Mathematical Underpinnings Lab 8 17.04.2024

Task 1 (VAE)

Load the data: `(x_train, y_train), (x_test, y_test) = mnist.load_data()`.
Next, scale the data so the value of each pixel is in $[0, 1]$ (and not $[0, 255]$). Make a plot of the first digit in the dataset. Transform the matrices in which each image of a digit is stored into vectors. The length of that vector will be denoted as `original_dim` and in our case be equal to $28 \cdot 28$.

a) Define a function computing ELBO. It should contain two parts: the expectation of $p(x|z)$ according to q (input: a vector of observed pixels and posterior probability of a pixel being black after reparametrization trick) and Kullback-Leibler divergence of q and p (input: the means and log-variances of the latent variable z).

b) Define sampling function using the reparametrization trick.

c) Define the encoder (input: a vector of pixels, output: a vector with concatenated mean and log-variance of the hidden variable z).

d) Define the decoder (input: a vector of length that is equal to latent space dimension, output: a vector of `original_dim` length with numbers in $[0, 1]$).

e) Combine the encoder, sampling, decoder and the risk function and fit the weights using training data.

f) Visualise the results - compare the true digits with artificially generated using test data. For artificially generated image, take a true image, then find its representation in terms of a latent space, decode the representation using decoder, and then transform the result into matrix and draw it.

g) Find latent representation of the digits from the test dataset. Next, draw a scatterplot (if your latent space has more dimensions than 2 use an algorithm which will map it into 2 dimensions). Color the points according to the digit they represent.