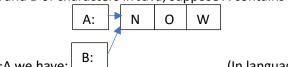
# Shareable Data Structures

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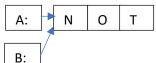
## 1 Introduction

The study of Data Structures is an essential early stage in any Computing programme, and needs to be revisited later on when the student has mastered threading. Early exercises on threading use many example programs containing data structures such as arrays to illustrate the need for locking. Students quickly learn that the standard string data type in modern languages such as Java and C# is immutable, but are often not told why: even if they are taught that this is to enable sharing of strings, they may fail to appreciate that other data structures can also benefit from being made safe in this sense, that is, immutable and shareable. Telling them that in these languages "strings are values" only serves to obscure the real issue.

Using arrays A and B of characters in Java, suppose A contains the characters NOW. After the

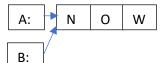


assignment B=A we have: (In languages such as Java, C#, or Python, the assignment of a whole data structure just involves updating a pointer.)

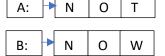


After A[2]='T' we have: \_\_\_\_ may be what the programmer intended.

, so that the value of B has been changed too. This



With a safe array, or a string, the assignment B=A would still give  $\Box$ . The assignment A[2]='T' might need to be rewritten as A=A.Set(2,'T'), but would give a new safe array or



string, so that we would end with . This behaviour is called value semantics. With safe arrays or strings the assignment operation for the whole structure still involves updating a single pointer. At the very least the programmer needs to be clear which behaviour is intended.

#### Value Semantics

Strings as values were mentioned above. In fact, the notion of value semantics is key to our approach in this booklet. Whatever type S you are working with, S should have value semantics if possible. This means that if I place a value x of type S into a variable v, any subsequent changes to x will not be visible from v. Thus, every assignment of a value of any shareable type S assigns a snapshot of that value, as at the time of the assignment. This is just what happens when we assign an integer or a string.

But importantly, as we are told when we begin programming, this is not the case with structured data. When we use such everyday structures as lists, arrays, stacks, trees, or enumerators/iterators, we have to be careful to copy the data elementwise ("cloning") when a snapshot is what we want. Sharing these data structures is a nuisance in a multi-threaded program (such as one with a graphic user interface) as they need to be locked by any method that modifies them. We even have to be careful when a parameter is passed into a method "by value". If the parameter is an array in C# or

Java, there is nothing to stop the method from modifying it, so it is "the programmer's responsibility" to ensure that nothing unexpected happens as a result. These habits make the task of programming unnecessarily complex.

In this little book I want to present a useful set of data structures that all have value semantics but still cover the same needs as the standard collection types listed above. To emphasise that these are all shareable in the above sense the type names will begin with S. We will also use generics a lot for strong typing: so we will have classes such as SList<T>, SDict<K,V> etc. As we will see they all have the property that their fields are all *readonly* or *final*.

But, as with strings, that does not make them less useful. If you manipulate a string in C# or Java, you get a new string. In just the same way, the method to change the nth entry in an SList will return a new SList. This ensures that the previous value remains accessible as long as anyone has a copy of it. And as we shall see, because any substructure will also be shareable, much of the substructure will be the same as in the previous value, so that this strategy ends up reducing memory allocation operations.

There is a place for unsafe data structures such as arrays. They can be used as local variables and where needed for using standard library methods. For example, array sorting algorithms can be safely done using unsafe arrays provided the arrays are copied beforehand. There is also nothing wrong with using an unsafe data structure as a parameter for a constructor of a shareable type.

In this booklet we will study the advantages and disadvantages of using dense arrays compared with more complex shareable indexed structures. Unless otherwise signalled, everything in this booklet applies equally to Java and C#, and nearly always also to other imperative programming languages. For the code examples though we need to choose a language, and C# is used in these notes. In most cases the only change needed to convert the code from C# to Java is to replace the keyword readonly by final. Where the Java version contains interesting differences this is explained in the text. The associated software repository on github.com (github.com/MalcolmCrowe/ShareableDataStructures) has versions of these classes for C# and Java.

There is a package from Microsoft called System.Collections.Immutable for C#. Unfortunately, it uses unsafe Enumerators, with dangers outlined below. The collection described here also defines operators such as += and -= for adding and removing collection items, which makes for elegant and intuitive programming in C#.

## Object Orientation and Identity

For decades, data structures have been associated with object orientation, with methods seen as sending messages to the object. In some systems every object has an object identity, and may even be the reference for accessing the object. "Value objects" such as integers and strings are generally seen as too low-level to have object identity, Some languages (e.g. C#) allow value objects to be dereferenced and placed in collections, while others provide parallel object classes (e.g. Boolean) that can be placed in collections. All languages allow value objects to be placed in arrays. If a data structure is shareable in our sense, it can still have an object identity, provided we understand that every time it is modified we get a new version of the object, keeping the same identifier. This mirrors our real-life experience where the same person gets a new photograph, name or address. In principle there could be an archive of all the successive versions of their information.

Accordingly, in this booklet, different versions of such a collection object can be in volatile memory at any one time, but the object identity can be used to retrieve the latest committed version from a persistent store. This is related to the notion of Temporal Patterns (Martin Fowler). We return to this idea when we implement databases later.

## Threads and Locking

With a shared unsafe data structure, locking is needed to manage concurrent access. For example, in the case of the arrays A and B above, if another thread changes B, there is in principle a race between the two threads to see which update succeeds. This is not a problem for an array of characters as the simple assignment shown is thread-safe. But, without locking, a more complex change might lead to inconsistency. In elementary courses it is a standard exercise to have two threads with a shared file such as standard output (or input!) to demonstrate that locking is needed to disentangle the operations.

If there are many structures to lock, deadlock can be a problem. Using safe structures will not avoid locking altogether, but can greatly reduce it, as we will show in this booklet. We will still need locking somewhere, as our application will surely have some variable data. Any method that modifies the fields of an object containing such data should still lock the object, as otherwise some important changes will get lost sooner or later. Obviously this does not apply if all of the fields are readonly as they can only be modified in the constructor, and nobody else has access to the object at this time. (C# and Java both prevent some uses of **this** inside a constructor.)

The repository contains a number of Test programs that compare the performance of these data structures with standard, non-shareable data structures. The occurrence of garbage collection affects the timings for both sorts of data structure, and as may be expected, the shareable structures make the memory allocator work harder. Otherwise, the timings are broadly similar. In considering the results of these tests, bear in mind that the reasons for using shareable data structures are (a) thread-safety (b) quick snapshots (c) use of bookmarks instead of iterators. This last point will be discussed later.

# Application to Database technology

By the time we get to the end of this booklet we will have enough shareable data structures to implement something major like a DBMS. Rather obviously, a DBMS server will have a List of Database objects that are in use, and we can ensure that these are shareable (SDatabase, say). The list of databases itself can be mutable: it contains the current committed state of each database, and is modifiable only by the transaction manager (the main thread of the DBMS). Someone (a client service thread) wanting to work with one of these simply copies the SDatabase they want (in all of these languages, this copy is just of the 64-bit pointer to the structure). This will be a snapshot of the current database state and they can start to modify this as they please (it is private to them). If they want their changes to be made permanent, they need to keep a note of the changes they want.

If the client service thread aborts or rolls back, their copy of the database can be simply forgotten, since no-one else knows about it. If they request for their copy to be committed, the DBMS will examine their list of changes against the current state of the database in its list – this will no longer match the snapshot the client was given. If the proposed changes do not conflict with changes that other threads have made, they can be applied to the master copy, and the SDatabase value in the master list of databases will be modified to be this new value. As the assignment of a single SDatabase pointer is an atomic operation nothing needs to be locked, and the commit process need not interfere with the operation of any other threads.

However, if we also want durability, the changes need to be written to disk (say in a transaction log) as part of the commit process. Since the disk file will be busy, providing copies of records being worked on by other threads, we need to lock it during commit and while seeking a record. This is the only locking that we need to perform for this case where commit is just for a single database. This solves the entire database-deadlock problem at a stroke.

# First Steps: a safe Linked List

We begin with an introduction to the terms used in the above discussion, by means of the simplest structures encountered by students: List and Linked List. As normally defined, both are unsafe, even though documentation tells us that their methods are thread-safe.

Instances or values are stored in one or more memory locations, e.g.



Where a value occupies more than one location (as here) it is helpful to show the origin or starting position. A variable is something that refers to a value: a variable also needs a memory location to hold it: what it holds is either null or a reference to the current value.

```
String s = "Fred";
```

If I now have another variable, String t and assign t= s, both of these will refer to the same place in memory. There are no methods that allow the value Fred to change. But either t or s can be made to refer to a different string, or be null (not point to anything). We will illustrate a reference with an arrow, and a null pointer with a diagonal.

Java and C# go to quite a lot of trouble to pretend that the same sort of picture is accurate for simple values such as int. But it is not really the case. If I have int x=17; and set int y=x; there are two different memory locations, both with the value 17. But as with strings there are no methods that can change the value 17. If I now say x = x+1, it will place the value 18 into x. This has exactly the same effect as if x now pointed to a location containing 18. Even though Java also has a reference class Integer that really does point to an integer value, the behaviour will appear to be the same.

**Example 1:** Java always (and C# by default) passes method arguments by value. We can add a new entry to a List **a** either by passing it as a value parameter **x** to a function that calls **x**.Add(**n**), or by simply calling **a**.Add(**n**). In both cases **a** will have been modified. If someone else has a copy of the list **a**, it will be modified as well. If we want to avoid this behaviour, we need to make an elementwise copy of the List (this used to be called a clone).

We will return to the List structure later. We turn first to consider the linked list.

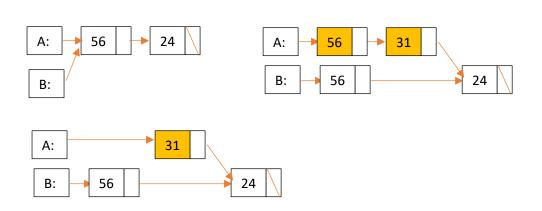
## **Example 2:** Consider a simple Linked List data structure:

```
class ListOfInt {int element; ListOfInt next;}
```

If I have an instance **a** of this class (so **a** is not null), it is the head of a list of integers, and contains at least one. But if I let anyone have a copy of this list, there is nothing to stop them changing the value of the first element, or any of the links.

## **Example 3:** Consider a *shareable* Link data structure:

```
public class SListOfInt {
    public readonly int element;
    public readonly SListOfInt next;
    public SListOfInt(int e, SListOfInt n) { element = e; next = n; }
    // more methods may get added here
}
```



Readonly is an access modifier that ensures that a value can be assigned in the constructor, but then cannot be changed. It is well implemented in C# and Java.

Now I can share an instance of this class in the sure knowledge that no-one can change it. Moreover, no method of this class can change the list represented by an instance. However, if I have a variable of this class, such as SListOfInt  $\bf a$ , I can always change  $\bf a$  to refer to a different instance. I can even make a refer to a longer list by a statement such as  $\bf a$  = new SListOfInt(22, $\bf a$ ); But importantly, anyone with a copy of my old list will see no changes.

Now in either of these linked list structures we need to allocate a piece of memory for each element of the list. This does require some extra resources provided by our programming library and/or operating system: a memory allocator and a garbage collector. If we have a very large linked list we might be tempted to "save resources" by using an array of int instead. But notice that when we added a new entry at the head of a long list, we did not need to copy the whole thing. We would probably have had to do so if it was an array (especially if someone else wanted to keep the original version).

A major advantage with the shareable list is that we never need to check for a cycle. The constructor always creates a new head and no next-pointer can be altered to point to it.

## **Example 4:** A function to test if a given int is in the list:

```
public bool Contains(int x)
{
    return x == element || (next?.Contains(x) ?? false);
}
```

This definition is recursive, and in some systems its execution will cause the stack to grow temporarily in the worst case by N stack frames, where N is the length of the list. But it is a very special sort of recursion, called tail recursion, and many programming language implementations will automatically replace it by an equaivalent loop. If ours does not, we could program the loop ourselves:

# (Example 4a)

```
public bool Contains(int x)
{
    for (var a = this; a != null; a = a.next)
        if (x == element)
            return true;
    return false;
}
```

We will leave this alterative mechanism to further examples of tail recursion as an exercise for the reader.

**Example 5:** Let us add a method to the SListOfInt class, to remove the **n**th element (n>=0).

```
public SListOfInt RemoveAt(int n) {
    if (n==0)
        return next;
    return new SListOfInt(element,next.RemoveAt(n-1));
}
```

Now, anyone who has a copy of the list will see no change, but I have a new list. Importantly, we did not make a copy of the whole thing first, and if the list is long we can see that most of the links are common to both lists. Again we could replace the tail recursion with a loop if we wanted.

**Example 6:** Another method could insert a new element at position n in the list (n>=0):

```
public SListOfInt InsertAt(int x, int n) {
    if (n==0)
        return new SListOfInt(x,this);
    return new SListOfInt(element,next.InsertAt(x,n-1));
}
```

In both of these examples, there is a point ( $\mathbf{n}$ ) in the list where we have made changes: we had to allocate  $\mathbf{n}$  or  $\mathbf{n}+1$  new pieces of memory, for the path from the head of the list to where we resume the old list. The statements that we use when  $\mathbf{n}==0$  (at this stage the recursion has reached the head of the list) simply reuse the pieces of memory that made up the previous value.

Programmers familiar with the usual implementations of List<T> and other collection types need to remember that is x is an SList<T>, then

```
x.RemoveAt(1);
or
y.InsertAt(64,2);
```

will do nothing (except possibly throw an exception if x or y is too short). It is important to remember to write

```
x = x.RemoveAt(1);
and
y = y.InsertAt(64,1);
```

This error is so common that in C# we will always use operators += and -= instead of Add and Remove (so that we write e.g. x+=(64,2)). We enforce this by making the above operations protected, and defining operator+ and operator -. The mechanism is standard and the details are in the code files. For Java we have to stick with the public InsertAt and RemoveAt methods, and remember to be careful.

Reminder: if x or y is a mutable field in another data structure a, you must lock the data structure before doing this (a is the problem, not x!), e.g.

It is important that you never have to do this if all of your data structures are shareable!

Note that this method does not allow us to create the first entry in a list. For that we need a special constructor. Since an empty list would be null, for a method that works even for an empty list you will want either (a) a static method (and as we will see that will cause problems in Java for generic classes), (b) use a two stage mechanism where SListOfInt is an object containing a readonly pointer to the start of the list, or (c) define a special Empty instance of the collection (which for a generic collection in Java would need to contain a sample element of the right type). We will come back to these options later.

**Example 7:** We need at least one method that accesses the list. In C# we can implement a subscript method:

```
public int this[int n]
{
    get { return (n == 0) ? element : next[n - 1]; }
}
```

And something similar in Java (would need to be called something like getAt(int i) . Note some C# 2017+ shortcut tricks here:

**Example 8:** Here is another tail-recursive property to get the length of the list<sup>1</sup>:

```
public int Length
{
    get { return 1 + (next?.Length ?? 0); }
}
```

The Length property uses a shortcut mechanism for C#. The ?. means if the left-hand side is not null, evaluate the right hand side, otherwise return null. The ?? means if the left hand side is null, use the right hand side. So the method is equivalent to

```
public int Length
{
    get { return (next!=null)? next.Length +1 : 1; }
}
```

In Java this is a method getLength() as Java does not have properties.

Some people like facilities such as Length and subscripts: they can make a complicated data structure look encouragingly like an array, which may be more familiar. But it might lead a tired programmer to write a very inefficient version of the Contains function:

```
public bool Contains(int x)
{
   for (var i = 0; i < Length; i++)
        if (this[i]== x)
            return true;
   return false;
}</pre>
```

In fact, we are so used to loops of this sort that we should pause and make some criticisms of the foreach (Iterator) concept found in many programming languages. Typically the iterator programming paradigm implements foreach loops by having a method on the data structure (called GetEnumerator() or begin()) return an iterator; and once we have an iterator there should be methods such as hasNext() or end() to say if we have reched the end of the list, ++ or next() or MoveNext() to advance the iterator, and \* or .Current to give the object at the current position. The details vary from

<sup>&</sup>lt;sup>1</sup> In the library, we make Length a common property of all Shareable data structures as most of our shareable structures have a readonly length.

one language to another (the above examples are variously from C++, C# and Java), but in no published case is there any way of implementing a shareable iterator of any sort, as all of them assume the same iterator object is kept during the iteration.

On the other hand, the pattern we have for traversing our list with a loop (Example 4a) looks great. Instead of an Iterator, we will use Bookmarks. We will have a function First() giving a Bookmark for the first element (if any) of a structure, and if we have a Bookmark we can have a function Next() that gives the a Bookmark for the next item in the structure (if any). Either of these can be null indicating that the list is empty or has no more entries.

We could formalise this by the following interfaces:

```
public interface Shareable
{
    Bookmark First();
}
public interface Bookmark
{
    Bookmark Next();
    Shareable Value();
    int Position();
}
```

Then the pattern for traversing a list (whenever we need to) becomes

```
for (var b = list?.First(); b != null; b = b.Next())
DoSomethingWith(b.Value());
```

We will do something very like this using generic definitions starting with the next section.

Summary: The SListOfInt class.

```
public class SListOfInt
    public readonly int element;
    public readonly SListOfInt next;
    public SListOfInt(int e, SListOfInt n) { element = e; next = n; }
    public bool Contains(int x)
        return x == element || (next?.Contains(x) ?? false);
    }
    public int this[int n]
        get { return (n == 0) ? element : next[n - 1]; }
    }
    public int Length
        get { return 1 + (next?.Length ?? 0); }
    public SListOfInt InsertAt(int x, int n)
        if (n == 0)
            return new SListOfInt(x, this);
        return new SListOfInt(element, next.InsertAt(x, n - 1));
    }
    public SListOfInt RemoveAt(int n)
        if (n == 0)
            return next;
        return new SListOfInt(element, next.RemoveAt(n - 1));
    }
```

#### Classes or Structs?

Despite everything found in books if a struct has mutable fields then it can never truly have value semantics. In this booklet the distinction between classes and structs is not important. But for best results use classes for shareable types: assignment of a value (creation of a snapshot) is a single machine instruction. Unfortunately both C# and Java have some peculiarities that make it impossible² to implement a class Shareable<K> that works the same way for a class K and a struct K.

A small advantage of using structs for your own shareable types in C# is that you can declare them as readonly struct, and this restriction will also apply to any structure inheriting from your shareable type. But there is currently no way to impose this as a constraint on a generic type parameter, so for the reason given above, you may prefer to stick to shareable class types as suggested here.

## Using the code in this booklet

I don't like having to include a huge library in my executable software when I only want one or two classes. All of the class definitions in the remaining sections of this booklet will be made available on shareabledata.org in separate class files. The C# files will conform to Windows line termination, and the Java files will conform to Linux line termination (this policy will be kept under review).

All of the code uses the latest versions of C# and Java at the time of writing.

Importantly, the classes are made freely available for you to use. I would love to receive suggestions for improving them: email me at <a href="Malcolm.crowe@uws.ac.uk">Malcolm.crowe@uws.ac.uk</a>.

# 2 Stack, List and Array

In this chapter we give generally-useful shareable SList<T> class that we can also use as a stack. Instances of the SList<T> class will be shareable lists whose elements have type T: T should be a shareable type<sup>3</sup>.

## Collection<T>4

As we are going to use generics, let us provide some abstract base classes:

```
public abstract class Collection<T>
{
    public readonly int? Length;
    protected Shareable(int c = null) { Length = c; }
    abstract Bookmark<T> First();
    public T[] ToArray()
    {
        if (Length == null)
            return null;
        var r = new T[Length.Value];
        for (var b = First(); b != null; b = b.Next())
            r[b.Position] = b.Value;
        return r;
    }
}
public abstract class Bookmark<T>
```

<sup>&</sup>lt;sup>2</sup> Java disallows it for a struct, while C# has different mechanisms for default values and nullability of classes and structs (even in C# 2019). In the distribution we skirt around these problems for the price of some warnings from the compiler.

<sup>&</sup>lt;sup>3</sup> Unfortunately, there isn't currently a way of enforcing this.

<sup>&</sup>lt;sup>4</sup> This class was called Shareable<T> in early versions of this booklet. But a class is called shareable if all its fields are public readonly. This is not the same as being an instance of the abstract class.

```
{
    public readonly int Position; // >=0
    protected Bookmark(int p) { Position = p; }
    public abstract Bookmark<T> Next();
    public abstract T Value { get; }
}
```

int? is a nullable integer class in C#. In Java we will not be able to use it (and Length will just be an int). It is here because nearly all of our data structures will have a known length: if we are sure, we will write Length. Value to get the int value of Length. But later on we will have row-sets obtained by applying a search-condition to a potentially very large data structure, and we don't want to traverse the whole thing first. C# is clever about nullable integers, so Length+1 is allowed – if Length is null, so will Length+1. The parameter for the Shareable constructor gives the default unknown null value for Length. In Java we can't currently use default parameters and we will use the impossible Length of -1 when we need to indicate an unknown length.

We have made these into abstract classes so that we automatically have the ToArray() method. And the Length field ensures among other things that even an empty Shareable is not actually null (this helps with Java).

On the other hand we don't want millions of empty nodes so we will often make a special Empty Collection<T> node and protect the constructor that makes an empty node so that everything shares the one Empty node. Empty is more useful than it would appear: it will be a single memory pointer, se we won't be calling a constructor to make it each time, and testing to see if this is Empty is really fast.

We can now see that ToArray will have the same code no matter what T is, so it might as well go in the base class.

The only new bits are the properties Length and Position. Note that these are not recursive (and so they are cheaper). The idea with Position is that First() should give a bookmark with position 0, and the Next() should have a position 1 higher. With more complex infrastructure we could make this setting automatic, but it would complicate things unduly. As with checking that all fields are declared public-readonly we will simply ask implementors to get things right.

```
SList<T>
namespace Shareable
{
    public class SList<T> : Collection<T>
        /// <summary>
        /// An empty list is Empty.
        /// SLists are never null, so don't test for null. Use Length>0.
        /// </summary>
        /// <typeparam name="T"></typeparam>
        public readonly T element;
        public readonly SList<T> next;
        public static readonly SList<T> Empty = new SList<T>();
        protected SList() { element = default(T); next = null; }
        internal SList(T e, SList<T> n) : base(n.Length+1)
            element = e;
            next = n;
        public static SList<T> New(params T[] els)
            var r = Empty;
            for (var i = els.Length - 1; i >= 0; i--)
```

```
r = new SList<T>(els[i], r);
            return r;
        }
        public SList<T> Push(T x)
            return new SList<T>(x, this);
        }
        public SList<T> InsertAt(T x, int n)
            if (Length==0 || n == 0)
                return new SList<T>(x, this);
            return new SList<T>(element, next.InsertAt(x, n - 1));
        }
        public SList<T> Pop()
        {
            return next;
        }
        public SList<T> RemoveAt(int n)
            if (Length==0)
                return Empty;
            if (n == 0)
                return next;
            return new SList<T>(element, next.RemoveAt(n - 1));
        }
        public SList<T> UpdateAt(T x,int n)
            if (Length==0)
                return Empty;
            if (n == 0)
                return new SList<T>(x, next);
            return new SList<T>(element, next.UpdateAt(x, n - 1));
        }
        public Bookmark<T> First()
            return (Length==0)?null:new SListBookmark<T>(this);
        }
    public class SListBookmark<T> : Bookmark<T>
        internal readonly SList<T>_s;
        internal SListBookmark(SList<T> s, int p = 0) :base(p) { _s = s; }
        public override Bookmark<T> Next()
        {
            return (_s.Length <= 1) ? null</pre>
                     : new SListBookmark<T>(_s.next, Position+ 1);
        public override T Value => _s.element;
    }
}
```

This is not much to add about the three methods Push, Pop, InsertAt, RemoveAt and UpdateAt, which are very similar to the two methods of the last chapter<sup>5</sup>. However, we can see a disadvantage with this class since insertion Is linear in the length of the list. In our database implementation we will often use SDict instead (see B-Trees below) as this has logarithmic behaviour.

<sup>&</sup>lt;sup>5</sup> In C# we will write s+=6 instead of s=s.Push(6) and s-- instead of s=s.Pop(). Some languages make Pop(0 return the top value while popping the stack as a side effect: Python returns a pair consisting of top and Pop().

The New method is not available in Java. It provides a way of creating an SList<T> from a (possibly empty) array T[] .The params keyword means that a client program can write SList<T>.New(a,b,c) if a, b, and c are of type T, as well as SList<T>.New(t) where t is an array of type T. Without the keyword params you would need to write SList<T>.New(new T[]{a,b,c}) . The New method is declared static because we don't need to have an SList<T> already, and it can't be a ordinary constructor unless we are sure the parameter T[] is non-empty.

In Java it turns out to be easier to use null for an empty SList, and use the constructor to create a list with one element.

Finally, the ToArray() method is similar to the corresponding method in the usual List<T> class in returning an ordinary array of T. When compiling the Java version we need to suppress warnings as Java does not like generic array creation and we need an unchecked type conversion instead.

#### SCList<T>

This is a version of SList for the case where elements of the list can be compared. There is not much to say about it. We override methods to ensure that things have the right types.

```
/// <summary>
/// An empty list is Empty.
/// SCLists are never null, so don't test for null. Use Length>0.
                                                                      ///
/// </summary>
/// <typeparam name="K"></typeparam>
public class SCList<K> : SList<K>, IComparable where K : IComparable
    public new static readonly SCList<K> Empty = new SCList<K>();
    SCList() { }
    public SCList(K el, SCList<K> nx) : base(el, nx) { }
    public new static SCList<K> New(params K[] els)
    {
        var r = Empty;
        for (var i = els.Length - 1; i >= 0; i--)
            r = new SCList<K>(els[i], r);
        return r;
    }
    public int CompareTo(object obj)
        if (obj == null)
           return 1;
        var them = obj as SList<K> ?? throw new Exception("Type mismatch");
        SList<K> me = this;
        for (; me.Length > 0 && them.Length > 0; me = me.next, them = them.next)
            var c = me.element.CompareTo(them.element);
            if (c != 0)
                return c;
        return (me.Length > 0) ? 1 : (them.Length > 0) ? -1 : 0;
    }
    public override Bookmark<K> First()
        return (Length == 0) ? null : new SCListBookmark<K>(this);
    }
}
public class SCListBookmark<K> : SListBookmark<K> where K : IComparable
    internal new readonly SCList<K> s;
    internal SCListBookmark(SCList<K> s, int p = 0) : base(s, p) { _s = s; }
    public override Bookmark<K> Next()
```

```
{
    return (_s.Length <= 1) ? null :
        new SCListBookmark<K>((SCList<K>)_s.next, Position + 1);
}
```

## SArray<T>

For completeness we include a shareable array class SArray<T>, but it is not very useful in practice. It is obvious that a lot of copying is required to implement shareable behaviour. It is better to use SDict instead of arrays (see B-Trees below).

```
public class SArray<T> : Collection<T>
    public readonly T[] elements;
    public SArray(params T[] els)
        elements = new T[els.Length];
        for (var i = 0; i <els.Length; i++)</pre>
            elements[i] = els[i];
    }
    public SArray<T> InsertAt(int n,params T[] els)
        var x = new T[elements.Length + els.Length];
        for (int i = 0; i < n; i++)</pre>
            x[i] = elements[i];
        for (int i = 0; i < els.Length; i++)</pre>
            x[i + n] = els[i];
        for (int i = n; i < elements.Length; i++)</pre>
            x[i + els.Length] = elements[i];
        return new SArray<T>(x);
    }
    public SArray<T> RemoveAt(int n)
        var x = new T[elements.Length - 1];
        for (int i = 0; i < n; i++)
            x[i] = elements[i];
        for (int i = n + 1; i < elements.Length; i++)</pre>
            x[i - 1] = elements[i];
        return new SArray<T>(x);
    }
    public SArray<T> UpdateAt(T x, int n)
        var a = new T[elements.Length];
        for (int i = 0; i < n; i++)
           a[i] = elements[i];
        a[n] = x;
        for (int i = n+1; i < elements.Length; i++)</pre>
            a[i] = elements[i];
        return new SArray<T>(a);
    Public override Bookmark<T> First()
        return (Length==0)? null : new SArrayBookmark<T>(this,0);
public class SArrayBookmark<T> : Bookmark<T>
    internal readonly SArray<T> _a;
    internal SArrayBookmark(SArray<T> a,int p) : base(p)
    {
        _a = a; Position = p;
```

There is a lot of copying going on here with loops, and wasteful reallocation of memory. If the length of the array is likely to be large and/or there are a lot of changes, it will be generally faster and less memory-intensive to use SList<T> instead.

Unusually, this bookmark retains a memory of the snapshot it is working on, and the current position in the array. This enables the bookmark to continue to traverse the snapshot it has been given even if the original source of the array has modified it. Moreover, anyone we give this Bookmark to will be able to continue using the snapshot it was created for.

An objection might be that any shareable copy of the SArray<T> is immutable, and we don't need a separate Bookmark class to traverse it. This is perfectly true: but we provide a Bookmark class (conforming to our standard pattern) for the same reason that ordinary arrays in Java have iterators. It means that we can get used to traversing any Shareable structure the same way. For example, in the ToArray() implementation.

# 3 Binary Search Trees

Recall that a search tree can be used to sort an array of items. The code for an unbalanced search tree is very simple, but it will perform very badly for sorting if the data is already in order, so that we use B-Trees instead for our database implementation. The code for the corresponding bookmark is more tricky and will be explained below.

```
/// <summary>
/// Implementation of an UNBALANCED binary search tree
/// </summary>
/// <typeparam name="T"></typeparam>
public class SSearchTree<T> :Collection<T>
    where T : System.IComparable
{
    public readonly T node;
    public readonly SSearchTree<T> left, right;
    public static readonly SSearchTree<T> Empty = new SSearchTree<T>();
    SSearchTree() { node = default(T); left = null; right = null; }
    internal SSearchTree(T n,SSearchTree<T> lf,SSearchTree<T> rg)
        : base(1+lf.Length+rg.Length)
        node = n;
        left = 1f;
        right = rg;
    public static SSearchTree<T> New(params T[] els)
        var r = Empty;
        foreach (var t in els)
           r = r + t;
        return r;
    }
    protected SSearchTree<T> Add(T n)
    {
```

```
if (this == Empty)
                return new SSearchTree<T>(n, Empty, Empty);
            var c = n.CompareTo(node);
            if (c <= 0)
                return new SSearchTree<T>(node, left.Add(n),right);
            else
                return new SSearchTree<T>(node, left, right.Add(n));
        }
        public static SSearchTree<T> operator+( SSearchTree<T> t,T n)
            return t.Add(n);
        }
        public bool Contains(T n)
            if (this == Empty)
                return false;
            var c = n.CompareTo(node);
            return (c == 0) ? true : (c < 0) ? left.Contains(n) : right.Contains(n);</pre>
        }
        public override Bookmark<T> First()
            return (this==Empty)?null:new SSearchTreeBookmark<T>(this, true);
        }
    }
   public class SSearchTreeBookmark<T> : Bookmark<T> where T : System.IComparable
        internal readonly SSearchTree<T> s;
        internal readonly SList<SSearchTree<T>> stk;
        internal SSearchTreeBookmark(SSearchTree<T> s, bool doLeft,
            SList<SSearchTree<T>> stk= null,
            int p = 0) : base(p)
        {
            if (stk==null) stk = SList<SSearchTree<T>>.Empty;
            for (;doLeft && s.left != SSearchTree<T>.Empty; s = s.left)
                stk = stk.InsertAt(s, 0);
            _s = s; _stk = stk;
        }
        public override T Value => _s.node;
        public override Bookmark<T> Next()
        {
            return (_s.right != SSearchTree<T>.Empty)?
                new SSearchTreeBookmark<T> (_s.right, true, _stk, Position + 1)
                : (_stk == SList<SSearchTree<T>>.Empty) ? null
                : new SSearchTreeBookmark<T> (_stk.First().Value, false,
                     _stk.RemoveAt(0), Position + 1);
        }
    }
The Bookmark class traverses the tree. In books you we see a traversal written as
       void Traverse(Tree n, ref int i)
       {
              If (n==null) return;
              Traverse(n.left,ref i);
             DealWith(n.node,i++);
              Traverse(n.right,ref i);
```

This means that the First() node visited will involve moving down the left branches as far as possible, and the recursion builds a stack of nodes to be visited later. We see this in the above code, where the First() method builds a stack of nodes to visit later. This stack \_stk is included in the bookmark.

The Next() method takes up the story just after the DealWith line in the traversal pseudocode, and involves going right from current node if possible, otherwise taking the top off the stack and using it without first going down its left branches. This is why there is a Boolean doLeft in the bookmark.

The New method isn't allowed in Java, and we can't define an operator+ either, so in the Java version of this code the Add method needs to be public.

For now we observe that in the worst case Add-ing a node to a tree of size N might create N new nodes. This means that the worst case for building a tree of size N will involve N(N-1)+1 steps. This is not good.

On the other hand the code is very safe, as cycles cannot occur.

In the next chapter we will present a B-tree implementation for SDict<K,V>, and we will find that SDict<T,bool> gives a more efficient sorting algorithm than SSearchTree<T>.

## 4 B-Trees

The next example is a shareable implementation of B-trees called SDict. B-Trees are not binary trees: we avoid calling them BTree in the code to avoid confusion. These manage sorted lists of key-value pairs using an n-ary tree (so we have two type parameters SDict<K,V>). The design is quite standard and described in many textbooks, but is rather complex to code: it usually includes a self-balancing part of the algorithm called spilling. We avoid spilling in our implementation: this is because without spilling the only new nodes (for add or remove) are on the path from the root to the key's location; whereas with spilling all of the nodes on the same level as the key's location might get replaced. Without spilling we continue to avoid the worst-case scenario mentioned above, but not all paths from root to leaves have the same length.

In a binary search tree described above each node has a value and two child nodes: we can think of such a node as (k,T1,T2) where T1 is a subtree of values less than k and T2 is a subtree of values greater than k (T1 and/or T2 may be empty).

With the B-tree, nodes can have a number of children up to a fixed maximum S, which is usually  $2^n$  for leaf nodes and  $2^n+1$  for inner nodes, for some n. At lower values of S the memory allocator works harder, and at higher values there is less sharing of nodes between old and new trees, in experiments these aspects tend to balance out so that values of S in the range 8 to 33 are satisfactory. The number of comparisons needed to locate an entry is  $O(\log N)$  in any case because binary search is used within nodes  $O(\log n)$  and the depth of the B-tree will be  $O(\log (N/n))$ .

Leaf nodes contain key-value pairs (k,v) say. Inner nodes contain an ordered group of pairs (k,T) and a subtree G; where each T is a non-empty subtree (either an inner node or a leaf node) whose greatest key value is k, and G is a subtree containing larger than the last k. All nodes apart from the root have at least S/2 children; the root can have fewer (if the tree is empty it will have none).

If the tree has fewer than S keys, there will be just one node in the tree (the root). At this point the addition of another key will cause the node to split, and the two resulting nodes will become children of a new root node. When nodes are removed, the reverse process happens, and nodes with fewer than S/2 children will be coalesced, reducing the depth of the tree.

For consistency with the previous chapter, let us consider an SDict<K,V> to be a Shareable list of pairs (K,V), so we define an auxiliary class called SSlot<K,V> (whose fields key and val are public readonly of course). This makes SDict<K,V> a subclass of Shareable<SSlot<K,V>>, and if we apply our bookmark machinery we will have a bookmark of type SBookmark<SSlot<K,V>>, so for convenience we create a subclass SDictBookmark<K,V>

It is clear from this discussion that there should be two classes of nodes, for inner and leaf nodes, and it makes sense to have a common base class, which we call SBucket<K,V>. Again all of these classes will have public readonly fields.

In C# it also helps to have an common interface, which we call IBucket: it helps the SDictBookmark to access the current position in the bucket as well as the number of slots in a bucket.

#### SDict<K.V>

From the above discussion we have quite a group of new classes just for implementation of SDict<K,V>, so we will not give full listings of everything in this document. SDict<K,V> will have an API rather like the classic Hashtable<K,V> or Dictionary<K,V> classes in standard libraries, and its implementation will have methods that call the methods of the Shareable base class (and SSlot).

SDict<K,V> itself is clear enough:

```
public class SDict<K,V> : Shareable<SSlot<K,V>> where K: IComparable
{
   /// <summary>
   /// Size is a system configuration parameter:
   /// the maximum number of entries in a Bucket.
   /// It should be an even number, preferably a power of two.
   /// It is const, since experimentally there is little impact on performance
   /// using values in the range 8 to 32.
   /// </summary>
   public const int Size = 8;
   public readonly SBucket<K, V> root;
   protected SDict(SBucket<K,V> r):base(r?.total??0) { root = r; }
   internal SDict(K k, V v) : this(new SLeaf<K, V>(new SSlot<K, V>(k, v))) { }
   /// <summary>
   /// Avoid unnecessary constructor calls by using this constant empty tree
   /// </summary>
   public readonly static SDict<K, V> Empty = new SDict<K, V>(null);
   /// <summary>
   /// Add a new entry or update an existing one in the tree
   /// </summary>
   /// <param name="k">the key</param>
   /// <param name="v">the value to add</param>
   /// <returns>the modified tree</returns>
   protected virtual SDict<K,V> Add(K k,V v)
        return (root == null || root.total == 0)? new SDict<K, V>(k, v) :
            (root.Contains(k))? new SDict<K,V>(root.Update(k,v)) :
            (root.count == Size)? new SDict<K, V>(root.Split()).Add(k, v) :
           new SDict<K, V>(root.Add(k, v));
   }
   public static SDict<K,V> operator+( SDict<K,V> d,ValueTuple<K,V> x)
   {
       return d.Add(x.Item1, x.Item2);
   /// <summary>
   /// Remove an entry from the tree (Note: we won't have duplicate keys)
   /// </summary>
   /// <param name="k"></param>
   /// <returns></returns>
   protected virtual SDict<K,V> Remove(K k)
        return (root==null || root.Lookup(k)==null) ? this :
            (root.total == 1) ? Empty :
           new SDict<K, V>(root.Remove(k));
   public static SDict<K,V> operator-( SDict<K,V> d, K k)
```

```
{
        return d.Remove(k);
   public bool Contains(K k)
       return (root == null) ? false : root.Contains(k);
   }
   public V Lookup(K k)
       return (root==null)?default(V):root.Lookup(k);
   }
   /// <summary>
   /// Start a traversal of the tree
   /// </summary>
   /// <returns>A bookmark for the first entry or null</returns>
   public override Bookmark<SSlot<K, V>> First()
        return (SBookmark<K, V>.Next(null, this) is SBookmark<K,V> b)?
            new SDictBookmark<K, V>(b):null;
   }
}
```

We see in the above Add and Remove methods some indications of the tree restructuring dicsussion above. The SDictBookmark class is also simple enough:

```
public class SDictBookmark<K, V> : Bookmark<SSlot<K, V>> where K : IComparable
{
   public readonly SBookmark<K, V> _bmk;
   public override SSlot<K, V> Value =>
        ((SLeaf<K,V>)_bmk._bucket).slots[_bmk._bpos];
   internal SDictBookmark(SBookmark<K,V> bmk) :base(bmk.position())
        { _bmk = bmk; }
   public K key => Value.key;
   public V val => Value.val;
   public override Bookmark<SSlot<K, V>> Next()
   {
        return (_bmk.Next() is SBookmark<K,V> b)? new SDictBookmark<K,V>(b):null;
   }
}
```

We can see an advantage in defining this class in that we have shortcut properties key and val that access the corresponding fields in the Value SSlot. More importantly note that the standard First() and Next() methods return instances of the SDictBookmark class instead of Bookmark<SSlot<K,V>>.

For further details, see the code files.

## 5 Multi-level indexes

The next sample is a shareable implementation of multi-level indexes, allowing duplicate and null keys, here called SMTree<K,int>, since for simplicity we assume the records being indexed are in a list somewhere (e.g. on disk), and retrieved using an integer address. The multi-level key consists of several columns (an SCList), and the implementation uses a B-Tree for each one with the final level leading to an integer value identifying the record. If a level allows duplicate key values, there will be an extra SDict<pos,true> that can be traversed to find all of the rows with matching keys (pos gives the disk location of the record). From the outside, the MTree is a SDict<SList,int>, where the key type is the multi-column key. In principle, the lookup for a given key (k0,k1,..,kn) proceeds as T1 = M.Lookup(k0), T2=T1.Lookup(k1), and so on until pos=Tn.Lookup(kn) where the desired row is then SA[pos], however, the partially-ordered stage if any complicates this picture, as do null key values (e.g. if we allow a short key where later fields of a key might be null).

#### **SMTree**

From this description we can already see that the implementation will require a number of auxiliary classes and enumerations. We will already need an SList<TreeInfo> to describe the multicolum key class. Then a single level of the implementation can be done by an internal class SITree<K1,V1>, say. But its K1 and V1 types can't be just K and int, since the intermediate objects T1, T2, .. in the above discussion can be trees of at least two kinds (partial and compound) and the last value will be an integer. So we invent a Variant class to handle this variety and an enumeration Variants for the case that arise.

To implement the recursive process described here, it is convenient to generalise the SMTree to use the Vaiant class we have just introduced. A shortened (and non-generic!) version of the SMTree class can be represented as follows (the full details are online):

```
public class SMTree<K> : Shareable<SSlot<SCList<Variant>, long>>, IComparable
where K:IComparable
    {
        public class SITree ...
        public readonly SITree _impl;
        public readonly SList<TreeInfo<K>> _info;
        SMTree(SList<TreeInfo<K>> ti, SITree impl,int c) :base(c)
        {
            _info = ti;
            _impl = impl;
        public SMTree(SList<TreeInfo<K>> ti) : base(0) { }
        public SMTree(SList<TreeInfo<K>> ti,SList<Variant> k,long v)...
        public override Bookmark<SSlot<SCList<Variant>,Variant>> First() ...
        public MTreeBookmark PositionAt(SCList<Variant> k) ...
        public SMTree Add(int v,params Variant[] k)
            var r = Add(SCList<Variant>.New(k), v,out TreeBehaviour tb);
            return (tb == TreeBehaviour.Allow) ? r :
              throw new Exception(tb.ToString());
        public SMTree Add(SCList<Variant> k,int v, out TreeBehaviour tb) ...
        protected SMTree<K> Add(SCList<Variant> k, Variant v) ...
        public static SMTree<K> operator+( SMTree<K> t,
                                            ValueTuple<SCList<Variant>,long> x)
        {
            return t.Add(x.Item1,x.Item2);
        }
    }
```

The MTreeBookmark class implements the recursion described in the text, with an outer Bookmark for the current level, and an MTreeBookmark for the inner recursion if any.

Full details can be found in the code files.

## 6 Files and Streams

It is natural to ask how files and streams can be handled with a shareable infrastructure. To start the discussion we observe that a sequential file generally may have multiple readers but at most one writer. We will wish to avoid any kind of file that allows arbitrary changes to file contents, so we will limit our discussion to the "append storage" model where the only changes to a disk file involve the addition of new data at the end, and "Direct access" to the file contents is allowed for the readers. We ould easily implement shareable access classes to disk files that embody these ideas, such as SBuffer, which is a shareable contiguous block of data read from the file, with a readonly integer giving the current position within it.

Streams can be viewed as a special kind of SLIst: an input stream is like a queue of objects to be processed (so that taking off the head object gives a new input stream), and an output stream is like a list where new objects are added at the end. Standard input, standard output, and web services are the most common uses of streams. It is quite common for these streams to be passed between threads, and making them into shareable classes is not a large step.

There are choices to be made as to the level of abstraction: does the file/stream consist of a sequence of characters, or strings, or records? This means that the shareable class for files and streams should also have have a generic type parameter for the objects in the file or stream. Generally, these types will be serialisable Strangely, the seralisation classes in .NET do not support shareability, so we provide our own Seralisable framework in this section.

Since the implementation of Serialisable will be based on low-level framework classes for input/output and network access, the implementation of i/o will need to use locking to ensure the shareable behaviour we require. We will create a subclass of FileStream called AStream with the methods we need for the implementation.

Our Buffers will be shareable for stream i/o, and all our serialsable classes will be shareable. However, in this work we will NOT use shareable classes for the underlying files and streams themselves. (We could do so, but that would not remove the need for locking the underlying operating system file object, so there seems little point.)

## **Types**

The enumeration Types gives a list of 18 types starting with Serialisable, which will be enough for us to implement a simple DBMS with Tables, Columns, Views and Transactions.

```
public enum Types
{
    Serialisable = 0,
    STimestamp = 1,
    SInteger = 2,
```

```
SNumeric = 3,
    SString = 4,
    SDate = 5,
    STimeSpan = 6,
    SBoolean = 7,
    SRow = 8,
    STable = 9,
    SColumn = 10,
    SRecord = 11,
    SUpdate = 12,
    SDelete = 13,
    SAlter = 14,
   SDrop = 15,
    SView = 16,
    SIndex = 17,
   SSearch = 18 ...
}
```

Entries 1-8 in this list can be the primitive data types for a database implementation.

#### Serialisable

Serialisable is a base class for the Serialisable types. All the other types listed in the Types enumeration have the following basic operations:

- A public constructor to create a new instance of the Serialisable class (not associated with a file).
- A static Get method to deserialise an instance of the type from a stream. There will be a
  protected constructor with parameter StreamBase to create this new instance. There will be
  two sorts of Stream: AStream which is a database file, and AsyncStreams which are for
  communication with the database client.
- A virtual Put method to serialise an instance to a stream. Other serializable classes will call the base. Put first and then serialise their extra fields to the stream.

```
public class Serialisable : IComparable
    public readonly Types type;
    public readonly static Serialisable Null
                 = new Serialisable(Types.Serialisable);
    protected Serialisable(Types t)
    {
        type = t;
    public Serialisable(Types t, StreamBase f)
        type = t;
    }
    public static Serialisable Get(StreamBase f)
    {
        return Null;
    }
    public virtual void Put(StreamBase f)
    {
        f.WriteByte((byte)type);
    }
}
```

The simple types such as SInteger, SString, etc have rather obvious subclasses based on the Serialsable class (details online). We give details below for the database objects as these are more interesting and their usefulness is more obvious.

### StreamBase, Buffer, Reader and SocketReader

StreamBase is a standard subclass of FileStream with methods such as PutInt for implementing the serialisable types. It also has a Buffer class for buffering the input and output. It is not a shareable data structure. Again, the details are very straightforward and can be found online.

StreamBase defines an internal class called Buffer. This is not shareable, but is designed in such a way that if we fetch objects from different parts of the database file, they are fetched into different Buffers. So for the database file there is one Buffer for writing, and a number of Readers each with its own Buffer: a Reader consists of a Buffer and a position within it. So input methods are actually methods of Reader rather than Buffer, so that when the Reader get to the end of a Buffer it gets another Buffer to continue.

StrongDBMS and StrongLink have subclasses of AStream that use Sockets instead of Files. To support this, the Shareable namespace has a SocketReader class that is a subclass of Reader. All of these classes are non-shareable.

#### **AStream**

AStream is mutable and not shareable, and is a subclass of StreamBase. With the help of this class we start to define what we mean by a database object (see SDbObject below). AStream has methods for obtaining SDbObjects from the database file (GetOne, GetAll, etc), a Commit method for appending new objects and data to the stream, and a constructor that obtains exclusive access (FileShare.None) to the underlying FileStream object. All calls to its Get and Put methods are protected within the Commit and Get/GetOne methods (Commit will be protected in Transaction, see next chapter).

```
public class AStream : StreamBase
 {
     public readonly string filename;
     internal Stream file;
     long position = 0;
     public long length = 0;
     internal SDict<long, long> uids = null; // used for movement or SDbObjects
     public AStream(string fn)
         filename = fn;
         file = new FileStream(fn,FileMode.OpenorCreate,FileAccess.ReadWrite,
                FileShare.None);
         length = file.Seek(0, SeekOrigin.End);
         file.Seek(0, SeekOrigin.Begin);
     public Serialisable Get(long pos)
         rbuf = new Buffer(this, position);
         Types tp = (Types)ReadByte();
         Serialisable s = null;
         switch (tp)
         {
             case Types.Serialisable: s = Serialisable.Get(this); break;
         return s;
     Serialisable Lookup(SDatabase db,long pos)
```

```
return db.Lookup(Fix(pos));
}
internal long Fix(long pos)
    if (uids.Contains(pos))
        pos = uids.Lookup(pos);
    return pos;
}
public SDbObject GetOne()
    lock (file)
        if (position == file.Length)
            return null;
        var r = _Get(position);
        position = rbuf.start + rbuf.pos;
        return (SDbObject)r;
    }
}
public Serialisable Get(long pos)
    lock (file)
    {
        return _Get(pos);
}
```

We will look at Commit in a later section below (see Conflicts).

## 7 Databases and Transactions

A relational database begins with a list of tables, and a file for persisting their state to disk. If the disk file is the transaction log, then it provides a guarantee that transactions are serialisable, so we follow this design. If our database is shareable, then we automatically get snapshot isolation. We will build on these two simple properties to implement a strong DBMS based on shareable objects wherever possible.

As a preparation for this development, the list of Seralisable types given in the last section contains useful shareable objects such as STable, SColumn etc. But before it is snesible to give the full details, we need to sketch out the database and transaction structure.

## **SDatabase**

Asd the name implies, SDatabase is shareable. It has a list of database objects accessible by name (such as STables), and the same list of STables accessible by uid. Uids are long integers consisting of file positions in the transaction log.

The class defines a lock for keeping the list of AStream files and the list of databases in sync, but it not needed in normal use.

```
public static SDatabase Open(string fname)
{
    if (dbfiles.Contains(fname))
        return databases.Lookup(fname)
        ?? throw new System.Exception("Database is loading");
    var db = new SDatabase(fname);
    dbfiles = dbfiles + (fname, new AStream(fname));
    db = db.Load();
    databases = databases + (fname, db);
    return db.Load();
}
...
}
```

A word of explanation is necessary here. We do not use lock(files) in the Open method. Both the Add methods being called are threadsafe. The Load() operation is potentially time-consuming. Instead we immediately enter a new exclusively-opened file for a new database, but if it is not yet in the databases list it is because it is still loading. Other potential clients (or threads) trying to access the same database will be told to wait because the database is loading: we also have to wait until is loaded.

There are protected constructors for adding each kind of database object to the database (or transaction!).

As is probably obvious by now, the database file contains serialised database object definitions including the details of inserted records and the specification of indexes. The volatile state of indexes etc is not written to disk, but reconstructed when the database is loaded. On the other hand, there is no need to maintain copies of the disk record data itself in memory, and the \_Get method given above can be used to fetch them.

## **STransaction**

A database transaction is about new or updated objects and data, and committing the transaction involves writing these to the transaction log. This results in an updated Database. With a shareable Database structure this means a new Database instance, but of course as discussed above the new instance will contain only a few new or changed allocated memory locations.

A Transaction object will evolve through addition of objects with transaction uids (i.e. above some large number), to be committed to the AStream. The Commit method is simple if there is no concurrency: it will go through the list of the transaction uids, and write them to the transaction log (with a uid consisting of the position in the log file). If there are concurrent transactions there will be more code in this method to check for conflicts with the Serialisables recorded since the start of the transaction.

Since every STransaction is also an SDatabase we define a virtual property called Committed that helps to distinguish them. (If we have a SDatabase variable for which Committed appears to be false, it is really an STransaction.)

```
public class STransaction : SDatabase
{
    // uids above this number are for uncommitted objects
    public static readonly long _uid = 0x4000000000000000;
    public readonly long uid;
    public readonly bool autoCommit;
    public readonly SDatabase rollback;
    protected override bool Committed => false;
    public Transaction(SDatabase d,bool auto) :base(d)
    {
        autoCommit = auto;
    }
}
```

```
rollback = (d is STransaction t)?t.rollback:d;
        uid = _uid;
        steps = SDict<int,Serialisable>.Empty;
    }
    public STransaction(STransaction tr, Serialisable s)
          :base(tr.name,tr.objects.Add(tr.uid+1,s),tr.names,tr.curpos)
    {
        autoCommit = tr.autoCommit;
        rollback = tr.rollback;
       uid = tr.uid+1;
    }
    /// <summary>
    /// We will single-quote transaction-local uids
    /// </summary>
    /// <returns>a more readable version of the uid</returns>
    internal static string Uid(long uid)
        if (uid > _uid)
           return "'" + (uid - _uid);
        return "" + uid;
    }
    public override STransaction Transact(bool auto=true)
        return this; // ignore the parameter
    }
    public override SDatabase Rollback()
        return rollback;
    public SDatabase Commit()
     }
}
```

We discuss the Commit() method below. The basic idea is clear: when we Commit a transaction we get a modified SDatabase, obtained by serialising the Serialsables to the (possibly updated) database file. We only do this once we are sure that the database has received no updates that conflict with the steps of the transaction. For details see the Conflicts section below.

# **SDbObject**

For database objects such as STable, we will want to record a unique id based on the actual position in the transaction log, so the Get and Commit methods will capture the appropriate file positions in AStream – this is why the Commit method needs to create a new instance of the Serialisable. The uid will initially belong to the Transaction. Once committed the uid will become the position in the AStream file.

There is an opportunity for making other uses of the uid field. We assume that the database file is smaller than 0x400000000000000, so uids above this can be used for transaction-local objects. Negative uids will be used for system identifiers (such as \_Log), and identifiers from the client will be given a negative uid before identification in the database. The system uids will grow downwards from -0x700000000000000. We distinguish the last case by whether the Reader is a SocketReader or an AStream.

```
public abstract class SDbObject : Serialisable
{
   public readonly long uid;
   /// <summary>
   /// For system tables and columns, with negative uids
   /// </summary>
```

```
/// <param name="t"></param>
/// <param name="u"></param>
protected SDbObject(Types t,long u) :base(t)
    uid = u;
/// <summary>
/// For a new database object we set the transaction-based uid
/// </summary>
/// <param name="t"></param>
/// <param name="tr"></param>
protected SDbObject(Types t,Transaction tr) :base(t)
   uid = tr.uid+1;
}
/// <summary>
/// A modified database obejct will keep its uid
/// </summary>
/// <param name="s"></param>
protected SDbObject(SDbObject s) : base(s.type)
   uid = s.uid;
}
/// <summary>
/// A database object got from the file will have
/// its uid given by the position it is read from
/// For AStream we subtract 1 to account for the Types byte.
/// </summary>
/// <param name="t"></param>
/// <param name="f"></param>
protected SDbObject(Types t,Reader f) : base(t)
{
   uid = (f is SocketReader)?-1:f.Position-1;
}
/// <summary>
/// During commit, database objects are appended to the
/// file, and we will have a (new) modified database object
/// with its file position as the uid.
/// We remember the correspondence between new and old in the AStream
/// temporarily (we reinitialise the uids on each Commit)
/// </summary>
/// <param name="s"></param>
/// <param name="f"></param>
protected SDbObject(SDbObject s,AStream f) :base(s.type)
    if (s.uid < STransaction. uid)</pre>
        throw new Exception("Internal error - misplaced database object");
   uid = f.Length;
   f.uids = f.uids + (s.uid, uid);
   f.WriteByte((byte)s.type);
}
/// <summary>
/// This little routine provides a check on DBMS implementation
/// </summary>
/// <param name="committed"></param>
internal void Check(bool committed)
{
    if (committed != uid < STransaction._uid)</pre>
        throw new Exception("Internal error - Committed check fails");
internal string Uid()
```

#### **SQuery**

Queries have a list of select columns, so STable will be a subclass of SQuery (this is not unreasonable: in SQL "table A" is the same as "select \* from A", and we will have table expressions later).

There are methods for adding and dropping columns and rows. Changes to cols happen comparatively rarely, so we cache column information by name and position as well as by uid.

There are also a couple of clever abstract methods that we will come to in the section on Queries and RowSets.

```
public abstract class SQuery : SDbObject
   public readonly SDict<int, SSelector> cpos;
   public readonly SDict<string, SSelector> names;
   public readonly SDict<long, SSelector> cols;
   public SQuery(Types t, long u) : base(t, u)
       cols = SDict<long, SSelector>.Empty;
       cpos = SDict<int, SSelector>.Empty;
       names = SDict<string, SSelector>.Empty;
   }
   /// <summary>
   /// Queries come to us with client-local SDbObjects
   /// instead of STransaction SDbObjects.
   /// We need to look them up
   /// </summary>
   /// <param name="db">A database or transaction</param>
   /// <returns>A version of this with correct references for db</returns>
   public abstract SQuery Lookup(SDatabase db);
   /// <summary>
   /// Construct the Rowset for the given SDatabase
   /// (may have changed since SQuery was built)
   /// </summary>
   /// <param name="db">The current state of the database or transaction</param>
   /// <returns></returns>
   public abstract RowSet RowSet(SDatabase db);
```

## STable

Apart from the columns information in the SQuery class, the STable object has a name. and a set of rows. Rows are data and will be retrieved from the disk when required (with true direct access SSDs this is really fast today), but when a row is updated the new version will have a different position in the transaction log, so the list of rows is actually an SDict of pairs (defpos, current).

In addition to the methods shown here, there will be methods for detecting conflicts with other serialisables. For example, an STable Serialisable defines a new table by name, so a conflict should be detected if another transaction created a table with the same name since the start of our transaction. We return to this point at this end of this section.

```
public class STable : SQuery
   public readonly string name;
   public readonly SDict<long, long> rows; // defpos->uid of latest update
   public STable(Transaction tr,string n) :base(Strong.STable)
        if (tr.names.Contains(n))
            throw new Exception("Table n already exists");
       name = n;
       rows = SDict<long, long>.Empty;
   }
   protected STable Add(SColumn c)
       return new STable(this,cols+(c.uid,c),cpos+(cpos.Length.Value,c),
            names+(c.name,c));
   }
   public static STable operator+(STable t,SColumn c)
       return t.Add(c);
   }
   protected STable Add(SRecord rec)
       return new STable(this,rows+(rec.Defpos, rec.uid));
   }
   public static STable operator+(STable t, SRecord r)
   {
       return t.Add(r);
   }
   public STable Remove(long n)
        if (cols.Contains(n))
            var k = 0;
            var cp = cpos;
            var sc = cols.Lookup(n);
            for(var b=cpos.First();b!=null;b=b.Next(),k++)
                if (b.Value.uid==n)
                    cp = cp.RemoveAt(k);
                    break;
            return new STable(this, cols-n, cp, names-sc.name);
        }
       else
            return new STable(this, rows-n);
   STable(STable t,SDict<long,SColumn> c,
                SDict<int, SSelector> p,SDict<string,SColumn> n) :base(t,c,p,n)
   {
       name = t.name;
       rows = t.rows;
   STable(STable t,SDict<long, long> r) : base(t)
       name = t.name;
        rows = r;
   }
```

```
STable(StreamBase f):base(Types.STable,f)
{
    name = f.GetString();
    rows = SDict<long, SRecord>.Empty;
}
STable(STable t,AStream f):base(t,f)
{
    name = t.name;
    f.PutString(name);
    rows = t.rows;
}
public new static STable Get(AStream f)
{
    return new STable(f);
}
...

public override string ToString()
{
    return "Table "+name+"["+Uid()+"]";
}
```

public abstract class SSelector : SDbObject

### **SSelector**

As we saw in the SQuery class, columns are SSelectors. There will be other sorts including literals and expressions to go in select lists.

As with SQueries we will have an abstract method for looking up a selector received from the client.

```
public readonly string name;
        public SSelector(Types t, string n, long u) : base(t, u)
        {
            name = n;
        }
. . .
        public abstract SSelector Lookup(SQuery qry);
    }
SColumn
    public class SColumn : SSelector
        public readonly Types dataType;
        public readonly long table;
        /// <summary>
        /// For system or client column
        /// </summary>
        /// <param name="n"></param>
        /// <param name="t"></param>
        /// <param name="u"> will be negative</param>
        public SColumn(string n,Types t,long u) :base(Types.SColumn,n,u)
        {
            dataType = t; table = -1;
        }
        public SColumn(Transaction tr,string n, Types t, long tbl)
              : base(Types.SColumn,n,tr)
        {
```

```
dataType = t; table = tbl;
   }
   internal SColumn(SColumn c,long t) :base (c)
   {
        dataType = c.dataType;
       table = t;
   SColumn(StreamBase f) :base(Types.SColumn,f)
       dataType = (Types)f.ReadByte();
       table = f.GetLong();
   }
   public SColumn(SColumn c,AStream f):base (c,f)
       dataType = c.dataType;
       table = f.Fix(c.table);
       f.WriteByte((byte)dataType);
       f.PutLong(table);
   }
   public new static SColumn Get(StreamBase f)
       return new SColumn(f);
   }
   public override SSelector Lookup(SQuery qry)
   {
        return qry.names.Lookup(name);
   }
   public override string ToString()
   {
        return "Column " + name + " [" + Uid() + "]: " + dataType.ToString();
   }
}
```

### SRecord

As mentioned above, we will not be keeping all SRecords in memory: they will be refetched from disk when we want them.

```
public class SRecord : SDbObject
{
   public readonly SDict<long, Serialisable> fields;
   public readonly long table;
   public SRecord(Transaction tr,long t,SDict<string,Serialisable> f)
          :base(Types.SRecord,tr)
   {
        fields = f;
       table = t;
   }
   public virtual long Defpos => uid;
   protected SRecord(SRecord r,long tb) :base(r)
       fields = r.fields;
       table = tb;
   }
   public SRecord(SDatabase tr, SRecord r, AStream f) : base(r,f)
       table = f.Fix(r.table);
       var fs = r.fields;
       f.PutLong(table);
       var tb = (STable)db.Lookup(table);
       f.PutInt(r.fields.Length);
       for (var b=fs.First();b!=null;b=b.Next())
```

```
{
                var oc = b.Value.key;
                var v = b.Value.val;
                var c = oc;
                if (f.uids.Contains(c))
                    c = f.uids.Lookup(c);
                    fs = fs-oc+(c, v);
                f.PutLong(c);
                v.Put(f);
            fields = fs;
        }
        protected SRecord(Types t, SDatabase d,StreamBase f) : base(t,f)
            table = f.GetLong();
            var n = f.GetInt();
            var tb = d.tables.Lookup(table);
            var a = SDict<string, Serialisable>.Empty;
            for(var i = 0;i< n;i++)</pre>
            {
                var k = f.GetLong();
                a = a + (k, f.\_Get(d));
            fields = a;
        }
        public static SRecord Get(SDatabase d, StreamBase f)
        {
            return new SRecord(d,f);
        }
        protected void Append(StringBuilder sb)
            sb.Append(" for "); sb.Append(Uid());
            var cm = "(";
            for (var b = fields.First(); b != null; b = b.Next())
            {
                sb.Append(cm); cm = ",";
                sb.Append(b.Value.key); sb.Append("=");
                sb.Append(b.Value.val.ToString());
            sb.Append(")");
        }
        public override string ToString()
            var sb = new StringBuilder("Record ");
            sb.Append(Uid());
            Append(sb);
            return sb.ToString();
        }
    }
SUpdate
```

The SUpdate record preserves the Defpos of the updated SRecord. For efficiency all of the current fields are recorded in the SUpdate.

```
public class SUpdate : SRecord
    {
        public readonly long defpos;
        public SUpdate(Transaction tr,SRecord r) : base(tr,r.table,r.fields)
            defpos = r.Defpos;
```

```
}
        public override long Defpos => defpos;
        SUpdate(SUpdate u,long tbl,long dp) :base(u,tbl)
            defpos = u.defpos;
        }
        public SUpdate(SDatabase db, SUpdate r, AStream f) : base(db,r,f)
            defpos = f.Fix(defpos);
            f.PutLong(defpos);
        SUpdate(SDatabase d,StreamBase f) : base(d,f)
            defpos = f.GetLong();
        }
        public static SRecord Get(SDatabase d, StreamBase f)
            return new SUpdate(d,f);
        }
        public override string ToString()
            var sb = new StringBuilder("Update ");
            sb.Append(Uid());
            sb.Append(" of "); sb.Append(Transaction.Uid(defpos));
            Append(sb);
            return sb.ToString();
        }
    }
SDelete
public class SDelete : SDbObject
    {
        public readonly long table;
        public readonly long delpos;
        public SDelete(Transaction tr, long t, long p) : base(Types.SDelete,tr)
        {
            table = t;
            delpos = p;
        internal SDelete(SDelete u, long tbl, long del) : base(u)
        {
            table = tbl;
            delpos = del;
        }
        public SDelete(SDelete r, AStream f) : base(r,f)
        {
            table = f.Fix(r.table);
            delpos = f.Fix(r.delpos);
            f.PutLong(table);
            f.PutLong(delpos);
        SDelete(StreamBase f) : base(Types.SDelete,f)
        {
            table = f.GetLong();
            delpos = f.GetLong();
        public new static SDelete Get(StreamBase f)
        {
            return new SDelete(f);
        }
. . .
```

```
public override string ToString()
{
    var sb = new StringBuilder("Delete ");
    sb.Append(Uid());
    sb.Append(" of "); sb.Append(Transaction.Uid(delpos));
    sb.Append("["); sb.Append(Transaction.Uid(table)); sb.Append("]");
    return sb.ToString();
}
```

## Conflicts

This is not another class, just the additions to the above code to detect conflicts. Note that the definitions of Conflicts should be such that a.Conflicts(b) iff b.Conflicts(a).

In AStream:

```
public SDatabase Commit(SDatabase db,SDict<int,SDbObject> steps)
        {
            wbuf = new Buffer(this);
            uids = SDict<long, long>.Empty;
            for (var b=steps.First();b!=null; b=b.Next())
                switch (b.Value.val.type)
                    case Types.STable:
                        {
                             var st = (STable)b.Value.val;
                             var nt = new STable(st, this);
                             db = db + (nt, Length);
                             break;
                         }
              }
In STransaction.Commit:
        public SDatabase Commit()
            var f = dbfiles.Lookup(name);
            SDatabase db = databases.Lookup(name);
            var rdr = new Reader(f, curpos);
            var since = rdr.GetAll(db,db.curpos);
            for (var i = 0; i < since.Length; i++)</pre>
                for (var b = steps.First(); b != null; b = b.Next())
                    if (since[i].Conflicts(b.Value.val))
                         throw new Exception("Transaction Conflict on " + b.Value);
            lock (f)
                since = rdr.GetAll(this, f.Length);
                for (var i = 0; i < since.Length; i++)</pre>
                    for (var b = steps.First(); b != null; b = b.Next())
                         if (since[i].Conflicts(b.Value.val))
                             throw new Exception("Transaction Conflict on " + b.Value);
                db = f.Commit(db,steps);
            Install(db);
            return db;
        }
In Serialisable:
```

```
public virtual bool Conflicts(Serialisable that)
            return false;
        }
In STable:
        public override bool Conflicts(Serialisable that)
        {
            switch (that.type)
            {
                case Types.STable:
                    return ((STable)that).name.CompareTo(name) == 0;
            return false;
In SColumn:
        public override bool Conflicts(Serialisable that)
            switch (that.type)
            {
                case Types.SColumn:
                        var c = (SColumn)that;
                        return c.table == table && c.name.CompareTo(name) == 0);
            return false;
In SRecord:
        public override bool Conflicts(Serialisable that)
        {
            switch(that.type)
            {
                case Types.SDelete:
                    return ((SDelete)that).delpos == Defpos;
            return false;
        }
In SUpdate:
        public override bool Conflicts(Serialisable that)
        {
            switch (that.type)
                case Types.SUpdate:
                    return ((SUpdate)that).Defpos == Defpos;
            return base.Conflicts(that);
        }
In SDelete:
        public override bool Conflicts(Serialisable that)
            switch(that.type)
                case Types.SUpdate:
                    return ((SUpdate)that).Defpos == delpos;
```

```
case Types.SRecord:
          return ((SRecord)that).Defpos == delpos;
}
return false;
}
```

### 8 Queries and RowSets

We introduced the abstract SQuery class in the last section. There will be quite a few subclasses. For example, we can handle selection of some columns from a table with a where-condition (consisting for now of a list of simple column values) with the following:

```
SSearch
```

```
public class SSearch : SQuery
    {
        public readonly SQuery sce;
        public readonly SDict<SSelector,Serialisable> where;
        public SSearch(SDatabase db, StreamBase f):base(Types.SSearch,f)
        {
            sce = f._Get(db) as SQuery ?? throw new Exception("Query expected");
            var w = SDict<SSelector, Serialisable>.Empty;
            var n = f.GetInt();
            for (var i=0;i<n;i++)</pre>
                var k = f._Get(db) as SSelector ?? throw new Exception("Selector
expected");
                k = k.Lookup(sce);
                w = w.Add(k, f._Get(db));
            }
            where = w;
        public SSearch(SSearch s,SQuery q,SDict<SSelector,Serialisable> w)
            :base(s)
            sce = q;
            where = w;
        public override void Put(StreamBase f)
            f.WriteByte((byte)type);
            sce.Put(f);
            f.PutInt(where.Length);
            for (var b=where.First();b!=null;b=b.Next())
                b.Value.key.Put(f);
                b.Value.val.Put(f);
            }
        }
        public override SQuery Lookup(SDatabase db)
            var s = sce.Lookup(db);
            var w = SDict<SSelector, Serialisable>.Empty;
            for (var b = where.First(); b != null; b = b.Next())
                w = w.Add(b.Value.key.Lookup(s), v.Value.val);
            return new SSearch(this,s,w);
        public override RowSet RowSet(SDatabase db)
            return new SearchRowSet(db, this);
        }
    }
```

#### RowSet

This abstract class is for the results of connect.Get, before they are sent to the client in Json format. This is also useful for intermediate results, as we will see.

An SQuery is built using an SDatabase or STransaction, but by the time we evaluate to get the RowSet the database may have moved on. So a RowSet uses a snapshot of the current state of the SDatabase and its bookmarks will give the successive records in the results of the given SQuery.

```
public abstract class RowSet : Shareable<Serialisable>
{
    public readonly SQuery _qry;
    public readonly SDatabase _db;
    public RowSet(SDatabase d, SQuery q)
    {
        _db = d; _qry = q;
    }
}
```

It is natural to ask why we use Serialisable here instead of SRecord. It is not a very good reason: but the contents of system tables are not SRecords.

#### RowBookmark

It is convenient to have a RowBookmark class, which is also abstract:

```
public abstract class RowBookmark : Bookmark<Serialisable>
{
    public readonly RowSet _rs;
    public readonly Serialisable _ob;
    protected RowBookmark(RowSet rs, Serialisable ob, int p) : base(p)
    {
        _rs = rs; _ob = ob;
    }
    public override Serialisable Value => _ob;
}
```

We are now in a position to build our binary Query API:

```
TableRowSet
    public class TableRowSet : RowSet
        public readonly STable _tb;
        public TableRowSet(SDatabase db,STable t) : base(db,t)
            _{tb} = t;
        }
        public override Bookmark<Serialisable> First()
            return TableRowBookmark.New(this);
        internal class TableRowBookmark : RowBookmark
            public readonly TableRowSet _trs;
            public Bookmark<SSlot<long, long>> bmk;
            protected TableRowBookmark(TableRowSet
trs,Bookmark<SSlot<long,long>>bm,int p)
                :base(trs,trs._db.Get(bm.Value.val),p)
            {
                _{trs} = trs; _{bmk} = bm;
            internal static TableRowBookmark New(TableRowSet trs)
```

```
{
    return (trs._tb.rows.First() is Bookmark<SSlot<long, long>> b) ?
        new TableRowBookmark(trs, b, 0) : null;
}
public override Bookmark<Serialisable> Next()
{
    return (_bmk.Next() is Bookmark<SSlot<long,long>> b)?
        new TableRowBookmark(_trs,b,Position+1):null;
}
}
```

#### SearchRowSet

In the constructor we can see that we look to see if we have an index for the source query. Details of the IndexRowSet are given below.

```
public class SearchRowSet : RowSet
    {
        public readonly SSearch _sch;
        public readonly RowSet _sce;
        public SearchRowSet(SDatabase db,SSearch sc) :base (db,sc)
            _{sch} = sc;
            sce = ( sch.sce is STable tb && db.GetPrimaryIndex(tb.uid) is SIndex ix)
?
                new IndexRowSet(db, tb, ix, _sch.where) :
                _sch.sce.RowSet(db);
        }
        public override Bookmark<Serialisable> First()
            return SearchRowBookmark.New(this);
        internal class SearchRowBookmark : RowBookmark
            public readonly SearchRowSet sch;
            public RowBookmark bmk;
            protected SearchRowBookmark(SearchRowSet sr,RowBookmark bm,int p):
                base(sr, bm._ob, p)
            {
                _{sch} = sr; _{bmk} = bm;
            }
            internal static SearchRowBookmark New(SearchRowSet rs)
                for (var b = rs.First(); b != null; b = b.Next())
                    if (((SRecord)((RowBookmark)b)._ob).Matches(rs._sch.where))
                        return new SearchRowBookmark(rs, (RowBookmark)b, 0);
                return null;
            }
            public override Bookmark<Serialisable> Next()
                for (var b = _bmk.Next(); b != null; b = b.Next())
                    if (((SRecord)((RowBookmark)b)._ob).Matches(_sch._sch.where))
                        return new SearchRowBookmark(_sch, (RowBookmark)b,
Position+1);
                return null;
            }
        }
    }
IndexRowSet
        internal class IndexRowBookmark : RowBookmark
        {
```

```
public readonly IndexRowSet _irs;
            public readonly MTreeBookmark<long> _mbm;
            protected IndexRowBookmark(IndexRowSet irs,Serialisable
ob,MTreeBookmark<long> mbm,int p) :base(irs,ob,p)
                _irs = irs; _mbm = mbm;
            }
            internal static IndexRowBookmark New(IndexRowSet irs)
                for (var b = irs._ix.rows.PositionAt(irs._key);b!=null;b=b.Next() as
MTreeBookmark<long>)
                {
                    var r = irs._db.Get(b.Value.val);
                    if (r.Matches(irs._wh))
                        return new IndexRowBookmark(irs, r, b, 0);
                }
                return null;
            }
            public override Bookmark<Serialisable> Next()
                if (_irs._unique)
                    return null;
                for (var b = _mbm.Next(); b != null; b = b.Next())
                    var r = _irs._db.Get(b.Value.val);
                    if (r.Matches(_irs._wh))
                        return new IndexRowBookmark( irs, r, b as MTreeBookmark<long>,
Position+1);
                return null;
            }
        }
    }
```

# References

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