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Organised by
University of Ballarat
and
University of Technology, Sydney

31st July & 1st August 1998

Mid City Motor Inn, Ballarat



Please return to:

ARC Workshop, School of Engineering, University of Ballarat, PO Box 663, Ballarat Victoria 3353 . . . Fax: (03) 5327 9137

Expression of Interest

I plan to attend the Workshop. Please send registration form

I would like to receive further details on the Workshop

Please note my mailing details for future related activities

Name: _____

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Research Interests _____

What's behind the technology?

- Nuclear medicine:
gamma camera imaging; X-ray CT
(transmission tomography); emission
tomography (SPECT and PET); MRI
- Inverse problem formulated
Radon-transforms (CT); Poisson
point processes (SPECT, PET,
dynamics)
- *Physics* specification
camera *geometry* and transport physics
- Algorithms
Filtered back projection (FBP) with
Radon transform model
Large systems of (linear) equations
Shepp-Vardi EM algorithm

Goal: discuss better incorporation of Physics

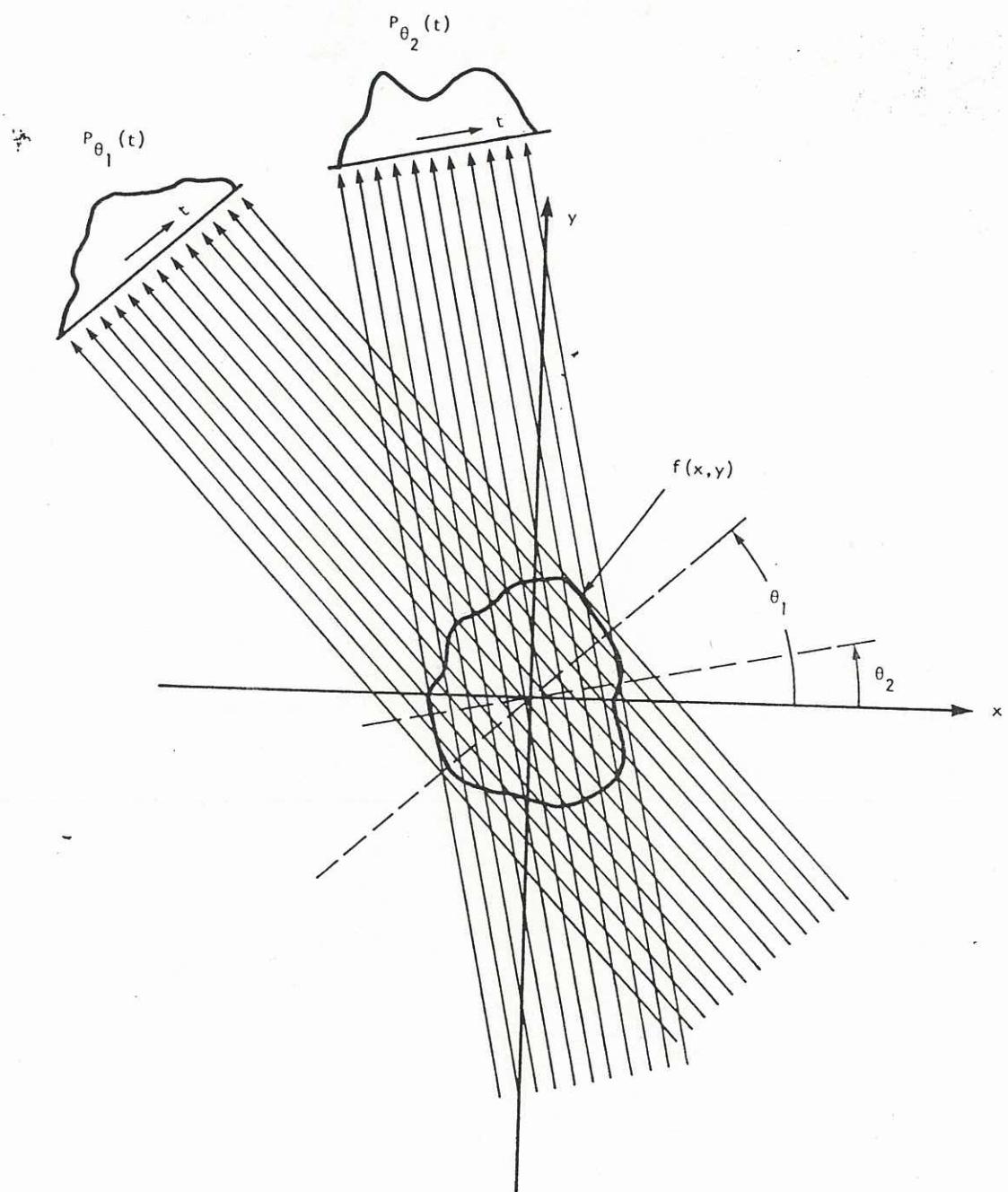


Fig. 2 This figure illustrates parallel projections.

shadows of denser regions of ~~biscuo~~ material

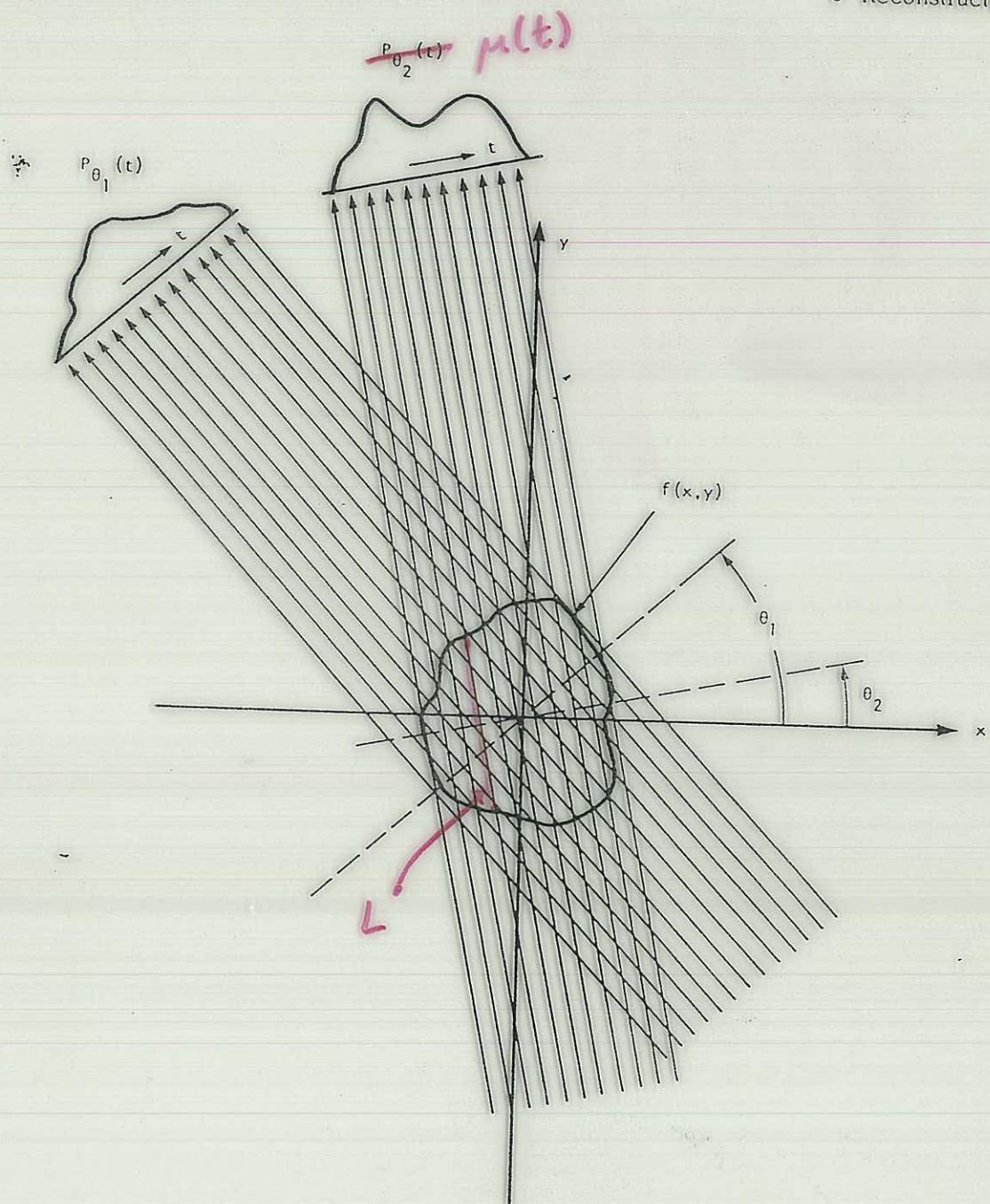


Fig. 2 This figure illustrates parallel projections.

$$\mu(t) = \int_L f(x, y) ds$$

TOMOGRAPHY

Continuous formulation

- transmission

initial intensity of rays passing through attenuated by density h.

$$I = I_0 \exp[- \int_L h(s) ds] \quad (1)$$

or

$$-\log g(t) = -\log \left(\frac{I}{I_0} \right) = \int_{L_t} h(s) ds, \quad (2)$$

- SPECT

$$g(t) = \int_{L_t} \exp[- \int_{L_{st}}^t h(u) du] f(s) ds \quad (3)$$

source of often as ray passes out activity at location in body.

- PET

$$g(t) = \exp \left[- \int_{L_t} h(u) du \right] \int_{L_t} f(s) ds. \quad (4)$$

Linear equations are formed by approximating line integrals by a weighted (ray) sum:

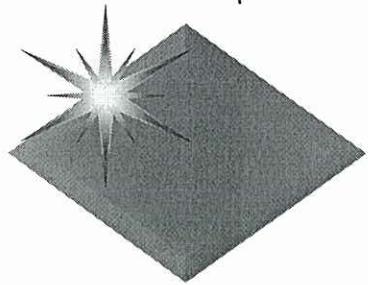
	y_t	x_s	$a(s, t)$
T	$-\log g(t) = -\log \frac{I}{I_0}$	$h(s)$	pixel/ray weights δ_{st}
$SPECT$	$g(t)$	$f(s)$	$\delta_{st} \exp[-\int_{L_{st}} h(u) du]$
PET	$g(t)$	$f(s)$	$\delta_{st} \exp \left[- \int_{L_t} h(u) du \right]$

Note:

g is assumed to be an observable distribution; when subject to noise, measured projections y_t are assumed to form a Poisson point process with intensity g . The distributions specified in discrete form as x are unknown.

Attenuation hazard (density) h is the *unknown* parameter in Transmission, but is assumed *known* for Emission.

Observed distribution: Poisson point process intensity g .



EM algorithm

- ◆ *Development*
Dempster, Laird, Rubin 1978 - EM formulation and properties
Shepp-Vardi 1982 - introduced EM in tomography
 - ◆ *When do we stop?* Convergence when:
fitted projections agree with with *observed* counts!
- Chart Likelihood, CSQ minimum.
EM = Expectation / Maximization

NOISE

Inverse problem with $N < \infty$ photons emitted
 $(x_i, y_i) \quad i=1, \dots, N$

leads to recorded data "histogram" \mathbf{g}_N

A natural assumption for binned data
is the independent Poisson, with density

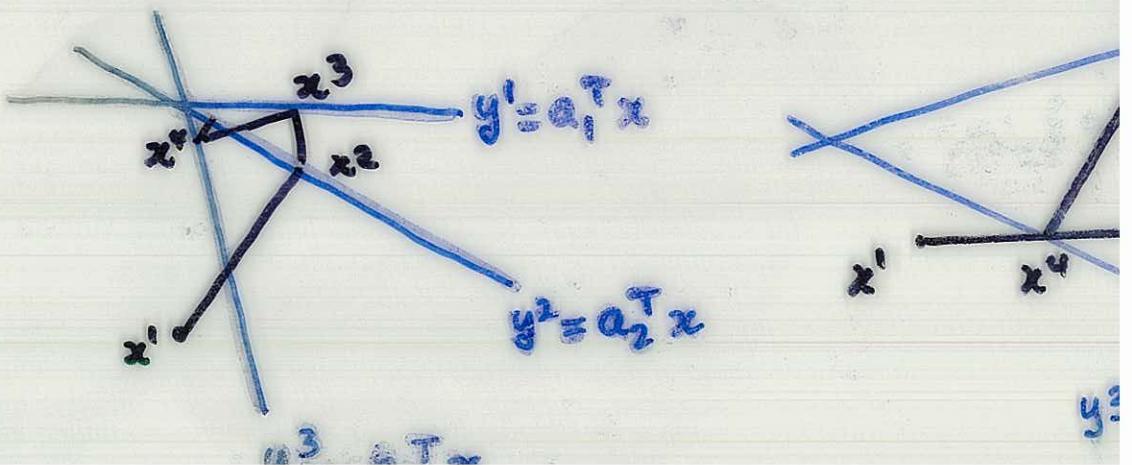
The discretized tomography model

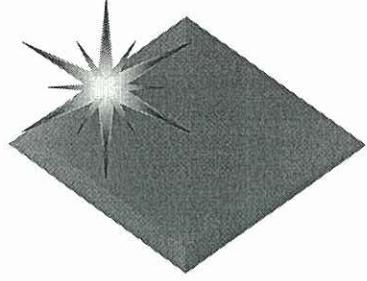
is $y_t \sim \text{Poisson}(\mu_t)$

with

$$\mu = \mathbf{A} \mathbf{x} .$$

- EM continues to increase Likelihood function
- Block-iterative methods (OS-EM) converge





Key Benefits:

- ◆ Generality in modelling Physics: DDR, scatter.
- ◆ Extensions of EM: patient movement, dynamics

STATISTICAL CONTRIBUTIONS

1. *Geometry*: of PET (Shepp and Vardi, 1982), SPECT (Lange & Carson 1983, Weir and Green 1994)
2. *Regularization*: encouraging local smoothness (Geman 1984)
3. Fusing images (e.g. MRI *anatomy* with SPECT *function*, Ardekani 1995)
4. Dynamics - mapping body metabolism; time changing emissions: a point process in space-time (O'Sullivan 1993)

TAC , curve fitting for local metabolic parameters

COMPUTING EM

Iterative, each iteration of two steps:

1. *Project* the current source estimate $f^n(v)$ to produce *fitted* projection data $g^n(u)$.
2. *Backproject* the ratio between *observed* and *fitted* projections, $g(v)$ and $g^n(v)$, to determine multiplicative corrections.

THE BLOCK-ITERATIVE KACZMARZ ITERATION

- Have proven convergence to the unique linearized solution of the ~~regular~~ MPL equations (with fixed W_k)
- use successive projections onto linear spaces corresponding to ~~the~~ successive data subsets - the partition of \mathbb{R}^n
- are expected to provide fast convergence ?

RE COMPUTATION OF WEIGHTS IN FISHER SCORING

- "No consensus about number of cycles ..."
- "The first iteration is usually the best .."
- "Great accuracy in weights is not necessary see discussion in Carroll & Rupert
"Transformation & Weighting in F