For linear elastic bonds undergoing small deformations, the tangential stiffness matrix, $[K]$, remains constant prior to failure. Its general form is derivable from the differential equations governing beam deformation by applying unit displacement theory for a Timoshenko beam.

$$\Delta F = [K] \cdot \Delta u \tag{1}$$

$$[K] = \begin{bmatrix} K_1 & -K_2 & -K_1 & -K_2 \\ K_2 & K_3 & -K_2 & K_4 \\ -K_1 & K_2 & K_1 & K_2 \\ K_2 & K_4 & -K_2 & K_3 \end{bmatrix} \tag{2}$$

where contains four effective shear stiffness coefficient matrices of size 3 × 3:

$$[K_1] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3(1+\varphi)} & 0 \\ 0 & 0 & \frac{12EI}{L^3(1+\varphi)} \end{bmatrix} \tag{3}$$

$$[K_2] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{-6EI}{L^2(1+\varphi)} \\ 0 & \frac{6EI}{L^2(1+\varphi)} & 0 \end{bmatrix} \tag{4}$$

$$[K_3] = \begin{bmatrix} \frac{EI}{L(1+v)} & 0 & 0 \\ 0 & \frac{EI(4+\varphi)}{L(1+\varphi)} & 0 \\ 0 & 0 & \frac{EI(4+\varphi)}{L(1+\varphi)} \end{bmatrix} \tag{5}$$

$$[K_4] = \begin{bmatrix} -\frac{EI}{L(1+v)} & 0 & 0 \\ 0 & \frac{EI(2-\varphi)}{L(1+\varphi)} & 0 \\ 0 & 0 & \frac{EI(2-\varphi)}{L(1+\varphi)} \end{bmatrix} \tag{6}$$

where E is the Young's modulus, A is the cross sectional area, I is the second moment of area of the contact and v is the Poisson's ratio of the material, and φ is the Timoshenko shear coefficient. Substituting the stiffness matrix into Eqs. (1) and (2) can get Eqs. (7) - (12):

$$\Delta F_{ix} = \frac{EA}{L_b}\left(v_{ix} - v_{jx}\right)\Delta t \tag{7}$$

$$\Delta F_{iy} = \frac{12EI_{iy}}{L_b^3(1+\eta)}(v_{jy} - v_{iy} + \frac{1}{2}L_b\omega_{iz} + \frac{1}{2}L_b\omega_{jz})\Delta t \tag{8}$$

$$\Delta F_{iz} = \frac{12EI_{iz}}{L_b^3(1+\eta)}(v_{jz} - v_{iz} + \frac{1}{2}L_b\omega_{iy} + \frac{1}{2}L_b\omega_{jy})\Delta t \tag{9}$$

$$\Delta M_{ix} = \frac{EI_{ix}}{L_b(1+\nu_b)}\left(\omega_{jx} - \omega_{ix}\right) \tag{10}$$

$$\Delta M_{iy} = \frac{EI_{iy}}{L_b}\left(\omega_{jy} + \omega_{iy}\right) - \frac{1}{2}L_b\Delta F_{iz} \tag{11}$$

$$\Delta M_{iz} = \frac{EI_{iz}}{L_b}(\omega_{jz} + \omega_{iz}) + \frac{1}{2}L_b\Delta F_{iy} \tag{12}$$