$$p(X \mid Y=1) = \lambda e^{-\lambda x} \, \mathbb{1}_{(0,\infty)}(x)$$

$$p(Y=k \mid X=x) = \frac{p(X=x \mid Y=k) \cdot \pi}{p(X=x \mid Y=-1) \cdot (1-\pi) + p(X=x \mid Y=1) \cdot \pi}$$

$$p(Y=1 \mid X=x) = \frac{p(X=x \mid Y=1) \cdot \pi}{p(X=x \mid Y=-1) \cdot (1-\pi) + p(X=x \mid Y=1) \cdot \pi} =$$

$$= \frac{1}{1+\exp(-\beta x - \beta_0)}$$

$$1+\exp(-\beta x - \beta_0) = \cancel{\frac{p(X=x \mid Y=1) \cdot \pi}{p(X=x \mid Y=1) \cdot \pi}} + \frac{p(X=x \mid Y=-1) \cdot (1-\pi)}{p(X=x \mid Y=1) \cdot \pi}$$

$$\frac{1-\pi}{\pi} \quad \frac{p(X=x \mid Y=-1)}{p(X=x \mid Y=1)} = \exp(-\beta x - \beta_0)$$

$$\frac{1-\pi}{\pi} \quad \frac{p(X=x \mid Y=-1)}{\lambda e^{-\lambda x}} = \exp(-\beta x - \beta_0) \qquad x > 0$$

$$\boxed{\lambda = 1}$$

$$p(X=x \mid Y=-1) = \frac{\pi}{1-\pi} \exp(-(\beta+1)x - \beta_0) = \qquad x > 0$$

$$= \frac{\pi}{1-\pi} e^{-\beta_0} \exp(-(\beta+1)x)$$

$$\Downarrow \quad \int = 1$$

$$\frac{\pi}{1-\pi} e^{-\beta_0} = \beta + 1$$

Q1, Q2

$$\beta_0 = -\ln\left(\frac{1-\pi}{\pi}(\beta+1)\right)$$

Q3 $\beta_1 = 1, \quad \pi = \frac{1000}{2000+1000} = \frac{1}{3}$ $\qquad \beta_0 = -\ln\left(\frac{1-\frac{1}{3}}{\frac{1}{3}}(1+1)\right) = -\ln(4) \approx$

$\approx -1{,}386$