1.1 NUMBERS – ROUNDING – SCIENTIFIC FORM

♦ NOTATION FOR SETS OF NUMBERS

Remember the following known sets of numbers:

N = {0, 1, 2, 3, 4, …}	**natural**	
Z = {0, ±1, ±2, ±3, …}	**integers**	
Q = $\{\frac{a}{b}: a,b\in Z,\ b\neq 0\}$	**rational**	(fractions of integers)
R = rational + irrational	**real**	

Known irrational numbers:

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and all $\sqrt{a}$ where a is not a perfect square

$\pi = 3.14159....$

$e = 2.7182818...$

To indicate particular subsets we use the indices +,-, * as follows:

$Z^+ = \{1, 2, 3, ...\}$	**positive integers**
$Z^- = \{-1, -2, -3, ...\}$	**negative integers**
$Z^* = \{\pm 1, \pm 2, \pm 3, ...\}$	**non-zero integers i.e.** $Z^* = Z-\{0\}$

Similar notations apply for the other sets above.

For intervals of real numbers we use the following notations:

$x\in[a,b]$	for $a \le x \le b$
$x\in]a,b[$ or $x\in(a,b)$	for $a < x < b$
$x\in[a,b[$ or $x\in[a,b)$	for $a \le x < b$
$x\in[a,+\infty[$ or $x\in[a,+\infty)$	for $x \ge a$
$x\in]-\infty,a]$ or $x\in(-\infty,a]$	for $x \le a$
$x\in]-\infty,a]\cup[b,+\infty[$	for $x \le a$ or $x \ge b$

I have to continue my notes with a – not so pleasant – discussion about rounding of numbers. The numerical answer to a problem is not always **exact** and we have to use some rounding.

- DECIMAL PLACES vs SIGNIFICANT FIGURES

Consider the number

$$123.4567$$

There are two ways to round up the number by using fewer digits:

- In a specific number of **decimal places (d.p.)**

in 1 d.p.	123.5
in 2 d.p.	123.46
in 3 d.p.	123.457

We can also round up before the decimal point:

to the nearest integer	123
to the nearest 10	120
to the nearest 100	100

- In a specific number of **significant figures (s.f.)**: for the position of cutting we start counting from the first non-zero digit:

in 4 s.f.	123.5
in 5 s.f.	123.46
in 6 s.f.	123.457

But also

in 2 s.f.	120
in 1 s.f.	100

Notice that the number at the critical position

remains as it is	if the following digit is 0, 1, 2, 3, 4
Increases by 1	if the following digit is 5, 6, 7, 8, 9

EXAMPLE 1

Consider the number

$$0.04362018$$

in decimal places		**in significant figures**	
in 2 d.p.	0.04	in 2 s.f.	0.044
in 3 d.p.	0.044	in 3 s.f.	0.0436
in 4 d.p.	0.0436	in 4 s.f.	0.04362
in 6 d.p.	0.043620	in 5 s.f.	0.043620

Important remark: In the final IB exams the requirement is to give the answers either in **exact** form or **in 3 s.f.** . For example

exact form	**in 3sf**
$\sqrt{2}$	1.41
2π	6.28
12348	12300

♦ THE SCIENTIFIC FORM $a\times10^k$

Any number can be written in the form

$$a\times10^k \quad \text{where} \quad 1 \le a < 10$$

We simply move the decimal point after the first non-zero digit. For example, the number

$$123.4567 \quad \text{can be written as} \quad 1.234567\times10^2$$

Indeed,

$$1.234567\times10^2 = 1.234567\times100 = 123.4567$$

Notice that

we moved the decimal point 2 positions **to the left**

$$\Rightarrow k = 2$$

Even for a "small" number, say

$$0.000012345$$

we can find such an expression:

$$1.2345\times10^{-5}$$

Notice that

we moved the decimal point 5 positions **to the right**

$$\Rightarrow k = -5$$

NOTICE:

- They may ask us to give the number in scientific form but also in 3 s.f. Then

$$1.2345\times10^{2} \cong 1.23\times10^{2}$$
$$1.2345\times10^{-5} \cong 1.23\times10^{-5}$$

- Most calculators use the symbol E±-- for the scientific notation:
  The notation 1.2345E+02 means 1.2345×10^{2}
  The notation 1.2345E-05 means 1.2345×10^{-5}

EXAMPLE 2

(a) Give the scientific form of the numbers
$x = 100000$ $y = 0.00001$ $z = 4057.52$ $w = 0.00107$

(b) Give the standard form of the numbers
$s = 4.501\times10^{7}$ $t = 4.501\times10^{-7}$

Solution

(a) $x = 1\times10^{5}$
$y = 1\times10^{-5}$
$z = 4.05752\times10^{3}$
$w = 1.07\times10^{-3}$

(b) $s = 45010000$
$t = 0.0000004501$

EXAMPLE 3

Consider the numbers

$x = 3\times10^7$ and $y = 4\times10^7$

Give $x+y$ and xy in scientific form.

Solution

$x+y = 7\times10^7$ [add 3+4]
[keep the same exponent]

$xy = 12\times10^{14}$ [multiply 3×4]
[add exponents]

$= 1.2\times10^{15}$ [modify a so that $1 \le a < 10$]

EXAMPLE 4

Consider the numbers

$x = 3\times10^7$ and $y = 4\times10^9$

Give $x+y$ and xy in scientific form.

Solution

For addition we must modify y (or x) in order to achieve similar forms

$x = 3\times10^7$

$y = 4\times10^9 = 400\times10^7$

$x+y = 403\times10^7$ [add 3+400]
[keep the same exponent]

$= 4.03\times10^9$ [modify a so that $1 \le a < 10$]

For multiplication there is no need to modify y:

$xy = 12\times10^{16}$ [multiply 3×4]
[add exponents]

$= 1.2\times10^{17}$ [modify a so that $1 \le a < 10$]