uzere.

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1)
a) $A = \begin{bmatrix} -7 & 6 & 6 \\ 0 & -1 & 0 \\ -12 & 12 & 11 \end{bmatrix} \Rightarrow \Delta_A(\lambda) = |\lambda I - A| = \begin{vmatrix} \lambda+7 & -6 & -6 \\ 0 & \lambda+1 & 0 \\ 12 & -12 & \lambda-11 \end{vmatrix} = (\lambda+1)\begin{vmatrix} \lambda+7 & -6 \\ 12 & \lambda-11 \end{vmatrix}$

$= (\lambda-5)(\lambda+1)^2$

$\lambda_1 = 5$, $\lambda_{2,3} = -1$ özdeğerlerdir

b) $\lambda_1 = 5 \Rightarrow W_1 = \left\{ X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} : AX = 5X \right\}$

$(5I - A) = \begin{bmatrix} 12 & -6 & -6 \\ 0 & 6 & 0 \\ 12 & -12 & -6 \end{bmatrix} \xrightarrow[R_3 \to 2R_2 + R_3]{R_1 \to R_1 + R_2} \begin{bmatrix} 12 & 0 & -6 \\ 0 & 6 & 0 \\ 12 & 0 & -6 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} 2x - z = 0 \\ y = 0 \end{matrix} \Big\}$ $3 - 2 = 1$ keyfi değ. sonsuz çöz.
$z = 2x$

$W_1 = \left\{ \begin{pmatrix} x \\ 0 \\ 2x \end{pmatrix} : x \in \mathbb{R} \right\} = \left\langle \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \right\rangle \to V_1$ özvektör

$\lambda_{2,3} = -1 \Rightarrow W_{-1} = \left\{ X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} : AX = (-1)X \right\}$

$((-1)I - A) = \begin{bmatrix} 6 & -6 & -6 \\ 0 & 0 & 0 \\ 12 & -12 & -12 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$x - y - z = 0 \Big\}$ $3 - 1 = 2$ keyfi değ. sonsuz çöz.
$x = y + z$

$W_{-1} = \left\{ \begin{pmatrix} y+z \\ y \\ z \end{pmatrix} : y, z \in \mathbb{R} \right\} = \left\langle \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle \to V_2, V_3$ özvektör

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c) $A\in\mathbb{R}^{3\times 3}$ ve 3 tane birbirinden lineer bağımsız özvektör old. köşegenleştirilebilir.

$$P=\begin{bmatrix}1&0&-1\\0&1&-1\\2&-1&0\end{bmatrix} \Rightarrow P^{-1}AP=\begin{bmatrix}5&0&0\\0&-1&0\\0&0&-1\end{bmatrix}.$$

2) a)

$$\langle u|v\rangle = 1.1+2.0+(-2).(-1)=3$$

$$\|u\| = \sqrt{1+4+4}=3$$

$$\|v\| = \sqrt{1+0+1}=\sqrt{2}$$

$$\cos\theta = \frac{\langle u|v\rangle}{\|u\|.\|v\|} = \frac{3}{3.\sqrt{2}} = \frac{1}{\sqrt{2}} \quad , \quad 0\leq\theta\leq\pi$$

$$\theta=\frac{\pi}{4}$$

b)
$$\begin{bmatrix}1&-3&0&2&-1\\2&-6&-1&3&-5\\-3&9&1&-5&6\end{bmatrix}\xrightarrow[R_3\to 3R_1+R_3]{R_2\to -2R_1+R_2}\begin{bmatrix}1&-3&0&2&-1\\0&0&-1&-1&-3\\0&0&1&1&3\end{bmatrix}\xrightarrow{R_3\to R_3+R_2}$$

$$\begin{bmatrix}1&-3&0&2&-1\\0&0&-1&-1&-3\\0&0&0&0&0\end{bmatrix}\Rightarrow H=\{(1,2,-3),(0,-1,1)\}\to \text{boy}H=2.$$

$$\{(1,2,-3),(2,3,-5)\}$$

$$\{(1,2,-3),(-1,-5,6)\}$$

$$\begin{bmatrix}1&0&1&0&0\\2&-1&0&1&0\\-3&1&0&0&1\end{bmatrix}\xrightarrow[R_3\to 3R_1+R_3]{R_2\to -2R_1+R_2}\begin{bmatrix}1&0&1&0&0\\0&-1&-2&1&0\\0&1&3&0&1\end{bmatrix}\xrightarrow{R_3\to R_2+R_3}$$

$$\begin{bmatrix}1&0&1&0&0\\0&-1&-2&1&0\\0&0&1&1&1\end{bmatrix}\Rightarrow\{(1,2,-3),(0,-1,1),(1,0,0)\}$$

$\Rightarrow\mathbb{R}^3$ ün bir bazıdır.

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3) $A=\begin{bmatrix}3 & -1\\ 1 & 1\end{bmatrix} \Rightarrow \Delta_A(\lambda)=|\lambda I-A|=\begin{vmatrix}\lambda-3 & 1\\ -1 & \lambda-1\end{vmatrix}=\lambda^2-4\lambda+4$

Cayley-Ham. Teo.'dan $A^2-4A+4I=0$

$$\Rightarrow 4I=-A^2+4A$$

$$I=\frac{1}{4}(4A-A^2)$$

$$\Rightarrow A^{-1}=\frac{1}{4}(4I-A)=I-\frac{1}{4}.A$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}3/4 & -1/4\\ 1/4 & 1/4\end{bmatrix}=\begin{bmatrix}1/4 & 1/4\\ -1/4 & 3/4\end{bmatrix}$$

4)
a) $\begin{bmatrix}2 & -1 & 3 & 1\\ -5 & 1 & 4 & -1\\ -1 & -1 & 10 & 1\end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix}-1 & -1 & 10 & 1\\ -5 & 1 & 4 & -1\\ 2 & -1 & 3 & 1\end{bmatrix} \xrightarrow[R_3 \to 2R_1+R_3]{R_2 \to -5R_1+R_2}$

$\begin{bmatrix}-1 & -1 & 10 & 1\\ 0 & 6 & -46 & -6\\ 0 & -3 & 23 & 3\end{bmatrix} \xrightarrow{R_2 \to 2R_3+R_2} \begin{bmatrix}-1 & -1 & 10 & 1\\ 0 & 0 & 0 & 0\\ 0 & -3 & 23 & 3\end{bmatrix} \Rightarrow \left.\begin{array}{l}-x-y+10z+t=0\\ -3y+23z+3t=0\end{array}\right\}$

$4-2=2$

$y=\frac{23}{3}z+t$

$x=-\frac{23}{3}z-t+10z+t$

$=\frac{7}{3}z$

$C.U=\left\{\left(\frac{7}{3}z,\ \frac{23}{3}z+t,\ z,\ t\right): z,t\in\mathbb{R}\right\}$

$=\left\langle\left(\frac{7}{3},\frac{23}{3},1,0\right),(0,1,0,1)\right\rangle$

boy(C.U.) = 2

b)

$|A|=\begin{vmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{vmatrix}=1.\begin{vmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{vmatrix}=\cos^2\theta+\sin^2\theta$

$=1$

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