SISC '96

Density deconvolution using spectral mixture models

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  Brian Hutton
  Medical Physics, Westmead Hospital (collaborative team)
  if you want more details
* Interface proceedings ~~for forthcoming~~

GUIDE

Models for time evolution in activity at a voxel

Reconstruction method (dynamics parameters)

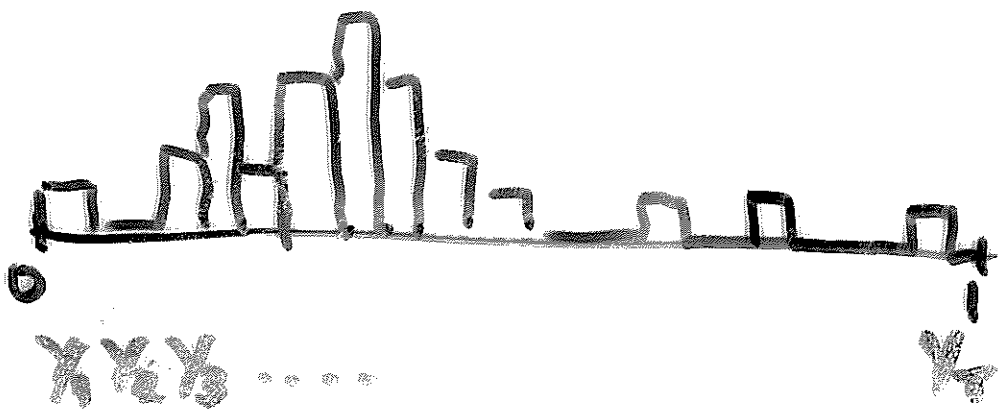
Evaluation by simulation

Data Representation

(observation in range 0 – 1)

I. n observations

II. Digitised data : (binned)



CONTEXT

As in earlier talks
Earlier talks have highlight again opportunities for Statisticians to model.
The opportunity for modelling new technologies.

Medical image SPECT - dynamics!

Introduction of radioisotope into bloodstream,
uptake in imaged body region (delay voxel specific)
scintillation *events* recorded in a sequence of time slices.

* Aim is to estimate dynamic parameters (key to functioning of body not anatomy)

Our approach involves reconstruction:

at every point (voxel) within the body region
from a time series of (activity) counts
mixture model parameter estimation (voxel specific)

Evaluation - desirable properties

interpretable
failsafe
fast computation
efficient

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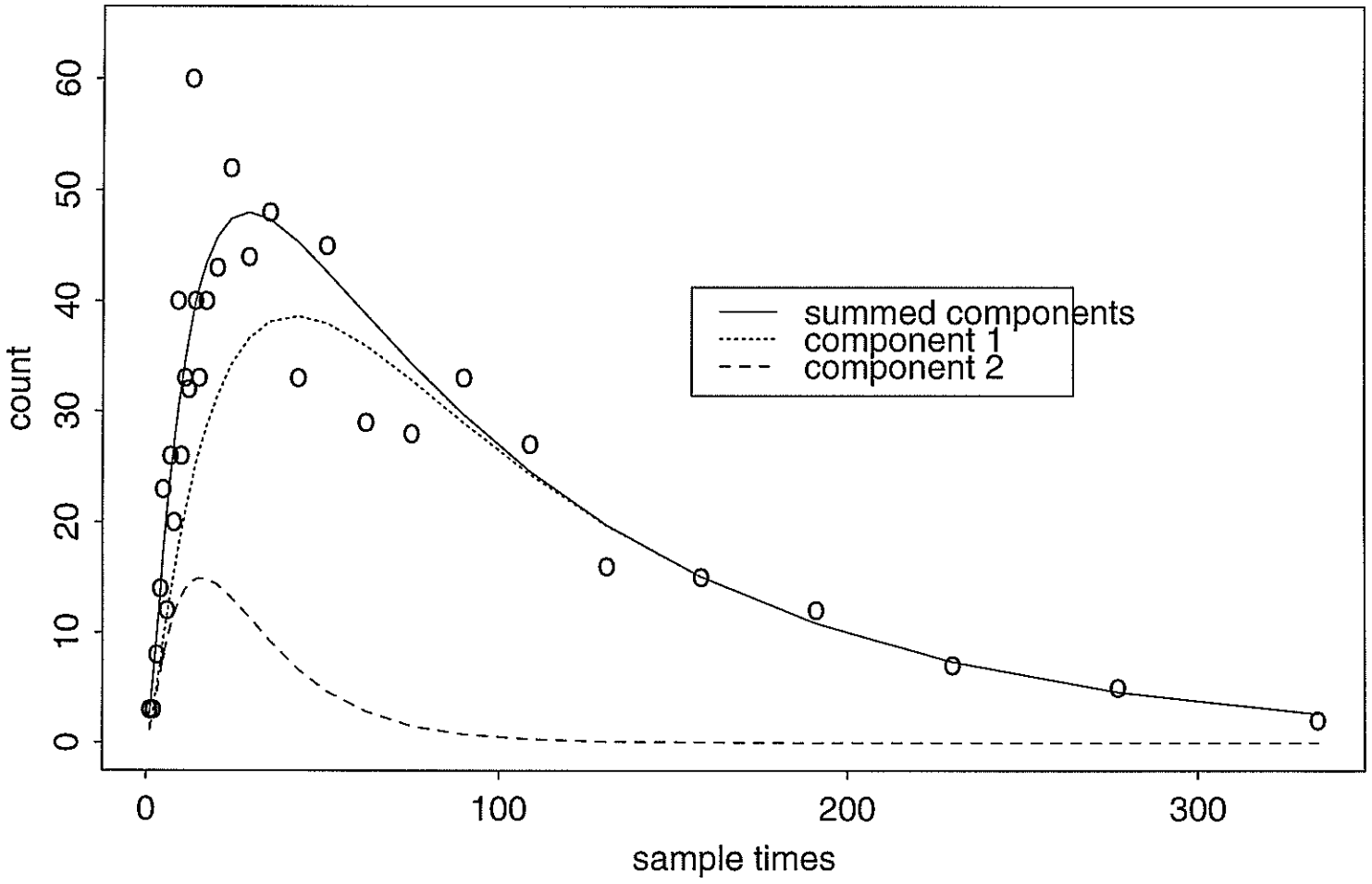
Count data and components of the distributio



NOTE: 32 observations, each a count.

TACs

Snyder (1984), Cunningham & Jones (1993)

Compartmental models parameterize f_j, involving a *mixture model* for the expected number of events:

$$\lambda_{jt} = f_j(t) = \sum_{k=1}^{K} \beta_{jk}\ x_k(t)$$

where

$$x_k(t) = x_0(t) \circledast (1/\sigma_k) exp(-t/\sigma_k)$$

β_k- weights of the K *modes*
$x_0(t)$ - known input function

Fitting parameters:

- EM algorithm for Maximum Likelihood (ML)
- Quadratic programming for Weighted Least Squares (WLS) subject to constraints

Approach:

Linear model for *digitised* event times.

Resulting model:

independent Poisson counts $Y_1, ..., Y_T$ with vector of expected values

$$EY = \lambda = X\beta$$

where X is a *known* TxK matrix, β is a Kx1 vector proportional to the mixing probabilities.

Non-zero β-coefficients correspond to modes present in the data: each column of X is the TAC for one particular *mode*.

Quadratic Programming

Minimize

$$S(\beta) = \sum_{1}^{T} w_i (Y_i - \mu_i(\beta))^2$$

where

$$\mu_i(\beta) = \lambda_i = x_i^T \beta$$

$w_1, \ldots, w_T$ are weights fixed in advance.

The minimum is found subject to *constraints* $\beta_k \geq 0$.

Active constraints are those parameters β_k set to zero. The corresponding modes (components of the mixture) are absent.

Implementation: Gill & Murray, NNLS

SKIP ?

How does convolution arise?

I. Errors in measurements

Model

$$y = x + z$$

Observables

1. We record independent sample observations $y_1, \ldots, y_n$, where
2. error distribution is known (e.g. normal distribution)

Objective is to reconstruct the d.f. of x (non parametric formulation).

II. Delayed or blurred signals

here we will take a semi-parametric formulation

SIMULATION DESCRIPTION

Event activity distribution the mixture of 2 components.

Sequences of 32 corresponding bin counts, *logarithmic* spacing.

Variables:

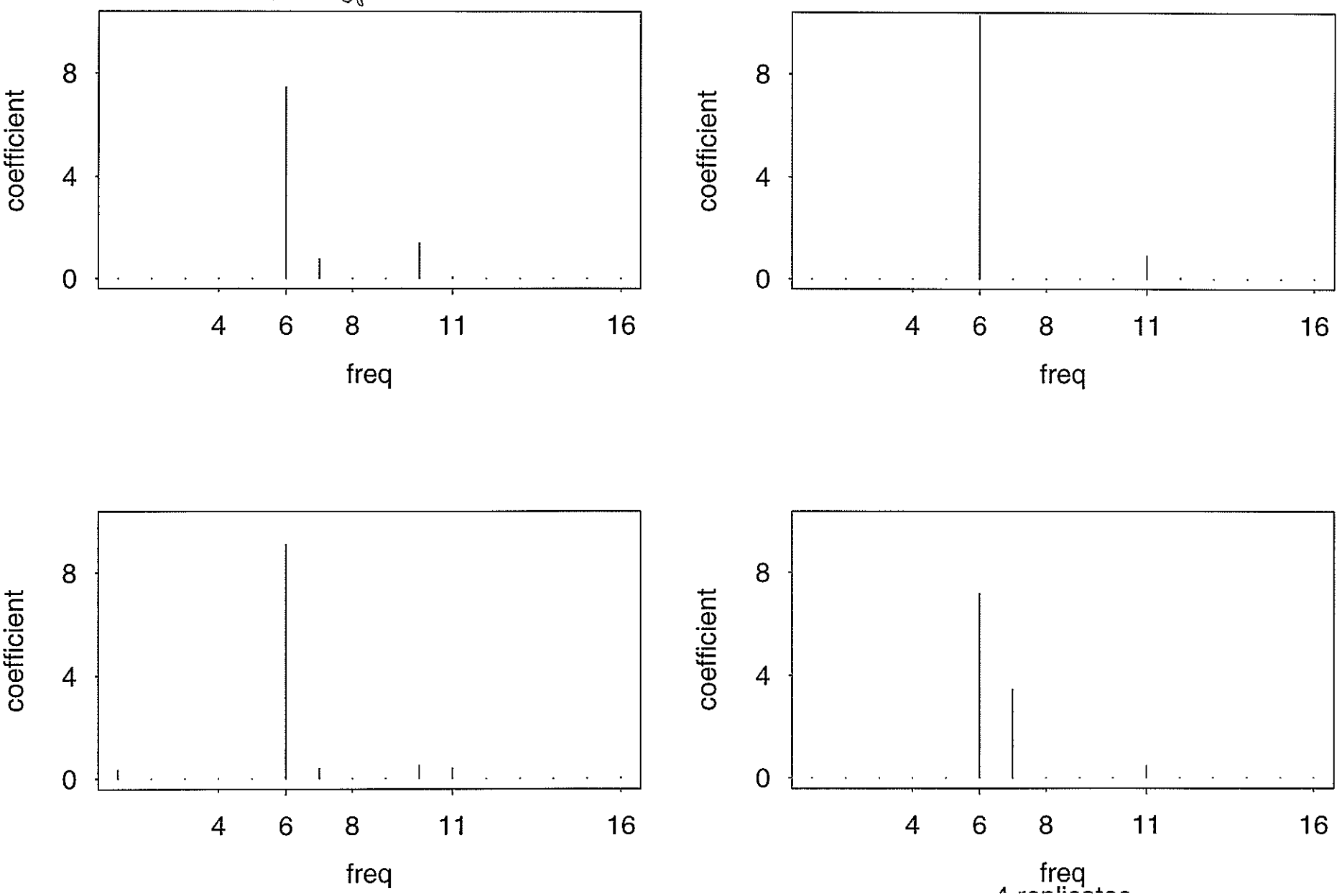
sample size: small/ large — for a single voxel in SPECT (not aggregated ROI)

mix of high and low frequency delays: 10: 1 and 1:10

reconstruction approach

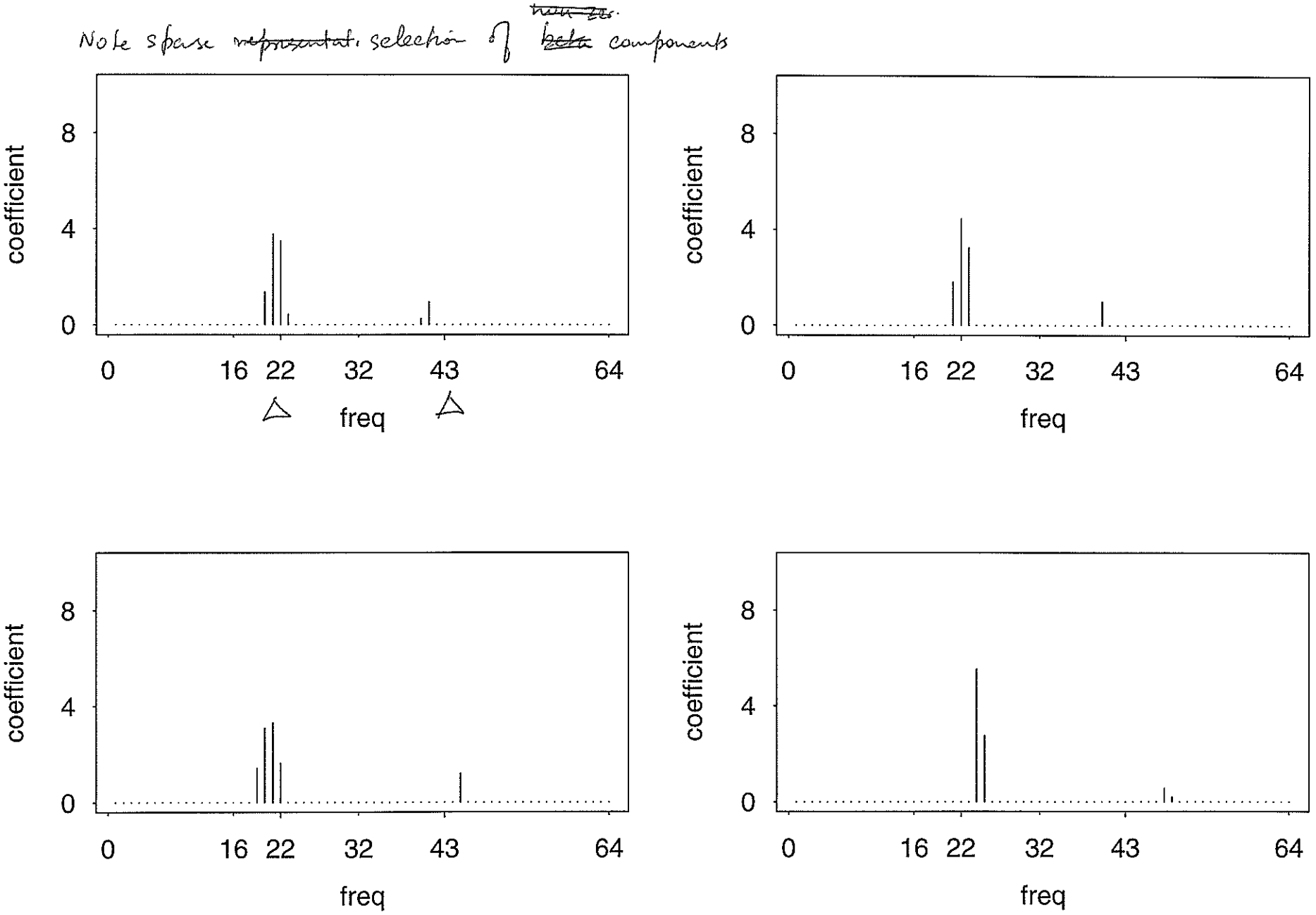
- basis dimension (K=16 /64) — remember 32 observations!
- variance estimate (true/ data)
- ridge adjustment constant

Coefficients: emprirical weights, N=32

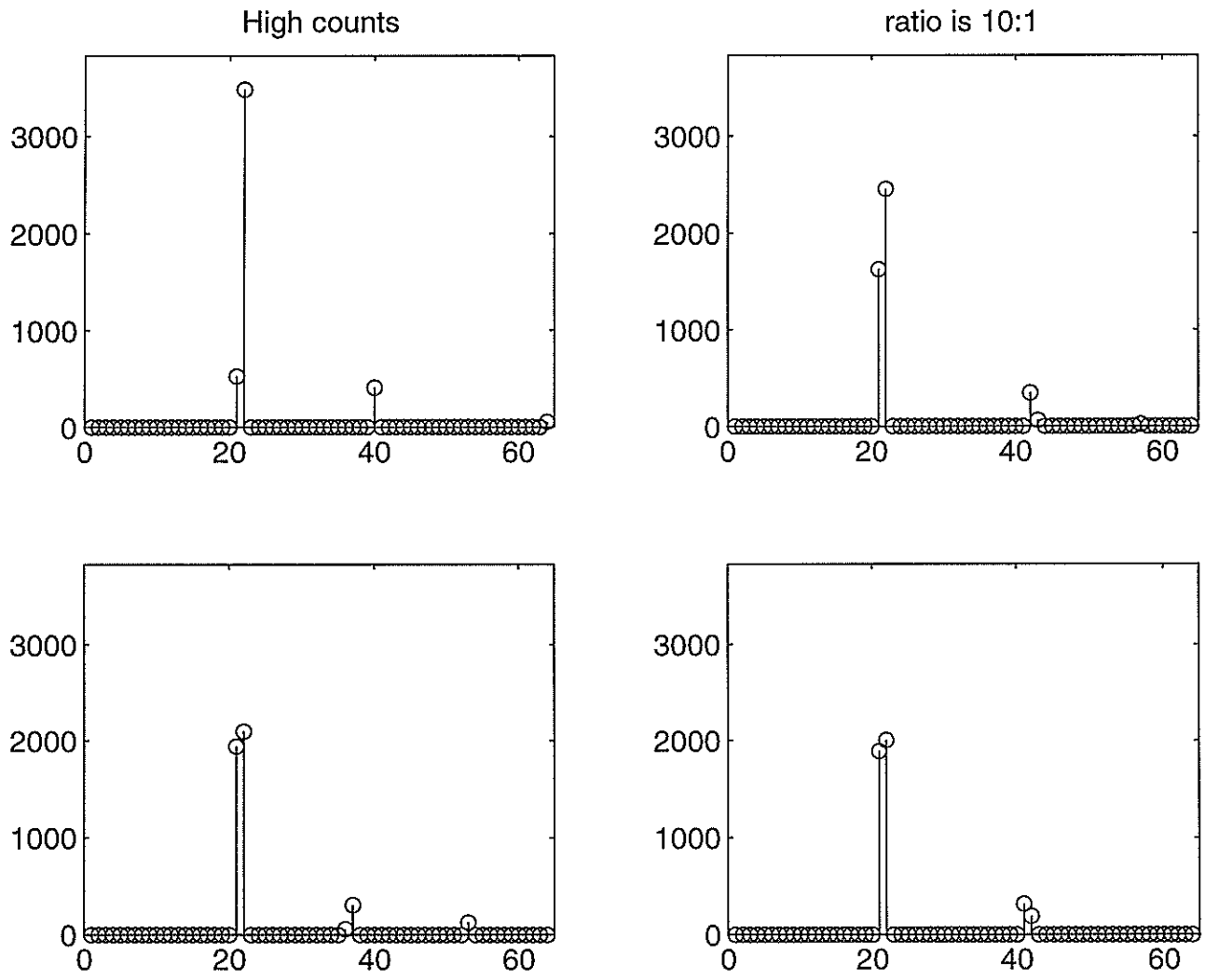


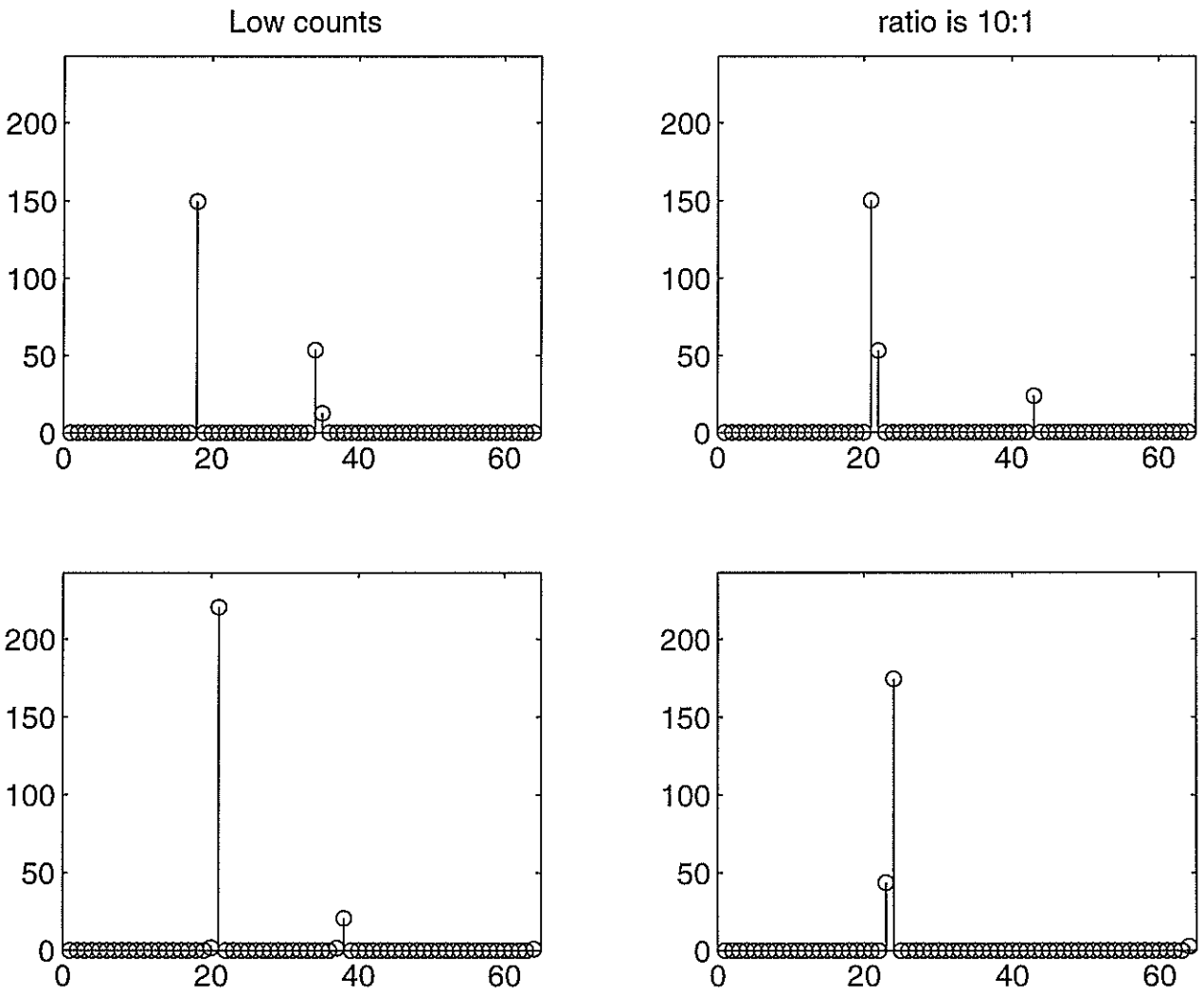
4 replicates, high noise

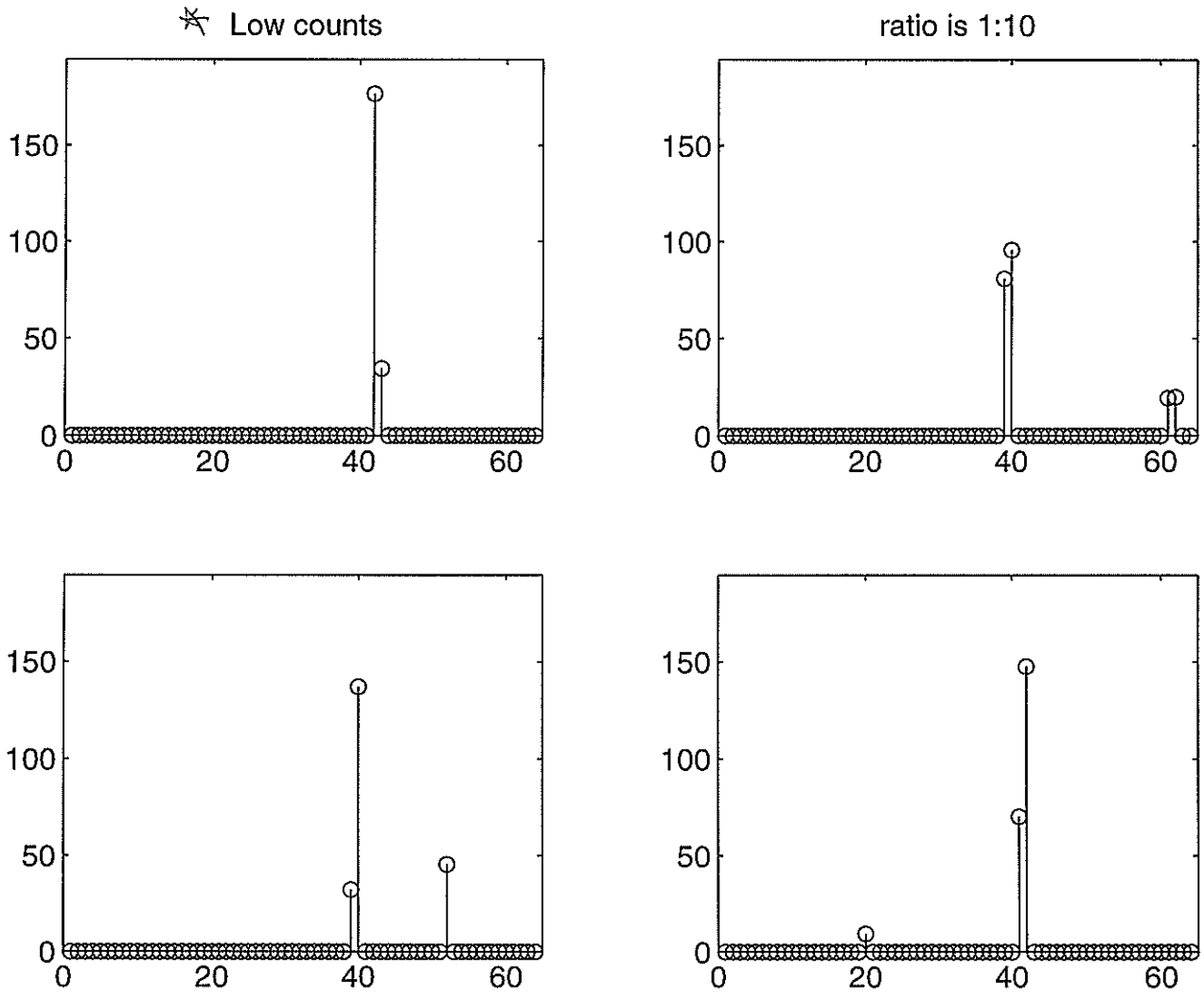
Note sparse ~~representation~~ selection of ~~beta~~ components



* NNLS
* Stability of high count results

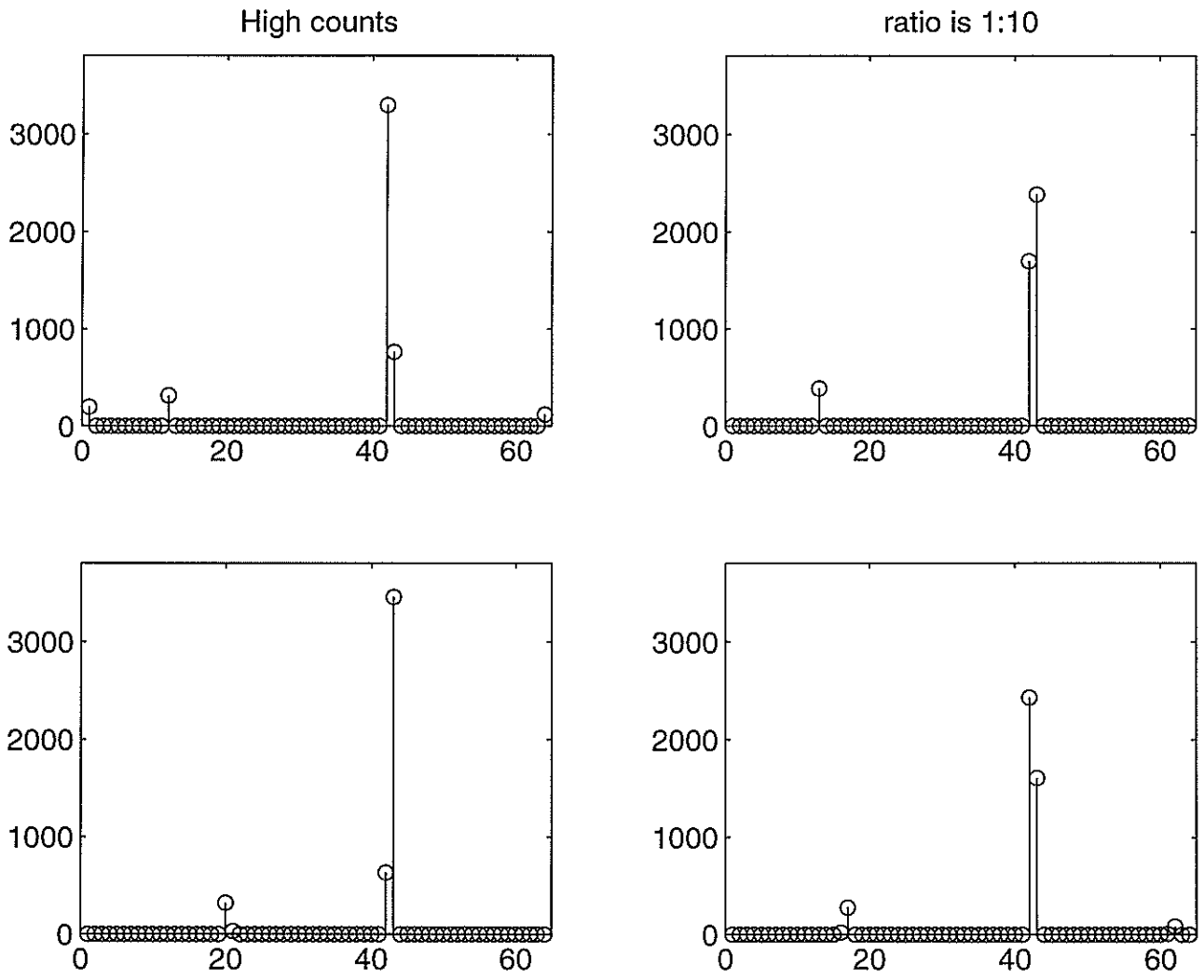



Low counts
ratio is 10:1
200
150
100
50
0
0
20
40
60


Low counts
ratio is 1:10
150
100
50
0
0
20
40
60

When High frequency predominant ~~in ch~~

* accurate results



Review:

- a multiple mode model was fitted to find different components of delay to a signal;
- we empirically determine the probabilities of modes (indexed by k); selecting amongst a dense basis
- ridge regression is effective in reducing high frequency artefacts.

Some observations/ extensions:

- convergence of the algorithm can be assessed by *leakage*;
- QP algorithm is faster than EM, but uses WLS (not ML), with weights affected by data variability;
- this doesn't seem to matter;
- Exploration tool — multiple images result; overspecify then postprocess by spatial and/or ~~frequency~~ temporal smoothing?
- following ML-EM reconstruction from *projections*, the Poisson space-time process is *NOT* a naive assumption.