

CSD2341

Computer Graphics Programming



Area Fills

Third Edition

- Hearn and Baker 3.15
 - polygon classifications
 - Inside-outside tests
- Hearn and Baker 4.10-4.13

Fourth Edition

- Hearn, Baker & Carithers 4.7
 - polygon classifications
 - Inside-outside tests
- Hearn, Baker & Carithers 6.10-6.13

Math review: Polygons

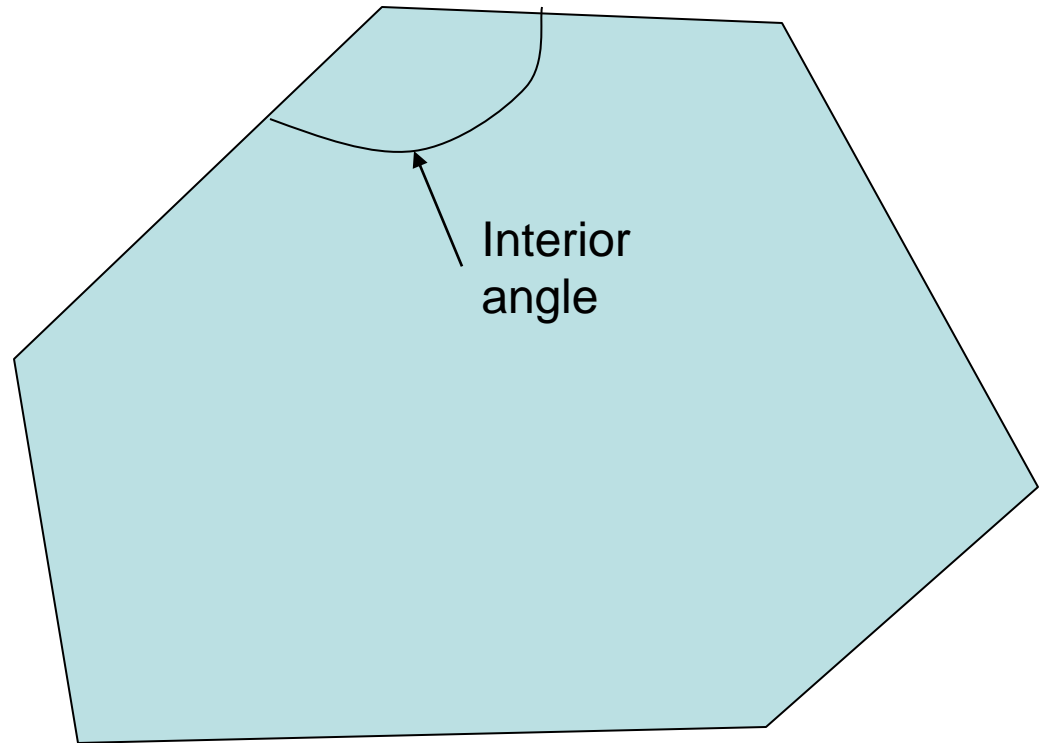
- A **polygon** is a shape defined by joining successive pairs of points (**vertices**) in a plane (including joining the last vertex to the first)
- Each successive pair of vertices is joined by a line segment – an **edge** or **side**.
- Your textbook calls this a **closed polyline**, and requires that a polygon does not intersect itself (but this is not always part of the definition).

Math review: Polygons

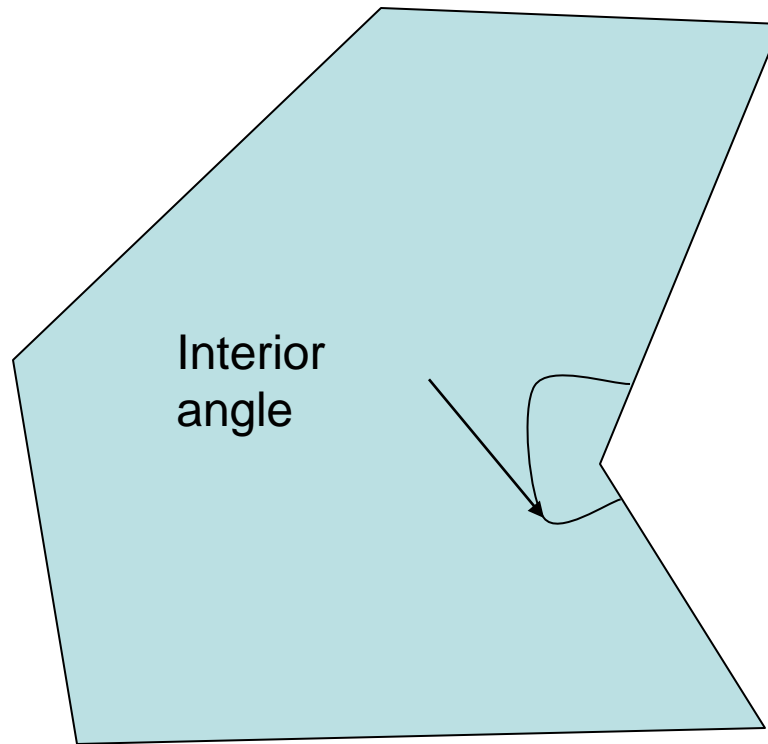
- Polygons divide the plane into three parts
 - The outside or **exterior**
 - The polygon itself
 - The inside or **interior**
- In graphics, we use polygons to define areas to be filled with a colour or pattern – we fill the interior
- In 3D graphics, we often use connected networks of polygons to describe the surface of an object.

Math review: Convex Polygons

- All interior angles $< 180^\circ$



Math review: Concave Polygons



Splitting concave polygons

- Lots of graphics algorithms only work on convex polygons.
- Therefore, concave polygons need to be broken down into a number of convex ones.
- Sometimes, polygons are even split into triangles – triangles are very easy and fast to calculate with.
- Text book discusses methods for doing this – read it, but this part is not examined.

Inside/outside tests for polygons

- Some graphics algorithms need a way to tell whether a point is inside or outside the polygon
- This needs to work for polygons with self-intersections too.
- There are a couple of different methods to determine this:

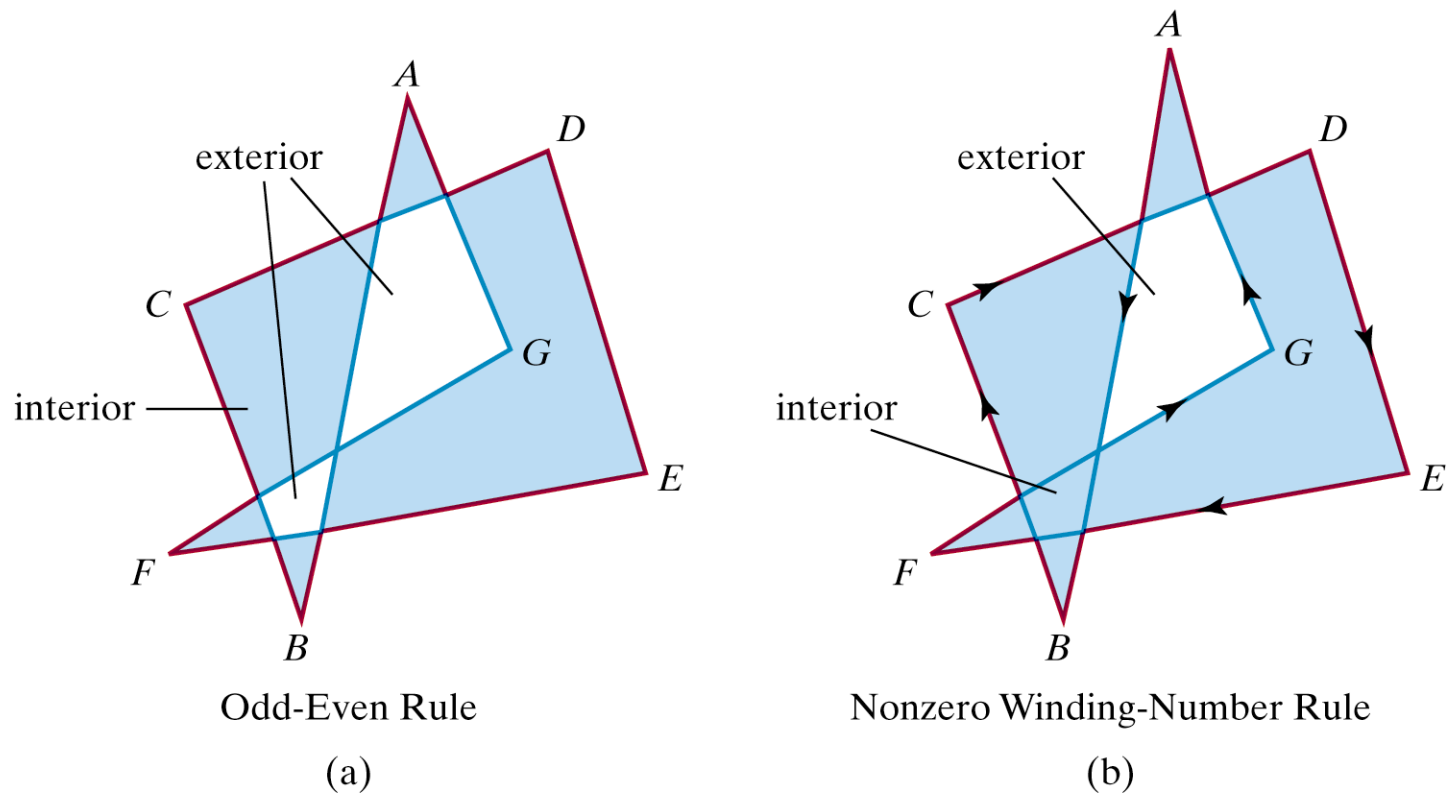


Figure 3-46

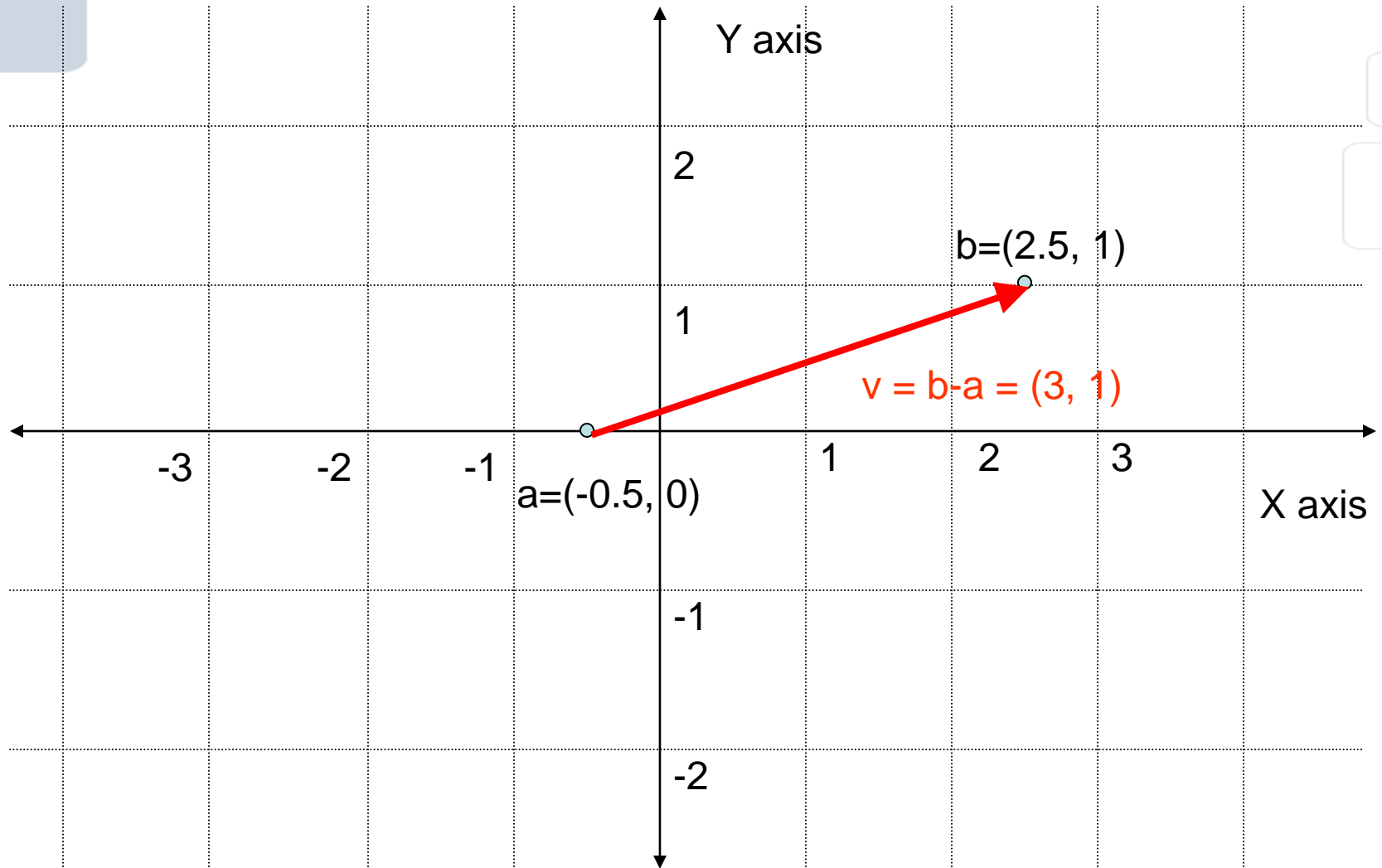
Identifying interior and exterior regions of a closed polyline that contains self-intersecting segments.

Working out directions for the winding number method

- We need to work out which direction each edge crosses the reference vector
- Two methods:
 - Using vector cross product
 - Using vector dot product

Math review: vectors

- A vector is the difference between two points
- Think of it as an arrow starting at one point and ending at the other.
- See Appendix A-2

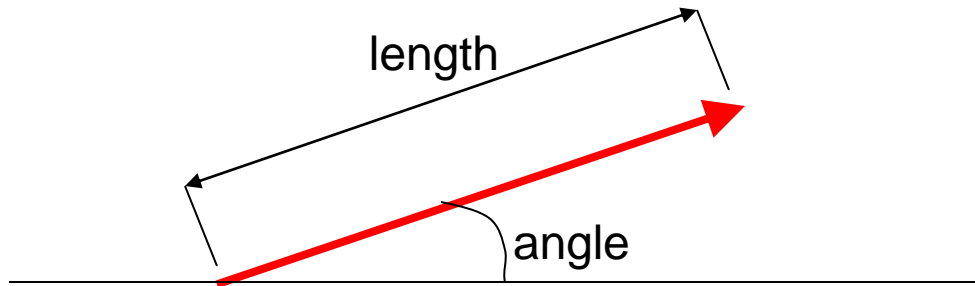


Vector calculations

- Vectors have coordinates e.g. (3, 1) means 3 units in the x-direction and 1 unit in the y-direction
- To calculate coordinates, just subtract the corresponding coordinates of the two end points.
- $(2.5, 1) - (-0.5, 0) = (2.5 - (-0.5), 1 - 0) = (3, 1)$

Vector calculations

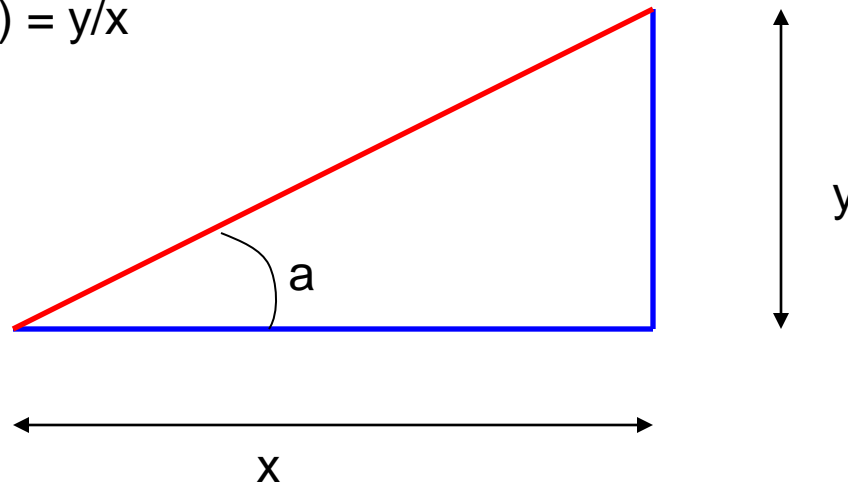
- Vectors have a direction and a length
- If $v = (x, y)$
 - Direction: angle = $\arctan(y/x)$ (in radians)
 - Length = $\sqrt{x^2 + y^2}$



arctan?

- What's arctan? First, remember what tan (short for tangent) is:

$$\tan(a) = y/x$$

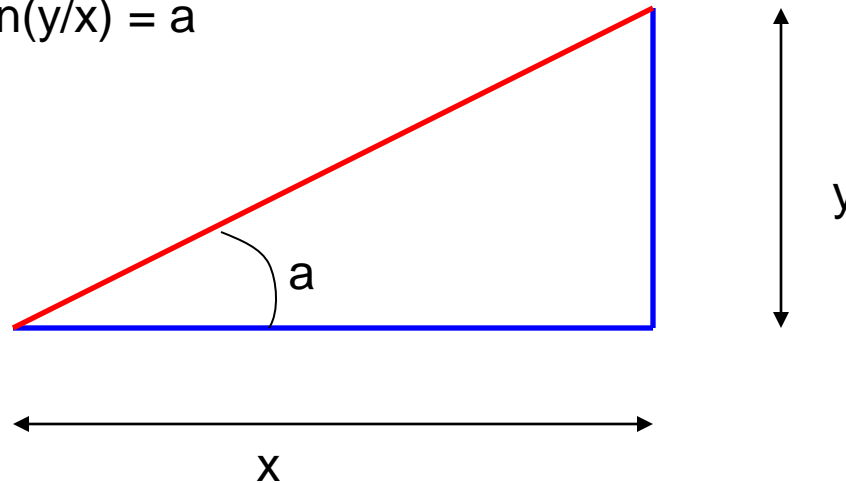


- That is: the tan of the angle “a” is y/x , i.e. the slope of the red line.

arctan?

- Another way of saying this is

$$\arctan(y/x) = a$$

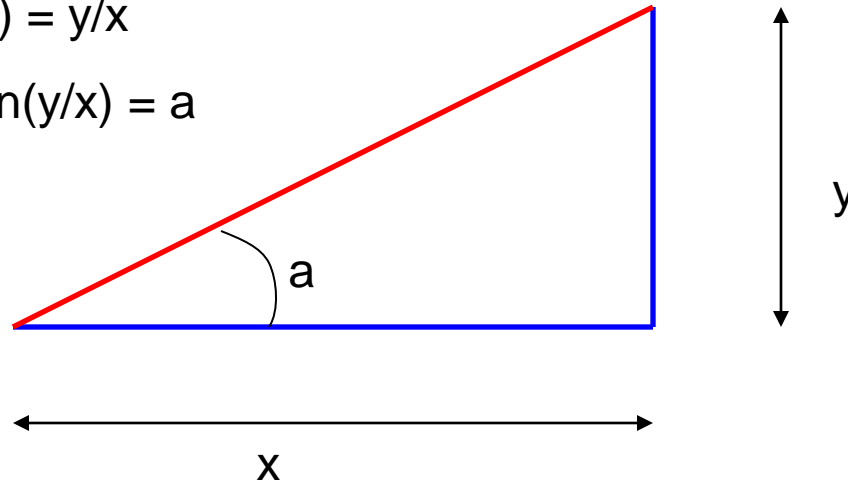


- That is: the angle that matches the slope of the red line (has a tan of y/x) is “a”.

arctan?

$$\tan(a) = y/x$$

$$\arctan(y/x) = a$$



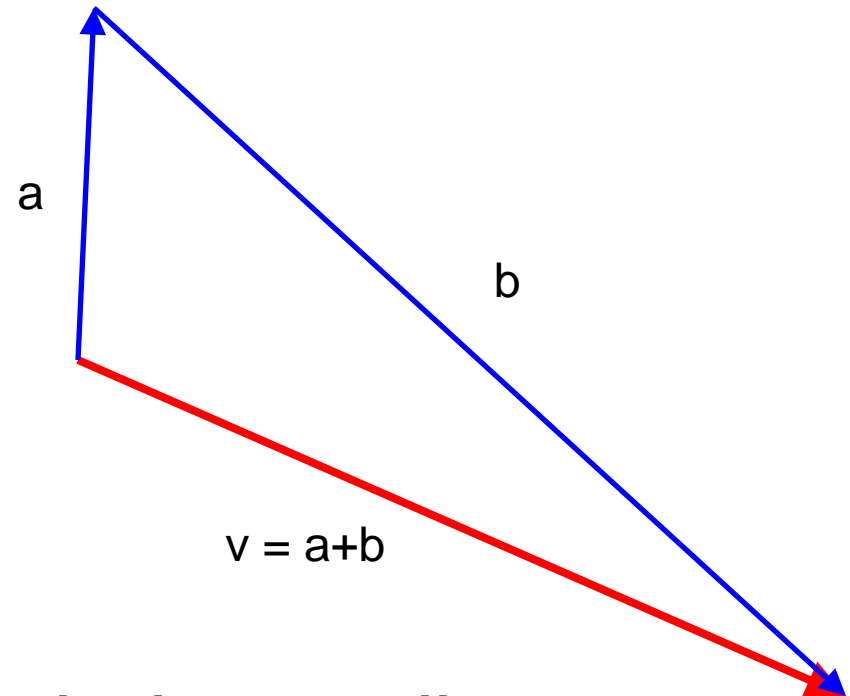
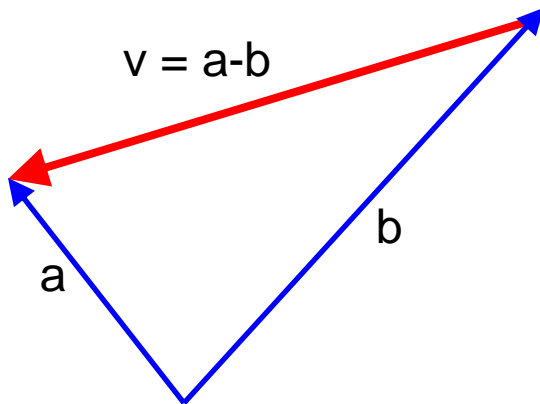
- Note that in these equations, a is measured in **radians**
- Radians can be converted to degrees:
 - Degrees = $180 \times \text{radians} / \pi$
 - Where π is about 3.1416...

arctan in Java

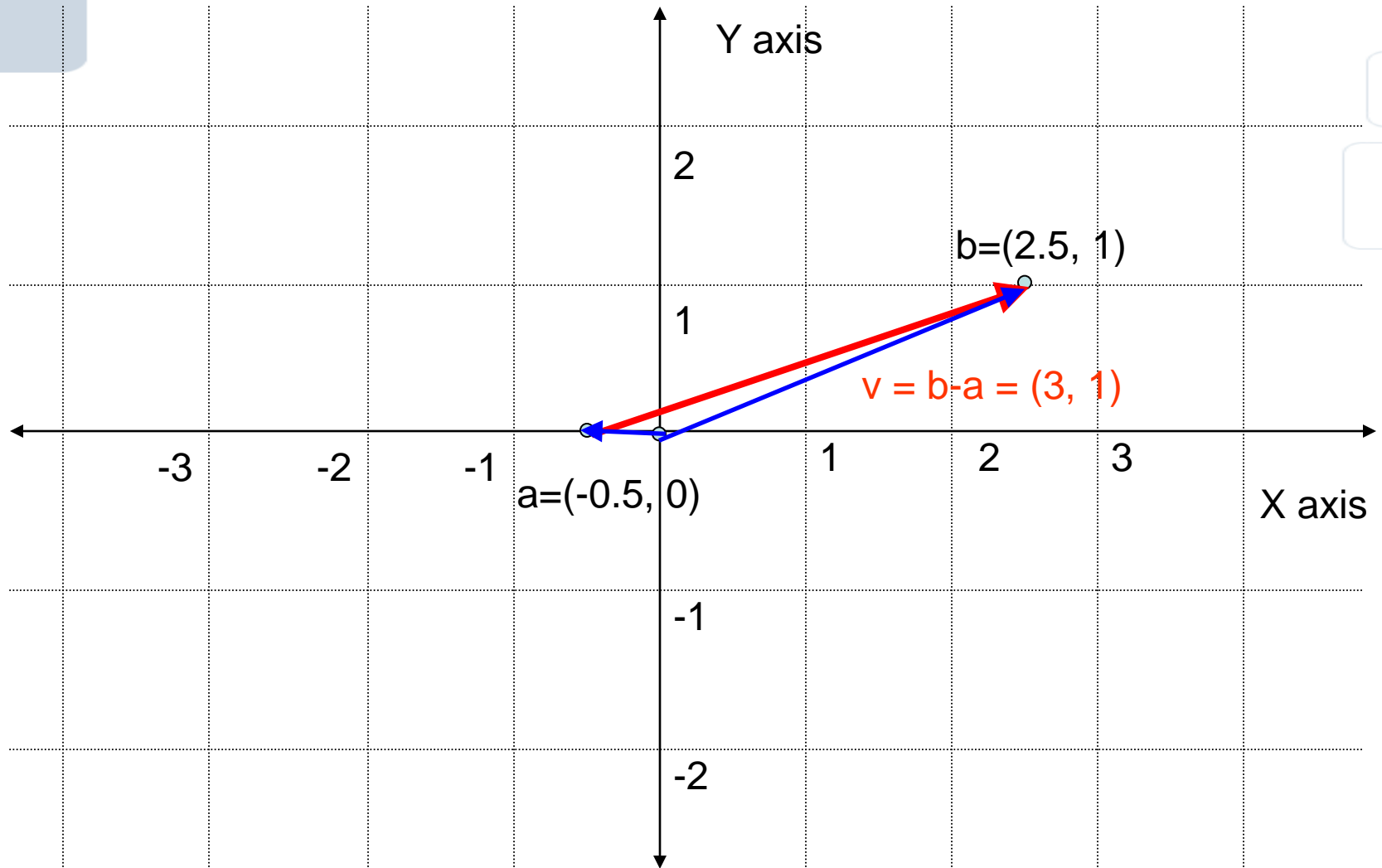
- To convert radians to degrees:
 - `Math.radiansToDegrees(x)`
- To convert degrees to radians:
 - `Math.degreesToRadians(x)`
- To get π :
 - `Math.PI`
- To calculate tan:
 - `Math.tan(x)` – if x is in radians
 - `Math.tan(Math.degreesToRadians(x))` – if x is in degrees
- To calculate arctan:
 - `Math.atan2(y, x)` – the result will be in radians

Vector calculations

- Vectors can be added and subtracted

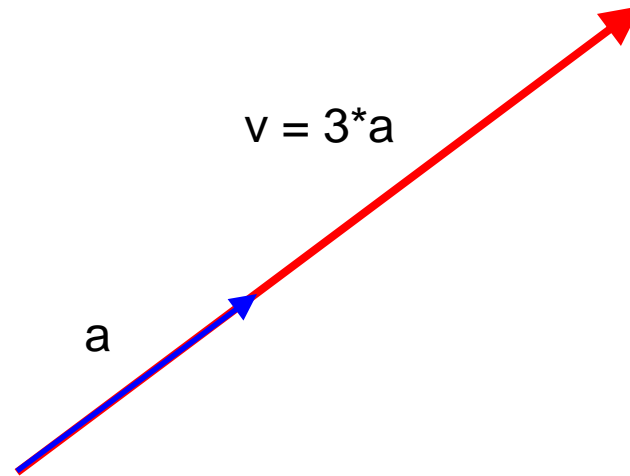


- Just add or subtract their coordinates



Vector calculations

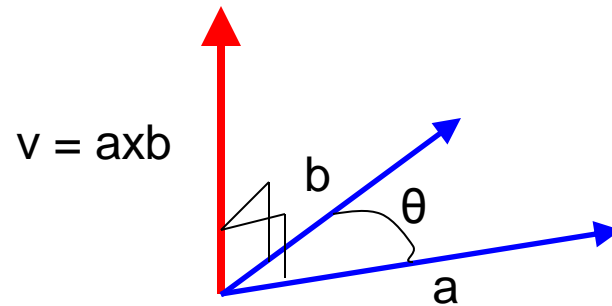
- A vector can be scaled



- Just multiply its coordinates by the scalar

Vector cross product

- Can be applied to two vectors in a plane
- The result is a vector pointing at right angles into or out of the plane (so it is a 3D vector)



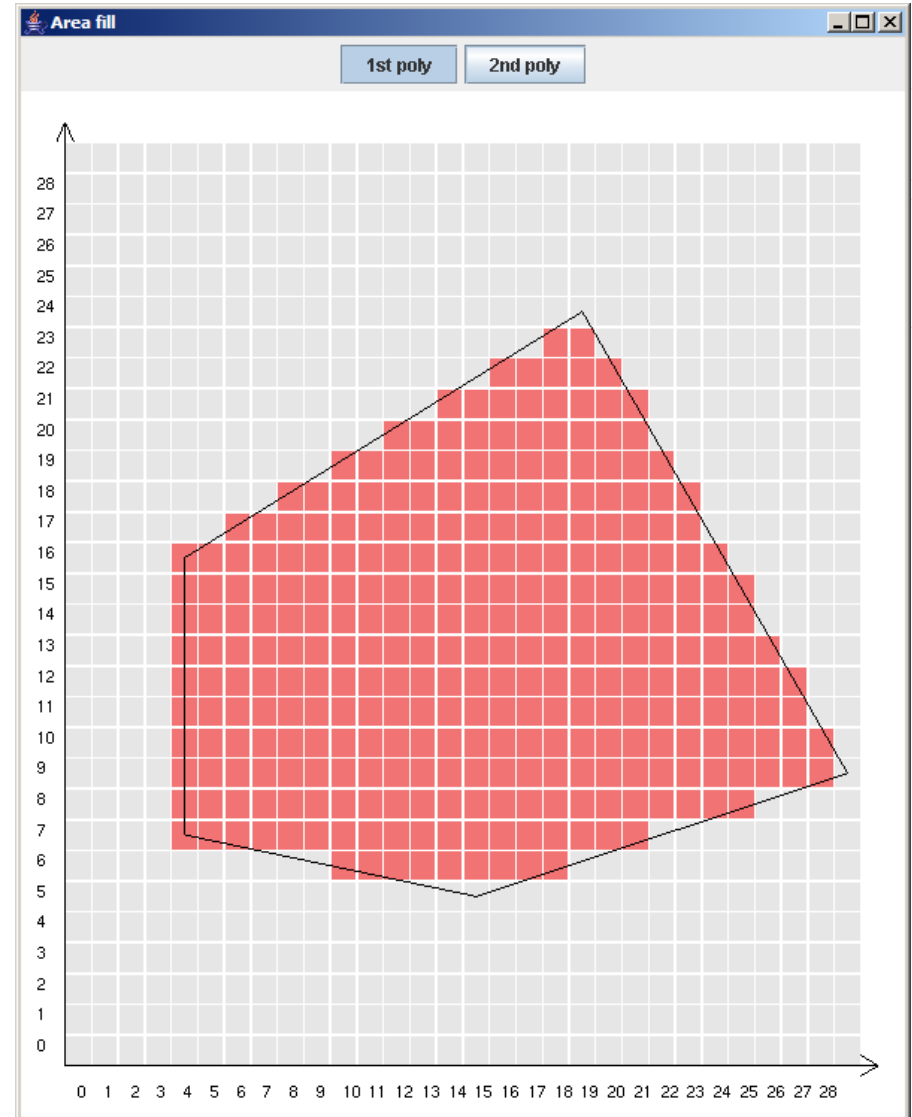
- Formula
 - $v = a \times b = u * |a| * |b| * \sin(\theta)$, where u is a unit vector whose direction is determined by the right-hand rule
 - $V = (a_2 * b_3 - a_3 * b_2, a_3 * b_1 - a_1 * b_3, a_1 * b_2 - a_2 * b_1)$

Vector scalar product

- Also called dot product, inner product
- Formula
 - $a \cdot b = |a| * |b| * \cos(\theta)$
 - $a \cdot b = a_1 * b_1 + a_2 * b_2 + a_3 * b_3$
- Now back to polygons....

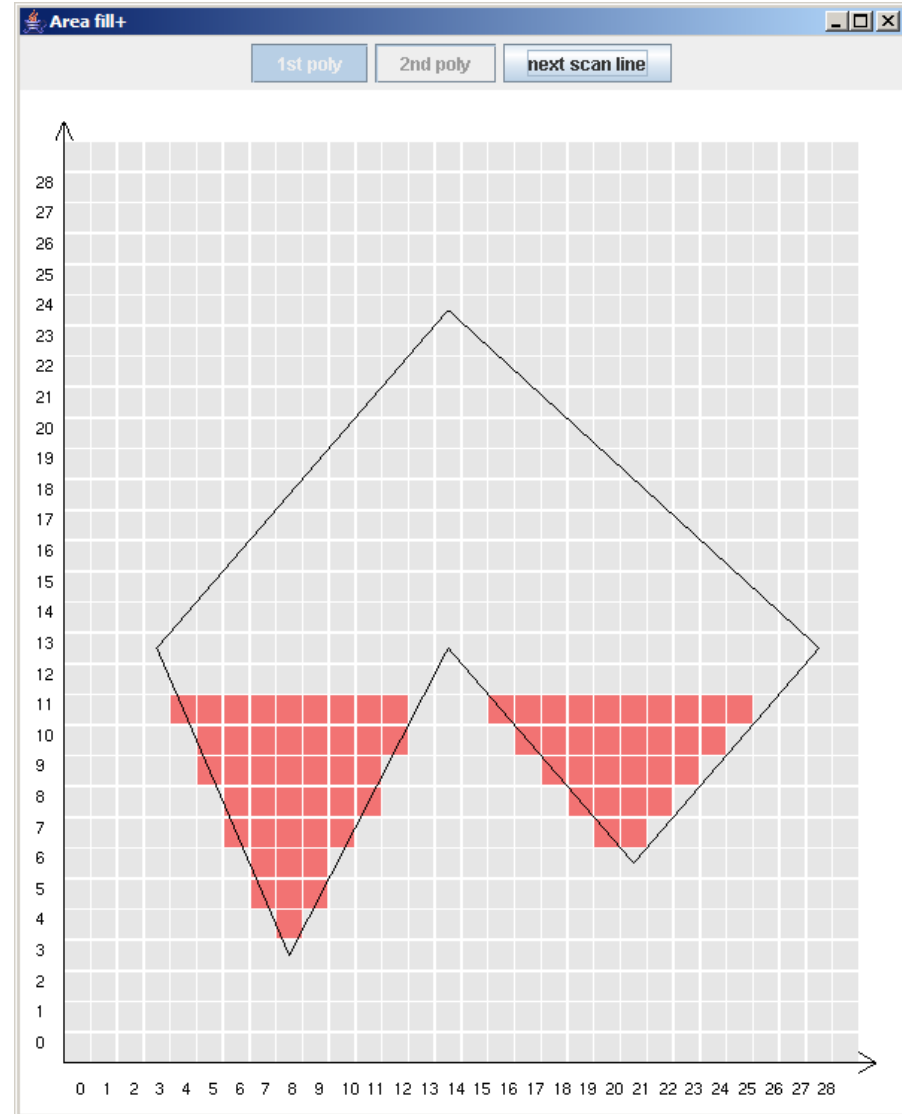
Polygon scan-conversion

- Say we want to “colour in” a polygon on a raster display
- Which pixels should we set?
- E.g. set pixel if the pixel centre is in the interior of the polygon (be careful about edges)
- We could do an inside-outside test for each pixel in the raster – this would be very inefficient.
- Instead....



Scan-line polygon fill algorithm

- Works by counting intersections of scan-lines with polygon edges
- Pixels between pairs of intersections are filled
- Starts at bottom and works up one scan line at a time
- Uses incremental calculations for efficiency (coherence)



Scan-line polygon fill algorithm

- Tricky points
 - Scan line passes through vertex
 - Depends whether edges change direction
 - Horizontal edges
 - Depends if on top or bottom
 - Vertical edges
 - Depends if on left or on right
- Explore these using AreaFill.java
 - Try different cases
 - Examine scanConvert() method
- Work through simple example
 - Sorted edge table
 - Active edge list

Pseudo-code for creating edge table

Initialise sortedEdgeTable (entry for each scan-line should be empty)

For each edge of the polygon

if slope $\neq 0$

Set edge = upwards copy of edge

Set edge = shorten the edge by 1 scan-line

Set index = scan-line for start of edge

add edge to sortedEdgeTable[index],
keeping sorted by starting x coordinate

Contents of sorted edge table

- Each entry contains
 - y coordinate of top of edge
 - x coordinate of start of edge
 - $1/m$ where m is the slope
- Update:
 - Set $x = x + 1/m$

Pseudo-code for scan-line algorithm

Create sortedEdgeTable (see previous slide)

Set row = scan line at bottom of polygon

Set activeEdgeList = sortedEdgeTable[row]

While activeEdgeList not empty

 Set inside = true

 for each edge in activeEdgeList

 if inside then

 start = round up x coord of edge

 else

 end = round down x coord of edge

 fill between start and end

 Set inside = ! inside

increment row

remove edges that finished on the last row

update x values for all edges in activeEdgeList

add sortedEdgeTable[row] into activeEdgeList

make sure activeEdgeList sorted by x coordinate

Filling with a pattern

- This can be done using a bit pattern array or texture pattern array
- If the array is n_x by n_y in size
 - Fill the pixel at (x, y) using the value at index $(x \bmod n_x, y \bmod n_y)$
 - Easy to incorporate into the scan-conversion algorithm

Gradient fill

- This is where the colour of pixels in the filled polygon varies in a smooth way.
- Can be done by associating a colour with each vertex of the polygon
 - Along an edge, interpolating the colours of the vertices at each end
 - Between edges, interpolate the colours two edges
- Can be incorporated into the scan-conversion algorithm and computed incrementally

Anti-aliasing

- Similar to the line drawing case – problems occur on edges of polygons
- Some possible anti-aliasing methods:
 - Pixel phasing (can be used for lines too)
 - Estimate how much of a pixel is inside the polygon
 - Calculate the area of the intersection between polygon and pixel (very expensive)
 - Supersampling – use sub-pixels
 - Pitteway and Watkinson – similar to Bresenham's algorithm – use along polygon edges

Polygons tables etc

- The rest of 3.15 (4.7, 4th ed.) is to do with polygons in 3D
- Take a look, but it is not examined.

Region filling

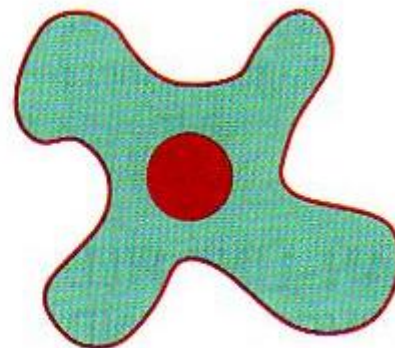
- Scan-conversion good for vector-based applications
- For bitmapped applications (e.g. a paint package), use
 - Flood fill
 - Boundary fill

Boundary fill

- Fill an area until a boundary of a particular colour is hit
- Start inside and search neighbouring pixels to fill until boundary is reached



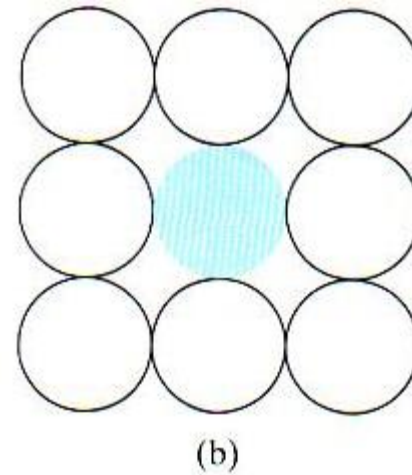
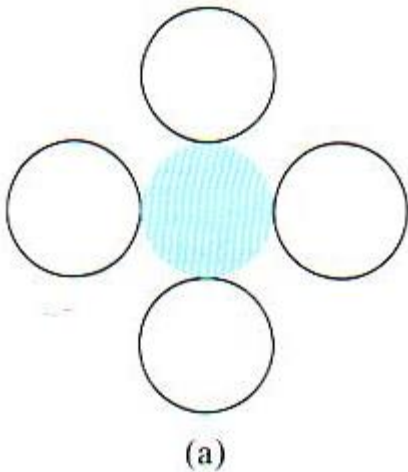
(a)



(b)

Boundary fill neighbours

- Two possible definitions:
 - 4-connected
 - 8-connected
- Look at some examples with BoundaryFill program



Boundary Fill Algorithm

- From your text – recursive version in C++

```
void boundaryFill4 (int x, int y, int fillColor, int borderColor)
{
    int interiorColor;

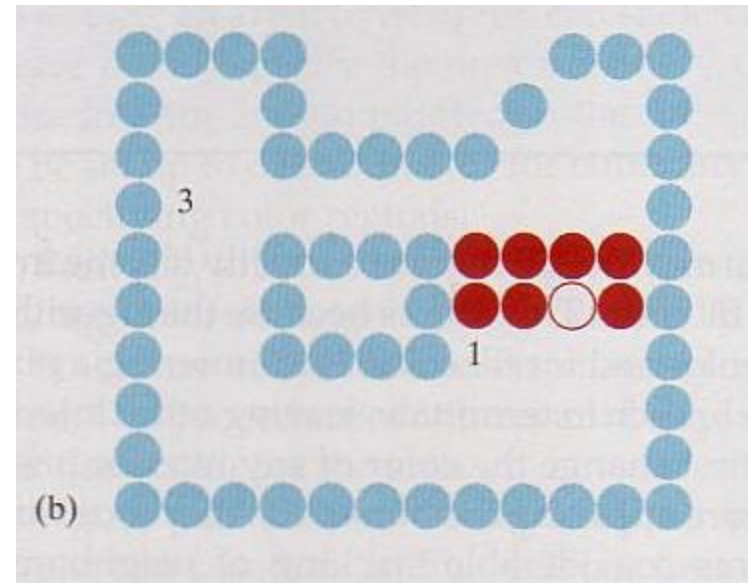
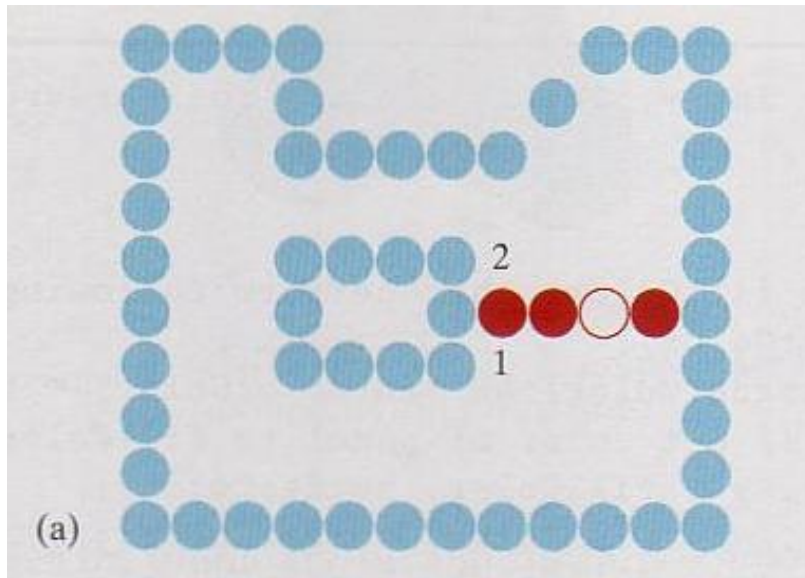
    /* Set current color to fillColor, then perform following oprations. */
    getPixel (x, y, interiorColor);
    if ((interiorColor != borderColor) && (interiorColor != fillColor)) {
        setPixel (x, y);    // Set color of pixel to fillColor.
        boundaryFill4 (x + 1, y , fillColor, borderColor);
        boundaryFill4 (x - 1, y , fillColor, borderColor);
        boundaryFill4 (x , y + 1, fillColor, borderColor);
        boundaryFill4 (x , y - 1, fillColor, borderColor)
    }
}
```

Boundary Fill problems

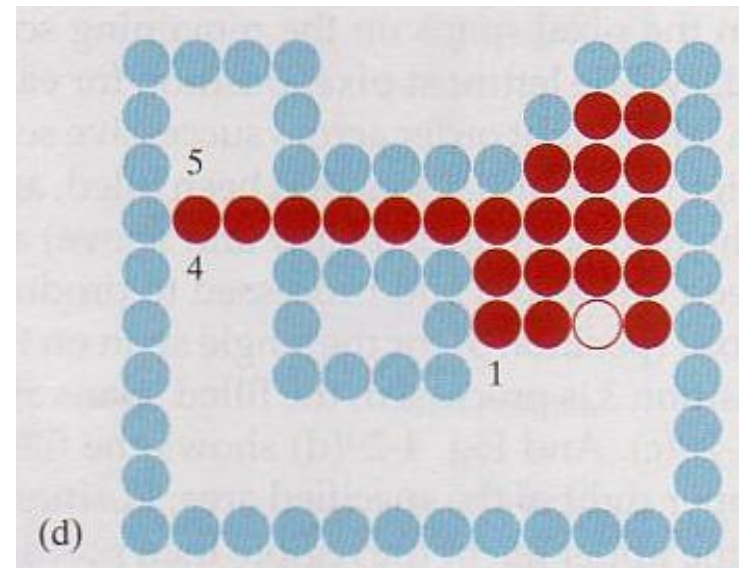
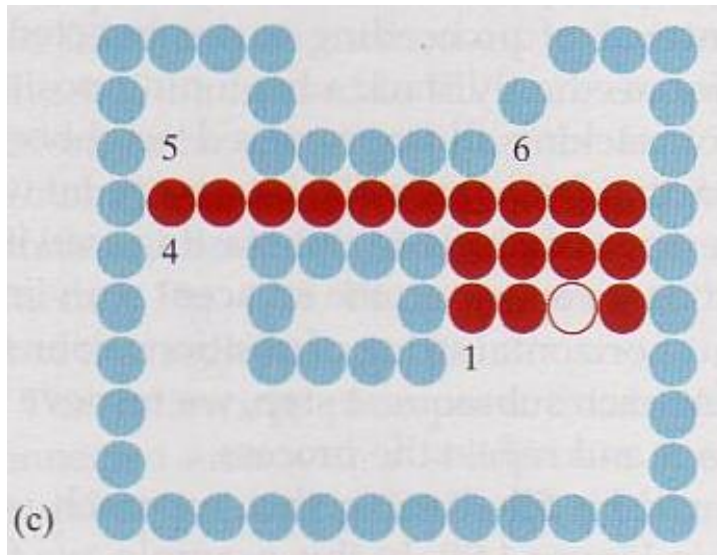
- Errors occur if any interior pixels already are same colour as fill colour
- This method uses lots of memory so you quickly run out of memory
- Although most texts give this as the standard algorithm, more efficient methods need to be employed
- Think about other ways that you could do it !

Better boundary fill

Work with “spans” of pixels to fill

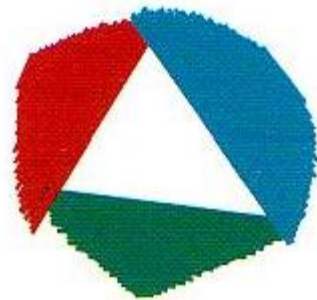


Better boundary fill



Flood fill

- Start with a “seed” pixel and change its colour
- Recursively change neighbouring pixels that also have the seed colour
- Keep going until neighbours with different colours reached



Flood fill

- Similar to Boundary Fill algorithm
- 4-connected or 8-connected search methods
- Recursive – same problem with memory as Boundary Fill
- Again, most texts give this approach as the standard method
- Need to use more efficient methods to avoid memory problems
- Again, think of solution similar to boundary filling

Flood Fill algorithm

```
/* Set current color to fillColor, then perform following operations. */  
getPixel (x, y, color);  
if (color = interiorColor) {  
    setPixel (x, y);    // Set color of pixel to fillColor.  
    floodFill4 (x + 1, y, fillColor, interiorColor);  
    floodFill4 (x - 1, y, fillColor, interiorColor);  
    floodFill4 (x, y + 1, fillColor, interiorColor);  
    floodFill4 (x, y - 1, fillColor, interiorColor)
```