Face modelling and recognition

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Objectives

- Introduce the strategies ¨top-down and bottom-up.
- Modelling the human face as a union of submanifolds according to its curvature.
- Study the evolution of the model created through flows, which also allow to analyse gestures.
- Inquire into data-driven strategies, exemplified by the eigenfaces method.

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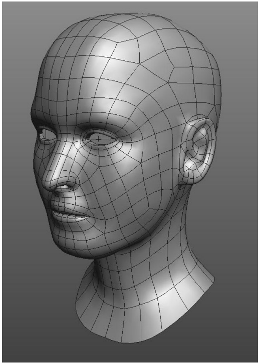
The face as a union of submanifolds

- A model is adopted based on appearances, without considering the underlying structure of bones and muscles.
- The face is initially modelled as a connected surface without holes.



Geometry of the human face

Tangent space

$$T_pM = \{[c]_p \mid \text{c is a tangent curve in p}\}$$

Vector bundle

Given an immersion $f : S \to \mathbb{R}^3$ of a surface S, the normal space at each point is $(T_p\mathbb{R}^3 |_S)/(T_pS)$. The gluing by local letters of all of them gives the total space of the vector bundle $\mathcal{N}_f$.

Vector fields

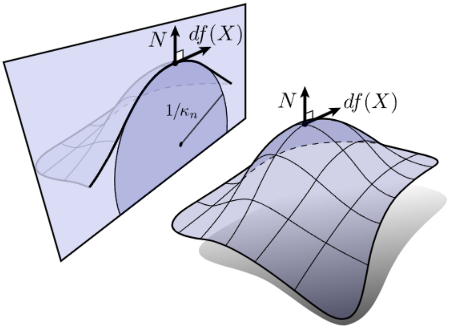
A **vector field** on M is a local section of the tangent fibration.
The set of vector fields on M of class C^k is denoted $mathcalX^k(M)$.

[1] Abraham, Marsden y Ratiu. Manifolds, tensor analysis, and applications. 1988.

Moving references on a surface

Gauss map

$$N : U \to \mathbb{S}^2 \ / \ N(p) = \frac{r_u \times r_v}{||r_u \times r_v||}$$



Algebraic approach

Directional or Lie derivative

$$\mathcal{L}_X f \equiv X[f] = df \cdot X = \sum_i \frac{\partial f}{\partial x_i} X_i \in \mathcal{X}^{k-2}(M)$$

Weingarten endomorphism

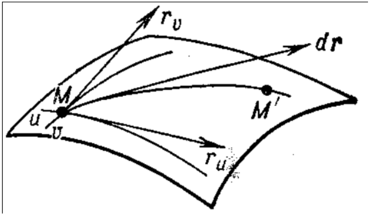
The differential of the Gauss map $dN : T_pS \to T_{N(p)}\mathbb{S}^2$ induces the Weingarten endomorphism L: $T_pS \to T_pS$.

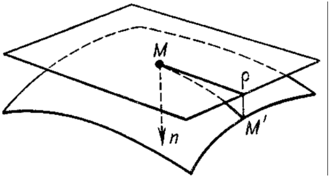
[2] Manfredo P. do Carmo. Geometría diferencial de curvas y superficies. 1990.

Fundamental forms

k-th fundamental form

Given a surface S and $X, Y \in T_pS$, we define the k-**th fundamental form** as $\langle L^{k-1}X, Y\rangle$; $k \in \mathbb{N}$.

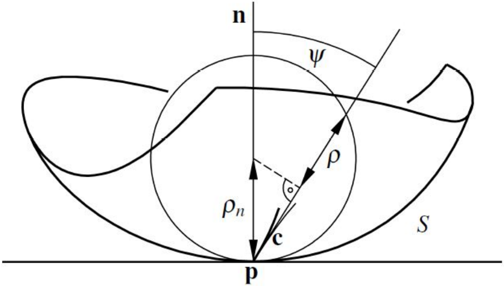




Normal surface sections

Meusnier's proposition

All the curves of a surface that pass through the same point and have the same tangent to it, have the same normal curvature at that point and the osculating circles form a sphere (**Meusnier sphere**).



Let K be the matrix of the differential of the Gaussian map $d_p N$, its algebraic invariants are determined by its eigenvalues κ_1 and κ_2, assuming $\kappa_1 \geq \kappa_2$.

- **Mean currvature**: $H \equiv \kappa_m = \frac{1}{2}\operatorname{tr}(K) = \frac{1}{2}(\kappa_1 + \kappa_2)$
- **Gaussian curvature**: $\mathbf{K} \equiv \kappa_t = |K| = \kappa_1 \kappa_2$

This allows a classification of the various points of the surface according to the sign of the Gaussian curvature, if any principal curvature is cancelled or in case of extreme values of these:

- Elliptic
- Umbilic
- Hyperbolic
- Flat
- Parabolic
- Ridge

Teorema Egregium (Gauss)

The Gaussian curvature of a regular surface is invariant by local isometries.

Compatibility equations

$$E\mathbf{K} = (\Gamma^2_{11})_v - (\Gamma^2_{12})_u + \Gamma^1_{11}\Gamma^2_{12} + \Gamma^2_{11}\Gamma^2_{22} - \Gamma^1_{12}\Gamma^2_{11} - (\Gamma^2_{12})^2,$$
$$e_v - f_u = e\Gamma^1_{12} + f(\Gamma^2_{12} - \Gamma^1_{11}) - g\Gamma^2_{11},$$
$$f_v - g_u = e\Gamma^1_{22} + f(\Gamma^2_{22} - \Gamma^1_{12}) - g\Gamma^2_{12}.$$

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Evolution of surfaces

For gesture analysis, we will study the evolution over time of a family of explicitly represented surfaces (Monge form); $z = f(x, y)$.

$$S_t = \{(x, y, z) \ | \ z = F(x, y, t)\}$$

Its deformation along the normal direction poses the initial value problem:

$$F_t(x, y, t) = \beta \cdot \sqrt{1 + F_x^2 + F_y^2}$$

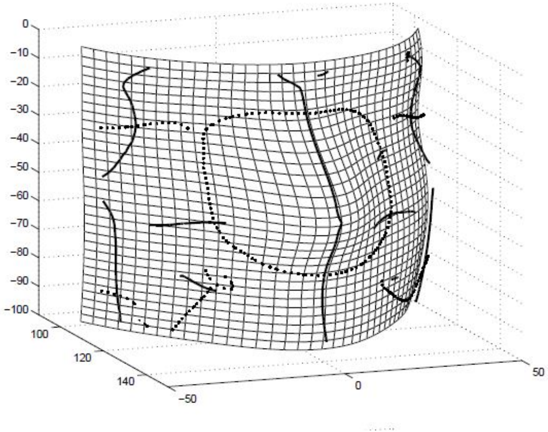
$$F(x, y, 0) = f(x, y)$$

The solution of this, for a small t, can be approximated by the flow:

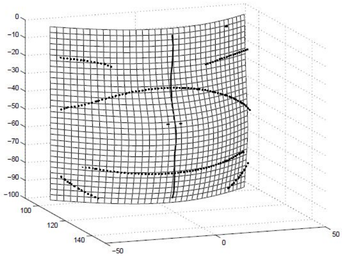
$$F(x, y, t) = f(x, y) + \beta \cdot \sqrt{1 + f_x^2 + f_y^2} \cdot t + O(t)$$

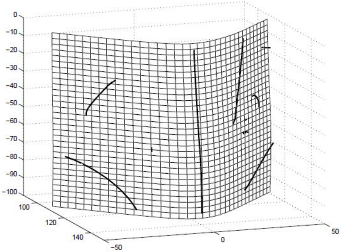
[3] Lu, Cao y Mumford. Surface Evolution under Curvature Flows. 2002.

Square mesh over the face scan



Principal curvature flows over the face





The face after passing through the K_1 (left) and K_2 (right) flows at $t = 1000$.

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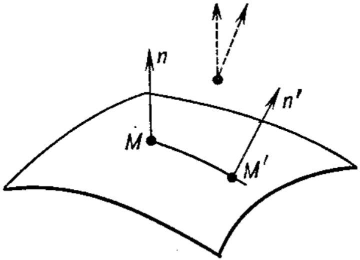
Willmore energy

Willmore energy

$$\mathcal{W}(S) = \int_S (H^2 - \mathbf{K})\, dA = \frac{1}{4}\int_S (\kappa_1 - \kappa_2)^2\, dA$$



[4] Jose Antonio Lorencio Abril. Superficies de Willmore en el espacio euclídeo. 2022.

Willmore energy functional

Gauss's theorem for spherical mappings

$$|\omega(D)| := \int_S \mathbf{K}\, dA = \iint_{\mathbb{S}^2} d\sigma$$

Willmore energy functional

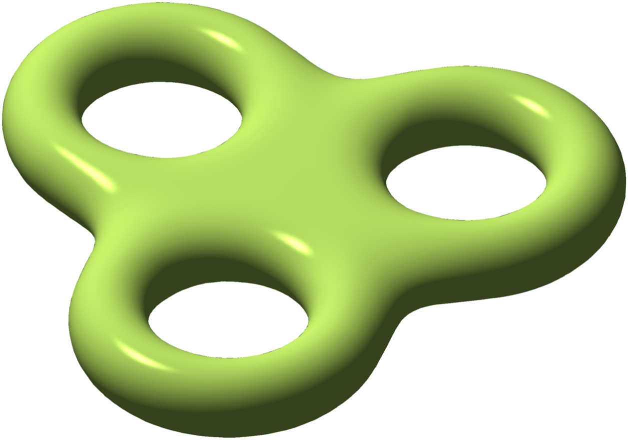
$$\mathcal{W}(S) = \int_S H^2\, dA$$

A Willmore surface is a critical point of the functional

$$\mathcal{W} : SCO(\mathbb{R}^3) \to \mathbb{R},$$

where $SCO(\mathbb{R}^3)$ is the set of compact and orientable surfaces.

Application to facial modelling



The human face as Willmore surfaces

Conformal invariance of the Willmore functional

Let $S \in SCO(\mathbb{R}^3)$ and Φ be a conformal and injective application. Then,

$$S' = \Phi(S) \in SCO(\mathbb{R}^3) \text{ y } \mathcal{W}(S) = \mathcal{W}(S').$$

Euler-Lagrange equation for Willmore surfaces

$$\Delta H + 2H(H^2 - 2\mathbf{K}) = 0$$

Geometric flows

- **Optical flow in video sequences**: Extraction of significant facts from extreme values of the intensity function,

$$I : R \times [0, D] \rightarrow [0, 255]$$

in specific frames. For this purpose, the latter is assumed constant at close instants and the associated optical flow is studied as:

$$\int_{\Omega} \nabla_t I = 0$$

- **Volume flows**: Analysis of the evolution of moving surfaces as three-dimensional varieties.

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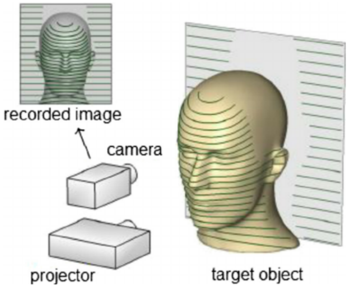
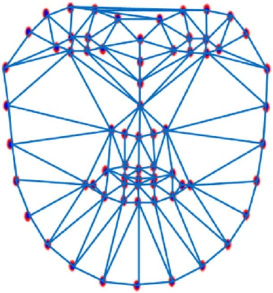
Data capture





Facial recognition methods

- **Holistic representation**: Analysis of the face as a single element, preserving the interrelationship between the parts.
- **Extraction of significant facts**: Decompose the face into regions of interest for individual analysis.

Because of the large volume of data to be handled, it is necessary to apply dimensionality reduction techniques such as **Principal Component Analysis** (**PCA**).

[5] Turk y Pentland. Eigenfaces for Recognition. 1991.

Eigenfaces

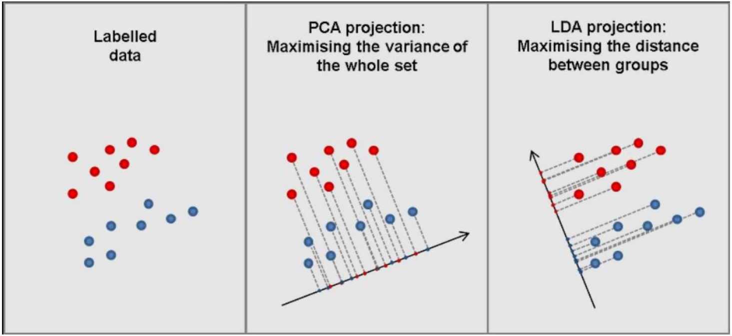
1. Obtain the training set of images.
2. Calculate and subtract the average image.
3. Calculate eigenvalues and eigenvectors of the covariance matrix.
4. Choosing the k principal components according to a threshold $\epsilon < \frac{\sum_{i=1}^{k} s_i^2}{\sum_{i=1}^{n} s_i^2}$.

Olivetti dataset

Olivetti dataset eigenfaces

LDA

Linear Discriminant Analysis (**LDA**) is a technique for classifying data into classes by creating hyperplanes.



Fisherfaces

(a)

(b)

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Conclusions

- Surface modelling of the face as a manifold is an appropriate approach.
- The face can be segmented into constant curvature regions.
- Curvature flows allow surface singularities to be simplified and gestures to be modelled.
- Other alternatives for gesture tracking are flows such as the Willmore energy flow.
- The ¨bottom-up strategies allow to complement and expand these theoretical models.

Future work

- Incorporate underlying geometry of muscle and skeleton into the appearance-based model.
- Perform a simulation of facial gestures using curvature maps and their relationships in terms of the Willmore energy functional.
- Develop the feedback between top-down and bottom-up strategies.
- Train and validate AI models for facial gesture recognition, tracking and simulation.

Questions