



University of Ballarat



## Organisers

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University of Ballarat

Prof. Binñ Pham, School of Information Technology  
& Mathematical Sciences, University of Ballarat

Dr. Michael Braun, School of Applied Physics,  
University of Technology, Sydney

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ARC Special Research Initiatives Program

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- Image & Video Research Infrastructure Facility
- Research Centre for Intelligent Tele-imaging
- Fiona Elsie Cancer Research Centre

University of Technology, Sydney

- Centre for Biomedical Technology

Australian Pattern Recognition Society  
(Victorian Branch)



## Contact Details

ARC Workshop

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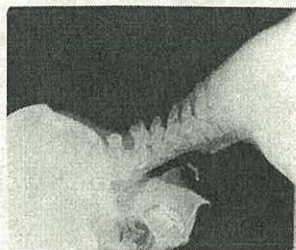
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Invitation to Participate



# Research Workshop on Automated Medical Image Analysis

Sponsored by  
ARC Special Research Initiatives Program

Organised by  
University of Ballarat  
and  
University of Technology, Sydney

31<sup>st</sup> July & 1<sup>st</sup> August 1998

Mid City Motor Inn, Ballarat



Please return to:

ARC Workshop, School of Engineering, University of Ballarat, PO Box 663, Ballarat Victoria 3353 ... Fax: (03) 5327 9137

## Expression of Interest

I plan to attend the Workshop. Please send registration form ☐

I would like to receive further details on the Workshop ☐

Please note my mailing details for future related activities ☐

Name: \_\_\_\_\_

Address: \_\_\_\_\_

Tel: \_\_\_\_\_

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Research Interests \_\_\_\_\_



*What's behind the technology?*

- Nuclear medicine:  
gamma camera imaging; X-ray CT  
(transmission tomography); emission  
tomography (SPECT and PET); MRI
- Inverse problem formulated  
  
Radon-transforms (CT); Poisson  
point processes (SPECT, PET,  
dynamics)
- *Physics* specification  
camera *geometry* and transport physics
- Algorithms  
  
Filtered back projection (FBP) with  
Radon transform model  
Large systems of (linear) equations  
Shepp-Vardi EM algorithm

*Goal: discuss better incorporation of Physics*

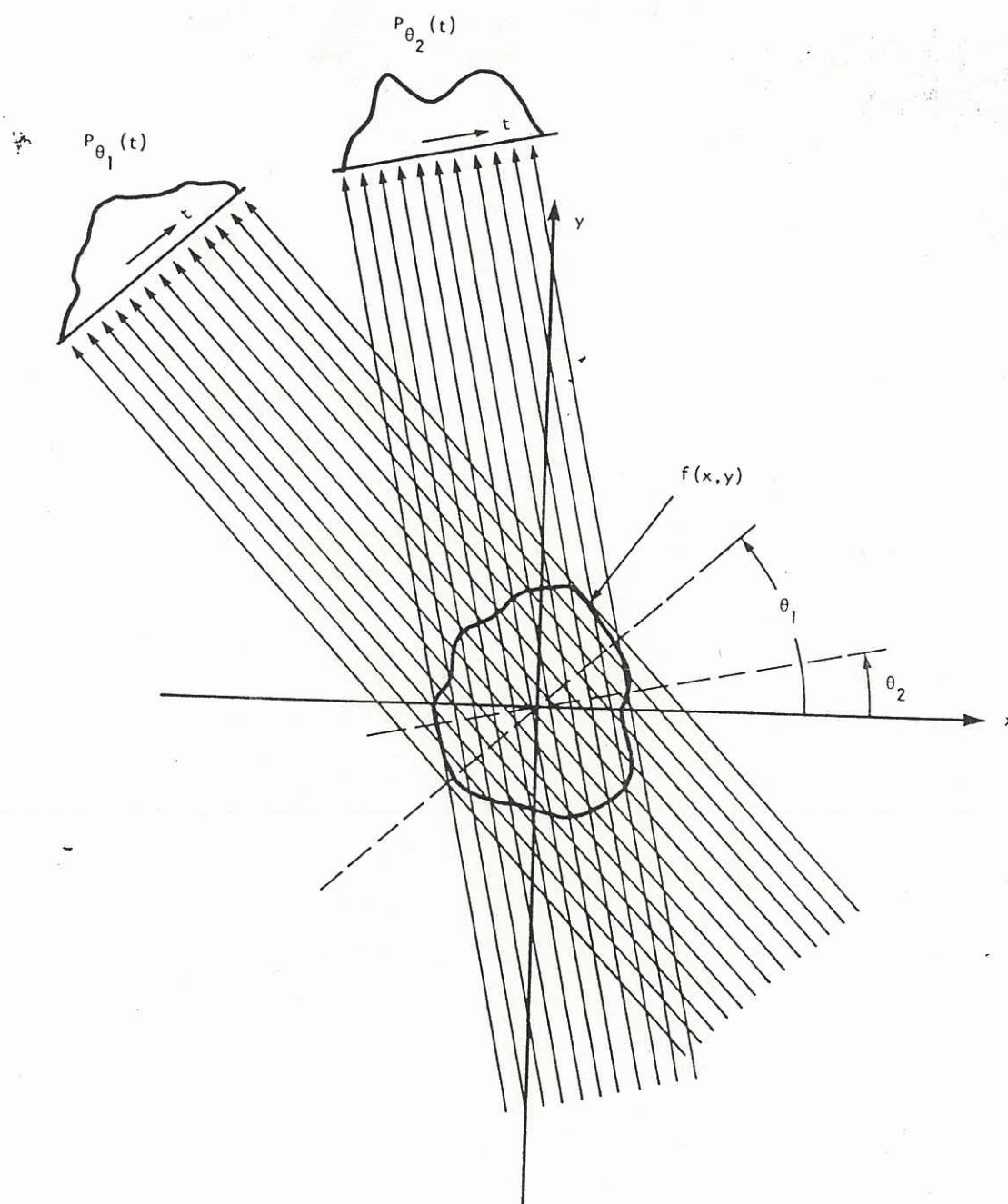


Fig. 2 This figure illustrates parallel projections.

shadows of denser regions of ~~beam~~ material .

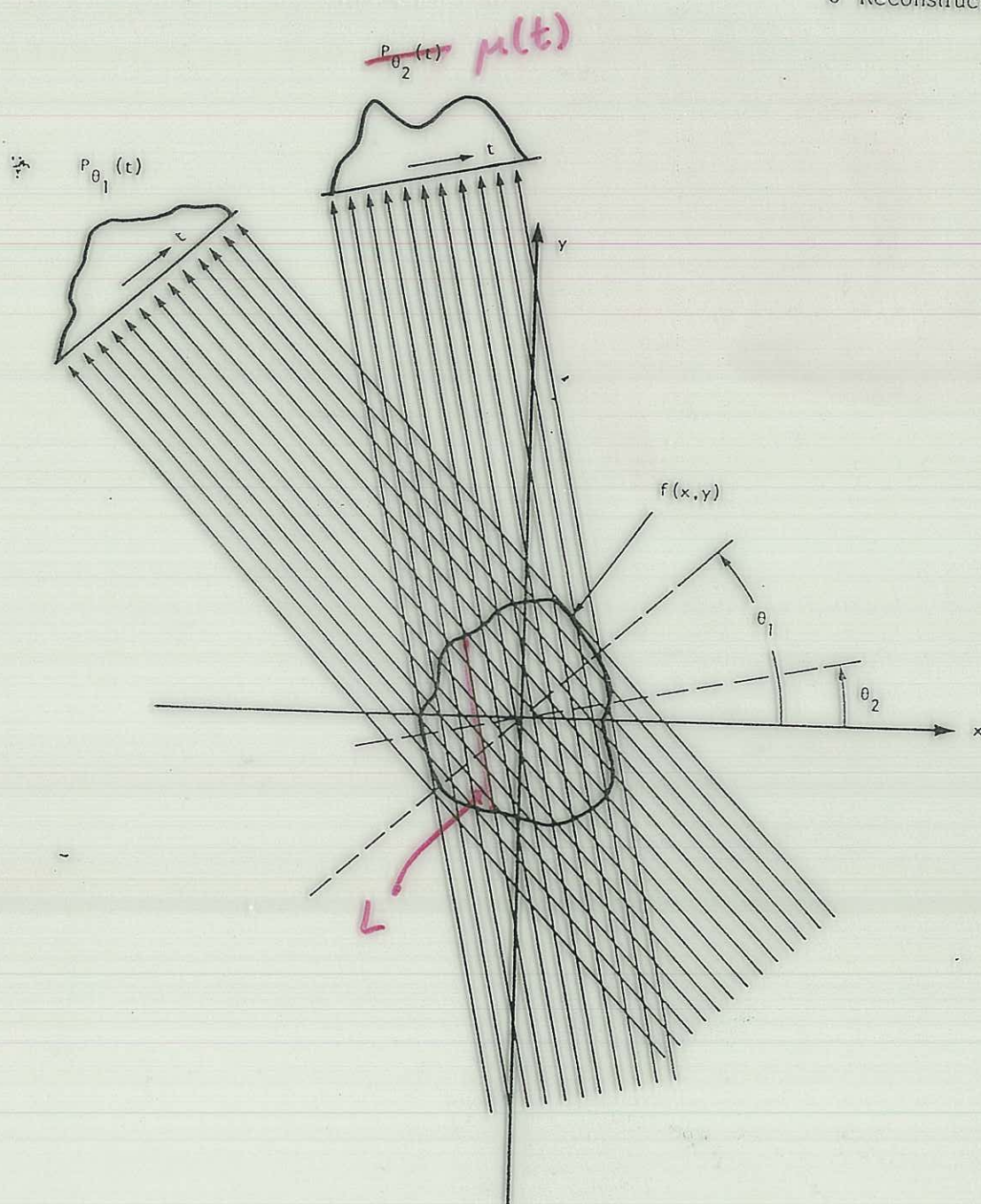


Fig. 2 This figure illustrates parallel projections.

$$\mu(t) = \int_L f(x, y) ds$$



## TOMOGRAPHY

### Continuous formulation

#### 1. transmission

$$I = I_0 \exp \left[ - \int_L h(s) ds \right] \quad (1)$$

*initial intensity of rays passing through  
attenuated by density  $h$ .*

or

$$-\log g(t) = -\log \left( \frac{I}{I_0} \right) = \int_{L_t} h(s) ds, \quad (2)$$

#### 2. SPECT

$$g(t) = \int_{L_t} \exp \left[ - \int_{L_{st}} h(u) du \right] f(s) ds \quad (3)$$

*same kind of atten. as ray passes out  
activity at location  $s$  inside body.*

#### 3. PET

$$g(t) = \exp \left[ - \int_{L_t} h(u) du \right] \int_{L_t} f(s) ds. \quad (4)$$

Linear equations are formed by approximating line integrals by a weighted (ray) sum:

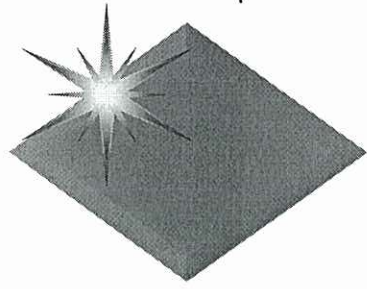
	$y_t$	$x_s$	$a(s, t)$
$T$	$-\log g(t) = -\log \frac{I}{I_0}$	$h(s)$	pixel/ray weights $\delta_{st}$
$SPECT$	$g(t)$	$f(s)$	$\delta_{st} \exp[-\int_{L_{st}} h(u) du]$
$PET$	$g(t)$	$f(s)$	$\delta_{st} \exp[-\int_{L_t} h(u) du]$

Note:

$g$  is assumed to be an observable distribution; when subject to noise, measured projections  $y_t$  are assumed to form a Poisson point process with intensity  $g$ . The distributions specified in discrete form as  $x$  are unknown.

Attenuation hazard (density)  $h$  is the *unknown* parameter in Transmission, but is assumed *known* for Emission.

Observed distribution: Poisson point process intensity  $g$ .



# *EM algorithm*

- ◆ *Development*

Dempster, Laird, Rubin 1978 - EM formulation and properties

Shepp-Vardi 1982 - introduced EM in tomography

- ◆ *When do we stop? Convergence when:*

*fitted* projections agree with *observed* counts!

Chart Likelihood, CSQ minimum.

EM = Expectation / Maximization



# NOISE

Inverse problem with  $N < \infty$  photons emitted

$$(x_i, y_i) \quad i=1, \dots, N$$

leads to recorded data "histogram"  $g_N$

A natural assumption for binned data is ~~the~~ independent Poisson, with density

Ex  
The discretized tomography model

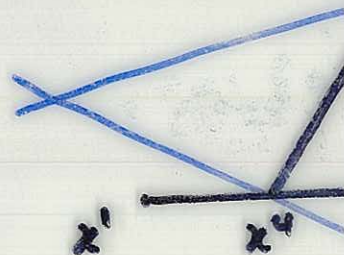
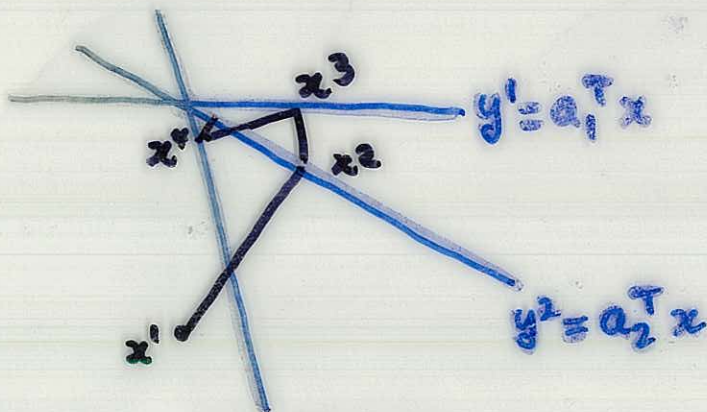
is  $y_t \sim \text{Poisson}(\mu_t)$

with

$$\mu = Ax$$



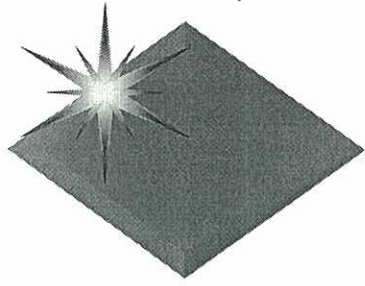
- EM continues to increase Likelihood function
- Block-iterative methods (OS-EM) cycle



$$y^3 = a_3^T x$$

$$y^3$$





## *Key Benefits:*

- ◆ Generality in modelling Physics: DDR, scatter.
- ◆ Extensions of EM: patient movement, dynamics

## STATISTICAL CONTRIBUTIONS

1. *Geometry*: of PET (Shepp and Vardi, 1982), SPECT (Lange & Carson 1983, Weir and Green 1994)
2. *Regularization*: encouraging local smoothness (Geman 1984)
3. Fusing images (e.g. MRI *anatomy* with SPECT *function*, Ardekani 1995)
4. Dynamics - mapping body metabolism; time changing emissions: a point process in space-time (O'Sullivan 1993)

TAC , curve fitting for local metabolic parameters



## COMPUTING EM

*Iterative*, each iteration of two steps:

1. *Project* the current source estimate  $f^n(v)$  to produce *fitted* projection data  $g^n(u)$ .
2. *Backproject* the ratio between *observed* and *fitted* projections,  $g(v)$  and  $g^n(v)$ , to determine multiplicative corrections.

## THE BLOCK-ITERATIVE KACZMARZ ITERATION

- Have proven convergence to the unique solution of the <sup>linearized</sup> ~~reg~~ MPL equations (with fixed  $W_k$ )
- use successive projections onto linear spaces corresponding to ~~the~~ successive data subsets — the partition of
- are expected to provide fast convergence ?

## RECOMPUTATION OF WEIGHTS IN FISHER SCORING

- "No consensus about number of cycles..."
- "The first iteration is usually the best..."
- "Great accuracy in weights is not necessarily achieved"
- see discussion in Carroll & Rupert
- "Transformation & Weighting in R"