AUSTRALIAN AND NEW ZEALAND SOCIETY OF NUCLEAR MEDICINE INC.

NSW BRANCH
CONTINUING EDUCATION SUBCOMMITTEE

Department Nuclear Medicine
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Hospital Rd
Concord NSW 2139

Prof. Malcolm Hudson
Department of Statistics
Macquarie University

23/9/97

Prof.Hudson,

Thankyou for accepting our offer of participating in the ANZSNM NSW Branch 1997 Continuing Education program. Your presentation is scheduled for Saturday 1st Novemeber at 14:25hrs. It is my understanding that between yourself and Brian Hutton the following material will be presented **" Overview of ordered subset methods"** and **" Improved efficiency of scatter modelling with OSEM"**. The Saturday afternoon program consists of two sessions plus afternoon tea. Other speakers in this session include Dennys Lau , with a presentation entitled **" Incorporation of resolution compensation and post reconstruction filtering"** , Seu Som , with a presentation entitled **" Properties of minimum cross - entropy reconstruction with a morphologically based prior "** , Mr Steve Meikle , Physicist , RPAH with a presentation entitled , **"Iterative Reconstruction in Dynamic Studies"** , Mr Roger Fulton , Physicist , RPAH with a presentation entitled **"Correction of head motion artifacts in SPECT and PET with iterative reconstruction"** .

Please complete the accompanying registration forms at your earliest convenience. I require details of your audiovisual requirements by October 3rd 1997. If you require duplication of written materials for participants please forward the material (1 copy in print and 1 copy on disk) by October 24th 1997.

Enquires may be directed to me on Phone: 9767 6339 o r Fax: 9767 7451 during normal working hours.

Yours Faithfully

Vivienne Bush
Honorary Secretary / Treasurer

Statistical reconstruction in Emission Tomography
Malcolm Hudson, Statistics Dept., Macquarie Univ.
(with B.F. Hutton)

Thank you
with fast evolving camera technologies
an opportunity ~~to improve~~ & need to develop alg. alternatives
Object of development is image quality.

What are ML and MAP estimates?

Implementation in ET

Advantages and disadvantages of statistical approaches

Improving convergence

The path on

Comparison

FBP

used in transmission tomography (low noise) and ET current generation hardware

analytic, non-iterative, fast

Statistical algorithms

iterative improvement on any initial ~~two~~ reconstruction

their advantage is the quality of reconstruction: better definition and noise suppression is observed

the model is flexible, better physics can be incorporated. e.g. Attenuation effects, scatter. (examples - Chornboy et al 1990)

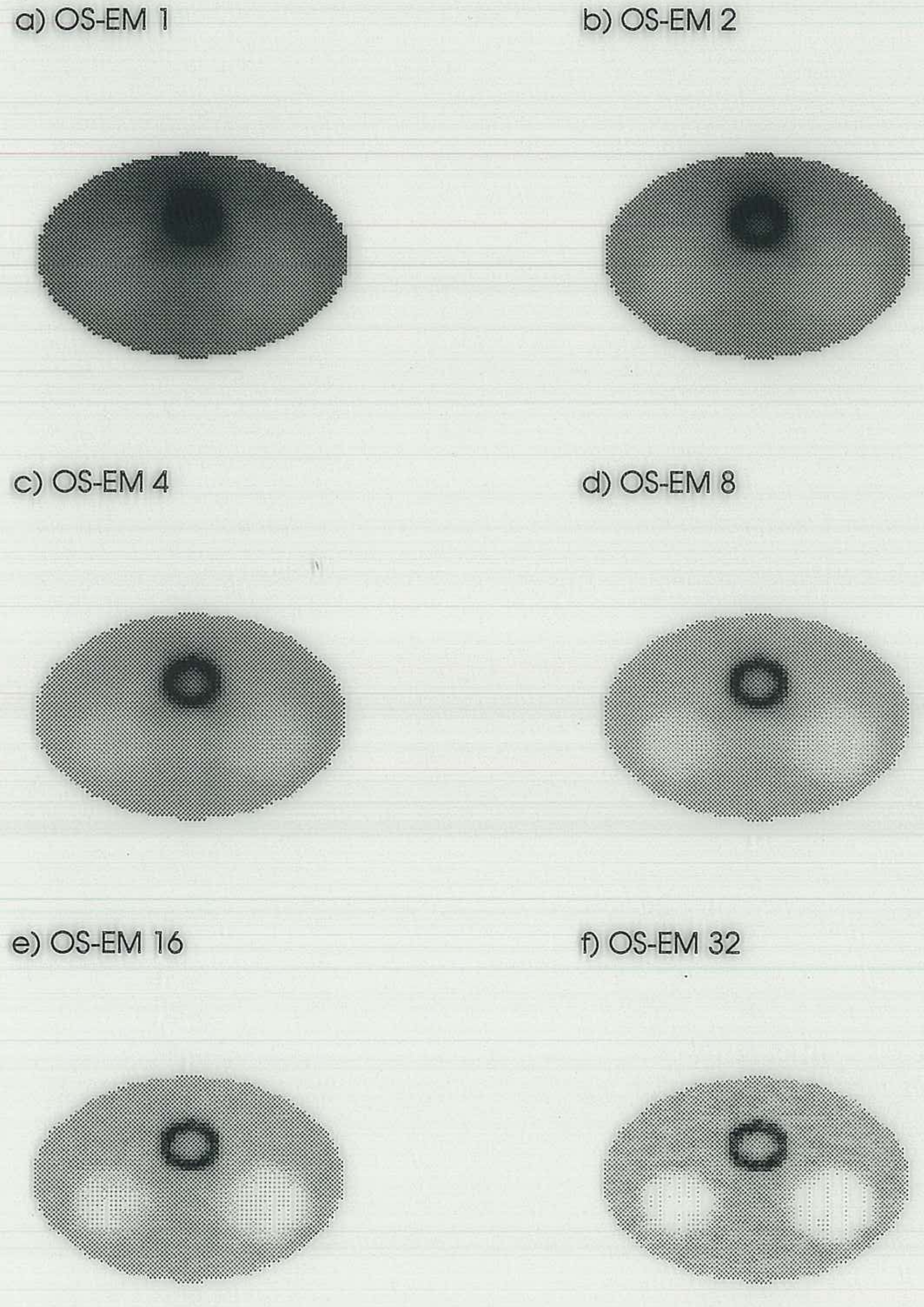
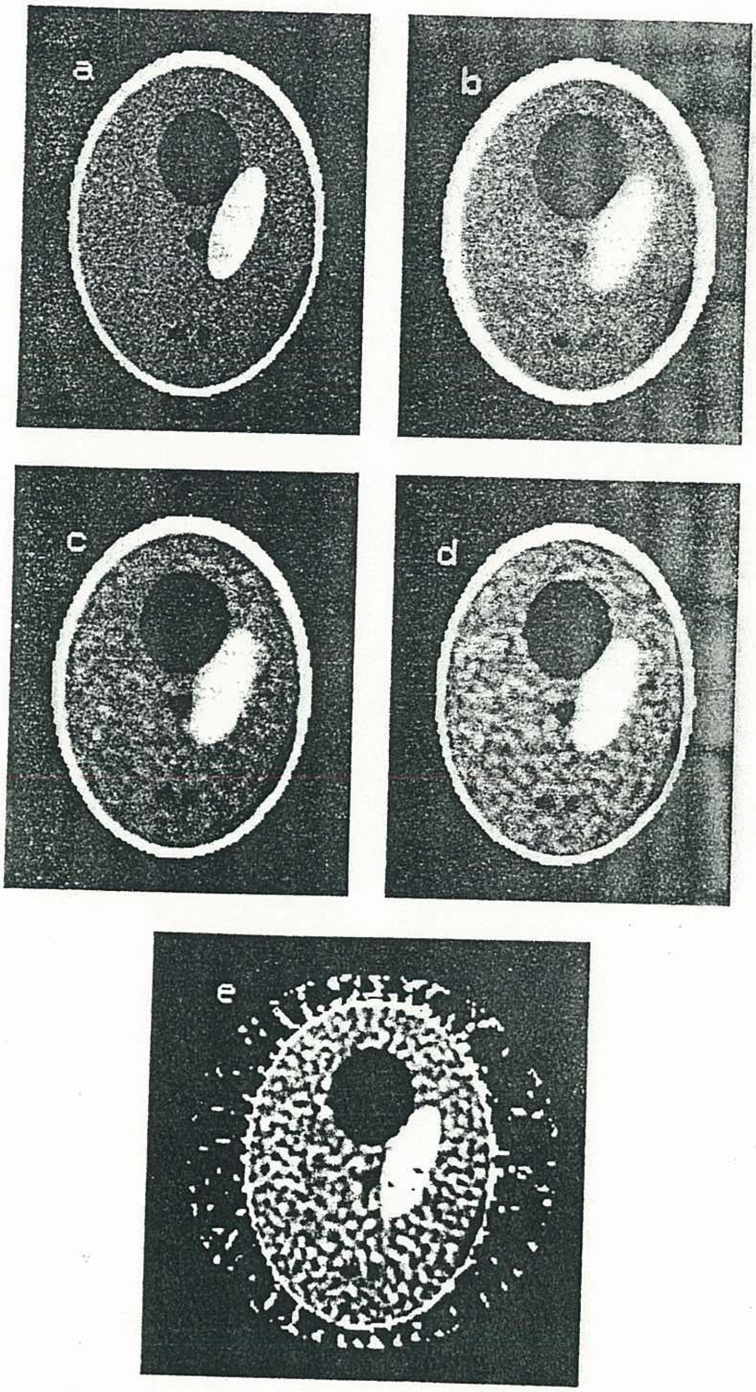


FIGURE 2: OS-EM Comparison After Single Iteration.



65x64 tube
128x128 param
~~counts~~

Figure 3. (a) The histogram $n(b)$, $b = 1, \ldots, B$ $(= 128^2)$, of the 10^7 counts drawn from the phantom of Figure 2 at a rate proportional to

ML and MAP reconstruction

Distinction between emitter activity and projections (Lung image)

ML principle:

Provide the reconstruction as the image that would, when projected, have the best overall agreement with the observed projection data.

i.e. maximise probability of observed data, a function of the emitter activity image $\langle x \rangle$

Penalized ML (Maximum A Posteriori) pragmatism:

Regularise ML to choose from many alternative images $\langle x \rangle$ with similar fit to projections the one which agrees with some prior requirement.

e.g. the "smoothest" image that agrees with data

Implementation

Filtered Back Projection for ML with Gaussian errors, all similar in scale, and exact projection of activity to cameras

ExpectationMaximisation algorithm (Shepp & Vardi) for ML with Poisson count data with arbitrary "projections" $\langle y \rangle = \langle A \rangle \langle x \rangle$

Adaption to penalized likelihood: Green 1990

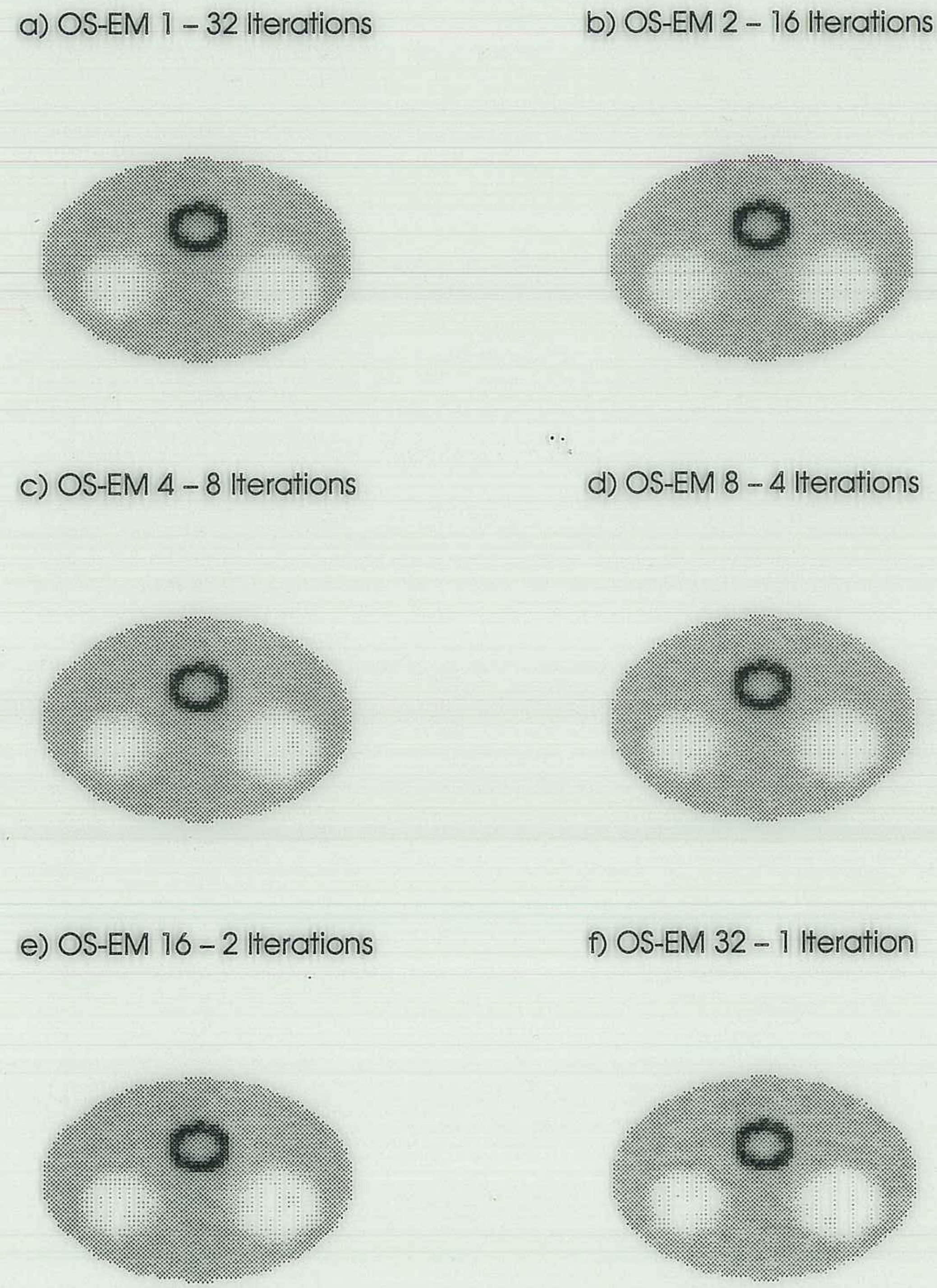


FIGURE 3: OS-EM Comparison After Matched Iterations.

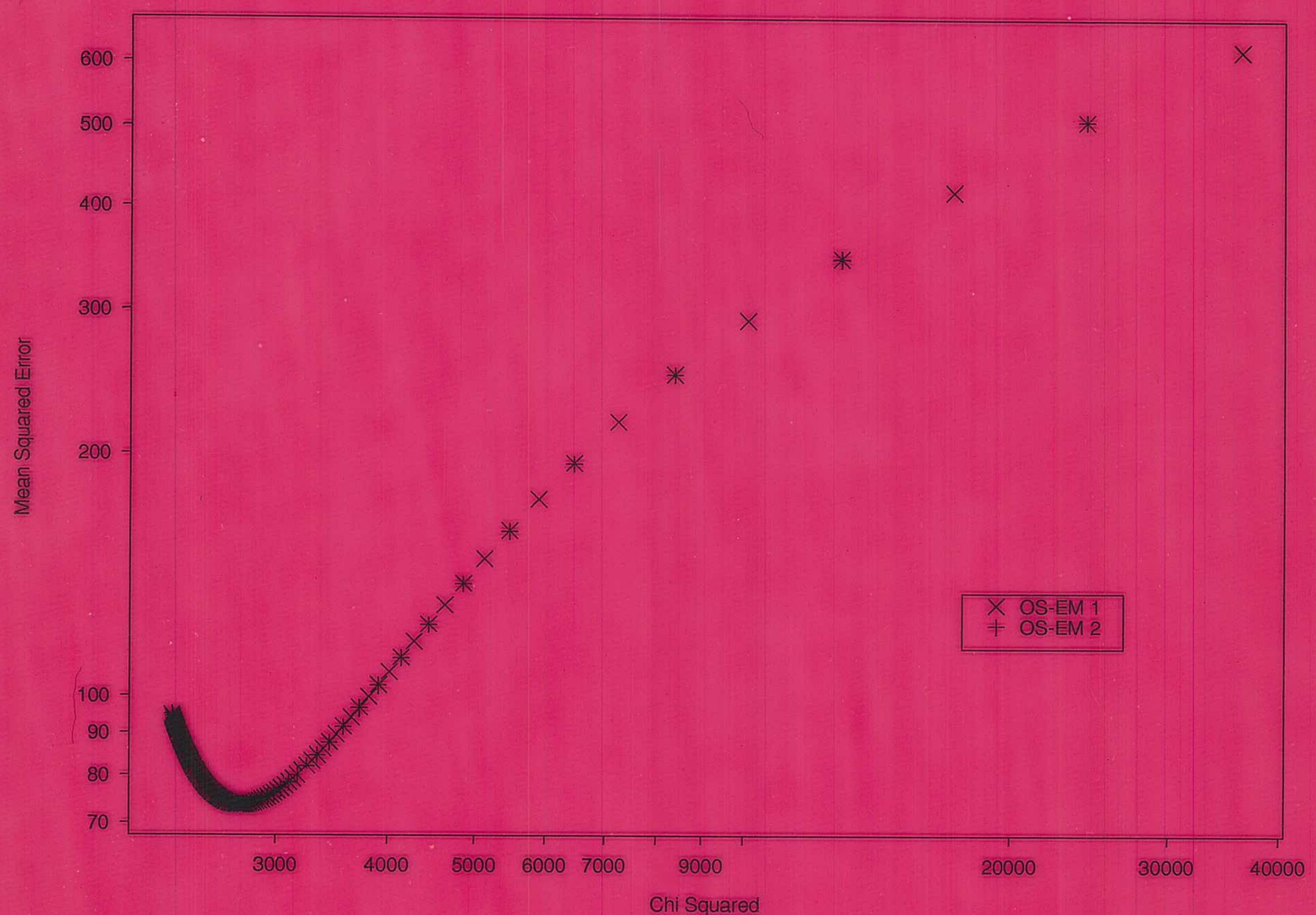


FIGURE 6: Mean Squared Error vs Chi Squared

PLAN FOR THIS TALK

- Motivation of use of ordered subsets (partition) of data in tomography.

- Kaczmarz projection methods for linear equations (iteratively by block iterative methods

- Fisher-scoring linearization of maximum penalized likelihood reconstruction.

x - unknown vector describing image activity :

x_j the activity from pixel j

y - known vector of count data from came

$A = (a_{tj})$ known probabilities specifying design matrix

$$= \begin{bmatrix} A_1 \\ \cdots \\ \vdots \\ \cdots \\ A_K \end{bmatrix}$$

W, W_k diagonal matrix of weights (at

R matrix determing roughness penal

EM algorithm

Let $\underline{x}$ be complete data

$\underline{y}$ incomplete data

$\underline{\theta}$ parameters

with log-likelihoods

$$L_x(\theta) = \log f(x|\theta)$$ complete

$$L_y(\theta) = \log g(y|\theta)$$ observable

Given current $\underline{\theta}^n$, define

$$Q(\underline{\theta}|\underline{\theta}^n) = E\left(\log f(x|\theta) \,\middle|\, y, \theta^n\right.$$

Estep: Compute $Q(\underline{\theta}|\underline{\theta}^n)$

Mstep: $\underline{\theta}^{n+1}$ chosen to maximize $Q($

regarded as a function of $\underline{\theta}$.

Shepp & Vardi application

Observable data counts

$$y_t \sim \text{Poisson}\Big(\overbrace{\sum_{j=1}^{J} a_{tj}\,\theta_j}^{\mu_t} \Big) \qquad t=$$

ind

Design matrix $A = (a_{tj})$ is known and h

<u>positive</u> (probability) elements.

It is also ~~___~~ extremely large.

Unobserved complete data

Let $x = \{x_{tj}\}$ where

$$x_{tj} \sim \text{Poisson}(a_{tj}\theta_j)$$

Then $Q(\theta|\theta^n) = \sum_t \sum_j \Big(-a_{tj}\theta_j + E(x_{tj}|y,$

where

$$E(x_{tj}|y,\theta^n) = y_t \frac{a_{tj}\theta_j}{\sum_k a_{tk}\theta_k} =$$

is the Back-Projection of H

Then

$$\theta_j^{n+1} = \theta_j^{n} \sum_{t=1}^{T} \frac{y_t \, a_{tj}}{\mu_t(\theta^n)}$$

$j = 1, \ldots$

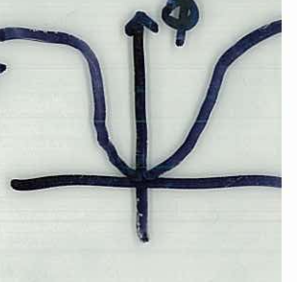
REGULARIZATION

With the number of parameters (J) approa
or exceeding the number of data observa
it is advisable to use a penalized likelih

Gibbs priors

$$S = -2 \log g(\underline{y} / \underline{\theta}) + \beta \sum_{i,j} w_{ij} \, \phi\left(\frac{\theta_i - \theta}{\delta}\right.$$

β is a smoothing parameter



Choose $\hat{\theta}$ to minimize $S(\theta)$.

How? An alteration to the M-step
EM is required

e.g. Green 1990 OS

EM algorithm: intro to accelerated reconstruction

Malcolm Hudson
Department of Statistics,
Macquarie University

Objective

- What is Likelihood? Why Maximum Likelihood?
- What is EM?
- Poisson assumptions - necessary?
- Convergence and properties of EM
- Regularization
- Other iterative algorithms

SPECT context

- Reconstruction problem

  $E(p)=A\lambda, \ \lambda>0.$

  The *transition* matrix A

  specifies the link between activity and projection

  ray sum + geometry + scatter

  Projection counts p are subject to Poisson noise.

  Given $p>0$ and A, estimate λ.

Objective

- What is Likelihood? Why Maximum Likelihood?
- What is EM?
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EM algorithm

- iterative improvement of reconstruction

EM

substitute (for all pixels/voxels j):

$$\lambda_j \leftarrow \lambda_j * \Sigma_i [a_{ij}\, p_i / \mu_i] / \Sigma_i a_{ij},$$

where *fitted projections* are

$$\mu_i = \Sigma_j a_{ij} \lambda_j$$

General form: *project*, then *back-project* discrepancies.
Normalized system: $\Sigma_i a_{ij} = 1$.
Repeat this substitution to improve further!

OSEM

projection counts in *subsets* (e.g. different camera views); introducing each subset p^k in turn, substitute

$$\lambda_j \leftarrow \lambda_j * \Sigma_{i \text{ in subset}} [a_{ij}\, p^k{}_i / \mu_i] / \Sigma_{i \text{ in subset}} a_{ij}.$$

Subset balance when denominators equal for all subsets.

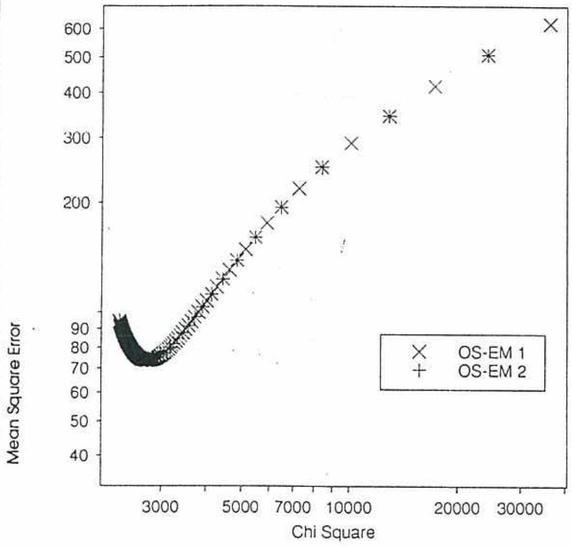
Study results [slides].

- Reconstruction quality observed to be better than CBP.
- *Noise artifact* appears after "too many" iterations.

RESULTS

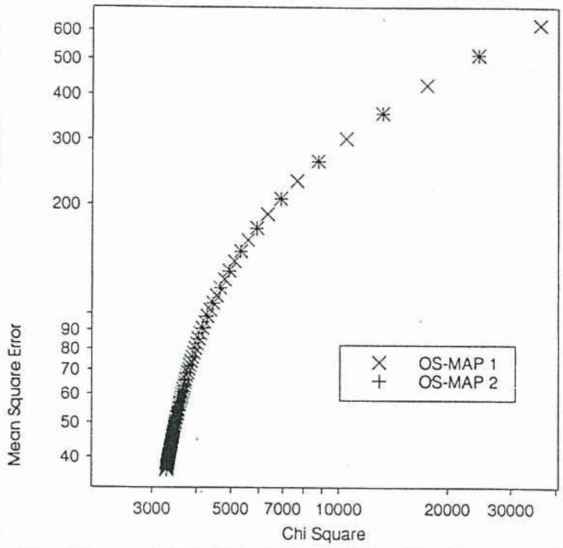
COMPARISON OF SINGLE SUBSET AND TWO SUBSET METHODS

MAXIMUM LIKELIHOOD



SUBSETS	ITERATIONS	CHI-SQUARE	MSE
1	32	3108	78.4
2	16	3105	78.4
32	2	3421	78.7
1	2	36275	621.2

MAXIMUM A POSTERIORI



SUBSETS	ITERATIONS	CHI-SQUARE	MSE
1	32	3774	68.7
2	16	3767	68.5
32	2	3883	62.5
1	2	36275	621.2

SIMULATED DATA

Regularize ML

- many alternatives with similar fits to projections
- choose one which agrees with some prior requirement.
- e.g. the "smoothest" image that agrees with projection counts.

Regularized reconstruction

- Benefit from smoothing (spatially) within EM iterations?
- Best use of prior information

  Ardekani incorporates MRI anatomy in priors.
- Bayes/MAP alternatives to ML

  Priors. Probability distribution of λ is given.

  Gibbs distributions - specify Prob(λ_j | its neighbours)

Other iterative reconstructions

- Criteria: WLS, ML, ME (KL)
- Algorithms:
  - SMART, a product form of EM
  - MART, similar approach to OSEM

EM $\lambda_j \leftarrow \lambda_j \sum_t a_{tj} \frac{p_t}{\mu_t}$ OS.

SMART $\lambda_j \leftarrow \lambda_j \prod_t \left(\frac{p_t}{\mu_t}\right)^{a_{tj}}$ M

Likelihood

$L(\lambda) = \text{Prob}(p)$ is used to compare reconstructions:

higher probability means a better λ;

ML principle: prefer the solution λ most consistent with data p

Probability is calculated under *Poisson* noise assumptions

ML solution is *unique* when the linear system is *inconsistent*.

EM Properties

- Always provide a non-negative ML solution:

  whether counts are Poisson or not;

  solve consistent systems;

  geometric mean data.

- Slow to converge. Ordered Subset EM.