Department of Computational Mathematics, Faculty of Information Technology
University of Moratuwa

Honours Degree of Bachelor of science in Information Technology & Management
Batch 21- Level 2 (Semester II)
CM 2111: Statistical Inference

Assignment 01

1. Let $X_1, X_2, \dots, X_n$ be a random sample from a population with pdf,

$$f(x|\theta) = \frac{1}{\theta} x^{\left(\frac{1-\theta}{\theta}\right)}, \qquad 0 < x < 1, 0 < \theta < \infty\,; \quad where\ \theta\ is\ a\ parameter$$

a) Find the maximum likelihood estimator (MLE) of θ.
b) Calculate an estimate using the MLE when $x_1 = 0.12, x_2 = 0.32, x_3 = 0.42, x_4 = 0.54$.
c) Obtain a method of moments estimator(MOM) for θ. $\left[\text{Hint : E(X)} = \frac{1}{\theta+1}\right]$
d) Calculate an estimate using the MOM when the same values are in part (a).

2. A Geiger counter is a device which is used to measure radioactivity. In order to check that the device is calibrated correctly, a measurement can be taken from a source of known radioactive strength. The counts recorded by the Geiger counter over 200 one second intervals were recorded and these can be represented by the random variables $X_1, \dots, X_{200}$.

For this particular radioactive source, if the Geiger counter is functioning correctly then the mean count in a one second interval should be 20.

Assume that the counts $X_1, \dots, X_{200}$ are independent and each follow a $Poi(\theta)$ distribution, and that the sum of the observed counts was $\sum_{i=1}^{200} x_i = 3654$.

Write down the log-likelihood function for θ. Use this to find the maximum likelihood estimate of θ in terms of the observed values $X_1, X_2, \dots, X_n$, making sure to check that you have, indeed, found a maximum likelihood estimate.

3. a) Let $X_1, X_2, \dots, X_n$ be a iid from a population with pdf,

$$f(x|\theta) = \frac{\theta}{x^2}\,; \quad 0 < \theta \le x;$$

Obtain the maximum likelihood estimator with the reasons.

4. Let $X_1, X_2, \ldots, X_n$ be a random sample (iid) with pdf $f(x|\theta)$ and let $W(\boldsymbol{X}) = W(X_1, X_2, \ldots, X_n)$ be any estimator satisfying ,

$$\frac{dE_\theta(W(\boldsymbol{X}))}{dx} = \frac{d}{dx}\int_{\mathcal{X}} W(\boldsymbol{X})\, f(\mathbf{x}|\theta)\, d\mathbf{x} = \int_{\mathcal{X}} \frac{\partial}{\partial\theta} W(\boldsymbol{X})\, f(\mathbf{x}|\theta)\, d\mathbf{x}$$

and

$$V_\theta\big(W(\boldsymbol{X})\big) < \infty.$$

$$V_\theta\big(W(\mathbf{X})\big) \geq \frac{\left(\frac{dE_\theta\left(W(\mathbf{X})\right)}{d\theta}\right)^2}{-nE_\theta\left[\frac{\partial^2}{\partial\theta^2}\log f(X|\theta)\right]}.$$

Suppose a random sample $X_1, X_2, \ldots, X_n$ from a normal distribution $N(\mu, \theta)$ with μ given and the variance θ unknown.

Calculate the lower bound of variance for any estimator, and compare to that of the sample variance S^2.

$\left[\text{Hint: } Var(S^2) = \frac{2\theta^2}{(n-1)}\right]$