

LINEER CEBİR ARASWAYI

Grup Analizi

(6.11.2022)

$$1) \left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 4 \\ 2 & -3 & 2 & -3 & -1 \\ 3 & -5 & 3 & -4 & 3 \\ -1 & 1 & -1 & 2 & 5 \end{array} \right] \xrightarrow{\substack{R_2 \rightarrow -2R_1 + R_2 \\ R_3 \rightarrow -3R_1 + R_3 \\ R_4 \rightarrow R_1 + R_4}} \left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 4 \\ 0 & 1 & 0 & -1 & -9 \\ 0 & 1 & 0 & -1 & -9 \\ 0 & -1 & 0 & 1 & 9 \end{array} \right] \xrightarrow{\substack{R_3 \rightarrow -R_2 + R_3 \\ R_4 \rightarrow R_2 + R_4}} \left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 4 \\ 0 & 1 & 0 & -1 & -9 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 4 \\ 0 & 1 & 0 & -1 & -9 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \Rightarrow \begin{cases} x_1 - 2x_2 + x_3 - x_4 = 4 \\ x_2 - x_4 = -9 \end{cases} \begin{cases} 4 - 2 = 2 \\ \text{keyfi degerli} \\ \text{sonuç 0'dır} \end{cases}$$

$$x_2 = x_4 - 9, \quad x_1 = -x_3 + 3x_4 - 14$$

$$G.K. = \{ (-x_3 + 3x_4 - 14, x_4 - 9, x_3, x_4) : x_3, x_4 \in \mathbb{R} \}$$

$$2) a) A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

$$S = \frac{1}{2} \left(\begin{bmatrix} 1 & 9 & 2 \\ 2 & 3 & 6 \\ 8 & 7 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 8 \\ 9 & 3 & 7 \\ 2 & 6 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 11/2 & 5 \\ 11/2 & 3 & 13/2 \\ 5 & 13/2 & 1 \end{bmatrix}$$

$$T.S = \frac{1}{2} \left(\begin{bmatrix} 1 & 9 & 2 \\ 2 & 3 & 6 \\ 8 & 7 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 8 \\ 9 & 3 & 7 \\ 2 & 6 & 1 \end{bmatrix} \right) = \begin{bmatrix} 0 & 7/2 & 3 \\ -7/2 & 0 & -1/2 \\ 3 & 1/2 & 0 \end{bmatrix}$$

$$b) AB = \begin{bmatrix} -2 & -1 & -6 \\ 3 & 2 & 9 \\ -1 & -1 & -4 \end{bmatrix} \begin{bmatrix} 1 & x & 3 \\ 3 & 2 & y \\ -1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 4-2x & -y \\ 0 & 3x-5 & -2y \\ 0 & 2-x & 1-y \end{bmatrix}$$

$$BA = \begin{bmatrix} 3x-5 & 2x-4 & 9x-18 \\ -y & 1-y & -4y \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{cases} 2-x=0 \Rightarrow x=2 \\ -y=0 \Rightarrow y=0 \end{cases}$$

3) $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$a_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$, $a_{12} = (-1)^{1+2} \begin{vmatrix} 3 & 0 \\ 1 & 1 \end{vmatrix} = -3$, $a_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = 2$

$a_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} = 0$, $a_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = -1$, $a_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1$

$a_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 2 \\ 3 & 0 \end{vmatrix} = -2$, $a_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} = 6$, $a_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5$

$\text{Adj}(A) = \begin{bmatrix} 1 & 0 & -2 \\ -3 & -1 & 6 \\ 2 & 1 & -5 \end{bmatrix}$, $A^{-1} = \frac{\text{Adj}(A)}{|A|}$

$|A| = \begin{vmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} \xrightarrow{\substack{R_2 \rightarrow R_1 + R_2 \\ R_3 \rightarrow R_1 + R_3}} \begin{vmatrix} 1 & 2 & 2 \\ 0 & -5 & -6 \\ 0 & -1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 2 \\ 0 & -5 & -6 \\ 0 & -1 & -1 \end{vmatrix} = -1 \neq 0$

$A^{-1} = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & -6 \\ -2 & -1 & 5 \end{bmatrix}$

4) a) $\begin{vmatrix} x & 1 & 0 & x \\ 0 & x & x & 1 \\ 1 & x & x & 0 \\ x & 0 & 1 & x \end{vmatrix} \xrightarrow{C_1 \rightarrow C_1 + C_2 + C_3 + C_4} \begin{vmatrix} 2x+1 & 1 & 0 & x \\ 2x+1 & x & x & 1 \\ 2x+1 & x & x & 0 \\ 2x+1 & 0 & 1 & x \end{vmatrix} = (2x+1) \begin{vmatrix} 1 & 1 & 0 & x \\ 1 & x & x & 1 \\ 1 & x & x & 0 \\ 1 & 0 & 1 & x \end{vmatrix}$

$\xrightarrow{\substack{R_2 \rightarrow R_1 + R_2 \\ R_3 \rightarrow R_1 + R_3 \\ R_4 \rightarrow R_1 + R_4}} (2x+1) \begin{vmatrix} 1 & 1 & 0 & x \\ 0 & x-1 & x & 1-x \\ 0 & x-1 & x & -x \\ 0 & -1 & 1 & 0 \end{vmatrix} = (2x+1) \begin{vmatrix} x-1 & x & 1-x \\ x-1 & x & -x \\ -1 & 1 & 0 \end{vmatrix} \xrightarrow{R_2 \rightarrow -R_1 + R_2}$

$(2x+1) \begin{vmatrix} x-1 & x & 1-x \\ 0 & 0 & -1 \\ -1 & 1 & 0 \end{vmatrix} = (2x+1) \cdot \begin{vmatrix} x-1 & x \\ -1 & 1 \end{vmatrix} = (2x+1)(2x-1) = 0$
 $\Rightarrow x = -1/2 \text{ veya } x = 1/2$

4) b)

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \\ 1 & 3 & 1 \end{vmatrix} \xrightarrow{R_3 \rightarrow -3R_2 + R_3} \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \\ -2 & 0 & 4 \end{vmatrix} = 1 \begin{vmatrix} 1 & 1 \\ -2 & 4 \end{vmatrix}$$

$$= 6$$

$$|A^4| = |A A A A| = |A| \cdot |A| \cdot |A| \cdot |A| = |A|^4 \\ = 6^4 = 1296$$