Pattern Recognition practical 2

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1 Covariance matrix

1.1

Using the code given in the appendix we get the following mean vector:

$$\begin{bmatrix} 5.8000 \\ 5.0000 \\ 6.2000 \end{bmatrix} \tag{1}$$

And the following covariance matrix is yielded:

$$\begin{bmatrix} 3.2000 & 0.2500 & -0.4500 \\ 0.2500 & 2.5000 & -3.7500 \\ -0.4500 & -3.7500 & 5.7000 \end{bmatrix} \tag{2}$$

1.2

We computed the following probability densities:

```
1  >> mvnpdf([5;5;6], mean1, cov1)
2  ans =      0.0543
3  >> mvnpdf([3;5;7], mean1, cov1)
4  ans =      6.1287e-04
5  >> mvnpdf([4;6.5;1], mean1, cov1)
6  ans =      7.0300e-29
```

2 Covariance matrix, analytically

Every element in the covariance matrix is determined by:

$$\sigma_{ij} = \frac{1}{n-1} \sum_{n=1}^{N} (x_{in} - \mu_i)(x_{jn} - \mu_j) \tag{3}$$

Since $n = 2$ for all the covariance matrices calculated below, we leave that factor out (since $\frac{1}{2-1} = 1$).

2.1

First element:

$$\sigma_{1,1} = (a - \frac{a+c}{2})^2 + (c - \frac{a+c}{2})^2 \tag{4}$$
$$= (a^2 - (a^2 + c)) + (c^2 - (a + c^2)) \tag{5}$$
$$= -c - a \tag{6}$$

Similarly for $cov(2,2)$ we get $-d - b$. For the other two elements, $cov(1,2)$ and $cov(2,1)$ which are the same, we get the following:

$$cov(1,2) = ((a - \frac{a+c}{2})(b - \frac{b+d}{2}) + (c - \frac{a+c}{2})(d - \frac{b+d}{2})) \tag{7}$$
$$= (ab - \frac{ab+ad}{2} - \frac{ab+bc}{2} + \frac{ab+ad+bc+cd}{4}) + (cd - \frac{bc+cd}{2} - \frac{ad+cd}{2} - \frac{ab+ad+bc+cd}{4}) \tag{8}$$
$$= (ab - \frac{4ab+2ad+2bc}{4} - \frac{ab+ad+bc+cd}{4}) + (cd - \frac{4cd+2bc+2ad}{4} - \frac{ab+ad+bc+cd}{4}) \tag{9}$$
$$= (ab - \frac{5ab+3ad+3bc+cd}{4}) + (cd - \frac{ab+3ad+3bc+5cd}{4}) \tag{10}$$
$$= ab + cd - \frac{3}{2}(ab+ad+bc+cd) \tag{11}$$
$$= -\frac{1}{2}(ab+cd) - \frac{3}{2}(ad+bc) \tag{12}$$

This results in the following matrix:

$$\begin{bmatrix} -c-a & -\frac{1}{2}(ab+cd) - \frac{3}{2}(ad+bc) \\ -\frac{1}{2}(ab+cd) - \frac{3}{2}(ad+bc) & -d-b \end{bmatrix} \tag{13}$$

2.2

Appendix

Code for assignment 1:

```
1  v1 = [4,5,6];
2  v2 = [6,3,9];
3  v3 = [8,7,3];
4  v4 = [7,4,8];
5  v5 = [4,6,5];
6
7  m1 = [v1;v2;v3;v4;v5];
8
9  mean1 = [mean(m1(:,1)); mean(m1(:,2)); mean(m1(:,3))];
10
11 cov1 = cov(m1);
12
13 mean1
14 cov1
```