21/04/2024

Es 3 5) $f(x) = x^{-2} = \frac{1}{x^2}$

a) $\text{dom} f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, +\infty)$

b) $f(x) = \frac{1}{x^2} > 0 \quad \forall x \in \text{dom} f \implies \text{Im} f = (0, +\infty)$

$y = f(x) \implies y = \frac{1}{x^2}$ con $x \in \text{dom} f \implies y > 0$

$x^2 y = \frac{1}{\cancel{x^2}} \cancel{x^2} \to x^2 = \frac{1}{y} \xrightarrow[y>0]{} x = +\sqrt{\frac{1}{y}}$

$\inf f = 0$ $\sup f = +\infty$

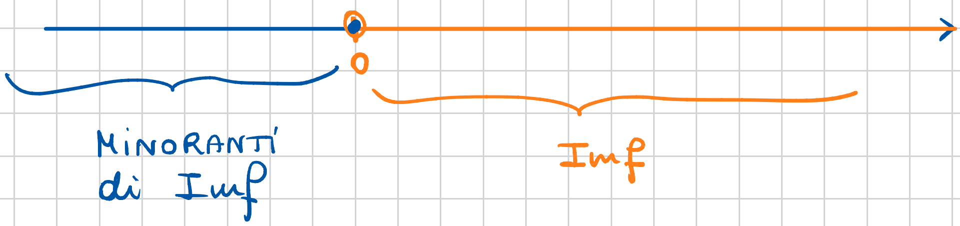
$\Downarrow$ $\Downarrow$

può esistere il $\min f$ se $\exists \bar{x} \in \text{dom} f$ t.c $f(\bar{x}) = 0$ ma tale non esiste

$\nexists \max(f)$

$f(\bar{x}) = 0 \to \frac{1}{\bar{x}^2} = 0 \to 1 = 0 \cdot \bar{x}^2$

$\to 1 = 0$ IMP



Quindi $\nexists \min(f)$

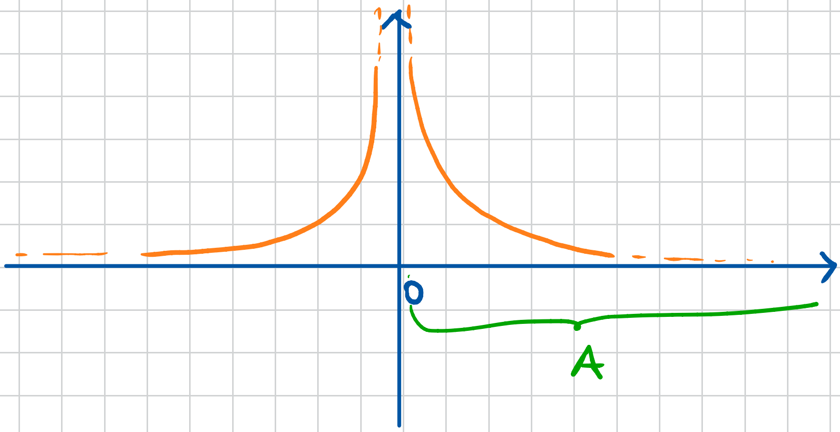
c) NON INIETTIVA $f(1) = 1 = f(-1)$

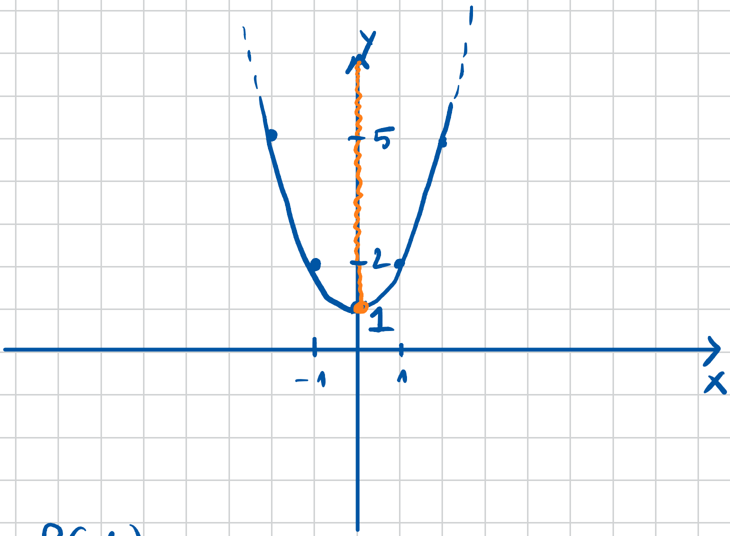
NON SURIETTIVA $\nexists x \in \text{dom} f$ t.c $f(x) = -2 \to \frac{1}{x^2} = -2$ IMP

d) Restringo la funzione f all'intervallo $A = (0, +\infty)$

Allora $f|_A$ è una funzione str. decrescente su I

e quindi iniettiva.





Es 4 1) $f(x) = x^2 + 1$

a) $\mathrm{dom} f = \mathbb{R}$

b) $\nexists \max f$, $\sup f = +\infty$

$\inf(f) = \min(f) = 1$

c) Iniettiva NO $f(1) = 2 = f(-1)$

non suriettiva $(-\infty, 1) \not\subseteq \mathrm{Im} f$

d) $g_1(x) = x^2$

$g_2(x) = x + 1$

$f(x) = x^2 + 1 = g_2(g_1(x))$

$g_2(g_1(x)) = g_2(x^2) = x^2 + 1$

$$\boxed{f(x) = \underset{\substack{\parallel \\ x^2}}{g_1(x)} + \underset{\substack{\parallel \\ 1}}{g_2(x)}}$$

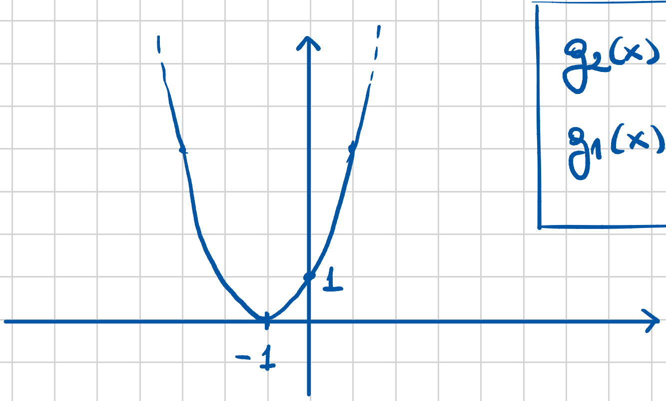
Es 4 2d) $f(x) = (x+1)^2$

$g_1(x) = x + 1$

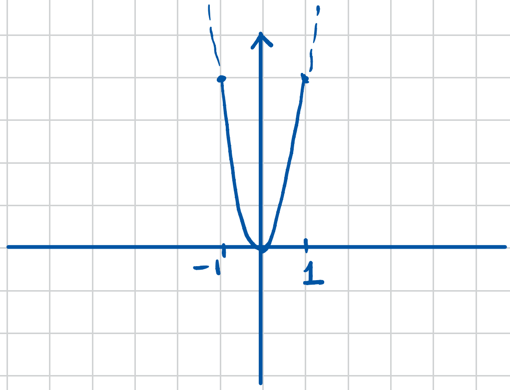
$g_2(x) = x^2$

$g_2(g_1(x)) = g_2(x+1) = (x+1)^2 = f(x)$

$$\boxed{\begin{array}{l} g_2(x) = x+1 \quad = g_1(x) \\ g_1(x)\, g_2(x) = f(x) \end{array}}$$



3d) $f(x) = 4x^2$



$g_2(x) = x^2$

$g_1(x) = 4$

$g_1(x) \cdot g_2(x) = 4x^2$

$g_2(x) = x$

$g_1(x) = 4x$

$g_1(x) = g_2(x) = 2x$

4d) $f(x) = (3x-1)^2 = 9x^2 - 6x + 1$

$g_1(x) = (3x-1) = g_2(x)$

•) $f(x) = g_1(x)\, g_2(x)$

•) $g_1(x) = 9x^2 \quad g_2(x) = 1-6x$

$f(x) = g_1(x) + g_2(x)$

685 1d) $g_1(x) = x^2 \quad g_2(x) = x \rightarrow f(x) = g_1(x) + g_2(x)$

$g_1(x) = x \quad g_2(x) = x+1 \rightarrow f(x) = g_1(x) \cdot g_2(x)$

$= x(x+1) = x^2 + x$

Es 5

$$f(x) = \frac{x}{x+3} = \frac{x+3-3}{x+3} = \frac{x+3}{x+3} - \frac{3}{x+3} = 1 - \frac{3}{x+3}$$

a) $\mathrm{dom}(f) = (-\infty, -3) \cup (-3, +\infty)$

b) $\lim_{x \to \pm\infty} f(x) = 1$ $\qquad \lim_{x \to -3^\pm} f(x) = 1 - \frac{3}{-3^\pm+3} = 1 - \frac{3}{0^\pm} = \mp\infty$

$\sup(f) = +\infty \qquad \inf(f) = -\infty$

$\Downarrow \qquad\qquad \Downarrow$

$\nexists \max(f) \qquad \nexists \min(f)$

c) **iniettiva** : $f(x_1) = f(x_2) \Rightarrow \cancel{1} - \frac{\cancel{3}}{x_1+3} = \cancel{1} - \frac{\cancel{3}}{x_2+3}$

$\rightarrow \frac{1}{x_1+3} = \frac{1}{x_2+3} \underset{\text{ho applicato il reciproco}}{\Rightarrow} x_1+3 = x_2+3 \Rightarrow x_1 = x_2$

Quindi è iniettiva

suriettiva : $\forall y \in \mathbb{R} \ \exists x \in \mathrm{dom} f$ tale che $f(x) = y$.

$y = f(x) \rightarrow y = \frac{x}{x+3} \xrightarrow[x \neq -3]{} (x+3)y = x$

$\rightarrow xy + 3y - x = 0 \rightarrow x(y-1) = -3y$

$\xrightarrow[y \neq 1]{} x = \frac{-3y}{y-1} = \frac{3y}{1-y}$ $\qquad$ NON SURIETTIVA

$y = 1$? $\rightarrow \frac{x}{x+3} = 1 \xrightarrow[x \neq -3]{} \cancel{x} = \cancel{x}+3$

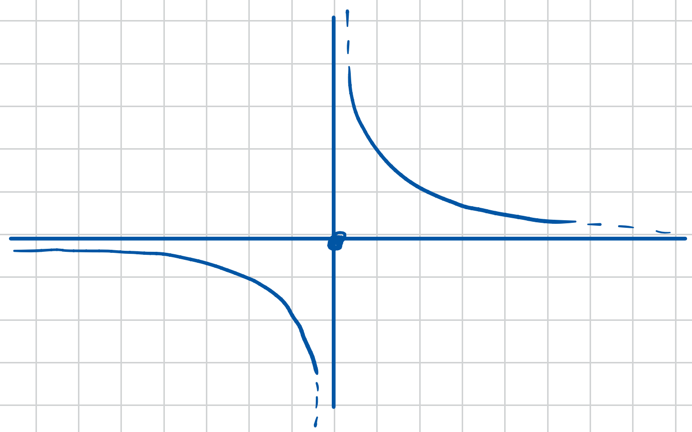
$\rightarrow 0 = 3$ $\underline{\underline{\text{IMP}}}$ $\Rightarrow 1 \notin \mathrm{Im} f$

$$(x+1)\cdot\frac{1}{x-1} \qquad\qquad \frac{x+1}{x-1} = \frac{x-1+2}{x-1} = 1+\frac{2}{x-1}$$

Es 2 $\quad f(x) = \begin{cases} 0 & \text{se } x=0 \\ \frac{1}{x} & \text{se } x\neq 0 \end{cases}$

$\operatorname{dom} f = \mathbb{R} = (-\infty,+\infty)$

- non è str. crescente su $\mathbb{R}$ perché se $1<2$ $\Rightarrow$ non è vero che $f(1)<f(2)$. infatti $f(1)=1$ e $f(2)=\frac{1}{2}$, quindi $1>\frac{1}{2}$ $\rightarrow$ $f(1)>f(2)$



- non è str decrescente su $\mathbb{R}$ perché se $-1<1$ ottengo che $f(-1)=-1<1=f(1)$ cioè $f(-1)<f(1)$

Non è monotona perché è str. decrescente in $(-\infty,0)$ e in $(0,+\infty)$

INIETTIVA $\quad$ Poiché in $(0,+\infty)$ f str decrescente $\Rightarrow$ f iniettiva in $(0,+\infty)$

Poiché in $(-\infty,0)$ f str decrescente $\rightarrow$ f iniettiva in $(-\infty,0)$

Siccome $f(x) \begin{cases} >0 & \text{in } (0,+\infty) \\ <0 & \text{in } (-\infty,0) \end{cases}$ $\Big|$ $\nexists\, x_1\in(0,+\infty),\ x_2\in(-\infty,0)$ t.c. $f(x_1)=f(x_2)$

$f(1) = 1 \neq -1 = f(-1)$ non è pari

$f(-x) = \frac{1}{-x} = -\frac{1}{x} = -f(x) \quad \forall$

$f(-x) = \frac{1}{-x} = -\frac{1}{x}$

$-f(-x) = -\left(-\frac{1}{x}\right) = \frac{1}{x} = f(x) \quad \forall x \in \mathbb{R} - \{0\}$

$-f(-x) = f(x)$?

se $x = 0$ $\quad -f(-0) = f(0)$?

$\underbrace{-f(0)}_{0} = \underbrace{f(0)}_{0}$

$0 = 0$

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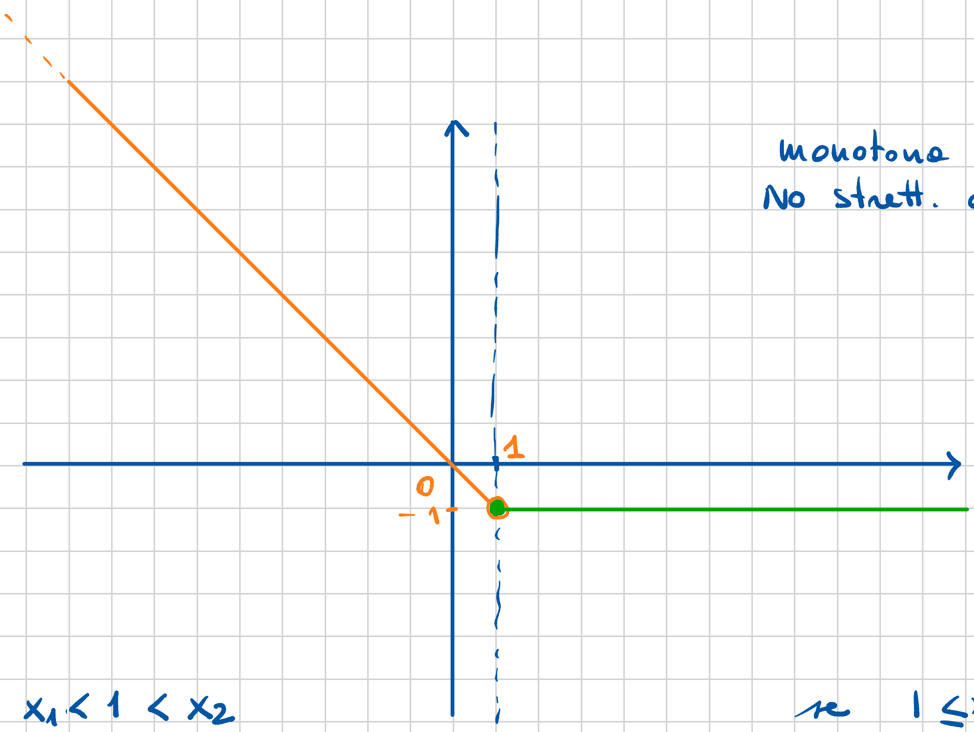
se $x = 0$ $\quad -f(-0) = f(0)$?

$-f(0) = f(0)$

$-1 = 1$ Falso

$$f(x) = \begin{cases} 1 & \text{se } x = 0 \\ \frac{1}{x} & \text{se } x \neq 0 \end{cases}$$



monotona decrescente

No strett. decrescente

se $x_1 < 1 < x_2$

$f(x_1) = -x_1 > -1 = f(x_2)$

se $1 \leq x_1 < x_2$

$f(x_1) = -1 = f(x_2)$

$x > 0$

$$f(-x) = -(-x) = x \overset{?}{=} f(x)$$

se $x \geq 1$ $f(x) = -1$

se $0 < x < 1$ $f(x) = -x$

$$\neq \begin{cases} -1 & \text{se } x \geq 1 \\ -x & \text{se } 0 < x < 1 \end{cases}$$

$f(-x) = -f(x)$?

$$f(-x) = \dots = x$$

$$-f(x) = \begin{cases} 1 & \text{se } x \geq 1 \\ x & \text{se } 0 < x < 1 \end{cases}$$

Se $x \geq 1$ $\quad f(-x) = -f(x)$? In generale NO basta considerare

$x = 47$ $\quad \underbrace{f(-47)}_{47} = \underbrace{-f(47)}_{+1}$ IMPOSSIBILE

Allora f non è dispari

Non è invertibile perché non è iniettiva essendo $f(1) = -1$ e $f(2) = -1$.

Ne posso invertire $f(x) = -x$ se $x < 1$

$y = -x$ con $x < 1$

$x = -y$ con $-y < 1 \to y > -1$

$g(x) = -x$ definita in $(-1, +\infty)$ è l'inversa di $f(x) = -x$ definita in $(-\infty, 1)$

$$f(g(x)) = f(-x) = -(-x) = x$$