}\beta \\ 4\beta - \frac{5}{2} - \frac{25}{4}\beta = -1 \end{cases} \Rightarrow \begin{cases} '' \\ '' \\ 4\beta - \frac{25}{4}\beta = -1 + \frac{5}{2} \end{cases}$$

$$\begin{cases} '' \\ '' \\ -\frac{9}{4}\beta = \frac{3}{2} \end{cases} \Rightarrow \begin{cases} \alpha = 0 \\ \gamma = -8/3 \\ \beta = \frac{2}{3} \end{cases}$$

CONDIZIONAMENTO MATRICI (ES 4)

$$A = \begin{pmatrix} 100 & -1 \\ -98 & 1 \end{pmatrix} \qquad x = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad \delta b \begin{pmatrix} 10^{-2} \\ 10^{-2} \end{pmatrix} \qquad b = A \cdot x$$

$$b = \begin{pmatrix} 100 & -1 \\ -98 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 101 \\ -99 \end{pmatrix}$$

$A_{11} = 1 \qquad A_{12} = 198$

$A_{21} = 1 \qquad A_{22} = 100$

$\det(A) = 100 - 98 = 2 \neq 0$

$$\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 98 & 100 \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ 49 & 50 \end{pmatrix}$$ VERIFICATO

b) $\|A\|_1 = \max(198, 2) = 198$

$\|A^{-1}\|_1 = \max(2, 99) = 99$

$M_1(A) = 198 \cdot 99 = 19'602$

$\|A\|_\infty = \max(99, 2) = 99$

$\|A^{-1}\|_\infty = \max(51, 50) = 51$

$M_\infty = 99 \cdot 51 = 5049$

c) $\|x\|_2 = \sqrt{1+1} = \sqrt{2}$

$\|x\|_\infty = 1$

$\|b\|_2 = \sqrt{101^2 + 99^2} = \sqrt{20'002}$

$\|b\|_\infty = 101$

$$\|\delta b\|_2 = \sqrt{(10^{-2})^2 + (10^{-2})^2} = \sqrt{10^{-4} + 10^{-4}} = \sqrt{2\,10^{-4}} = \frac{\sqrt{2}}{\sqrt{10^4}} = \frac{\sqrt{2}}{10^2}$$

$\|\delta b\|_\infty = 10^{-2}$

d) $\|\tilde{x} - x\|_\infty \qquad A\tilde{x} = b + \delta b$

$$\begin{cases} \\ \end{cases} \quad \frac{\|\delta b\|_\infty}{\|b\|_\infty} = \frac{10^{-2}}{101} = \frac{\frac{1}{10^2}}{101} = 0{,}000099$$

$\xi_x = \mu_{00}(A) \cdot \xi_b = 5049 \cdot 0{,}000099 = 0{,}50$

GRAFI

21-7-21

$$f(x) = \frac{2x-1}{2x+1} - \frac{x-2}{x+2}$$ VALORI MOLTO GRANDI x

AL POSTO DELLA $f(x)$ DATA USIAMO QUELLA DEL PUNTO b QUINDI:

$$f(x) = \frac{6x}{2x^2+5x+2}$$

$$f'(x) = \frac{6\cdot(2x^2+5x+2) - 6x(4x+5)}{(2x^2+5x+2)^2} = \frac{12x^2+30x+12-24x^2-30x}{(2x^2+5x+2)^2} =$$

$$= \frac{-12x^2+12}{(2x^2+5x+2)^2}$$

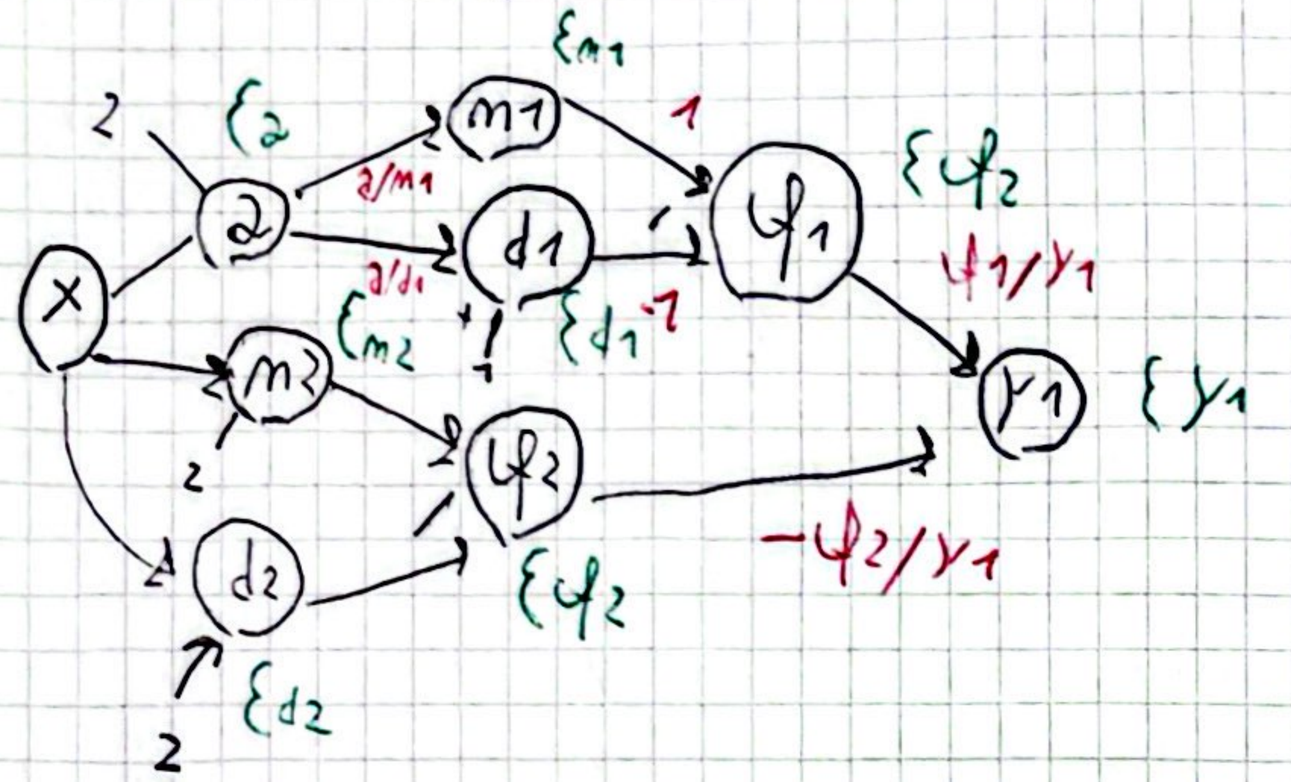
$$\varepsilon_f = \frac{x \cdot f'(x)}{f(x)} \Rightarrow x \cdot \left[\frac{-12x^2+12}{(2x^2+5x+2)^2}\right] \cdot \frac{1}{\frac{6x}{2x^2+5x+2}} \Rightarrow$$

$$\Rightarrow x \cdot \left[\frac{-12x^2+12}{(2x^2+5x+2)^2}\right] \cdot \frac{2x^2+5x+2}{6x} \Rightarrow \cancel{x} \cdot \frac{\cancel{6}(-2x^2+12)}{(2x^2+5x+2)^{\cancel{2}}} \cdot \frac{2x^{\cancel{2}}+5x+2}{\cancel{6}\cancel{x}}$$

$$= \frac{-2x^2+2}{2x^2+5x+2}$$

$$\lim_{x \to \infty} \frac{-2x^2+2}{2x^2+5x+2} = F.I. \overset{\text{DE HOPITAL}}{\Rightarrow} \frac{-4x}{4x+5} \overset{\text{DE HOPITAL}}{\Rightarrow} \frac{-4}{4} = -1$$ $\boxed{\text{STABILE}}$

b1) $\varphi_1 = \dfrac{2x-1}{2x+1}$ $\qquad \varphi_2 = \dfrac{x-2}{x+2}$ $\qquad y_1 = \varphi_1 - \varphi_2$



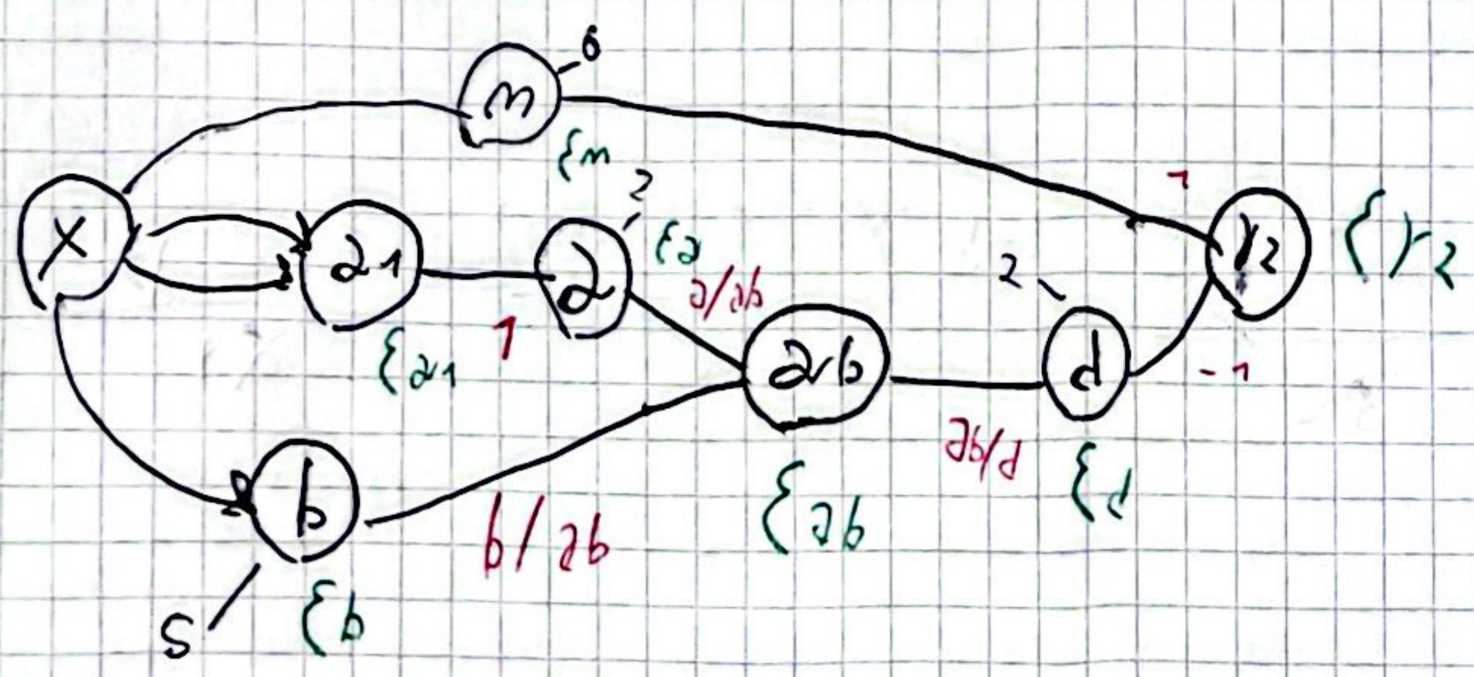
$$\{b_1 = \{y_1 + \{\varphi_2\left\{-\frac{\varphi_2}{y_1}\right\} + \{d_2\left\{(-1)\cdot\left(-\frac{\varphi_2}{y_1}\right)\right\} + \{m_2\left\{(1)\cdot\left(-\frac{\varphi_2}{y_1}\right)\right\}$$

$$+\{\varphi_1\left\{\frac{\varphi_1}{y_1}\right\} + \{d_1\left\{(-1)\cdot\left(\frac{\varphi_1}{y_1}\right)\right\} + \{m_1\left\{(1)\cdot\left(\frac{\varphi_1}{y_1}\right)\right\} + \{a\left\{\right.$$

$$\frac{a}{d_1}\cdot(-1)\cdot\frac{\varphi_1}{y_1} + \frac{a}{m_1}\cdot(1)\cdot\left(\frac{\varphi_1}{y_1}\right)\Bigg\}$$

SOMA $\qquad$ SOMA

b2) $m = 6x \qquad d = 2x^2 + 5x + 2 \qquad y_2 = m/d$



$$\{b_2 = \{y_2 + \{m\{1\} + \{d\{-1\} + \{ab\left\{(-1)\cdot\left(\frac{ab}{d}\right)\right\} +$$

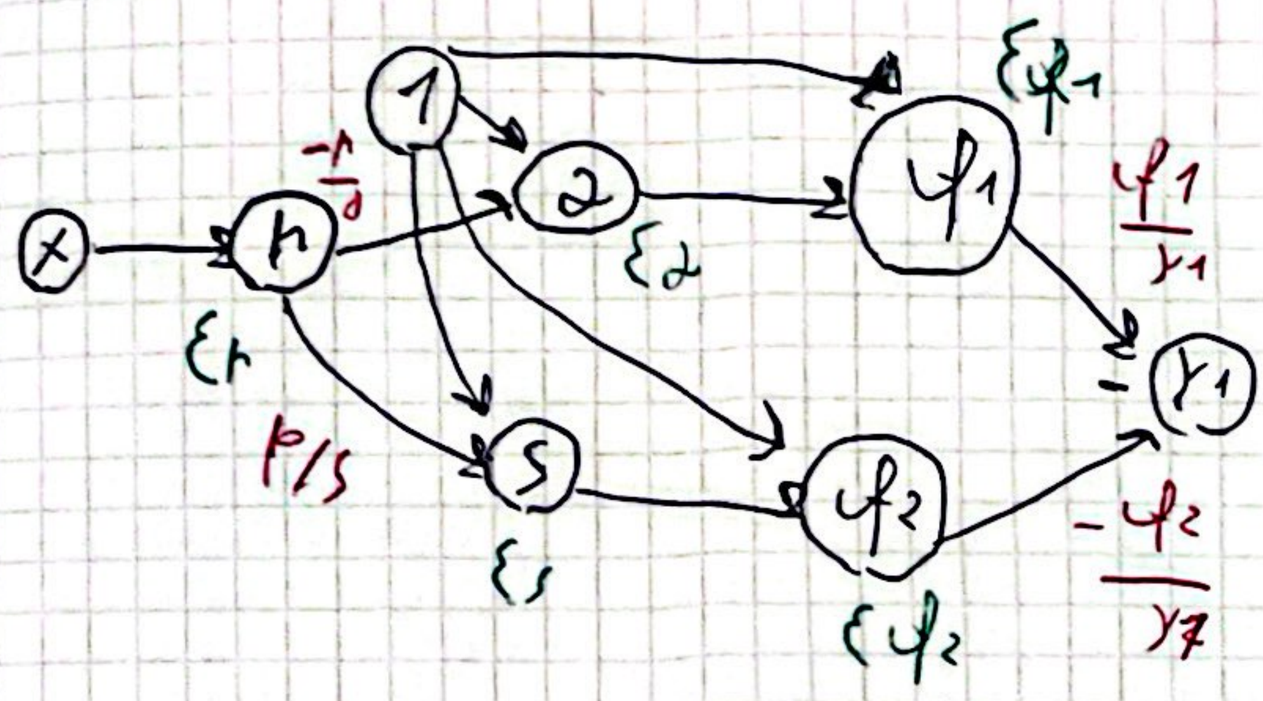
$$+\{b\left\{\frac{b}{ab}\cdot\frac{ab}{a}\cdot(-1)\right\} + \{a\left\{\frac{a}{ab}\cdot\frac{ab}{d}\cdot(-1)\right\} +$$

$$+\{a1\left\{1\cdot\frac{a}{ab}\cdot\frac{ab}{d}\cdot(-1)\right\}$$

[illegible] GABRI

1)

$$\varphi(x) = \frac{1}{1-\sqrt{x}} - \frac{1}{1+\sqrt{x}}$$

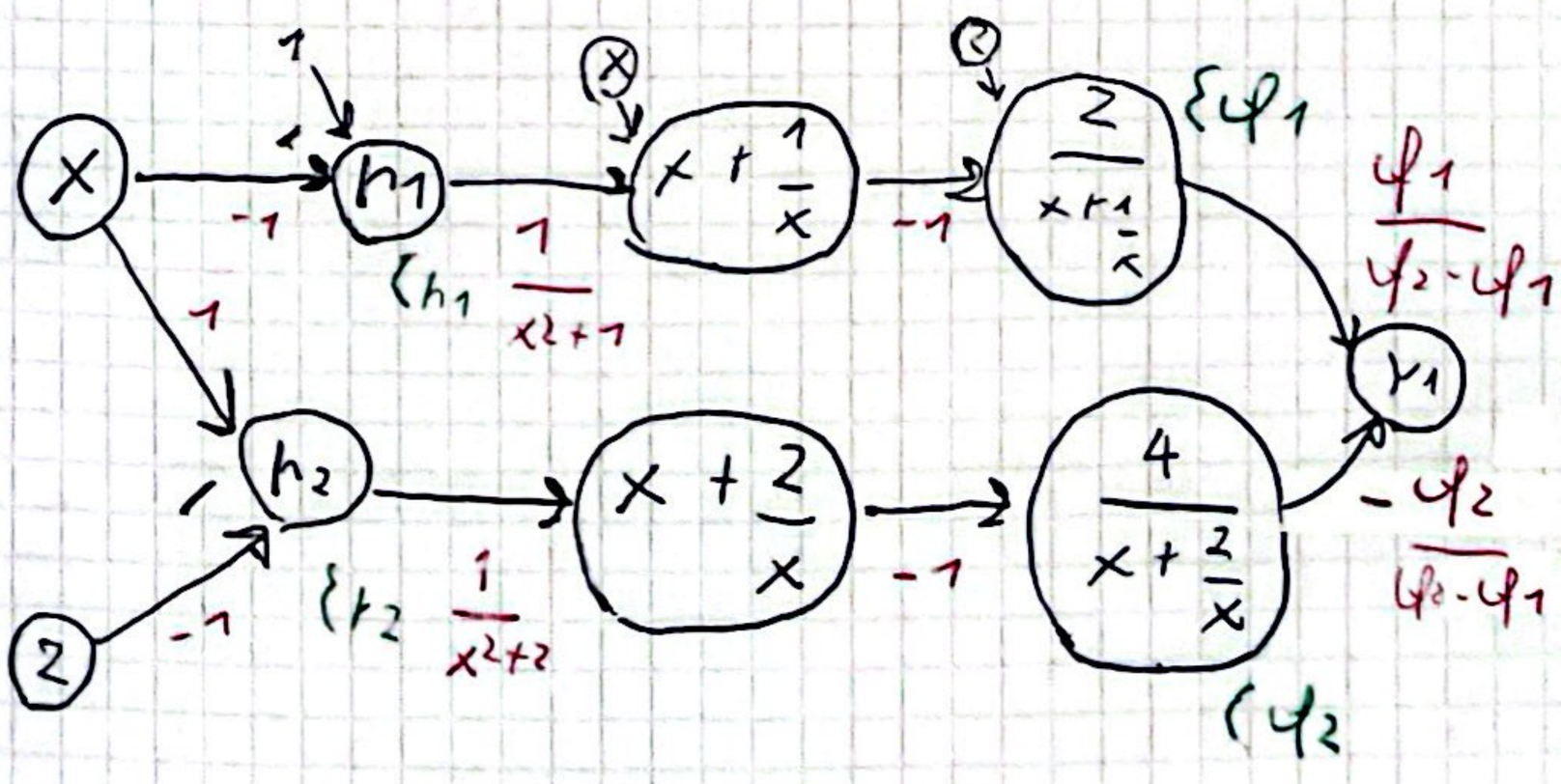


$$\{_{ALG1} = \{_r \left\{ \left(-\frac{r}{d}\right) \cdot (-1) \cdot \frac{\varphi_1}{y_1} + \frac{r}{s} \cdot (-1) \cdot \left(-\frac{\varphi_2}{y_1}\right) \right\} +$$

$$\{_s \left\{ (-1) \cdot \left(-\frac{\varphi_2}{y_1}\right) \right\} + \{_{\varphi_1} \left\{ \frac{\varphi_1}{y_1} \right\} + \{_{\varphi_2} \left\{ -\frac{\varphi_2}{y_1} \right\} + \{_{y_1}$$

3)

$r_1 = 1/x \qquad r_2 = 2/x \qquad \varphi_1 = \dfrac{2}{x+r_1} \qquad \varphi_2 = \dfrac{4}{x+r_2} \qquad y = \varphi_2 \cdot \varphi_1$



$$\{_{ALG} = \{_{r_1} \left\{ \frac{1}{x^2+1} \cdot (-1) \cdot \{_{\varphi_2} \right\} + \{_{r_2} \left\{ \frac{1}{x^2+2} \cdot (-1) \cdot \{_{\varphi_2} \right\} + \{_{\varphi_1} \left\{ \frac{\varphi_1}{\varphi_2 - \varphi_1} \right\} +$$

$$\{_{\varphi_2} \left\{ \frac{-\varphi_2}{\varphi_2 - \varphi_1} \right\} + \{_{y_1}$$

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