

Lab session 2

Dirac comb, short-term spectrum

Exercise 1 *Construction of the Dirac comb*

1. Write a program to plot the N first terms of the series:

$$1 + 2 \cos(2\pi t) + 2 \cos(4\pi t) + \cdots + 2 \cos(2\pi Nt) + \cdots$$

2. What happens when N tends to infinity $+\infty$?
3. Does this series converge point-wise?
4. Explain why the series could be written $\sum_{n \in \mathbb{Z}} e^{2i\pi nt}$.
5. How many points between 0 and 1 do we have to choose to plot correctly the last term in the truncated series? Check that this constraint is well fulfilled.
6. What happens if we consider the limit of

$$1 + 2 \cos\left(\frac{2\pi t}{Q}\right) + 2 \cos\left(\frac{4\pi t}{Q}\right) + \cdots + 2 \cos\left(\frac{2\pi Nt}{Q}\right) + \cdots$$

Give a theoretical argument to justify this behaviour. Write the corresponding program.

Exercise 2 *Short-term Fourier transform*

As frequency analysis is an operation that computes an average over the whole time span, some fundamental aspects can be completely hidden. But this does not mean the information is not there. Time-frequency analysis is essential in that matter.

1. Consider two sinusoids with respective frequencies $f_1 = 0.1$ Hz and $f_2 = 0.2$ Hz, the first one lasting $T_1 = 128$ s and the second one $T_2 = 64$ s. We thus obtain a signal lasting $T_1 + T_2$ s made of the first sinusoid in the first T_1 seconds et of the second sinusoid during the last T_2 seconds. Plot the modulus of DFT of this signal (having added some zeros on the initial sequence to obtain a sequence of size 256). You observe a symmetry in the representation, does that seem correct? What is the reason for such a behavior? Do the appropriate shifts to get an approximation of the Fourier transform.

2. If you add zeros to this initial sequence to obtain a sequence of size 512, what is the impact on frequency resolution. Give a detailed explanation.

3. Consider intervals on length N overlapping by $N/2$ and compute the modulus of the TFD on each such intervals (do not forget the appropriate shift). More precisely, the first interval corresponds to the first N indices of the signal f , the second interval to the indices between $N/2$ and $3N/2$, etc... Put the results in a matrix whose rows correspond to the different intervals. Associate to each interval a time corresponding to the center of the interval. Create a time-frequency mesh using the meshgrid command, the time index corresponding to the intervals and N corresponding to the N frequencies. Then plot the modulus on the above mentioned matrix using the contour command.