17/10

RICHIAMO:

$$\begin{matrix} a_{11}x_1 + \dots + a_{1m}x_m = b_1 \\ \vdots \\ a_{m1}x_1 + \dots \end{matrix} \quad \longleftrightarrow \quad Ax = b$$

$$A = \underbrace{\begin{pmatrix} * & \cdots & * \\ & \ddots & \vdots \\ 0 & & * \end{pmatrix}}_{m \times m} \rightsquigarrow \text{SOST. ALL'INDIETRO} \left(\frac{m^2}{2} \right)$$

A ARBITRARIA $\rightsquigarrow$ A

MATRICI ELEMENTARI

$$I_m \xrightarrow{R_i \leftrightarrow R_j} E_{ij}, \quad \text{ES. } E_{13} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$I_m \xrightarrow{R_i \leftarrow \lambda R_i} E_i(\lambda), \quad \text{ES. } E_2(3) \longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$I_m \xrightarrow{R_i \leftarrow R_i + \lambda R_j} E_{ij}(\lambda), \quad \text{ES. } E_{21}(4) = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

CALCOLIAMOCI I DETERMINANTI:

- $\det E_{13} = +1 \cdot \det \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = -1$
- $\det E_2(3) = 1 \cdot 3 \cdot 1 = 3$
- $\det E_{21}(4) = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 1 \cdot 1 \cdot 1 = 1$

MATRICI ELEMENTARI INVERTIBILI

$$A \longmapsto E_{ij} \cdot A$$

$$R_i \leftrightarrow R_j$$

$$A = \begin{pmatrix} 1 & 0 & -2 & 4 \\ 3 & -1 & 0 & 2 \\ -1 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} E_{12} \cdot A = \begin{pmatrix} 3 & -1 & 0 & 2 \\ 1 & 0 & -2 & 4 \\ -1 & 0 & 1 & 0 \end{pmatrix}$$

3×4

$$E_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$R_2 \leftarrow 3 \cdot R_2$

$$A \longmapsto E_2(3) \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -2 & 4 \\ 3 & -1 & 0 & 2 \\ -1 & 0 & 1 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & -2 & 4 \\ 9 & -3 & 0 & 6 \\ -1 & 0 & 1 & 0 \end{pmatrix}$$

$R_3 \leftarrow R_3 + 2 \cdot R_1$

$$A \longmapsto E_{31}(2) \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -2 & 4 \\ 3 & -1 & 0 & 2 \\ -1 & 0 & 1 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & -2 & 4 \\ 3 & -1 & 0 & 2 \\ 1 & 0 & -3 & 8 \end{pmatrix}$$

$\Rightarrow$ INVERSE: $E_{ij}^{-1} \cdot E_{ij} = I \to E_{ij}^{-1} = E_{ij}$

$$E_i(\lambda)^{-1} \cdot E_i(\lambda) = I \to E_i(\lambda)^{-1} = E_i(1/\lambda)$$

$$E_{ij}(\lambda)^{-1} \cdot E_{ij}(\lambda) = I \to E_{ij}(\lambda)^{-1} = E_{ij}(-\lambda)$$

MATRICI RIDOTTE (O A SCALINI)

PIVOT

$$\begin{pmatrix} * & * & * & * & * & 0 & * & & \\ 0 & 0 & * & 0 & * & 0 & * & & \\ 0 & 0 & 0 & * & * & 0 & 0 & * & \cdots \\ 0 & 0 & 0 & 0 & 0 & * & * & 0 & \cdots \end{pmatrix}$$

$$\downarrow$$

PIVOT $i+1$-ESIMA RIGA A DESTRA DEL PIVOT DELLA i-ESIMA.

$$\begin{pmatrix} 0 & 0 & 2 & 1 \\ 1 & 0 & 3 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \Rightarrow \text{NON RIDOTTA}$$

$$\begin{pmatrix} 0 & 0 & \boxed{2} & 1 \\ \boxed{1} & 0 & 3 & -1 \\ 0 & 0 & 0 & \boxed{1} \\ 0 & \boxed{1} & -1 & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} \boxed{1} & 0 & 3 & -1 \\ 0 & 0 & \boxed{2} & 1 \\ 0 & 0 & 0 & \boxed{1} \\ 0 & \boxed{1} & -1 & 0 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_4}$$

$$\longrightarrow \begin{pmatrix} \boxed{1} & 0 & 3 & -1 \\ 0 & \boxed{1} & -1 & 0 \\ 0 & 0 & 0 & \boxed{1} \\ 0 & 0 & \boxed{2} & 1 \end{pmatrix}$$

$$\Big\downarrow\ R4 \leftarrow R4 + (2)\, R_2$$

$$\begin{pmatrix} 1 & 0 & 3 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 1 \end{pmatrix}$$

$R3 \leftrightarrow R4$ $\begin{pmatrix} 1 & 0 & 3 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ RIDOTTA (1 PIVOT ∀ RIGA)

ALTRO ESEMPIO

$$A = \begin{pmatrix} \boxed{1} & 0 & -2 & 4 \\ \boxed{9} & -3 & 0 & 6 \\ \boxed{-1} & 6 & 1 & 0 \end{pmatrix} \xrightarrow{R2 \leftarrow R2 + \boxed{(-9)} R1} \begin{pmatrix} 1 & 0 & -2 & 4 \\ 0 & -3 & 18 & -30 \\ -1 & 6 & 1 & 0 \end{pmatrix}$$

IN GENERALE: PER AZZERARE a_{i1} AVENDO GIÀ IL PIVOT a_{11}, SI SCEGLIE $X = -a_{i1}/a_{11}$

$\downarrow R3 \leftarrow R3 + 1 \cdot R1$

$$\begin{pmatrix} \boxed{1} & 0 & -2 & 4 \\ 0 & \boxed{-3} & 18 & -30 \\ 0 & \boxed{6} & -1 & 4 \end{pmatrix}$$

$\downarrow R3 \leftarrow R3 + (2) R2$

$$\begin{pmatrix} \boxed{1} & 0 & -2 & 4 \\ 0 & \boxed{-3} & 18 & -30 \\ 0 & 0 & \boxed{35} & -56 \end{pmatrix}$$

RIDOTTA
1 PIVOT x RIGA, MA 4 COLONNA SENZA PIVOT

$$E_{32}(2) \cdot E_{31}(1) \cdot E_{21}(-9) \cdot A = A'$$

ESEMPIO 3

$$A = \begin{pmatrix} 0 & 0 & \boxed{1} & 1 & 1 \\ \boxed{2} & 2 & 1 & -5 & 5 \\ \boxed{3} & 3 & -1 & 0 & 0 \\ \boxed{1} & 1 & 0 & -1 & 1 \end{pmatrix} \xrightarrow{R_4 \leftrightarrow R_1} \begin{pmatrix} 1 & 1 & 0 & -1 & 1 \\ 2 & 2 & 1 & -5 & 5 \\ 3 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\xrightarrow{R_2 \leftarrow R_2 + (-2)R_1} \begin{pmatrix} 1 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & -3 & 3 \\ 3 & 3 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \xrightarrow{R_3 + (-3) \cdot R_1}$$

$$\rightarrow \begin{pmatrix} 1 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & -3 & 3 \\ 0 & 0 & -1 & 3 & -3 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$
\xrightarrow{r_3 + 1 \cdot r_2}
\begin{pmatrix}
1 & 1 & 0 & -1 & 1 \\
0 & 0 & 1 & -3 & 3 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1
\end{pmatrix}
\xrightarrow{r_4 - r_2}
\begin{pmatrix}
1 & 1 & 0 & -1 & 1 \\
0 & 0 & 1 & -3 & 3 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 4 & -2
\end{pmatrix}
$$

$$
\xrightarrow{r_3 \leftrightarrow r_4}
\begin{pmatrix}
1 & 1 & 0 & -1 & 1 \\
0 & 0 & 1 & -3 & 3 \\
0 & 0 & 0 & 4 & -2 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
= A' \Rightarrow
$$

RIDOTTA

4 RIGA SENZA PIVOT

2^a E 5^a COLONNA SENZA PIVOT

$$
= E_{34} \cdot E_{42}(-1) \cdot E_{32}(1) \cdot E_{31}(-3) \cdot E_{21}(-2) \cdot E_{14} \cdot A
$$

CASO SISTEMI LINEARI: $A \cdot x = b$

$$
A \xrightarrow[R_i \leftarrow R_i + \lambda R_j]{R_i \leftrightarrow R_j,\ R_i \to \lambda R_i} A'
$$
RIDOTTA x INVARIATA SE $b \rightsquigarrow b'$

"MATRICE COMPLETA": $\left(A \,\vdots\, b\right) \xrightarrow[m \times (n+1)]{} \left(A' \,\vdots\, b'\right)$

$$
Ax = b \qquad\qquad A'x' = b'
$$

ESEMPIO

$$A = \begin{pmatrix} 2 & -4 & 1 \\ 6 & -14 & 8 \\ -2 & 0 & 6 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \rightsquigarrow \left(\begin{array}{ccc|c} 2 & -4 & 1 & 1 \\ 6 & -14 & 8 & -1 \\ -2 & 0 & 6 & 1 \end{array}\right)$$

$$R_2 \leftarrow R_2 + (-3) R_1 \left(\begin{array}{ccc|c} 2 & -4 & 1 & 1 \\ 0 & -2 & 5 & -4 \\ -2 & 0 & 6 & 1 \end{array}\right)$$

$$R_2 \leftarrow R_2 + (-3) R_1 \left(\begin{array}{ccc|c} 2 & -4 & 1 & 1 \\ 0 & -2 & 5 & 4 \\ -2 & 0 & 6 & 1 \end{array}\right) \xrightarrow{R_3 \leftarrow R_3 + R_1}$$

$$\left(\begin{array}{ccc|c} 2 & -4 & 1 & 1 \\ 0 & -2 & 5 & -4 \\ 0 & -4 & 7 & 2 \end{array}\right) \xrightarrow{R_3 \leftarrow R_3 + (-2) R_2} \left(\begin{array}{ccc|c} 2 & -4 & 1 & 1 \\ 0 & -2 & 5 & -4 \\ 0 & 0 & -3 & 10 \end{array}\right)$$

$$= \left(A' \mid b' \right)$$

$$\begin{cases} 2x_1 - 4x_2 + x_3 = 1 \\ \quad\;\; -2x_2 + 5x_3 = -4 \\ \qquad\qquad -3x_3 = 10 \end{cases}$$

SOST. INDIETRO

$$\begin{cases} 2x_1 = 4x_2 - x_3 + 1 \Rightarrow x_1 = -\frac{21}{2} \\ -2x_2 = -4 - 5\left(-10/3\right) \Rightarrow x_2 = -\frac{19}{3} \\ x_3 = -10/3 \end{cases}$$

$$x = \begin{pmatrix} -21/2 \\ -19/3 \\ -10/3 \end{pmatrix}$$

ELIMINAZIONE DI GAUSS

(RIDUZIONE)