Periodic Hill Flow

– Numerical study comparison of flow over smooth constrictions.

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Abstract

Several turbulence models have been developed in order to resolve the flow physics at different levels of accuracy. The performance of the turbulence models can be investigated by simulating the periodic hill flow (PHF). The basic configuration of PHF depicts flow features like separation over a curved surface, reattachment, and complex turbulent flows. Thus it is used as a benchmark test case for studying the accuracy of different turbulence models. In this study, multiple computational fluid dynamics (CFD) simulations were carried out to compare the applicability, strengths, and weakness of the turbulence models RANS, URANS, Delayed Detached Eddy Simulation (DDES), and Scale Adaptive Simulation (SAS). The simulations were made using a 3D domain with periodic boundary conditions on the inlet and outlet, as well as the sides. The driving force behind the flow was a pressure gradient specified between the successive hills. The mesh used was a quad mesh, which was verified using RANS, investigating y^+, Δx^+, and Δz^+. Additionally, a two-point correlation, computed according to Gårdhagen [1], investigation was made along two vertical lines, in order to visualise how many mesh elements contain correlated flow structures. With the two-point correlation investigation being made using a DDES simulation. The results aims at discussing the accuracy between the scale resolving models and the RANS models, which are not known for resolving turbulence, by comparing to Large Eddy Simulation (LES) data from Breuer et al. [2].

The results showed that the RANS model was unable to capture any unsteady effects, and thus fails at capturing the chaotic flow near the bottom wall accurately. However, if only a rough approximation of the flow was needed, then the RANS model could be an option simply due to the comparatively extremely low computational cost. The URANS model gave completely nonphysical results, and was thus not presented. The DDES model showed superior results to SAS for most parameters. Especially for the $v'v'$ Reynolds stress, where the SAS model severely under predicts compared to both DDES and LES data. The SAS also predicted less mean vortex structures than DDES, as it only predicted the largest structures, whereas the DDES could resolve smaller structures. The flow near the bottom wall was also rather different between the three, as the SAS only seemed to act as a slightly more unstable RANS model.

The limiting factor for the accuracy of the results was found to be the mesh. As the scale resolving methods rely on the mesh size in terms of the size of the turbulent length scales which can be resolved, the mesh has a large implication on the accuracy. Though the mesh was verified using the RANS simulation, the correlation showed unsatisfactory results near the top of the upstream hill. Thus indicating that potentially badly resolved structures propagate downstream, causing further inaccuracies. Linked with the mesh is the sampled time, the mesh could not be increased due to the increase computational costs that it would bring, which would increase computational time. As time was limited, the mesh was subsequently also limited thereafter. The comparison between 15 and 30 flow through cycles of the DDES model showed that a longer time sampling would be needed. Similarly the mean q-criterion in the domain highlighted that aspect.

Conclusively the DDES showed to be the most accurate model. While time consuming, the level of accuracy it could provide was higher than the other models, and if accuracy is needed then it is the best of the tested models. With increased mesh density and longer time sampling accuracy could be improved further, however with penalties to computational costs.

Contents

1 Introduction

Predicting the behavior of flow by solving mathematical equations enclosed by boundary conditions is the potential usage of commercial based CFD solvers. The user has the freedom to choose the mathematical models in order to capture the necessary flow physics to overcome the issue. If higher detailing is demanded, then a turbulence model which can resolve the turbulent flow structures in transient state is needed. Several turbulence models have been developed aiming to solve more physics or solve necessary physics at cheaper computational power. Where different models resolve different ranges of turbulent structure sizes, so called length- or energy scales. In this study, a benchmark case of periodic hill flow is subjected to Scale Resolving Simulation (SRS) technique which includes SAS and DDES. The performance of these two models was studied and discussed along with RANS and URANS. The parameters used to investigate the performance was velocity and reynold stress profiles, as well as mean Q-criterion and wall shear stress on the flat bottom. The geometry of the periodic hill can be seen in figure 1. The mesh consisted of a fully quad-mesh, which was verified using a RANS simulation. Additionally the two-point correlation for the x-velocity at two lines was investigated, with data obtained from the DDES simulaiton, to further verify that the mesh was sufficiently fine.

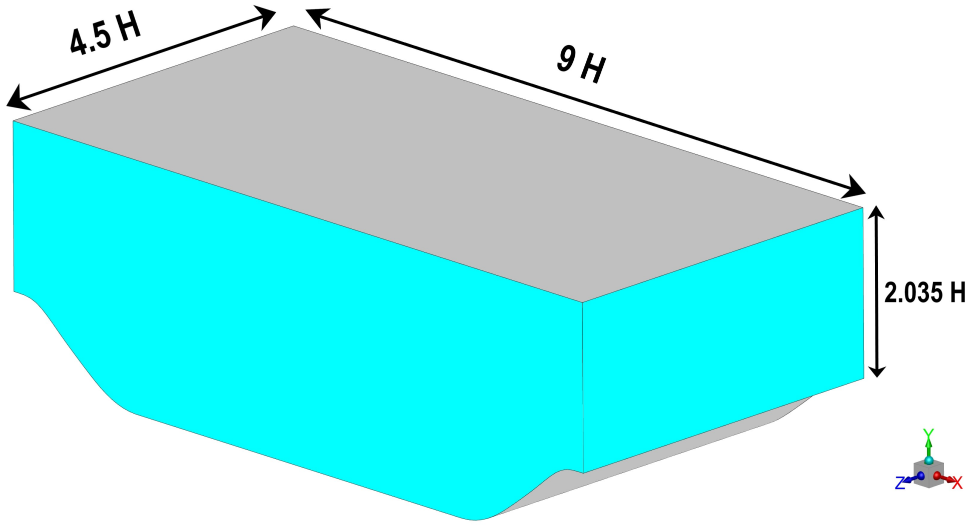


Figure 1: Periodic hill flow model used for the study, $\mathrm{H} = 1[\mathrm{m}]$

2 Method

In this section the simulation setup that includes meshing, fluid domain, and solver setup are presented.

2.1 Solver setup

In this section the solver setup used in Ansys Fluent is presented.

2.1.1 Steady state setup

As a matter to verify the mesh resolution a RANS simulation was carried out. A $k-\omega$ SST model was used for the RANS simulation with boundary conditions, fluid domain parameters and solution methods specified in table 1. Coupled PV coupling was used for faster convergence as SIMPLE was not time efficient. Second order upwind was used for spatial discretization for better accuracy.

2.1.2 Transient state setup

The transient simulations were carried out for URANS, DDES and SAS turbulence models. The fluid domain parameters and solution setup used can be seen in table 1. Default under relaxation factors were used to ensure faster convergence. RANS solution was used as an input for URANS simulations in order to reduce computation. Hybrid initialization was used for DDES and the SAS model was initialized with the solution of DDES. It was ensured that the initialization effects were removed for all turbulence models.

After the removal of initialization effects the time step was reduced to attain an average Courant-Freidrich-Lewy (CFL) number of ~ 1. For the URANS model there were no noticeable fluctuations in the instantaneous velocity, rather it was spiking over the time for URANS model. Considering that the URANS being unable to handle the unsteadiness, and thus giving a non-physical solution, it was not carried for further studies. However, DDES and SAS showed the expected chaotic fluctuations within the same range of amplitude, which signifies that the flow was stabilized in the domain and data sampling was turned on by selecting the necessary parameters uploaded with custom field functions for time averaging. The parameters include Reynolds stresses, raw-Q criterion, and SFC. A time step of 0.012 gave a CFl ~ 1 in 99% of the domain, hence this was used for DDES and SAS simulation. DDES results were captured after 15 and 30 flow cycles as the fluctuations of the monitored mean components were small enough. The time averaged velocity components and Reynolds stresses normalised with bulk velocity were compared to the LES data of Breuer et al.[2]. The bulk velocity is the area weighted average u-velocity component at the inlet of the hill. In order to visualize and compare the turbulent structures Q-criterion contour colored with velocity magnitude were used for a iso-value of 0.1 for all turbulence models [?].

Boundary Conditions		Simulation Parameters		Solution setup	
Top wall	**No Slip**	**Parameter**	**Value**	**Parameter**	**Method**
Hill	No Slip	Re_h h = 1[m]	10,400	PV coupling	DDES, SAS - Fractional step RANS, URANS - Coupled
Side- stremwise	Periodic	Dynamic Viscosity	1.85e-5 [Pa. s]	SD - Gradient	Least square cell based
Side- spanwise	Periodic	Density	0.195 Kg/m^3	SD - Pressure	Standard
		Pressure drop	-0.002262 [Pa/m]	SD - TKE SD - SDR	2nd order Upwind
				SD - Momentum	DDES,SAS - Bounded Central Diff. RANS, URANS - 2nd order upwind
PV - Pressure Velocity, SD- Spatial Discretization, *NITA - Non Iterative time Advancement*				Transient Formulation	SAS,DDES - 2nd order implicit NITA

Table 1: Simulation set up used for DDES, SAS simulations. note: Boundary condition and simulation parameter is same for all simulations.

In order to carry out a statistical measure of the length of an eddy and further verify the mesh, two vertical line was introduced in the domain stretching 19 cells. The instantaneous x-velocity was imported for every time step for determining the two point correlation in accordance with Gårdhagen [1]. This is a mathematical way of determining the spatial length of an eddy based on the fluctuating flow parameter(i.e. x-velocity).

2.2 Meshing

The given geometry of the hill was imported to Ansys meshing tool ICEM CFD. A hexahedral mesh was generated as it is computationally cheaper when compared to tetrahedral. By making use of blocking a higher mesh density was maintained around hill regions, this was similar to the mesh used by Breuer et al.[2]. The generated mesh can be seen in figure 2. In order to efficiently capture the boundary layer features the recommended first cell thickness had to be achieved. The first layer height of $\Delta x^+ < 40 - 100$, $y^+ \sim 1$ and $\Delta z^+ < 15 - 30$ in respective directions are the requirements for capturing wall bounded flow characteristics. Considering the Ansys academic node count limit of 512k nodes, the obtained mesh was within the limit. Node count used was $[X, Y, Z] = [88, 110, 52]$.

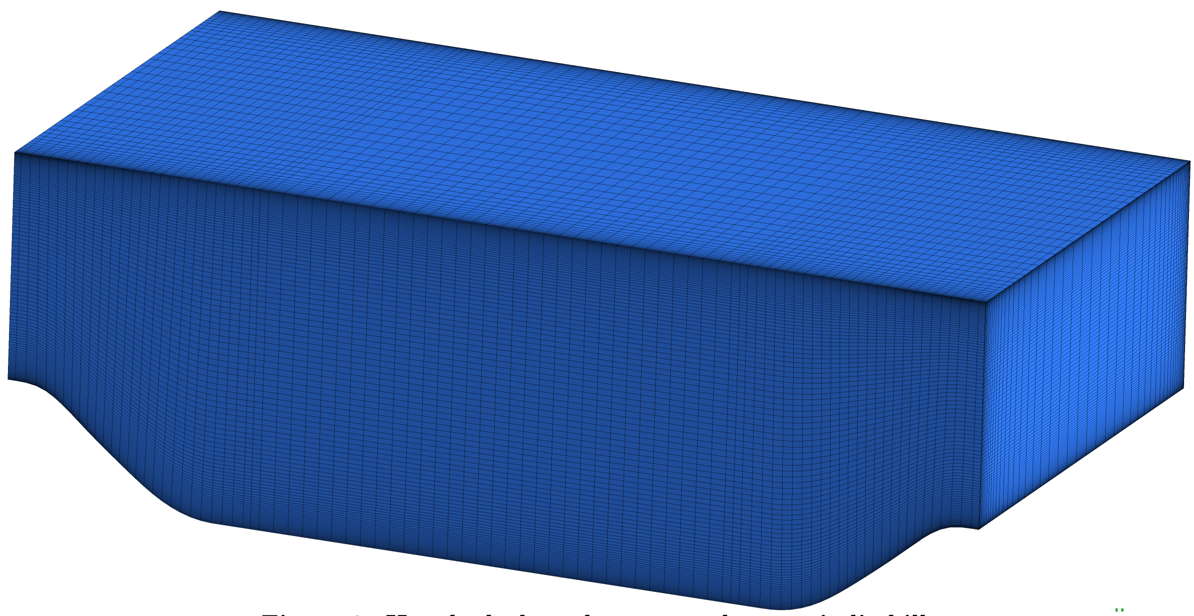


Figure 2: Hexahedral mesh generated on periodic hill.

2.2.1 Mesh Verification

The mesh was initially verified using the steady state RANS simulation. The concerned parameters were $\Delta x^+, y^+, z^+$ and ratio of the integral length scale (L_0) to Delta (Δ), where Δ is the (cell volume)$^{1/3}$ [3]. A rule of thumb for LES mesh is to achieve $L_o/\Delta < 5$, meaning a turbulent structure fits inside a 5x5x5 mesh structure. Thus, wherever the ratio is larger than 5, the structures fit inside a cube with sides larger than 5. Therefore the mesh is then likely to resolve 80% of turbulent kinetic energy (TKE) in the domain.[4] The integral length scale ratio on a 2D-slice of the domain can be seen in figure 3.

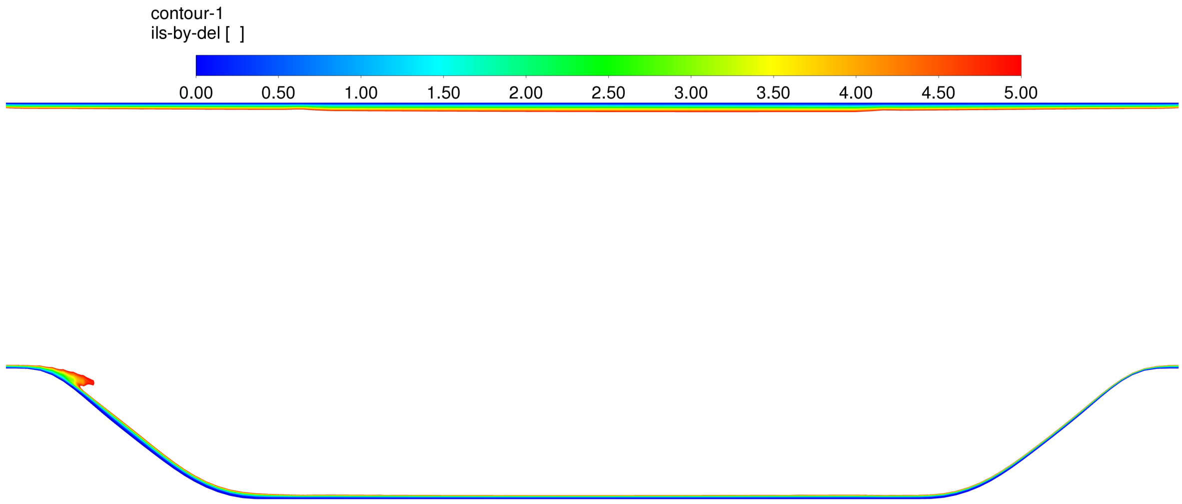


Figure 3: L_0/Δ **clipped** $0-5$ **contour on periodic hill and top wall.**

The $0 < L_0/\Delta < 5$ ratio contour was clipped to determine the turbulence resolving capability of the mesh. Figure 3 showed that mesh was satisfactory and capable of capturing $> 80\%$ of the TKE. In order to ensure that the boundary layer was captured, the $y^+ \sim 1$ on the majority of the hill and on the top surface was achieved, this can be seen in figure 4. Also the spatial discretization in the x and z-direction was investigated using Δx^+ and Δz^+ contours. Which show that the values are within the limits, in-fact they are lesser than the prescribed value. This can be seen in figure 5.

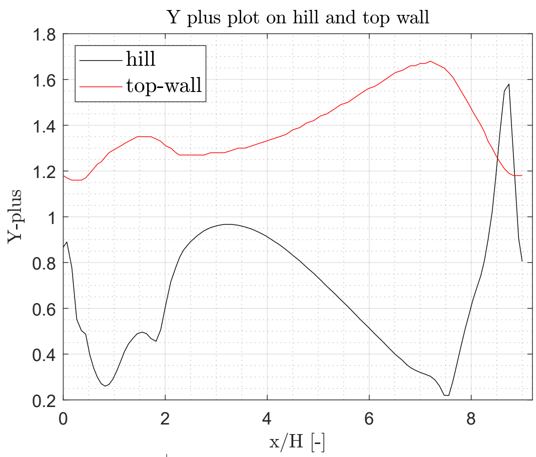
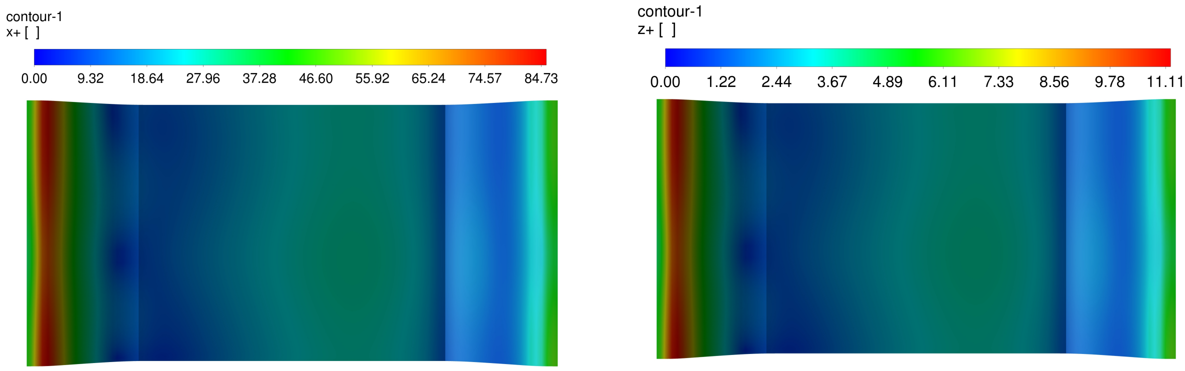


Figure 4: y^+ 1 **plot on hill and top surface.**

The metrics above proved the mesh sufficient to continue with the transient simulations. To further verify the mesh, a two-point correlation investigation was made along two vertical lines at $x/H = 1.5$ and $x/H = 1.7$. This was made using data from the DDES simulation for 15 flow cycles. The two-point correlation shows how correlated the x-velocity component at different



(a) Δx^+ **on the hill.** **(b)** Δz^+ **on the hill.**

Figure 5: Contours obtained after RANS simulation and used to study the resolution of mesh.

nodes along the line is to the first node, or the reference node. If the correlation is high, it means the flow in that cell has a similar behaviour to the first cell. It is thus beneficial to have the same flow structure through as many cells as possible, in order to properly resolve the structure. The resulting two-point correlation can be seen in figure 12. A standard value for determining de-correlation is 0.2 [5], meaning the flow is de-correlated in the node when the correlation value falls below 0.2.

3 Results

3.1 Performance Comparison

The figures 6, 7, and 8 show the velocity u, v profiles and reynolds stresses u′u′, u′v′, v′v′ at x/H = 0.5, 2, 4, 6 normalised with the bulk velocity (U_B). DDES I and II are the results obtained after 15 and 30 flow cycles of data sampling respectively.

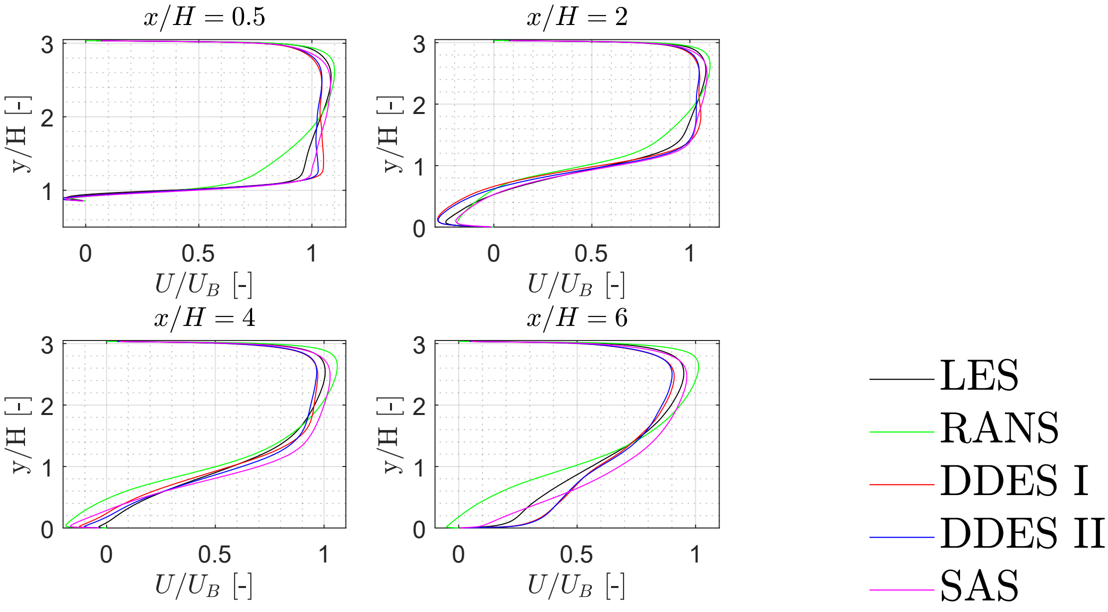


Figure 6: Velocity u component at different x/H locations

From the figures it is clear that the RANS lacks accuracy with most of the parameters. However, RANS shows quite accurate results away from wall at some points for some parameters, such as u-velocity at x/H = 4, see figure 6, or v-velocity at x/H = 6, see figure 7. Though the u- and v-velocity profiles from RANS shows some good agreement with the LES data at x/H = 2 and 4,

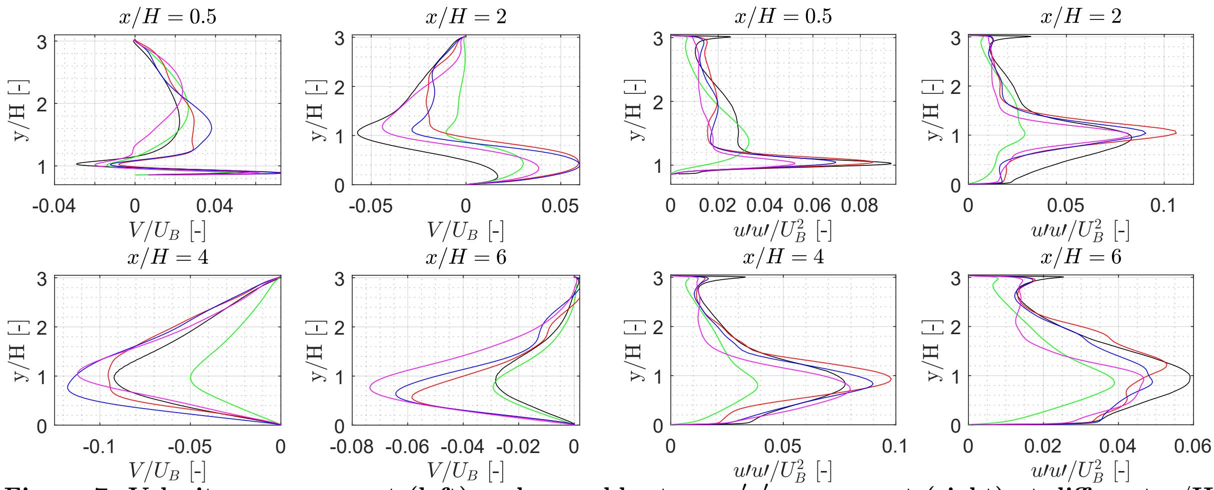


Figure 7: Velocity v component (left) and reynolds stress u′u′ component (right) at different x/H locations.

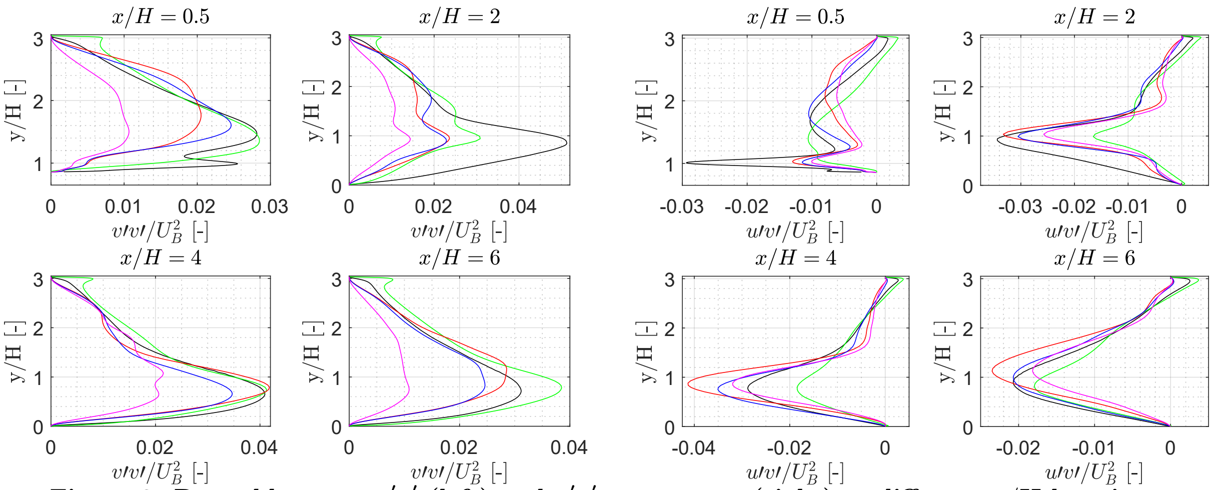


Figure 8: Reynolds stress v′v′ (left) and u′v′ component (right) at different x/H locations.

it under-predicts the values for other flow characteristics. The v-velocity profiles show quite good agreement, except at x/H = 2 and 4 where the velocity is overpredicted, see figure 7. The u′u′ component is in general underpredicted, while the v′v′ fluctuates from under- to over-predicting, see figure 8. The u′v′ component overpredicts slightly. Some results are thus somewhat accurate, but in general the predictions are not accurate.

DDES results captured after 30 flow cycles had better accuracy. The u-velocity profile in general is in good agreement with the LES data with minor lack of accuracy at x/H = 2 and 6. However, the v-velocity is overpredicted at x/H = 0.5 and 2, but under-predicted at the successive points. The obtained Reynolds stress u′u′ is mostly under predicted except at x/H = 2. The v′v′ is under predicted at all positions, also the u′v′ component at x/H = 0.5 and 2. The results of u′v′ are closer to LES data at x/H = 6.

As DDES gave better results after 30 flow cycles, data from SAS was also captured after 30 flow cycles. The velocity u-component is almost in a good agreement with the LES data, but it fails to capture minor details in the near wall region. The v-velocity component is overpredicted at x/H = 0.5 and 2 and underpredicted at the successive points. The Reynolds stress u′u′ is underpredicted at x/H = 0.5 and 6, but they are quite closer to LES data at other positions. The u′v′ profile is underpredicted at all positions, wherein v′v′ is overpredicted at x/H = 0.5, 2 and 6.

Figure 9, 10 shows the mean velocity of the vortex structures. RANS does not show much in terms of vortex structures, which is quite evident as it is not meant to resolve any turbulence.

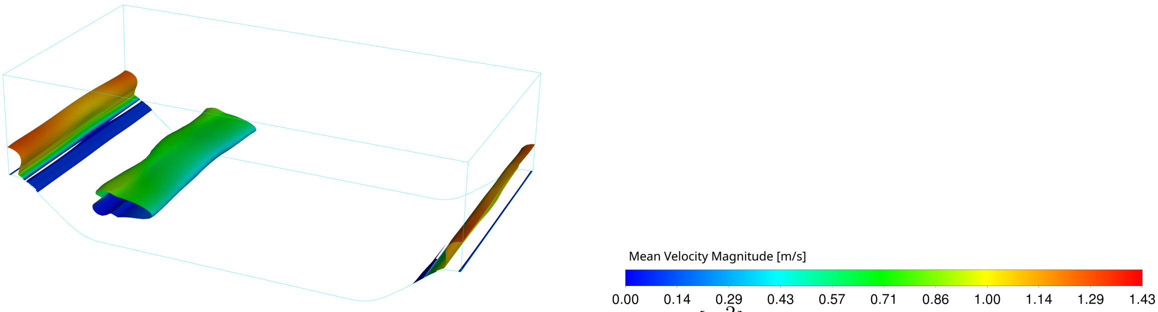


Figure 9: Q-criterion with iso value of $0.1[s^{-2}]$ for RANS model.

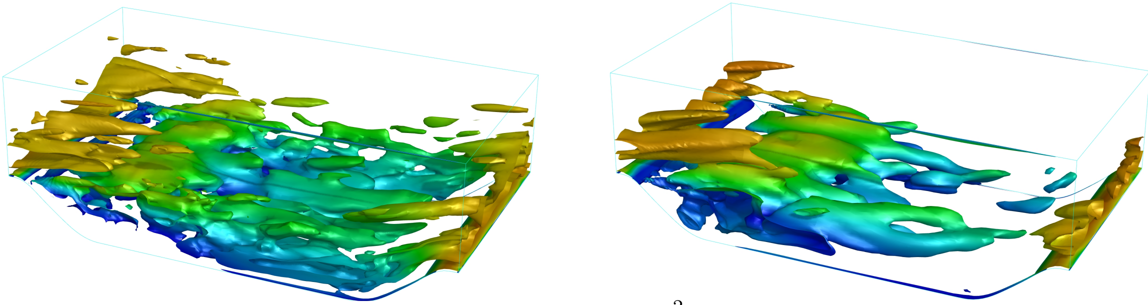


Figure 10: Q-criterion with iso value of $0.1[s^{-2}]$ for DDES and SAS model.

However, DDES shows more connected vortex structures in comparison to the SAS model. DDES has more number of smaller structures in comparison to the SAS, which signifies more turbulence being resolved.

A comparison of mean skin friction coefficient on the valley can be seen in figure 11. It is clearly visible that they show different characteristics, however the peak at x/H = 3 is almost the same for SAS and DDES which belongs to the re-circulation region. RANS is underpredicting the peak compared to the other models, still the nature of curve is similar to that of SAS. The results from DDES shows a different behaviour from x/H = 4.5 to x/H = 6.8, and in general a more detailed behaviour.

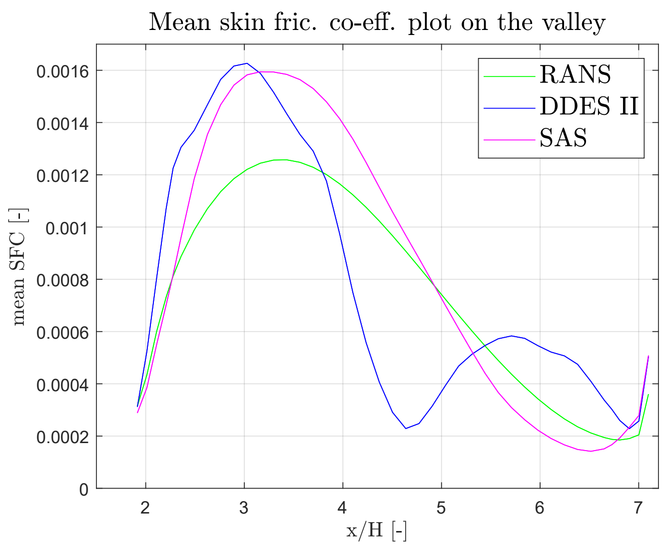


Figure 11: Mean skin friction coefficient on the valley.

Figure 12 shows that at x/H = 1.5 the flow de-correlates first at 4 nodes. Afterwards two correlated structures appear, which each de-correlate after 4 and 3 nodes, respectively. The structures follow a clear pattern of becoming less and less correlated at their peak when moving further away from the reference node. The structures indicate eddies which are entirely captured within 3-4 nodes. At x/H = 1.7 the correlation does not show any additional smaller eddies, and instead

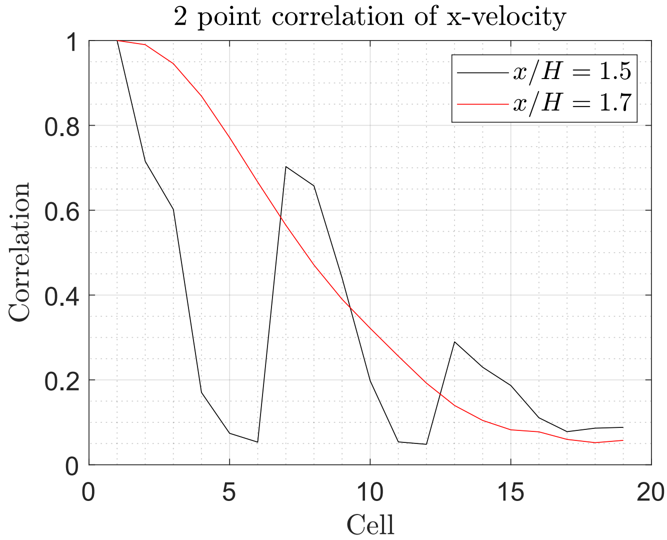


Figure 12: Two point correlation on a vertical line stretching 19 cells.

shows a smooth decline which de-correlates after 12 elements. Indicating that the flow structures is well captured within a large amount of nodes at x/H = 1.7.

4 Discussion

4.1 Mesh Verification

Resolution of mesh impacts the accuracy of results for the spatial discretization models, better the mesh more the resolved turbulence. Considering the separation and reattachment of flow the mesh density was increased at the hills, as seen in figure 2. A uniformly distributed high resolution mesh is always a better practice in general, but due to the limitation of cell nodes the mesh done was considered acceptable. The first cell thickness obtained were all within the limit, except for y^+ being slightly above 1 on the top wall, see figure 4. This was acceptable as the Δx^+ and Δz^+ compensates due to being lower than the prescribed limit. Breuer et al.[2] had similar issue and it did not affect their results negatively. The low y^+ is required as the SRS models use $k-\omega SST$ in the boundary layer. The L_0/Δ contour shows that mesh was capable of resolving most of the turbulent kinetic energy in the domain. With these two observations the mesh was considered to be satisfactory. However the low resolution of the mesh in the mid section could have some impact on the accuracy of the results. Another possible improvement is to have an equal spacing along the z-direction, or along the thickness of the domain, see figure 1. Due to the periodic boundary condition there is no need for the spacing, however, it was a mistake noticed too late to change. The mesh was still able to be verified, but this is one area of potential improvement without adding additional computational costs.

The correlation investigation showed that the mesh at x/H = 1.7 was good in terms of resolving the turbulent structures, as the x-velocity only de-correlates after 12 elements. Which when compared to Davidson's [3] recommendation of at least 8 elements for a coarse LES is rather satisfactory. However, the correlations at x/H = 1.5 shows that the first structure de-correlates after 4 elements, which falls short of the recommendation. Due to time constraints the mesh could not be refined enough to increase the element count until de-correlation. Thereby this potential cause for error remains and is likely a contributing factor to the deviations from the LES data which can be seen throughout the results, see section 3. According to figure 10 the mean vortex structures for DDES, which have the smallest structures, are slightly smaller near the inlet hill than the downstream hill. Thus it could be argued that the correlations should be lower at increasing x-positions.

4.2 Method

URANS setup was similar to that of RANS, however it was not capable of handling the flow case. Thus, the URANS gave nonphysical results, where the velocity kept increasing linearly. Therefore no URANS results were presented. The reason for this is likely that some poor modeling in the RANS part of the URANS model created a positive feedback loop over the periodic boundary conditions.

Transient cases are generally time consuming, a higher CFL used initially was to ensure the flow being developed in the entire domain. The time step size was lowered such that a maximum CFL of ~ 1 was obtained. Since a CFL above 1 can lead to incorrect solutions [6] the CFL was closely monitored. Due to the fluctuations in the flow, however, the maximum CFL sometimes peaked slightly above 1. Though as the max CFL was monitored, it would only be a very small region experiencing a CFL momentarily above 1, while the vast majority of the domain kept a CFL well below 1.

The simulations were run until the mean values of u-, v-, and reynolds stress-components at points on the $x/H = 0.5,\ 2$ and 6 lines showed sufficiently small fluctuations. The results of the DDES after 15 flow cycles was compared to that after 30, where both showed small fluctuations for most mean values. It can be seen in figures 6, 7, and 8 that the difference between DDES I and II is quite small, despite the latter being run for twice as many flow through cycles. For some parameters and position, for example $v'v'$ at $x/H = 4$ and 6, the DDES I shows more accurate results than DDES II. This could be due to structures with very low frequency being present in both, and thus over represented in DDES I, which then influence the time averaging greatly. Thus a larger amount of flow through would be needed for time averaging, which is coinfirmed by Breuer et al. [2] which used 140 flow through cycles. However, in the interest of time this was not possible to do, and 30 flow cycles was considered sufficient.

4.3 Performance Comparison

RANS is powerful and heavily used in the industry as it is computationally cheap and time efficient. However, it comes with certain drawbacks. It is limited to steady state, and only models turbulence i.e. only exhibits the existence of turbulence in the domain. RANS is the widely used model that solves the time averaged Navier Stokes equation by ignoring the time derivative term. In order to attain the closure property Bousinesq hypothesis is used which models the turbulent fluctuations. URANS is in contrast to the RANS models where U stands for unsteady state which includes the time derivative term in the N-S equation. As the periodic hill flow is completely turbulent, RANS approach is not a reliable option. This is proved by the Reynolds's stress components being underpredicted in most of the cases, referring to figure 7 and 8. The u-velocity profile offers closer results, refer figure 6 due to the presence of unsteadiness in the flow.

On the other hand, URANS is useful in the cases of periodic oscillations in a turbulent flow, however it cannot resolve the incoherent flow structures in complex flows [7]. It is only capable of resolving low frequency fluctuations.

This brings the need for the blend of Large Eddy Simulations (LES) and RANS models widely known as hybrid LES-RANS model. One of them is the Detached Eddy Simulation model. In order to balance the required accuracy and computational time this model resolves the turbulence production and separation using LES away from the wall in the largest part of the domain, and an eddy viscosity URANS model is used to model the near wall region. The DES model is a single equation model which has a drawback called the grid induced separation which causes a reduction in the skin friction and leads to early separation. In order to eliminate this problem, a two equation model is used in the near wall region which uses the molecular and turbulent viscosity information for switching mechanism, so that even in the case of grid space being smaller than the boundary layer the entire boundary layer is modeled by URANS [8]. The deviation in the turbulent stress profiles, referring figure 7 and 8, was due to the poor mesh and lack of flow cycles. Also, there might be chances of attaining better results with usage of other solver methods for discretization which can be subjected to solver dependent studies.

Scale Adaptive Simulation (SAS) also known as 'Improved URANS' was the other hybrid LES-RANS model included in the study. This model can adjust to the resolved structures in the domain via source terms. Unlike URANS there is no Reynolds averaging defined in the equation, rather the details of the turbulence spectrum is formulated by the eddy viscosity. An unstable flow condition is mandatory for this model so that the SAS formulation feeds eddy viscosity into the domain which breaks large turbulent structures [9]. A major drawback of the model is that if the flow has not reached a certain point of instability, it will not switch into scale resolving mode. Also, there is no mathematical relation that can be used to find the level of flow instability.[9]. The velocity profiles are somewhat similar to the DDES results, however the reynolds stresses seen in figure 8 were under-predicted. This could be due to the mesh, lack of flow cycles, or sufficient time was not given for the flow to attain unstable condition. Though the SAS was time averaged for the same number of flow cycles as DDES, this shows that the SAS model takes longer time to attain a good solution, compared to DDES. Hence, DDES is better compared to the SAS turbulence model, a similar conclusion was made by Davidson[10].

The bulk velocity obtained for all the models had some notable difference and as the skin friction coefficient is directly proportional to velocity, this could be the reason for the deviation in results as seen in figure 11. DDES showed higher capability of resolving vortex structures as it contains more number of small discretized structures when compared to the SAS, refer figure 10. Mesh plays a key role in this factor and letting more time for flow to be unstable for SAS would have provided better vortex structures.

5 Conclusions

In conclusion the RANS model proved to be incapable of capturing the unsteady nature of the flow, while the URANS model could not handle the flow case and gave un-physical results. The DDES model showed somewhat more accurate results, which in general agreed more with the LES data. The SAS model was quite similar to the DDES model for the u-velocities, however, severely under-predicted the reynolds stresses. It also showed quite poor resolution in terms of vortex structures when compared to the DDES model. Thus the DDES model was considered superior to the SAS model for this case. The main limiting factor for accuracy was the mesh, although it was verified using y^+, Δx^+, and Δz^+, see figures 4 and 5, and the integral length scale ratio, see figure 3. It was still unable to de-correlate the u-velocity near the peak of the hill within the recommended number of elements, see figure 12. Meaning flow structures potentially not resolved properly propagates through the domain, causing erroneous effects. The number of flow through cycles during the time averaging was also limited by the mesh, due to computational time being a limiting factor. Though the difference between 15 and 30 flow cycles gave quite similar results, the q-criterion, see figure 10, shows that more cycles would be needed. As the flow should be entirely homogeneous along the width of the domain, due to the periodic conditions. Thus, a finer mesh and more computational power or time would be needed to obtain even more accurate results.

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