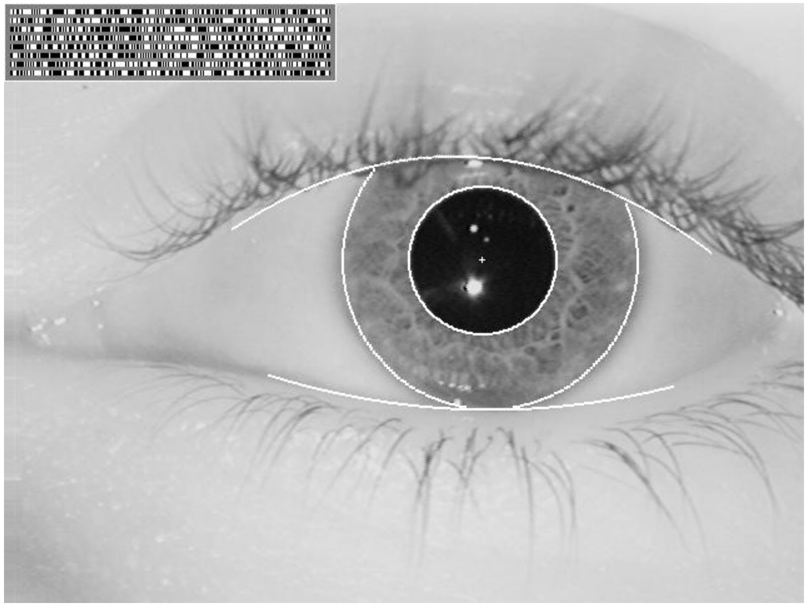
Missing binary features

In the framework of statistical decision theory

Origins of missing of data

- Occlusion by eyelids or eyelashes
- reflections from eye-glasses
- ring shadow by hard contact lenses
- local signal-to-noise ratio not good enough
- no good local iris texture



Iris code mask - takes value 1 (0) where data is considered good (bad)

Hamming distance of iris codes A and B with missing data masks

$$\text{Hamming Distance} = \frac{\|(codeA \otimes codeB) \cap maskA \cap maskB\|}{\|maskA \cap maskB\|}$$

- || maskA ∩ maskB || - number of bits *n* that are good in both iris codes (*n* can change from scan to scan)

Statistics reminder

- Let a tail (1) appear with a probability p in a coin tossing experiment
- Let the coin be tossed n times

The number of tails d is normally distributed:

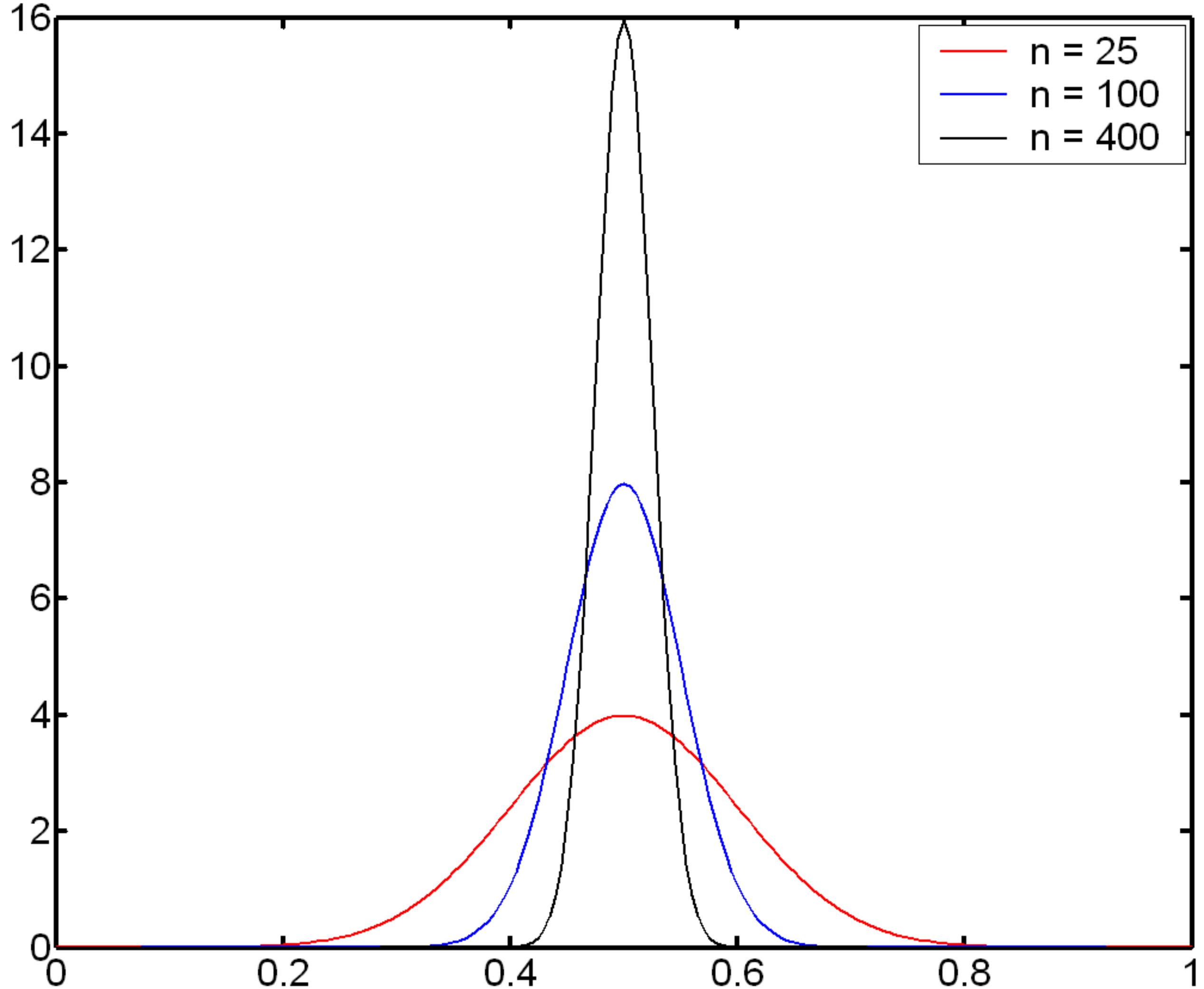
$$d \sim N(pn, p(1-p)n)$$

$$N(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{1}{2} \frac{(x-\mu)^2}{\sigma^2} \right)$$

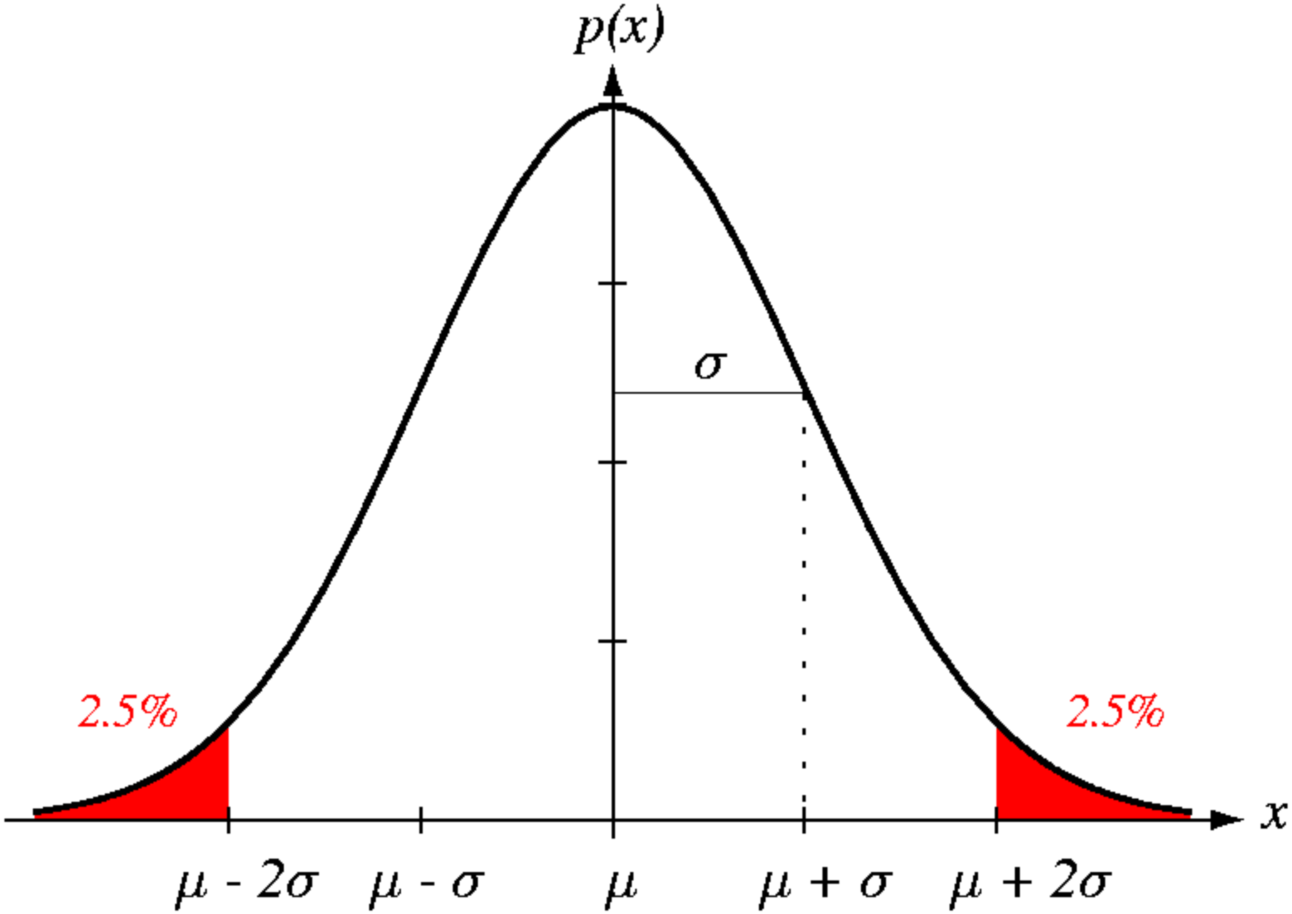
Normal distribution of normalized iris code HD

HD = d/n

HD ~ $N(p,p(1-p)/n)$



Normal distribution $N(\mu,\sigma^2)$



In 95% of the cases x is in the range $|x-\mu| \le 2\sigma$

Consequences for the choice of a decision criterion value

$$\mathrm{HD} \sim N(p,p(1\text{-}p)/n)$$

for $p = 0.5$

$$\mathrm{HD} \sim N(0.5,\sigma^2), \quad \sigma = 1/(2n^{1/2})$$

For error type I less than 0.025

$$\mathrm{HD}^* = 0.5 - 2\sigma$$

e.g.

$$n = 400 \;\rightarrow \sigma = 0.025 \rightarrow \; HD^* = 0.45$$

$$n = 100 \;\rightarrow \sigma = 0.05 \;\;\rightarrow \; HD^* = 0.4$$

$$n = \;\;25 \;\rightarrow \sigma = 0.1 \;\;\;\rightarrow \; HD^* = 0.3$$

Degrees of freedom

Degrees of freedom is (in general) the number of values used in the computation of a statistic that can vary independently

Example: n for the computation of the normalized Hamming distance of binary vectors of n statistically independent bits

$$\mathrm{HD} \sim N(p, p(1-p)/n) \qquad (\sigma^2 = p(1-p)/n)$$

(p - probability that two vectors differ in a given bit)

Binary degrees of freedom - the values used in the computation of a statistic are binary

Degrees of freedom and statistical independence

Are the $N \approx 2000$ bits of an iris code statistically independent? – No, there is certain correlation between bits corresponding to neighboring regions. Hence HD ~ $N(p,p(1\text{-}p)/N)$ does not hold. How many degrees of freedom are there then?

We know that the normalized HD of iris codes of different persons is normally distributed HD ~ $N(p, \sigma^2)$ with p and σ that can be determined empirically.

Find an n that corresponds to the measured σ.

Effective binary degrees of freedom

Theoretical HD ~ $N(p,p(1-p)/n)$, empirical HD ~ $N(p, \sigma^2)$

From the empirical values of σ and p we can compute some $n = p(1-p)/ \sigma^2$ ($n < N$) that represents the number of statistically independent bits needed to encode an iris pattern and that we call effective number of (binary) degrees of freedom.

In practice, $n \approx 244$ for iris recognition (for $\mu = 0.499$, $\sigma = 0.032$) .

The higher n, the larger the number 2^n of unique objects that can be represented and discriminated.

Generalized binary degrees of freedom

Problem: in other applications (e.g. finger print recognition) the values used to compute a dissimilarity are not binary. Then, how should we compare the discriminative power of different methods (e.g. iris vs. finger print recognition)?

Assumption: the dissimilarity is normally distributed D ~ $N(\mu, \sigma^2)$

Transform D to D' =D/ 2μ, D' ~ $N(½, (\sigma/2\mu)^2)$

We can now think of D' as the Hamming distance of two binary vectors of n statistically independent bits, D' ~ $N(p,p(1-p)/n)$

From $N(½, (\sigma/2\mu)^2) = N(p,p(1-p)/n)$ we get $n = (\mu/ \sigma)^2$

For finger print recognition $n \approx 35$.

The higher n, the larger the number 2^n of unique objects that can be discriminated.

More on Iris Scan

See: John Daugman

http://www.cl.cam.ac.uk/~jgd1000/

More on hypothesis testing

http://wikipedia.org

Summary of concepts and facts

- Authentication by iris pattern
- Iris code extraction
- Hamming distance of iris codes
- Histogram and probability density function
- Hypothesis testing
- Errors of type I and II
- Hamming distance of iris codes with missing bits
- Consequences of missing bits for the choice of a decision criterion value
- (Effective) number of (binary) degrees of freedom