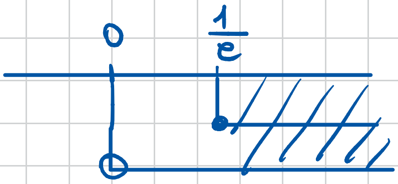
FOGLIO 4

Es 2

(3) $f(x) = \sqrt{\log x + 1}$

$$\begin{cases} \log x + 1 \geq 0 \\ x > 0 \end{cases} \rightarrow \begin{cases} \log x \geq -1 \\ x > 0 \end{cases}$$

$$\rightarrow \begin{cases} \overbrace{e^{\log x}}^{x} \geq e^{-1} \\ x > 0 \end{cases} \rightarrow \begin{cases} x \geq \frac{1}{e} \\ x > 0 \end{cases}$$



$x \geq \frac{1}{e}$

$\text{dom} f = \left[\frac{1}{e}, +\infty\right)$

$h(x) = \log x \qquad g(x) = e^x$

1) $x = (h \circ g)(x) = h(g(x)) = h(e^x)$
$= \log e^x$

$\log e^x = x$

2) $x = (g \circ h)(x) = g(h(x)) = g(\log x)$
$= e^{\log x}$

$$\boxed{e^{\log x} = x}$$

(4) $f(x) = \sqrt{e^x - 5}$

$e^x - 5 \geq 0 \rightarrow e^x \geq 5$
$\rightarrow e^x \geq e^{\log 5}$
$\rightarrow x \geq \log 5$

$5 = e^{\log 5}$

$a = e \simeq 2{,}7$

- se $0 < a < 1$ allora a^x è str. decrescente quindi se $a^{x_1} < a^{x_2}$ allora $x_1 > x_2$

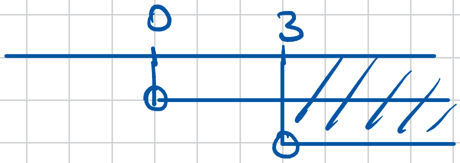
- se $a > 1$ allora a^x è str crescente quindi se $a^{x_1} < a^{x_2}$ allora $x_1 < x_2$

$\text{dom} f = \{x \in \mathbb{R} \mid e^x - 5 \geq 0\} = [\log 5, +\infty)$

Es 3 (4) $f(x) = \log x - (x+1)\log(x-3)$

$\begin{cases} x > 0 & \text{per definire il I logaritmo} \\ x - 3 > 0 & \text{" " " II "} \end{cases}$

$\begin{cases} x > 0 \\ x > 3 \end{cases}$



$x > 3$

$\text{dom} f = (3, +\infty)$

(3) $f(x) = 5 + \log|x+1|$

Per definire $\log|x+1|$ basta porre $|x+1| > 0$

$\rightarrow \quad \forall x \in \mathbb{R}, \quad x+1 \neq 0 \quad \rightarrow \quad \forall x \in \mathbb{R} \setminus \{-1\}$

$\text{dom} f = (-\infty, -1) \cup (-1, +\infty) = \mathbb{R} \setminus \{-1\}$

- $x^2 + 2 > 0 \quad \rightarrow \quad x^2 + 2 = 0 \quad \Delta = -8 < 0$

  $\hookrightarrow \quad \forall x \in \mathbb{R}$

- $x^2 - 2 > 0 \quad \rightarrow \quad x^2 - 2 = 0 \quad \Delta = +8 > 0$

  $\downarrow$

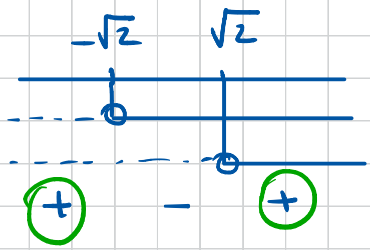
  $x_{1,2} = \pm\sqrt{2}$

  $\hookrightarrow \quad \cancel{x > \pm\sqrt{2}}$ ma $(x - \sqrt{2})(x + \sqrt{2}) > 0$

$x - \sqrt{2} > 0$, $x + \sqrt{2} > 0$

$\downarrow \qquad \downarrow$

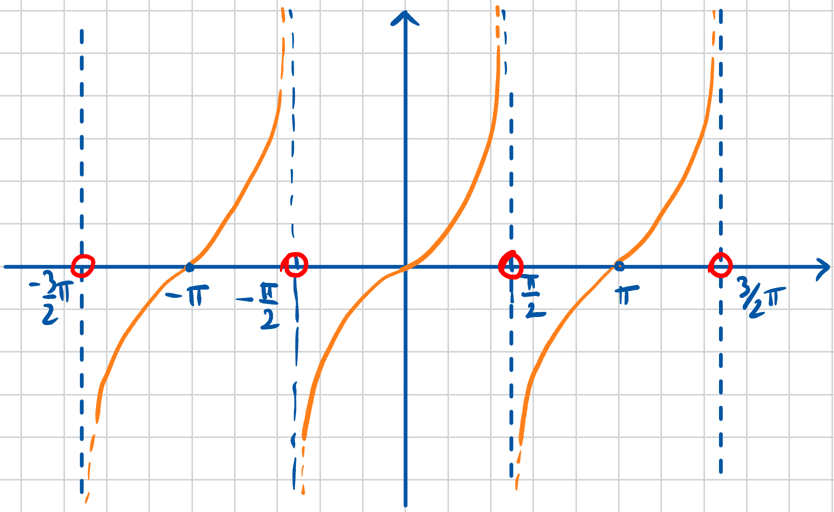
$x > \sqrt{2} \qquad x > -\sqrt{2}$



$\boxed{x < -\sqrt{2} \vee x > \sqrt{2}}$

Es 4 (1) $f(x) = \tan x + \arctan x$

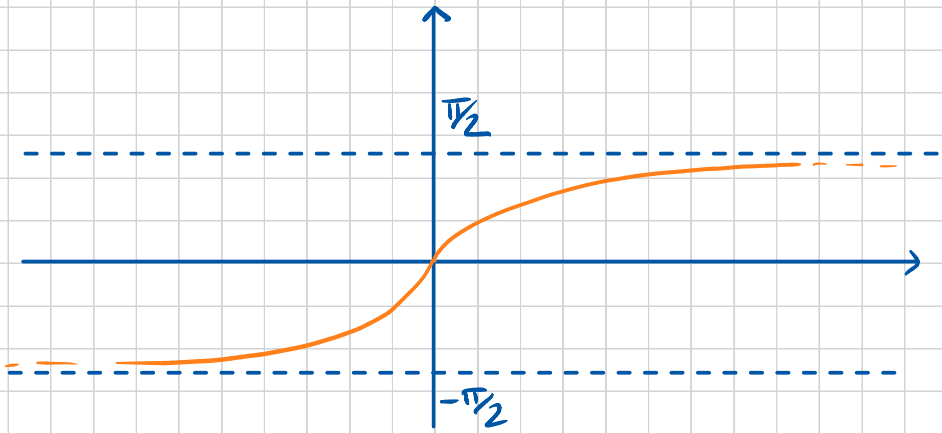
•) $y = \tan x$ ha dominio $\mathrm{dom} f = \mathbb{R} \setminus \{ \frac{\pi}{2} + k\pi \}$



NB $y = \tan x = \frac{\sin x}{\cos x}$ $\cos x \neq 0 \rightarrow x \neq \frac{\pi}{2} + k\pi$

•) $y = \arctan x$ $\mathrm{dom} f = \mathbb{R}$

FUNZIONE INVERSA DI $y = \tan x$



Allora $f(x) = \tan x + \arctan x$

$$\mathrm{dom} f = \mathbb{R} \setminus \{ \frac{\pi}{2} + k\pi \text{ con } k \in \mathbb{Z} \}$$

(2) $f(x) = \tan(3x - 2)$

$3x - 2 \neq \frac{\pi}{2} + k\pi \rightarrow 3x \neq \frac{\pi}{2} + k\pi + 2$

$\rightarrow x \neq \frac{\pi}{6} + k\frac{\pi}{3} + \frac{2}{3} \rightarrow x \neq \left(\frac{\pi}{6} + \frac{2}{3}\right) + k\frac{\pi}{3}$

$\rightarrow \quad x \neq \dfrac{\pi+4}{6} + k\pi$

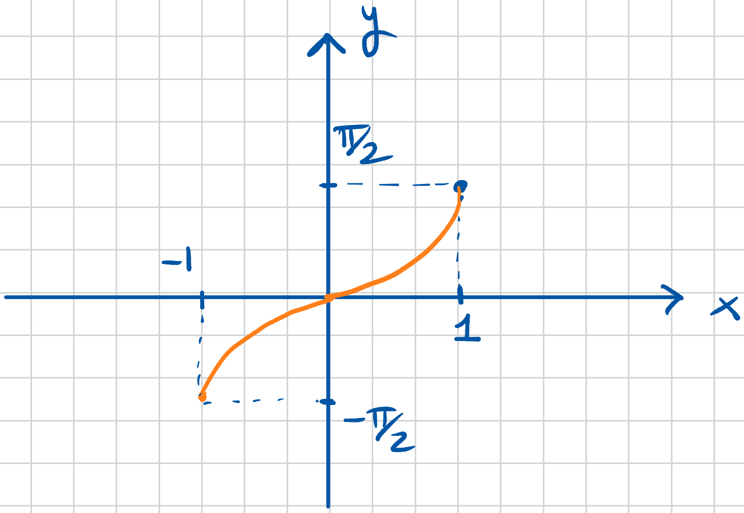
$$\text{dom} f = \mathbb{R} \setminus \left\{ \frac{\pi+4}{6} + k\pi \quad \text{con } k \in \mathbb{Z} \right\}$$

(3) $f(x) = \underbrace{\arcsin(x^2)}_{\text{inversa del seno}} + \underbrace{\arccos(x-1)}_{\text{inversa del coseno}}$

$y = \arcsin x$

$D = [-1, 1]$

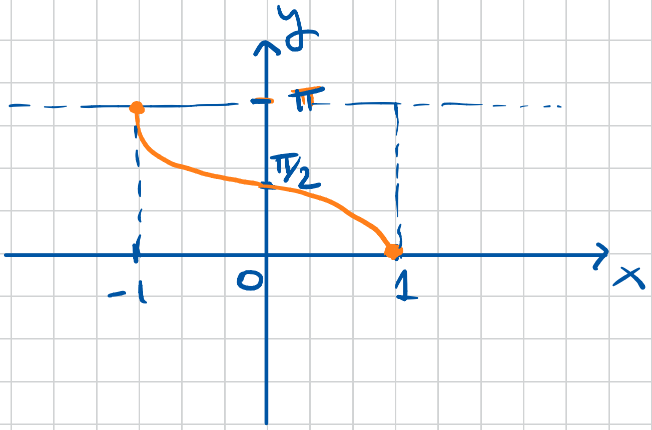
$C = [-\pi/2, \pi/2]$



$y = \arccos x$

$D = [-1, 1]$

$C = [0, \pi]$



$$\begin{cases} -1 \leq x^2 \leq 1 & (*) \\ -1 \leq x-1 \leq 1 & (**) \end{cases}$$

$(*)$ $\begin{cases} x^2 \leq 1 & -1 \leq x \leq 1 \\ x^2 \geq -1 & \forall x \in \mathbb{R} \end{cases}$

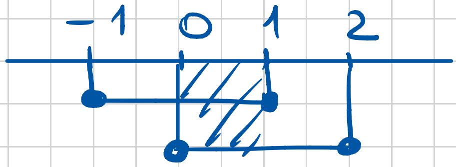
$\downarrow$

$-1 \leq x \leq 1$

$(**)$ $-1 \leq x-1 \leq 1$

$0 \leq x \leq 2$

$$\begin{cases} -1 \le x \le 1 \\ 0 \le x \le 2 \end{cases}$$



Soluzione: $0 \le x \le 1$ $\quad$ dom $f = [0,1]$

(4) $$f(x) = \frac{\arcsin(x^2+1)}{\arccos(x-1)}$$

$$\begin{cases} -1 \le x^2+1 \le 1 \\ -1 \le x-1 \le 1 \\ \arccos(x-1) \ne 0 \end{cases} \rightarrow \begin{cases} -2 \le x^2 \le 0 \\ 0 \le x \le 2 \\ x-1 = 1 \end{cases} \rightarrow \begin{cases} x = 0 \\ 0 \le x \le 2 \\ x = 2 \end{cases}$$

IMPOSSIBILE

dom $f = \emptyset$

$\arccos(t) =$ "angolo il cui coseno è t"

(5) $$f(x) = \arctan(x^2+1) \cdot \arccos(x^2-1)$$

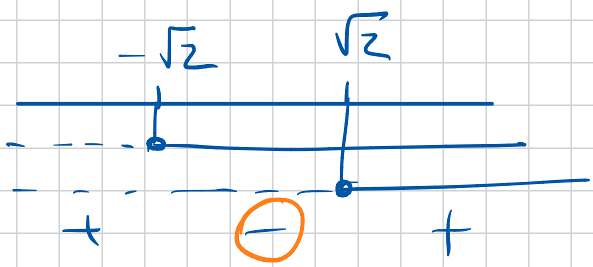
$$-1 \le x^2-1 \le 1 \quad \rightarrow \quad 0 \le x^2 \le 2$$

$$\begin{cases} x^2 \ge 0 \\ x^2 \le 2 \end{cases} \rightarrow \begin{cases} \forall x \in \mathbb{R} \\ -\sqrt{2} \le x \le \sqrt{2} \end{cases}$$

$$x^2 - 2 \le 0 \quad \rightarrow \quad (x-\sqrt{2})(x+\sqrt{2}) \le 0$$

$x \ge \sqrt{2}$

$x \ge -\sqrt{2}$



$\text{dom} f = [-\sqrt{2}, \sqrt{2}]$

(6) $f(x) = \arcsin\left(\dfrac{x^2+1}{x-2}\right)$

$$\begin{cases} -1 \le \dfrac{x^2+1}{x-2} \le 1 \\ x \neq 2 \end{cases}$$

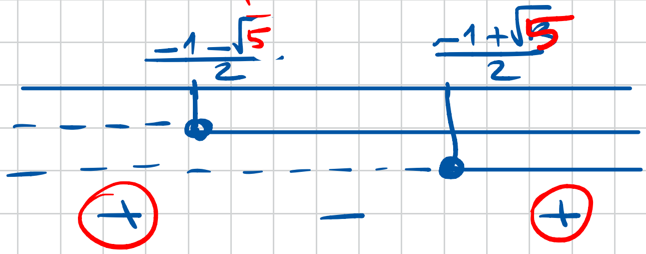
$$\begin{cases} \dfrac{x^2+1}{x-2} \geq -1 \\ \dfrac{x^2+1}{x-2} \le 1 \\ x \neq 2 \end{cases} \rightarrow \begin{cases} \dfrac{x^2+1}{x-2} \geq -\dfrac{x-2}{x-2} \\ \dfrac{x^2+1}{x-2} \le \dfrac{x-2}{x-2} \\ x \neq 2 \end{cases}$$

$$\begin{cases} \dfrac{x^2+1+x-2}{x-2} \geq 0 \\ \dfrac{x^2+1-x+2}{x-2} \le 0 \\ x \neq 2 \end{cases} \rightarrow \begin{cases} \dfrac{x^2+x-1}{x-2} \geq 0 & (1) \\ \dfrac{x^2-x+3}{x-2} \le 0 & (2) \\ x \neq 2 \end{cases}$$

(1) N: $x^2+x-1 \geq 0$

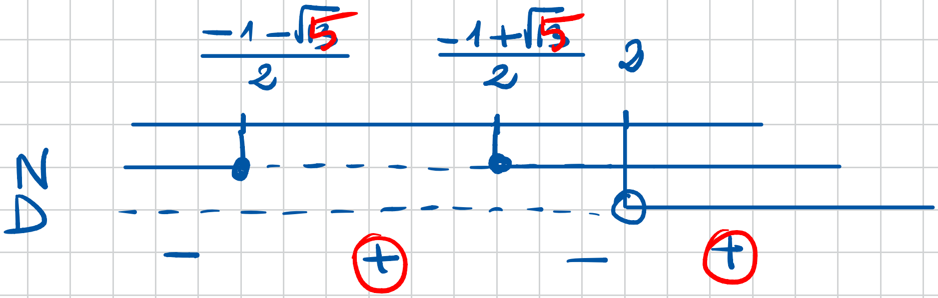
$x^2+x-1 = 0 \rightarrow \Delta = 1+4 = 5$

$x_{1,2} = \dfrac{-1 \pm \sqrt{5}}{2}$



$x \le -\dfrac{1-\sqrt{5}}{2} \vee x \geq \dfrac{-1+\sqrt{5}}{2}$

D: $x-2 > 0 \rightarrow x > +2$



$$\boxed{\frac{-1-\sqrt{5}}{2} \leq x \leq \frac{-1+\sqrt{5}}{2} \quad \vee \quad x > 2}$$

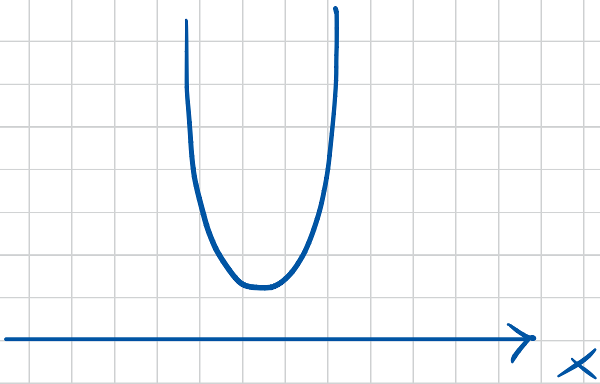
Soluzione di (1)

(2) $$\frac{x^2 - x + 3}{x-2} \leq 0$$

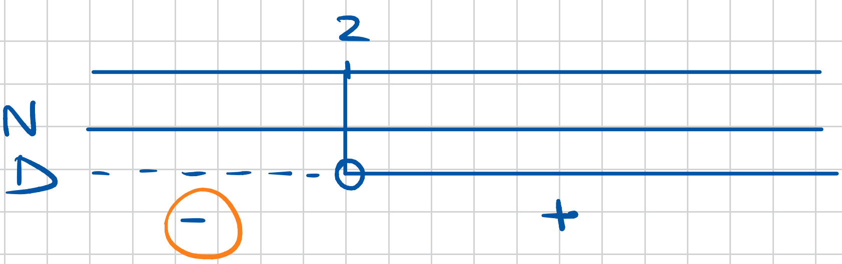
N: $x^2 - x + 3 \geq 0 \qquad \forall x \in \mathbb{R}$

Infatti: $x^2 - x + 3 = 0 \qquad \Delta = 1 - 12 = -11 < 0$

$y = x^2 - x + 1 \geq 0$



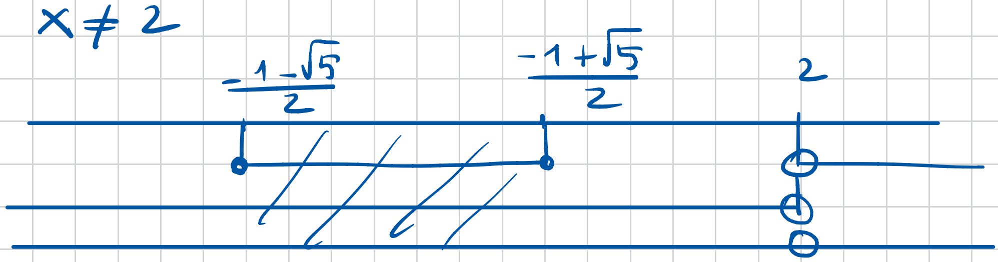
D: $x - 2 > 0 \quad \rightarrow \quad x > 2$



$$\boxed{x < 2}$$

Soluzione di (2)

$$\begin{cases} \dfrac{-1-\sqrt{5}}{2} \leq x \leq \dfrac{-1+\sqrt{5}}{2} \quad \vee \quad x > 2 \\ x < 2 \\ x \neq 2 \end{cases}$$



Soluzione : $\boxed{\dfrac{-1-\sqrt{5}}{2} \leq x \leq \dfrac{-1+\sqrt{5}}{2}}$

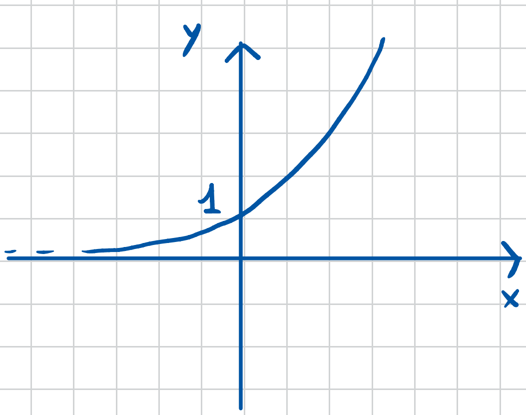
Es 6 (1) $\lim_{x \to 1} \dfrac{1}{(x-1)^2} = \dfrac{1}{(1-1)^2} = \dfrac{1}{0^2} = \dfrac{1}{0^+} = +\infty$

NB $\dfrac{k}{0^+} = \begin{cases} +\infty & se\ k > 0 \\ -\infty & se\ k < 0 \end{cases}$

$\dfrac{k}{0^-} = \begin{cases} -\infty & se\ k > 0 \\ +\infty & se\ k < 0 \end{cases}$

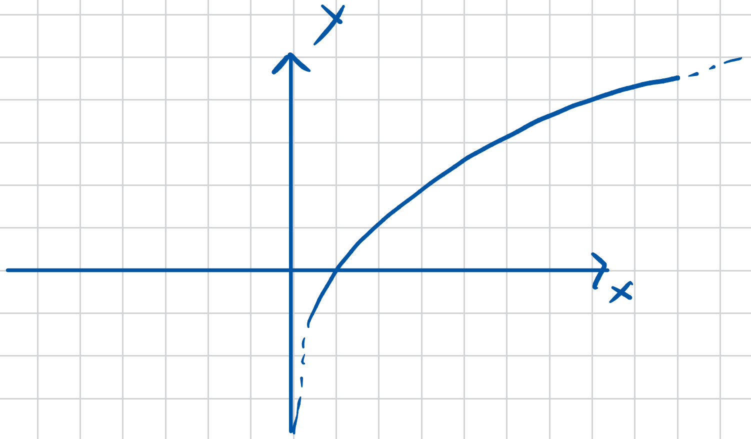
(10) $\lim_{x \to -\infty} \ln(1+e^{-x}) = \ln(1+e^{+\infty}) = \ln(1+\infty)$

$= \ln(+\infty) = +\infty$



$y = a^x$ con $a > 1$

$e \simeq 2{,}7$ NUMERO DI NEPERO



$y = \log_a x \qquad a > 1$

(11) $\lim_{x \to \frac{\pi}{2}^-} (\tan x)^x = \tan\left(\frac{\pi}{2}^-\right)^{\left(\frac{\pi}{2}\right)^-} = (+\infty)^{\pi/2} = +\infty$

Es 7 (1) $\lim_{x \to 0^+} \frac{1}{x} = \frac{1}{0^+} = +\infty$

$\lim_{x \to 0^-} \frac{1}{x} = \frac{1}{0^-} = -\infty$

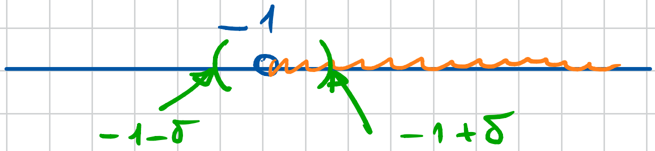
Poiché $\lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$ allora $\nexists \lim_{x \to 0} f(x)$

$\lim_{x \to +\infty} \frac{1}{x} = \frac{1}{+\infty} = 0$

$\lim_{x \to -\infty} \frac{1}{x} = \frac{1}{-\infty} = 0$

OSS $f(x) = \frac{1}{\sqrt{x+1}}$ $\qquad \text{dom} f = \{x \in \mathbb{R} \mid x+1 > 0\}$

$= (-1, +\infty)$



$\forall \delta > 0$

$\left[(-1-\delta, -1+\delta) \setminus \{-1\}\right] \cap \text{dom} f = (-1, -1+\delta) \neq \emptyset$

$$f(x) = \sqrt{x^4 - x^2}$$

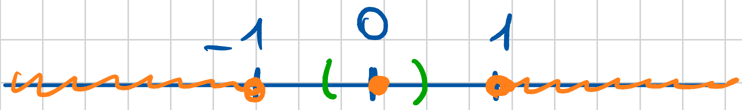
$$x^4 - x^2 \geq 0$$

$$x^2(x^2-1) \geq 0$$

$x^2 \geq 0 \quad \forall x \in \mathbb{R}$

$x^2 - 1 \geq 0 \quad \to \quad x \leq -1 \quad \vee \; x \geq 1$

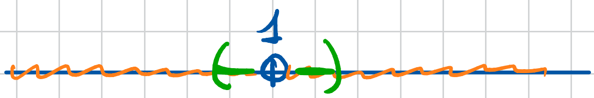
$$\text{dom} f = (-\infty, -1] \cup \{0\} \cup [1, +\infty)$$



Scelgo $\delta = \frac{1}{2}$ $\qquad (0 - \frac{1}{2}, 0 + \frac{1}{2}) \cap \left(\text{dom} f \setminus \{0\}\right) = \emptyset$

Es 8 $\quad$ (5) $\quad f(x) = \dfrac{1}{x-1} \qquad \text{dom} f = (-\infty, 1) \cup (1, +\infty)$

$x = 1$ punto di accumulazione ESTERNO al dominio



$$\lim_{x \to 1^{\pm}} \frac{1}{x-1} = \frac{1}{0^{\pm}} = \pm\infty$$