Es 5

(1) $f(x) = \dfrac{\sqrt{x^2+x+1}}{x+1}$

$\begin{cases} x^2+x+1 \geq 0 & \forall x \in \mathbb{R} \text{ perché } \Delta < 0 \\ x+1 \neq 0 \end{cases} \rightarrow \begin{cases} \forall x \in \mathbb{R} \\ x \neq -1 \end{cases}$

$\text{dom} f = (-\infty, -1) \cup (-1, +\infty)$

(2) $f(x) = \dfrac{x^5 - 32}{\log(x+1)}$

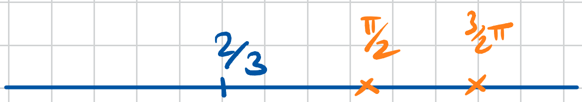
$\begin{cases} \log(x+1) \neq 0 \\ x+1 > 0 \end{cases} \rightarrow \begin{cases} x+1 \neq 1 \\ x > -1 \end{cases} \rightarrow \begin{cases} x \neq 0 \\ x > -1 \end{cases}$

$\text{dom} f = (-1, 0) \cup (0, +\infty)$

(3) $f(x) = \dfrac{\tan(x - 2\pi)}{\sqrt{3x-2}}$

$\begin{cases} x - 2\pi \neq \frac{\pi}{2} + k\pi \\ 3x - 2 > 0 \end{cases} \rightarrow \begin{cases} x \neq \frac{\pi}{2} + 2\pi + k\pi \\ x > 2/3 \end{cases}$

$\begin{cases} x \neq \frac{5}{2}\pi + k\pi \\ x > 2/3 \end{cases}$



$\text{dom} f = \{ x \in \mathbb{R} \mid x > \frac{2}{3} \wedge x \neq \frac{5}{2}\pi + k\pi \text{ con } k \in \mathbb{Z} \text{ e } k \geq -2 \}$

(4) $f(x) = \dfrac{x+1}{\arcsin(1-x^7)}$

$$\begin{cases} \arcsin(1-x^7) \neq 0 \\ -1 \leq 1-x^7 \leq 1 \end{cases} \rightarrow \begin{cases} 1-x^7 \neq 1 \\ -1 \leq 1-x^7 \leq 1 \end{cases} \rightarrow \begin{cases} x^7 \neq 0 \\ -2 \leq -x^7 \leq 0 \end{cases}$$

$$\rightarrow \begin{cases} x \neq 0 \\ 0 \leq x^7 \leq 2 \end{cases} \rightarrow \begin{cases} x \neq 0 \\ 0 \leq x \leq \sqrt[7]{2} \end{cases} \rightarrow \boxed{0 < x \leq \sqrt[7]{2}}$$

$x \geq 0 \qquad x \leq \sqrt[7]{2}$

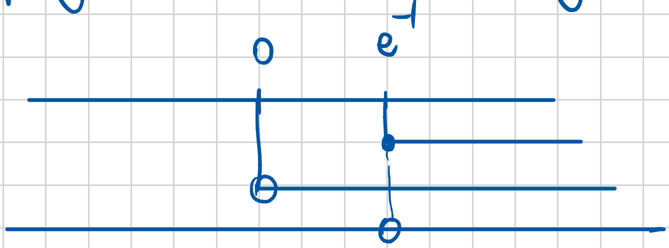
$\text{dom} f = (0, \sqrt[7]{2}]$

(5) $f(x) = \sqrt[3]{(x+1)\log(x-3)}$ $\qquad x-3>0 \rightarrow \boxed{x>3}$

$\text{dom} f = (3, +\infty)$

(6) $f(x) = \log\left(\sqrt{\log x + 1}\right)$

$$\begin{cases} \log x + 1 \geq 0 \\ x > 0 \\ \sqrt{\log x + 1} > 0 \end{cases} \rightarrow \begin{cases} \log x \geq -1 \\ x > 0 \\ \log x + 1 \neq 0 \end{cases} \rightarrow \begin{cases} x \geq e^{-1} \\ x > 0 \\ x \neq e^{-1} \end{cases}$$



$\boxed{x > e^{-1}}$

$\text{dom} f = (e^{-1}, +\infty)$

(7) $f(x) = \dfrac{\arcsin(5x^2+1)}{\sqrt{e^x+1}}$

$$\begin{cases} -1 \le 5x^2+1 \le 1 \\ e^x+1 > 0 \end{cases} \rightarrow \begin{cases} -2 \le 5x^2 \le 0 \\ \forall x \in \mathbb{R} \end{cases} \rightarrow \begin{cases} 5x^2 \le 0 \rightarrow x=0 \\ 5x^2 \ge -2 \rightarrow \forall x \\ \forall x \in \mathbb{R} \end{cases}$$

$$\rightarrow \boxed{x=0}$$

$\text{dom} f = \{0\}$

(8) $\log(\arcsin(e^x))$

$$\begin{cases} \arcsin(e^x) > 0 \\ -1 \le e^x \le 1 \end{cases} \rightarrow \begin{cases} 0 < e^x \le 1 \\ -1 \le e^x \le 1 \end{cases} \rightarrow e^x \le 1$$

$\underbrace{\ln e^x} \le \ln 1$

$x \underbrace{\ln e} \le \ln 1$

$x \cdot 1 \le \ln 1$

$x \le \ln 1$

$$\rightarrow x \le \ln 1 \rightarrow \boxed{x \le 0}$$

$\text{dom} f = (-\infty, 0]$

(9) $f(x) = \dfrac{\sqrt[5]{4x^3+1}}{2-\log|x-1|}$

$$\begin{cases} 2-\log|x-1| \ne 0 \\ |x-1| > 0 \end{cases} \rightarrow \begin{cases} \log|x-1| \ne 2 \\ \forall x \in \mathbb{R},\ x \ne 1 \end{cases}$$

$$\rightarrow \begin{cases} \log|x-1| \ne \log 10^2 \\ x \ne 1 \end{cases} \rightarrow \begin{cases} |x-1| \ne 100 \\ x \ne 1 \end{cases}$$

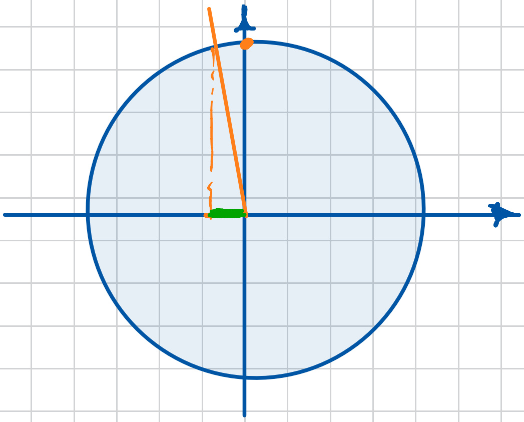
$$\rightarrow \begin{cases} x-1 \neq \pm 100 \\ x \neq 1 \end{cases} \rightarrow \begin{cases} x \neq 101 \vee x \neq -99 \\ x \neq 1 \end{cases}$$

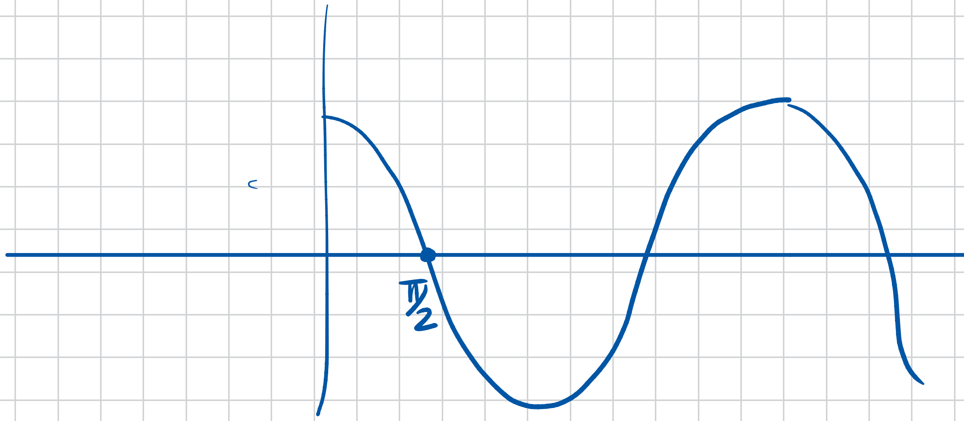
$$\text{dom} f = \mathbb{R} \setminus \{-99, 1, 101\}$$

10) $$f(x) = e^{\arctan(x^2+1)} \qquad \text{dom} f = \mathbb{R}$$

Es6

7) $$\lim_{x \to \frac{\pi}{2}^+} \frac{1}{\cos x} = \frac{1}{0^-} = -\infty$$





19) $$\lim_{x \to -\infty} \arcsin \frac{1}{1-x^2} = \arcsin\left(\frac{1}{-\infty}\right) = \arcsin 0 = 0$$

$$1-(-\infty)^2 = 1-\infty = -\infty$$

11) $$\lim_{x \to \frac{\pi}{2}^-} (\tan x)^x \qquad f(x)^{g(x)}$$

$$\lim_{x \to \frac{\pi}{2}^-} e^{\ln(\tan x)^x} = \lim_{x \to \frac{\pi}{2}^-} e^{x \ln(\tan x)} = e^{\frac{\pi}{2} \cdot \ln(+\infty)} = e^{+\infty} = +\infty$$

12) $\lim_{x \to -\infty} \tan(x)$ $\qquad f(x) = \tan x$

Devo cercare due sottosuccessioni a_m e b_m tali che

$a_m \to -\infty$ e $b_m \to -\infty$ e $\lim_{n \to +\infty} \tan(a_m) \neq \lim_{n \to +\infty} \tan(b_m)$

$$a_n = \pi/4 - 2m\pi \xrightarrow[n \to +\infty]{} -\infty$$

$$b_m = 0 - 2m\pi \xrightarrow[n \to +\infty]{} -\infty$$

- $\lim_{m \to +\infty} f(a_m) = \lim_{n \to +\infty} \tan\left(\pi/4 - 2n\pi\right) = \lim_{n \to +\infty} \tan(\pi/4) = 1$

  $\lim_{n \to +\infty} f(a_n) = 1$

- $\lim_{n \to +\infty} f(b_n) = \lim_{n \to +\infty} \tan\left(0 - 2m\pi\right) = 0$

I valori sono differenti cioè $\nexists \lim_{x \to -\infty} \tan x$

FOGLIO 5

Es 1 (4) $f(x) = \dfrac{\sqrt{|x-1|} - \sqrt{|x+1|}}{x-1}$

① $\begin{cases} |x-1| \geq 0 \\ |x+1| \geq 0 \\ x-1 \neq 0 \end{cases} \to \begin{cases} \forall x \in \mathbb{R} \\ \forall x \in \mathbb{R} \\ x \neq 1 \end{cases}$

$\text{dom} f = (-\infty, 1) \cup (1, +\infty)$

② • $\lim_{x \to -\infty} \dfrac{\sqrt{|x-1|} - \sqrt{|x+1|}}{x-1} = \dfrac{\sqrt{+\infty} - \sqrt{+\infty}}{-\infty} = \dfrac{+\infty - \infty}{-\infty}$

$= \lim_{x \to -\infty} \dfrac{(\overset{A}{\sqrt{|x-1|}} \overset{-}{-} \overset{B}{\sqrt{|x+1|}})(\overset{A}{\sqrt{|x-1|}} \overset{+}{+} \overset{B}{\sqrt{|x+1|}})}{(x-1)(\sqrt{|x-1|} + \sqrt{|x+1|})} =$

$= \lim_{x \to -\infty} \dfrac{|x-1| - |x+1|}{(x-1)(\sqrt{|x-1|} + \sqrt{|x+1|})} = \lim_{x \to -\infty} \dfrac{1 - \cancel{x} - (-1 - \cancel{x})}{(x-1)(\sqrt{1-x} + \sqrt{-1-x})}$

$|x-1| = \begin{cases} x-1 & \text{se } x \geq 1 \\ 1-x & \text{se } x < 1 \end{cases}$

$|x+1| = \begin{cases} x+1 & \text{se } x \geq -1 \\ -1-x & \text{se } x < -1 \end{cases}$

$\lim_{x \to -\infty} \dfrac{2}{(x-1)(\sqrt{1-x} + \sqrt{-1-x})} = \dfrac{2}{(-\infty)(+\infty + \infty)} = \dfrac{2}{-\infty} = 0$

• $\lim_{x \to +\infty} \dfrac{|x-1| - |x+1|}{(x-1)(\sqrt{|x-1|} + \sqrt{|x+1|})} = \lim_{x \to +\infty} \dfrac{x-1 - (x+1)}{(x-1)(\sqrt{|x-1|} + \sqrt{|x+1|})}$

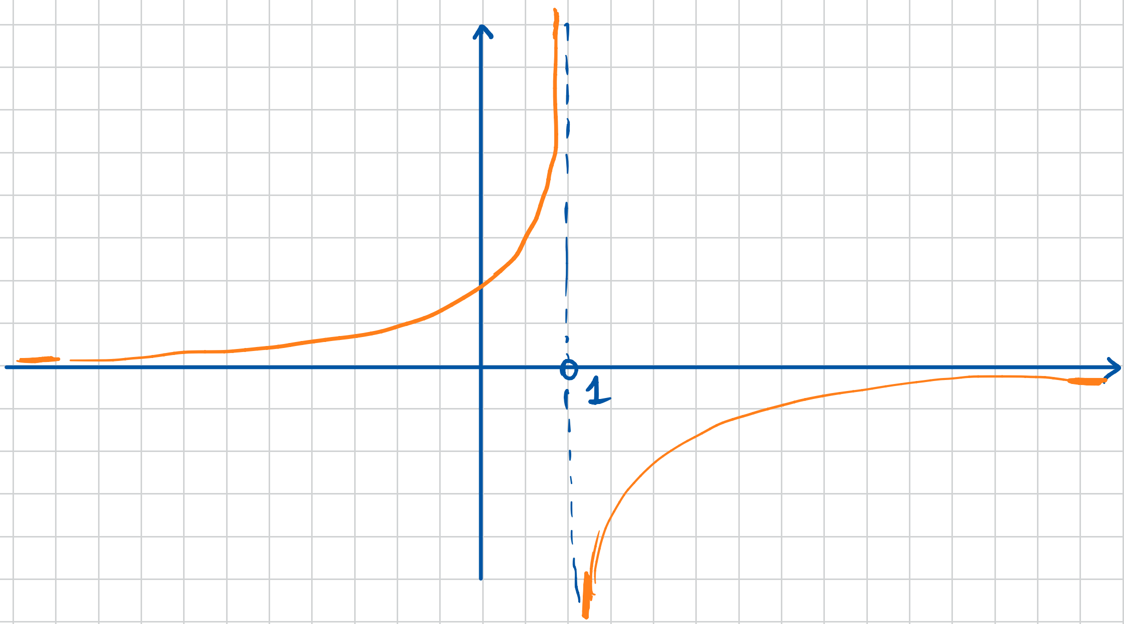
$$= \lim_{x\to+\infty} \frac{-2}{\underbrace{(x-1)}_{+\infty}\left(\underbrace{\sqrt{|x-1|}}_{+\infty}+\underbrace{\sqrt{|x+1|}}_{+\infty}\right)} = -\frac{2}{+\infty} = 0$$

$$\lim_{x\to 1^+} \frac{\sqrt{|x-1|}-\sqrt{|x+1|}}{x-1} = \lim_{x\to1^+}\frac{\sqrt{x-1}-\sqrt{x+1}}{x-1} =$$

$$= \lim_{x\to 1^+} \frac{\sqrt{0^+}-\sqrt{2}}{0^+} = -\frac{\sqrt{2}}{0^+} = -\infty$$

$$\lim_{x\to 1^-} \frac{\sqrt{|x-1|}-\sqrt{|x+1|}}{x-1} = \lim_{x\to1^-}\frac{\sqrt{1-x}-\sqrt{2}}{x-1} = \frac{-\sqrt{2}}{0^-}$$

$$= +\infty$$



PROLUNGABILE in $x = 1$?

1 Svolgo il $\lim_{x\to 1^\pm} f(x)$

2 Osservo che $\lim_{x\to1^\pm} f(x) = \pm\infty$ ← non è un numero

allora f non è prolungabile per continuità.

(2) $f(x) = \arctan\left(\frac{1}{x}\right) - \frac{\pi}{2} \cdot \frac{x}{|x|}$

dominio : $x \neq 0$ $\quad$ $\text{dom} f = (-\infty, 0) \cup (0, +\infty)$

$$f(x) = \begin{cases} \arctan \frac{1}{x} - \frac{\pi}{2} \cdot \frac{\cancel{x}}{\cancel{x}} & \text{se } x > 0 \\ \arctan \frac{1}{x} - \frac{\pi}{2} \cdot \frac{x}{(-x)} & \text{se } x < 0 \end{cases}$$

$$= \begin{cases} \arctan \frac{1}{x} - \pi/2 & \text{se } x > 0 \\ \arctan \frac{1}{x} + \pi/2 & \text{se } x < 0 \end{cases}$$

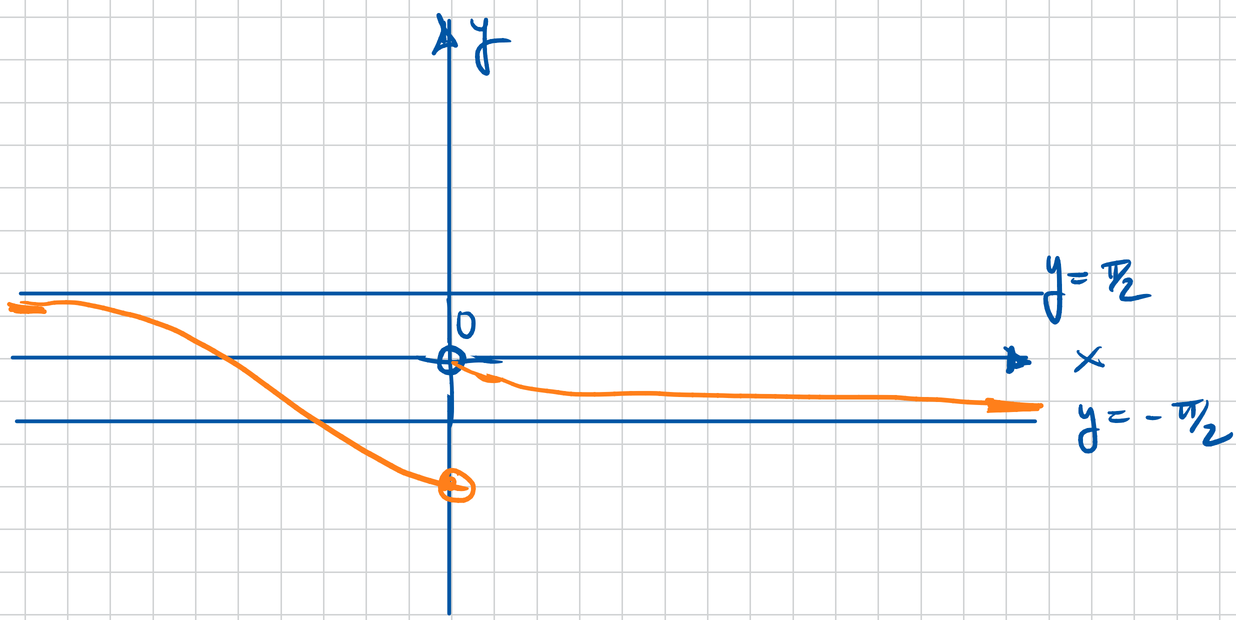
$$\lim_{x \to \pm\infty} \arctan \frac{1}{x} \mp \pi/2 = 0 \mp \pi/2$$

$y = -\pi/2$

$y = +\pi/2$

$$\lim_{x \to 0^\pm} \arctan \frac{1}{x} \mp \pi/2 = \begin{cases} \pi/2 - \pi/2 & \text{se } x \to 0^+ \\ -\pi/2 - \pi/2 & \text{se } x \to 0^- \end{cases}$$

$$= \begin{cases} 0 & \text{se } x \to 0^+ \\ -\pi & \text{se } x \to 0^- \end{cases}$$



f non è prolungabile per continuità

Es 2

$$f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{se } x>0 \\ x^2+a & \text{se } x\le 0 \end{cases}$$

f continua in $x=0$ $\iff$ $\lim\limits_{x\to 0} f(x) = f(0)$

$f(0) = 0^2 + a = a$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \dfrac{\sin x}{x} = 1$ (limite notevole)

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} x^2+a = a$

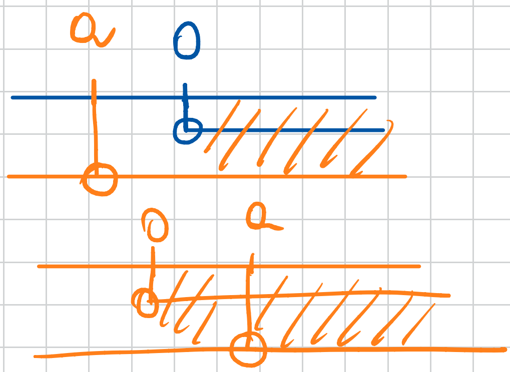
f è continua in $x=0$ $\iff$ $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^-} f(x) = f(0)$

$\iff$ $1 = a$ $\iff$ $a=1$

Quindi f è continua in $x=0$ $\iff$ $a=1$

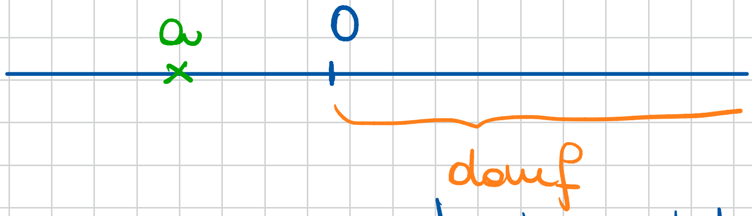
(6) $f(x) = \dfrac{\ln x}{x-a}$

$$\begin{cases} x>0 \\ x-a\neq 0 \end{cases} \to \begin{cases} x>0 \\ x\neq a \end{cases}$$



$$\operatorname{dom} f = \begin{cases} (0,+\infty) & a\le 0 \\ (0,a)\cup(a,+\infty) & a>0 \end{cases}$$

- se $a\le 0$ $\operatorname{dom} f = (0,+\infty)$ e f è continua in $(0,+\infty)$ perché quoziente di funzioni continue in $\operatorname{dom} f$



se $a<0$ $\quad x=a$ punto isolato

se $a=0$ $\quad x=0$ punto di accumulazione

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\ln x}{x-0} = \lim_{x\to 0^+} \frac{\ln x}{x} = -\infty$$

$\Rightarrow$ f non può essere estesa per continuità in $x=0$

<u>Se $a>0$</u>

$$\lim_{x\to a} f(x) = \lim_{x\to a^\pm} \frac{\ln x}{x-a} = \frac{\ln a}{0^\pm} = \infty$$

allora $\nexists\, a>0$ t.c. f sia prolungabile per continuità in $x=a$.