7/11

OPERAZIONI TRA MATRICI

$A \in \mathbb{R}^{m \times n}, \quad B \in \mathbb{R}^{m \times n} \longrightarrow \underbrace{(A+B)}_{\mathbb{R}^{m \times n}} = a_{ij} + b_{ij}$

$A = (a_{ij}), \quad B = (b_{ij})$

$A \in \mathbb{R}^{m \times n}, \quad \lambda \in \mathbb{R} \longrightarrow (\lambda A)_{ij} = \lambda a_{ij}$

$A \in \mathbb{R}^{m \times n}, \quad B \in \mathbb{R}^{n \cdot r} \longrightarrow (A\,B)_{ij} = R_i^A \, C_j^B = \sum_{k=1}^{n} a_{ik} b_{kj}$

$\boxed{AB \neq BA} \longrightarrow (A+B)(A-B) = A^2 + \overbrace{BA - AB}^{\neq 0} - B^2$

A, B DIAGONALI (MATRICI QUADRATE) $\Longrightarrow A \cdot B$ (DIAGONALE)

A, B TRIANGOLARI SUP. $\Longrightarrow A \cdot B$ TRIANGOLARI SUP

A, B SIMMETRICI $\not\Longrightarrow A, B$ SIMMETRICI

$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \longrightarrow A \cdot B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$$A \cdot B = 0 \quad \left(A = \begin{pmatrix}1 & -2\\ -1 & 2\end{pmatrix},\ B = \begin{pmatrix}2 & 4\\ 1 & 2\end{pmatrix} \to A \cdot B = \begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}\right)$$

$$A \cdot B = A \cdot C \not\Rightarrow B = C$$

$$A^2 = 0 \not\Rightarrow A = 0 \quad \left(A = \begin{pmatrix}2 & -4\\ 1 & -2\end{pmatrix}\right)$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

ESEMPIO:

$$\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix} \cdot \begin{pmatrix}0 & 1 & 2\\ 1 & 1 & 3\end{pmatrix} \cdot \begin{pmatrix}1\\ 4\\ 3\end{pmatrix}$$

$$\begin{pmatrix}2 & 3 & 8\\ 1 & 1 & 3\end{pmatrix} \begin{pmatrix}1\\ 4\\ 3\end{pmatrix} = \begin{pmatrix}38\\ 14\end{pmatrix}$$

$$= \begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix} \begin{pmatrix}10\\ 14\end{pmatrix} = \begin{pmatrix}38\\ 14\end{pmatrix}$$

$$A \cdot (B + B') = AB + AB'$$

$$(A + A') B = A \cdot B + A' B$$

$$(\lambda A) B = \lambda (A \cdot B)$$

$A(\lambda B) = \lambda(A \cdot B)$

$(A\,B)^t = B^t \cdot A^t$

$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ELEMENTO NEUTRO PRODOTTO

$$A\,I = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots \end{pmatrix} \begin{pmatrix} 1 & 0 & \dots & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \dots \\ a_{21} & \end{pmatrix} = A$$

$I \cdot A = A$

DATA $A \in \mathbb{R}^{m \cdot m}$, $\exists B \in \mathbb{R}^{m \cdot m}$ T.C. $AB = I$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \shortparallel$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad B \cdot A$

(A "INVERTIBILE", $B = A^{-1}$)
INVERSA

$$A = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} \longrightarrow A \cdot B = \begin{pmatrix} 6-5 & 3-3 \\ -10+10 & -5+6 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} 6-5 & -2+2 \\ 15-15 & -5+6 \end{pmatrix} = I \rightarrow B = A^{-1}$$

$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, CERCO $A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \longrightarrow A \cdot A^{-1} = \begin{pmatrix} c & d \\ 0 & 0 \end{pmatrix} \neq I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow$ A NON INVERTIBILE

A^{-1}, SE ESISTE, E' UNICA

$I^{-1} = I$

$(A^{-1})^{-1} = A \cdot (AB)^{-1} = B^{-1} A^{-1}$

$A \in \mathbb{R}^{n \times n} \longrightarrow$ DETERMINANTE ($\det A_{\in \mathbb{R}}$)

$n = 1 \rightarrow A = (a) \rightarrow \det A = a$

$n = 2 \rightarrow A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \xrightarrow[\text{CRAMER}]{\text{REGOLA DI}} \det A = + a_{11}a_{22} - a_{12}a_{21}$

ESEMPIO:

$\det \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} = 2 \cdot 5 - 4 \cdot 3 = -2$

$n > 2 \longrightarrow$ REGOLA DI LAPLACE

ESEMPIO:

$n = 3$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \longrightarrow \det A = +a_{11} \cdot \det A_{11}$$

$$= -a_{12} \cdot \det A_{12}$$

$$= +a_{13} \cdot \det A_{13}$$

A_{ij} = CANCELLO RIGA E COLONNA

$$A_{12} = \begin{pmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{pmatrix}$$

ESEMPIO

$$\det \begin{pmatrix} 1 & -2 & 0 \\ 1 & 1 & 2 \\ 0 & 2 & 3 \end{pmatrix} = +1 \cdot \det \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} - (-2) \cdot \det \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

$$+0 \det \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} = +1 \cdot (1\ 3 - 2\ 2) + 2(1\ 3 - 2\ 0) =$$

$$= -1 + 2 \cdot 3 = 5$$

IN GENERALE

$$\det A = \sum_{j=1}^{n} (-1)^{1+j} \cdot a_{1j} \cdot \det A_{1j}$$

$$\det \begin{pmatrix} 1 & -2 & 0 \\ 1 & 1 & 2 \\ 0 & 2 & 3 \end{pmatrix} = \overset{+}{0} \cdot \det() - 2\det\begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix} + 3\det\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$$

$$= 0 - 2(1 \cdot 2 - 0 \cdot 1 - 2) + 3(1 \cdot 1 - (-2) \cdot 1) =$$

$$= -2(2) + 3(1+2) = -4 + 9 = 5$$

$$\det(A^t) = \det A$$

$\det(A \cdot B) = \det A \cdot \det B$ (REGOLA DI BINET) $\forall A, B \in \mathbb{R}^{n \times n}$

A INVERTIBILE $\Longleftrightarrow \det A \neq 0$

$$A \cdot A^{-1} = I \implies \det(A \cdot A^{-1}) = \det(I)$$

$$\det A \cdot \det A^{-1}$$

$$= \det\begin{pmatrix} 1 & 0 & \cdots & 0 \\ & \ddots & & \\ & & & 1 \end{pmatrix} = 1 \cdot \det\begin{pmatrix} 1 & \\ & 1 \end{pmatrix} =$$

$$= \det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$$

$$\det\left(A^{-1}\right) = \frac{1}{\det A}$$