Binary Search Tree

Data Structures CS218

Outline

This topic covers binary search trees:

- Background
- Definition and examples
- Implementation details of BST operations

Background

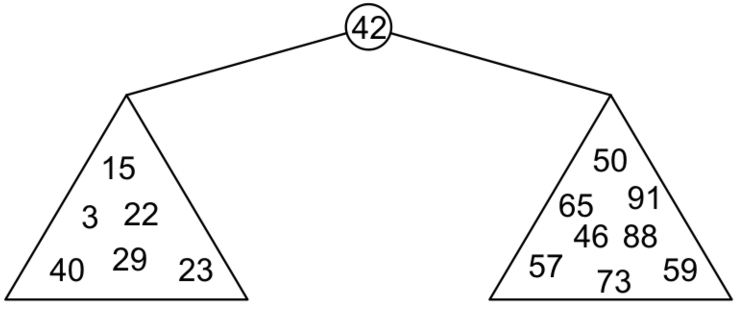
Recall that with a binary tree, we can dictate an order on the two children

We will exploit this order:

– Require all objects in the left sub-tree to be less than the object stored in the root node, and
– Require all objects in the right sub-tree to be greater than the object in the root object

Binary Search Trees

Graphically, we may relationship



– Each of the two sub-trees will themselves be binary search trees

Binary Search Trees

Notice that we have already used this structure for searching: examine the root node and if we have not found what we are looking for:

- If the object is less than what is stored in the root node, continue searching in the left sub-tree
- Otherwise, continue searching the right sub-tree

With a linear order, one of the following three must be true:

$$a < b \quad a = b \quad a > b$$

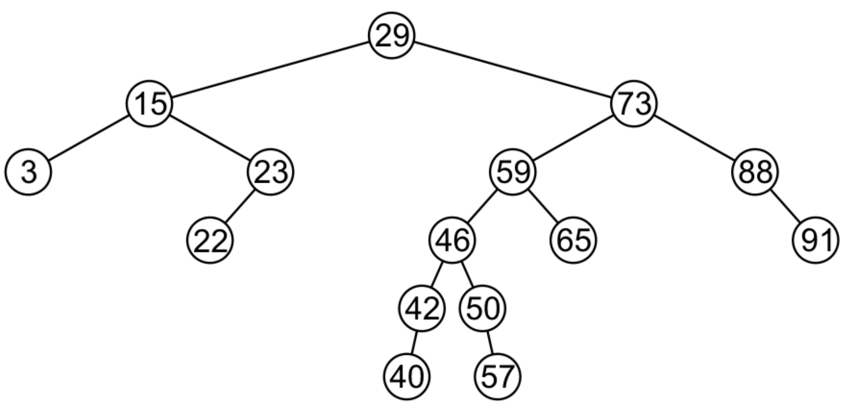
Definition

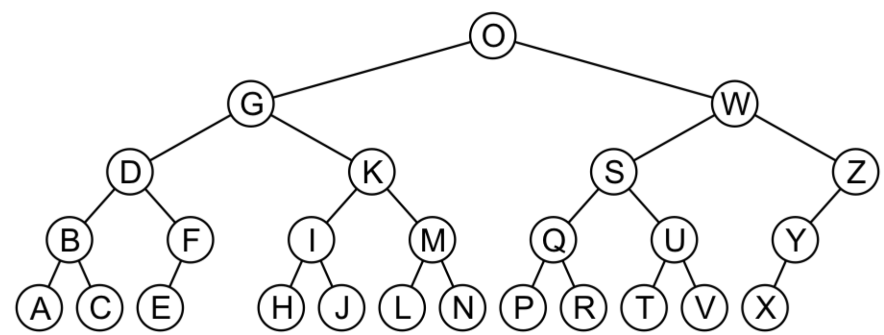
Thus, we define a non-empty binary search tree as a binary tree with the following properties:

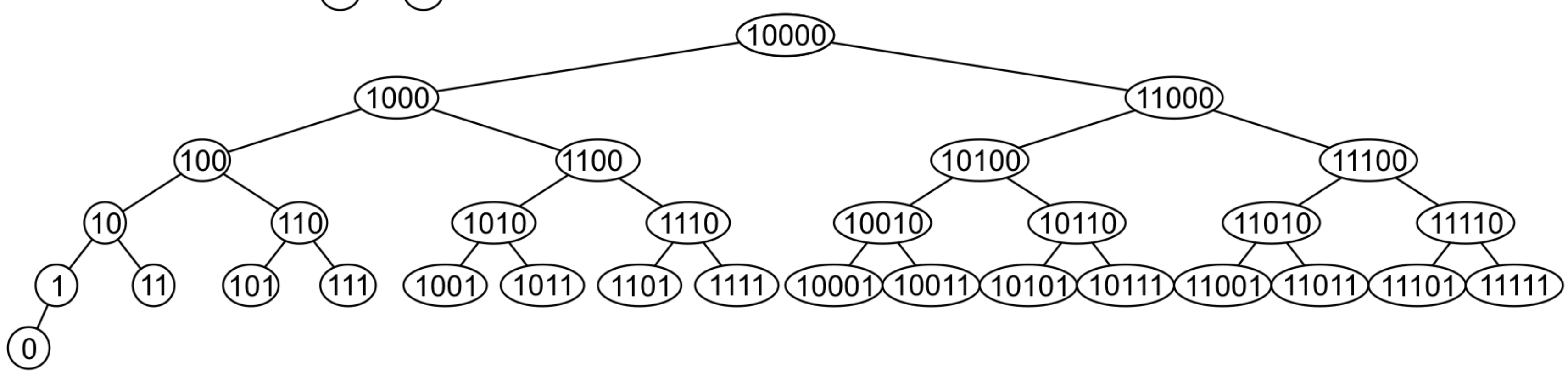
- The left sub-tree (if any) is a binary search tree and all values are less than the root value, and
- The right sub-tree (if any) is a binary search tree and all values are greater than the root value

Examples

Here are other examples of binary search trees:

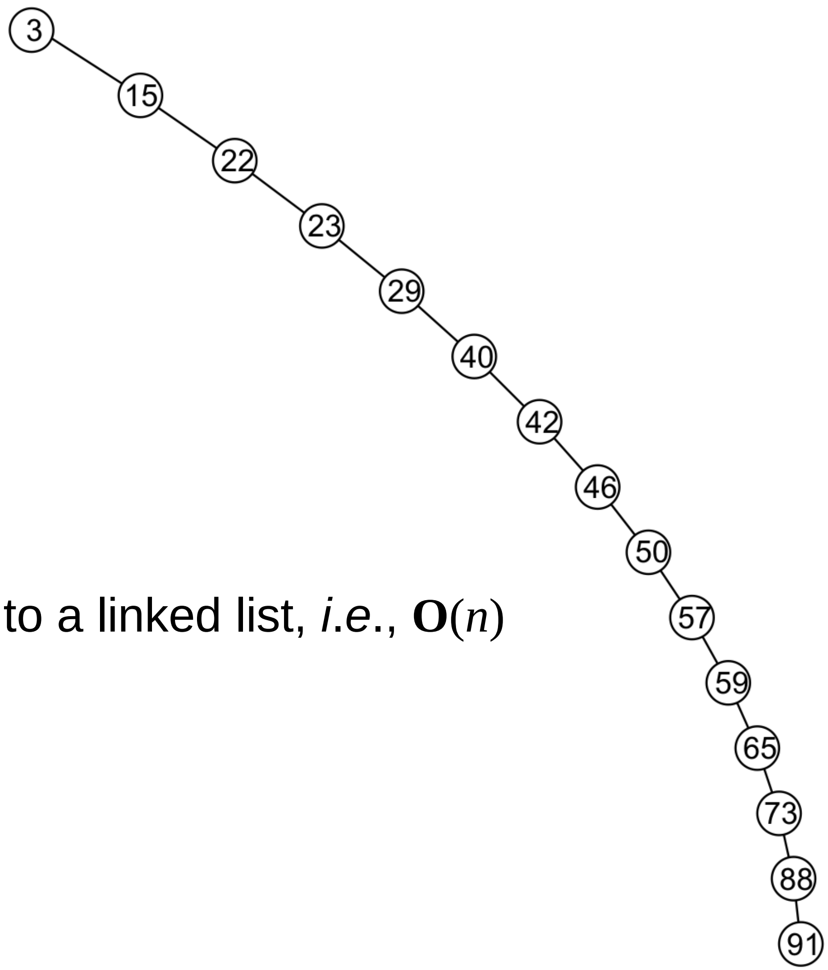






Examples

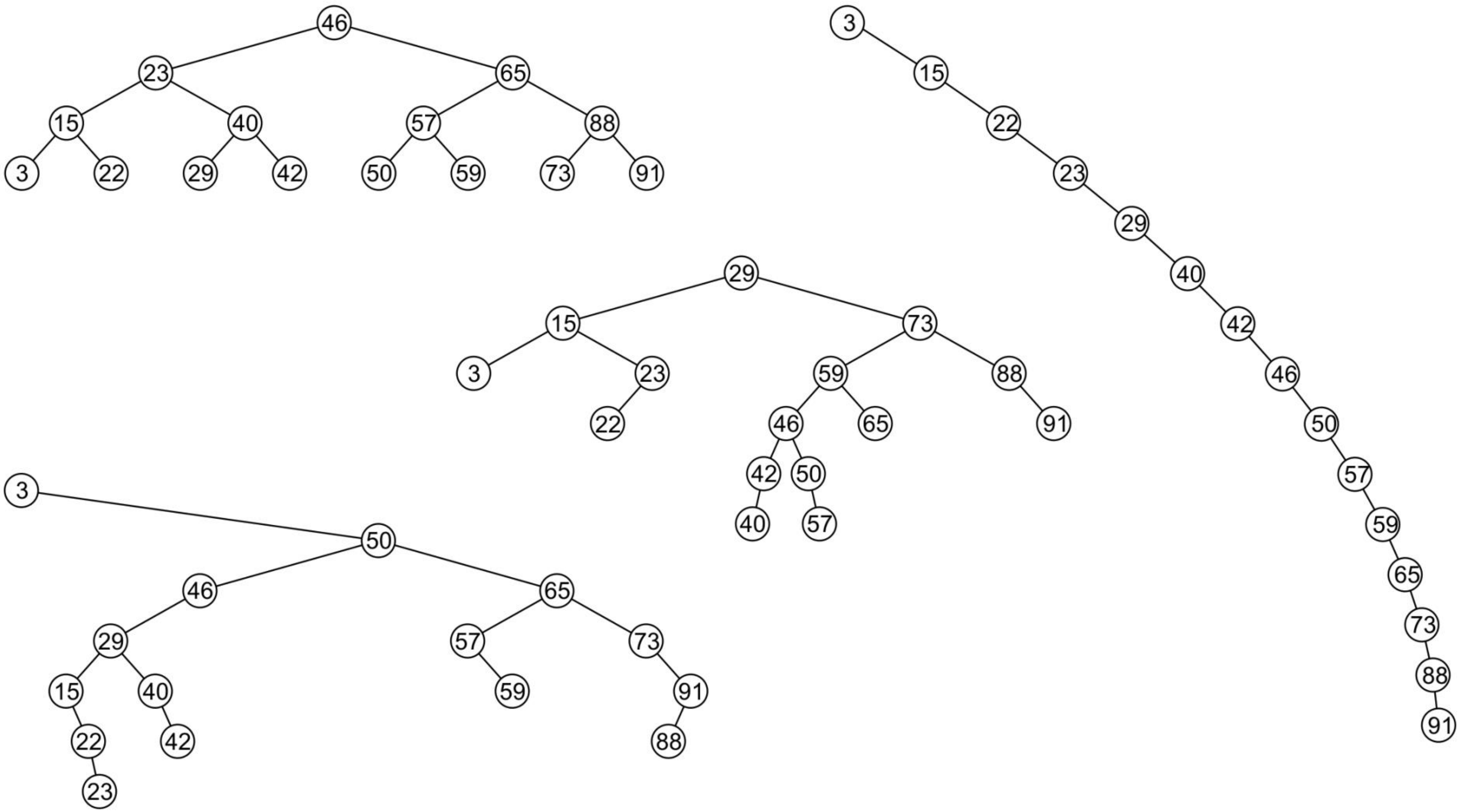
Unfortunately, it is possible to construct *degenerate* binary search trees



– This is equivalent to a linked list, *i.e.*, $\mathbf{O}(n)$

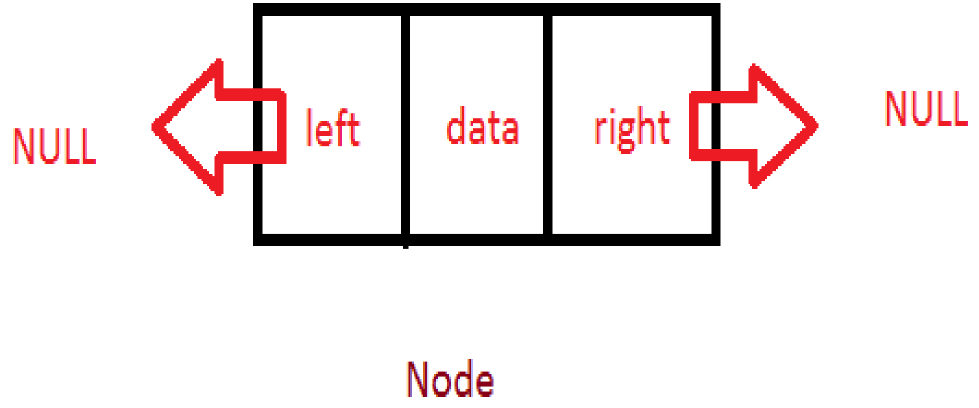
Examples

All these binary search trees store the same data



Structure of BST Node

```
Struct node{
        int data;
        node * left;
        node *right;
};
```



Operations on a BST

- Insert
- Update
- Search
- Delete
- Traversal
  - BFT
  - DFS
    - In-order, post-order, and pre-order
- Finding minimum
- Finding maximum
- Finding predecessor
- Finding Successor
- Finding height

Insert

- Let’s insert any random data into a BST (Discussion)

Implementation Details

```
Node* insert (int data, node *head)
{
        if(head == NULL)
        {
                head= new node;
                head->data= data;
                head->left= NULL;
                head->right= NULL;
        }
        else if (data < head->data)
                head->left= insert(data, head->left);
        else if(data > head->data)
                head->right= insert(data, head->right);
        return head;
}
```

Search

```
Node* search(node* head, int data){
        if(head == NULL)
                return NULL;
        else if( data < head->data)
                return search(head->left, data);
        else if (data > head->data)
                return search(head->right, data);
        else
                return head;
}
```

Finding minimum

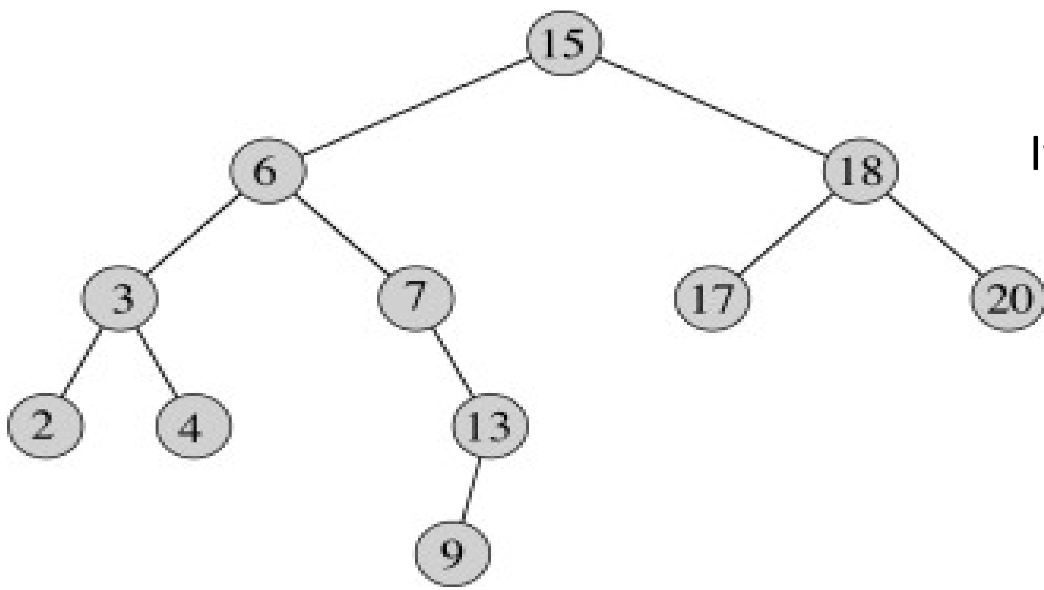
```
Node* min(node *head)
{
        if(head == NULL)
                return NULL;
        else if (head->left == NULL)
                return head;
        else
                return min(head->left);
}
```

Finding Maximum

```
Node* max(node *head)
{
        if(head == NULL)
                return NULL;
        else if (head->right == NULL)
                return head;
        else
                return max(head->right);
}
```

Finding Predecessor

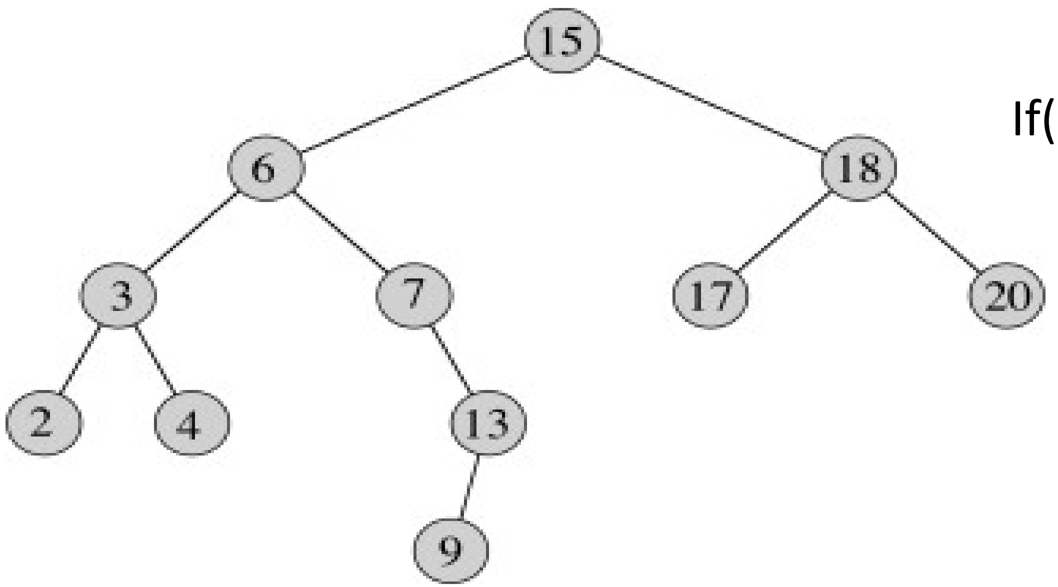
- A predecessor is a maximum value in a left sub tree
- A predecessor of a node is a right most element in a left sub tree.




```
If(head->left!=NULL)
        return max(head->left)
```

Finding Successor

- A successor is a minimum value in a right sub tree
- A successor of a node is a left most element in a right sub tree.



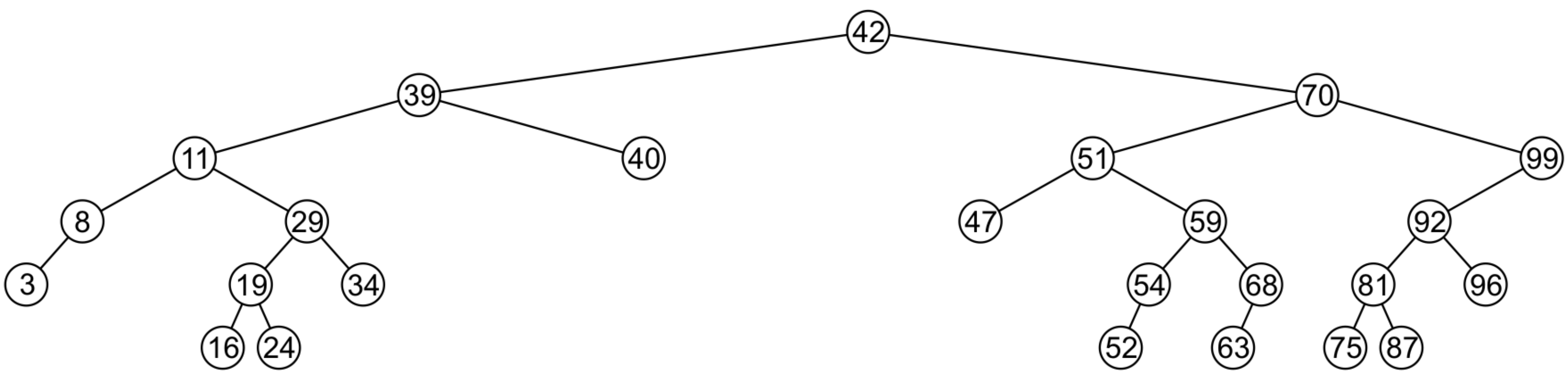

```
If(head->right!=NULL)
        return min(head->right)
```

Delete

A node being erased is not always going to be a leaf node

There are three possible scenarios:

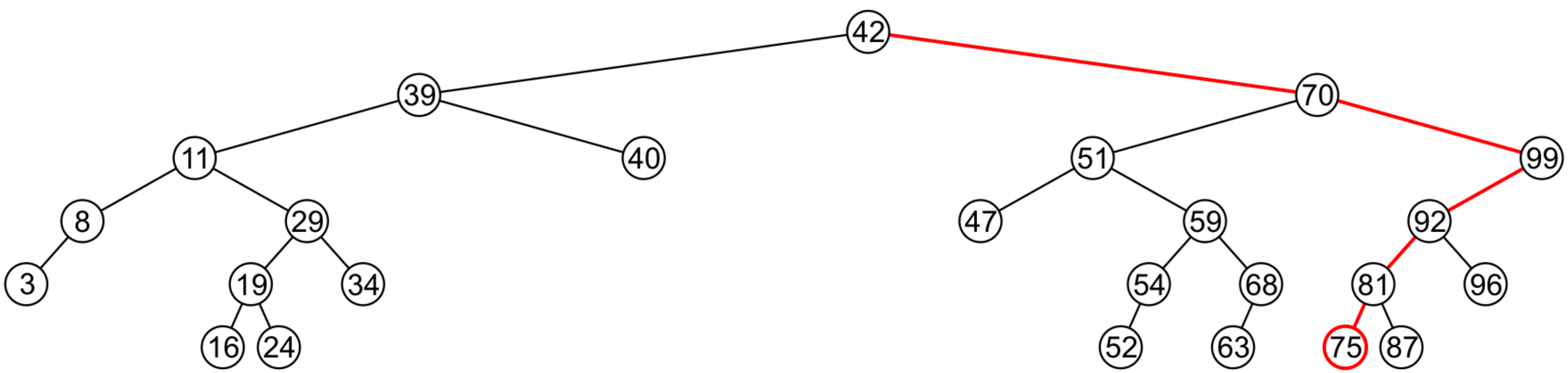
- The node is a leaf node,
- It has exactly one child, or
- It has two children (it is a full node)



Delete

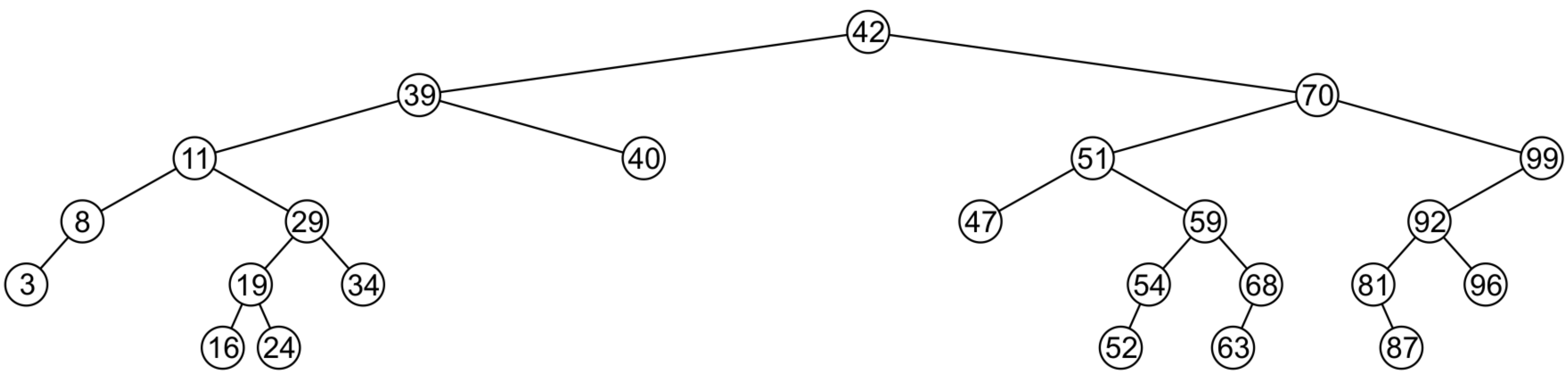
A leaf node simply must be removed and the appropriate member variable of the parent is set to `nullptr`

– Consider removing 75



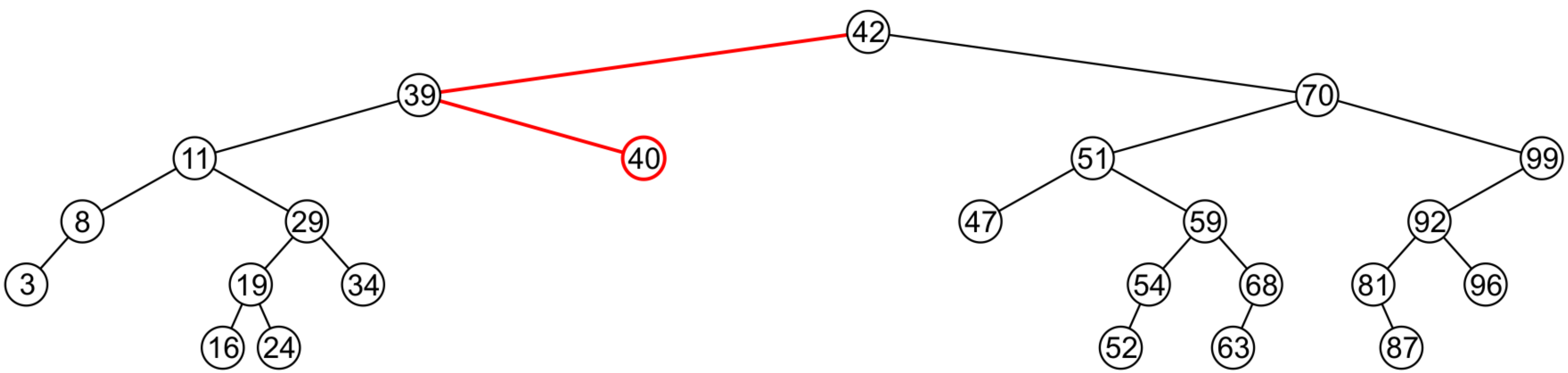
Delete

The node is deleted and `left_tree` of 81 is set to `nullptr`



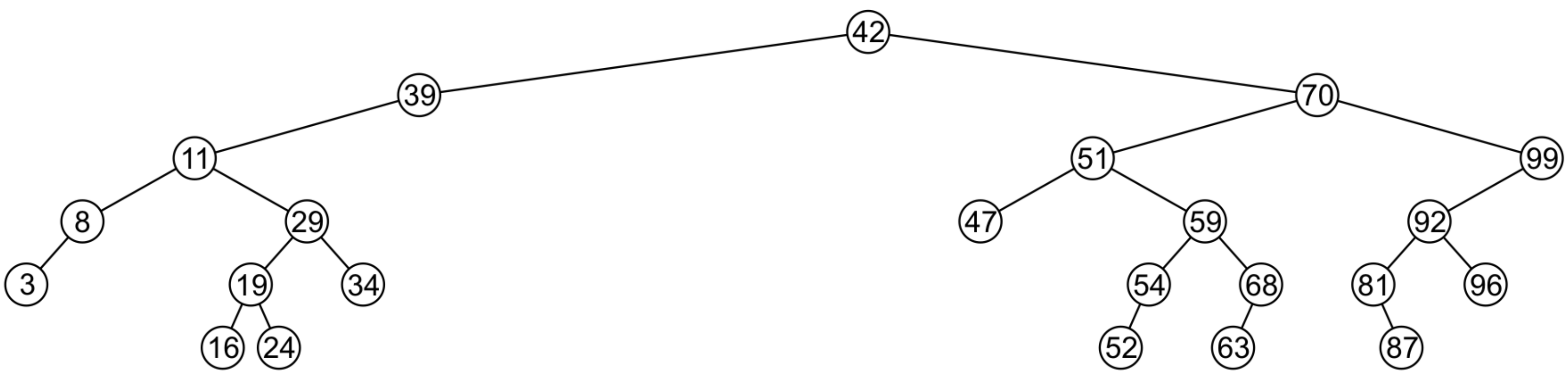
Delete

Erasing the node containing 40 is similar



Delete

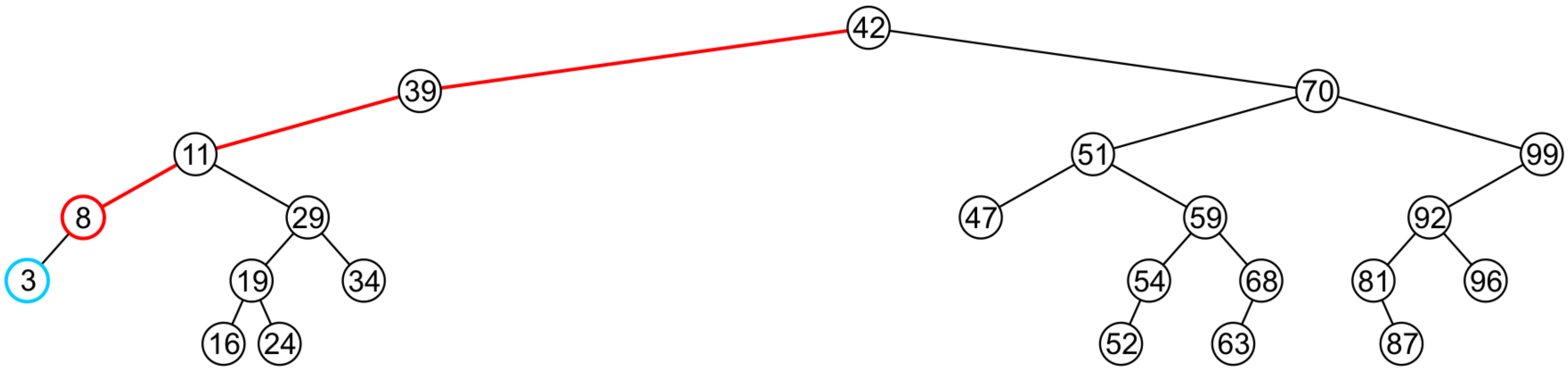
The node is deleted and `right_tree` of 39 is set to `nullptr`



Delete

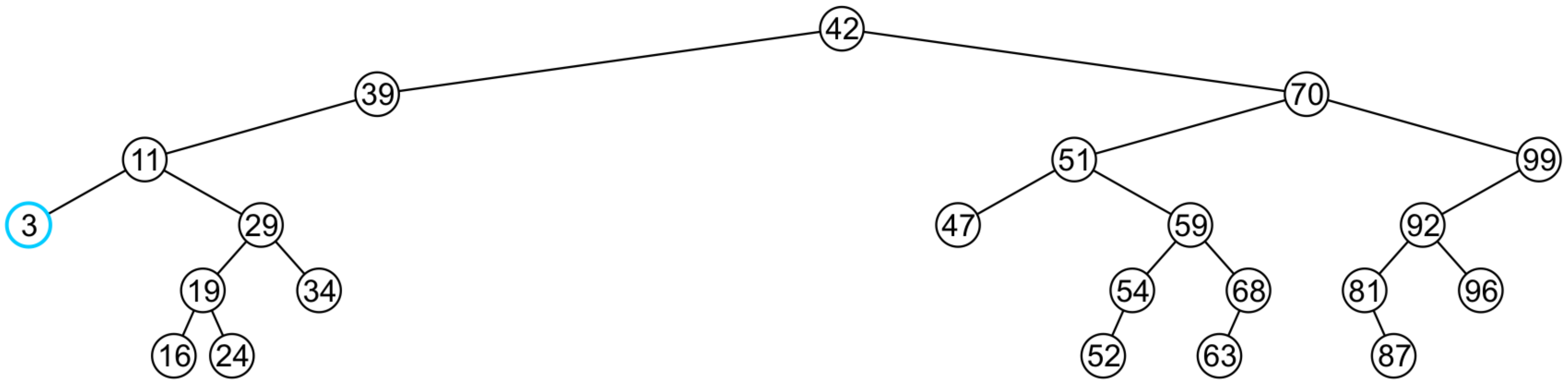
If a node has only one child, we can simply promote the sub-tree associated with the child

- Consider removing 8 which has one left child



Delete

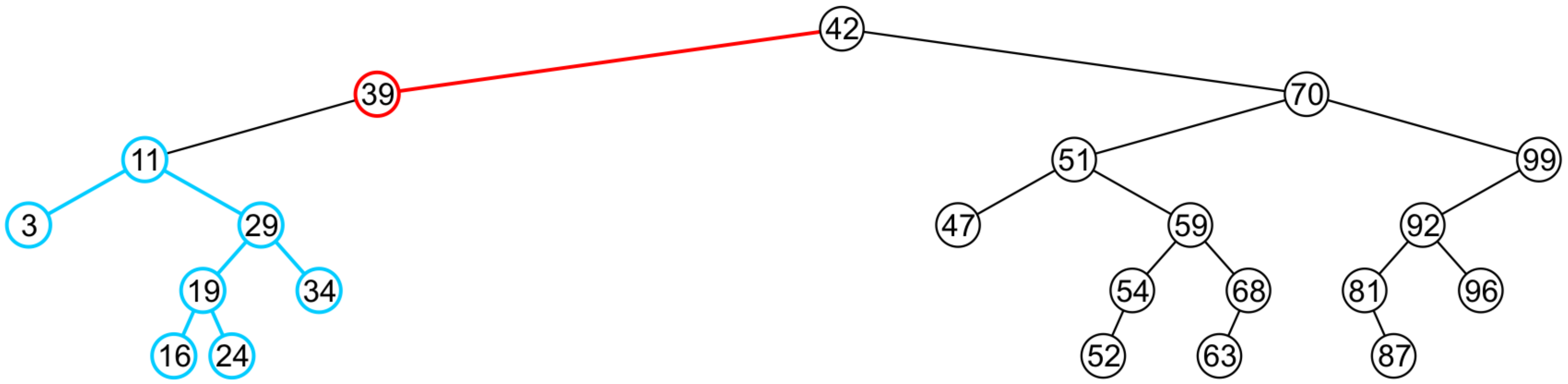
The node 8 is deleted and the `left_tree` of 11 is updated to point to 3



Delete

There is no difference in promoting a single node or a sub-tree

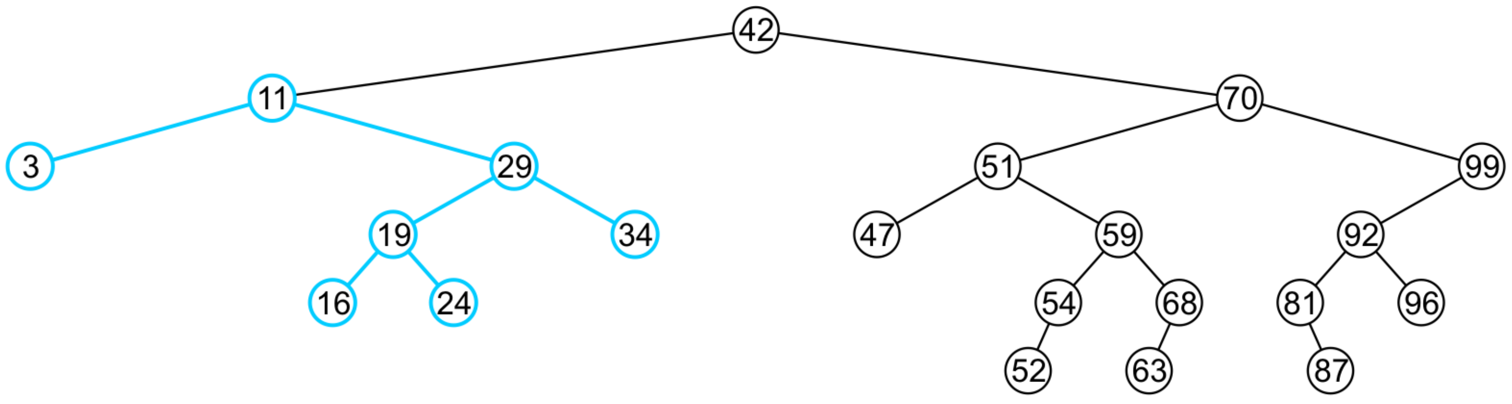
- To remove 39, it has a single child 11



Delete

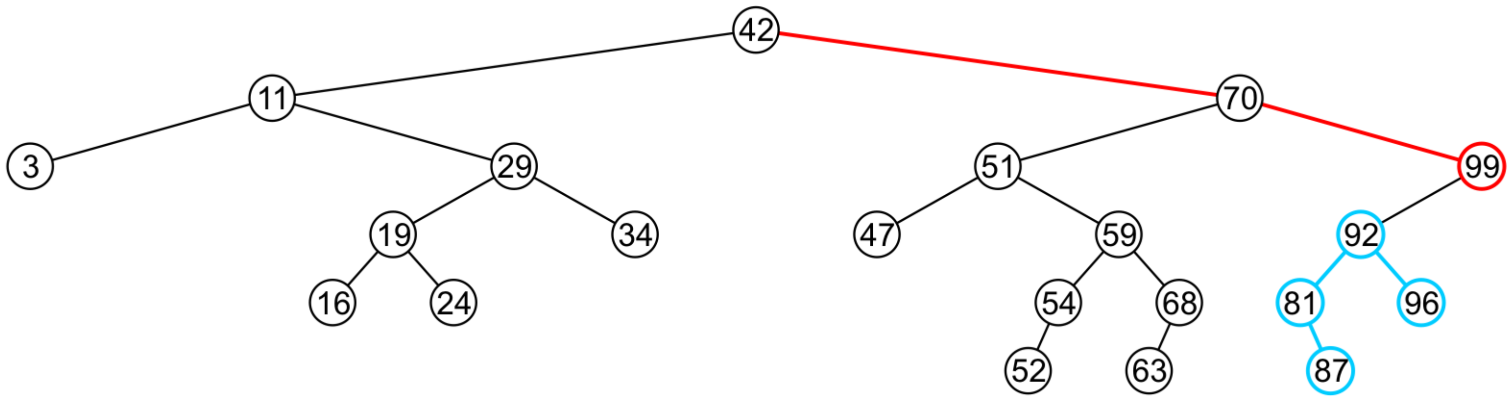
The node containing 39 is deleted and `left_node` of 42 is updated to point to 11

- Notice that order is still maintained



Delete

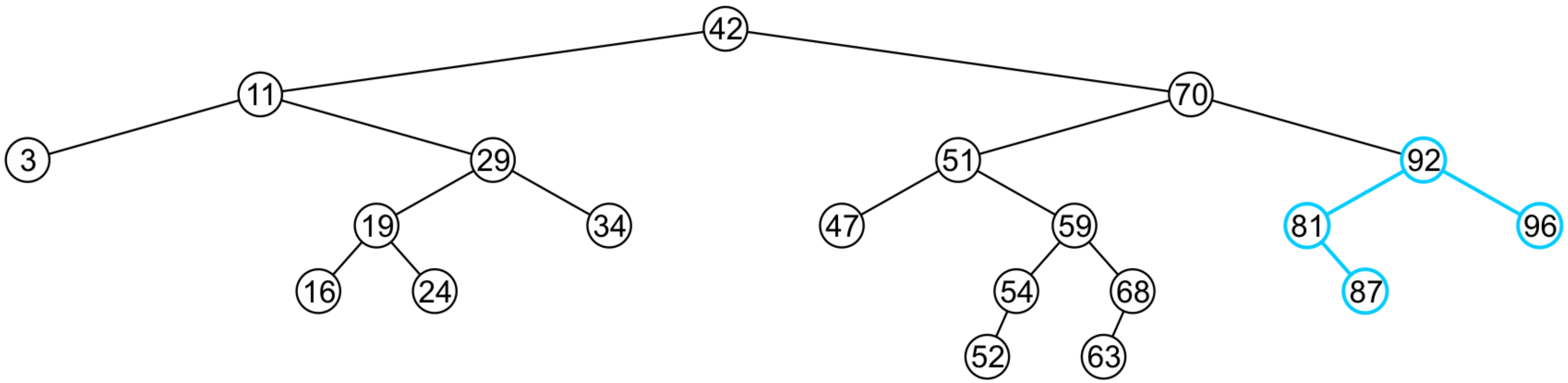
Consider erasing the node containing 99



Delete

The node is deleted and the left sub-tree is promoted:

- The member variable `right_tree` of 70 is set to point to 92
- Again, the order of the tree is maintained

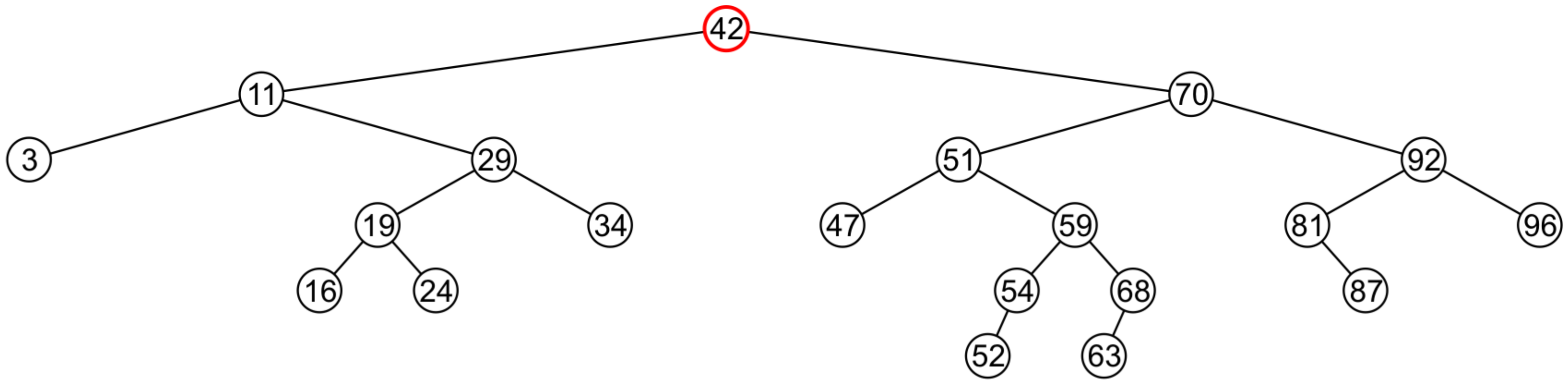


Delete

Finally, we will consider the problem of erasing a full node, *e.g.*, 42

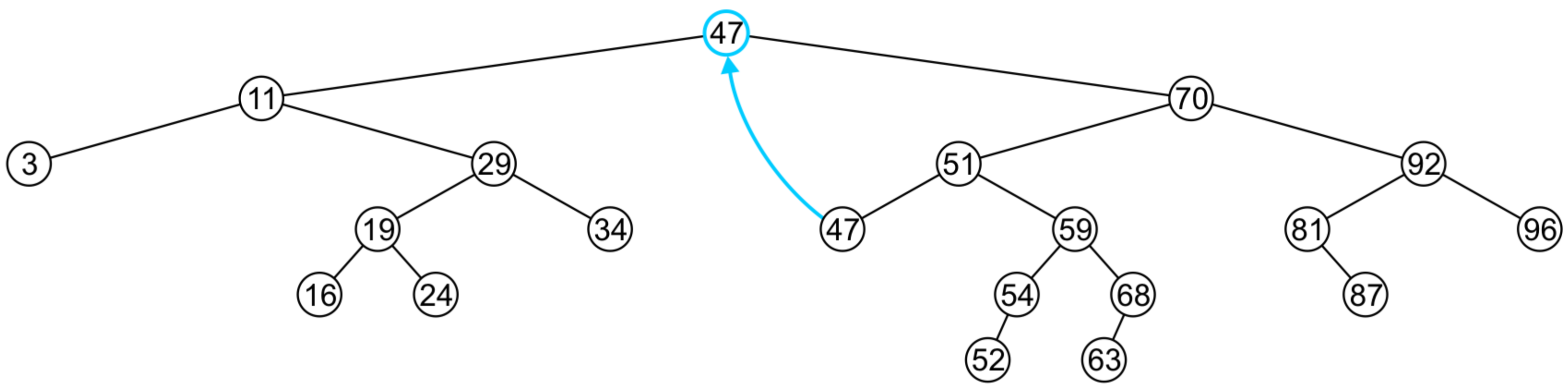
We will perform two operations:

- Replace 42 with the minimum object in the right sub-tree
- Erase that object from the right sub-tree



Delete

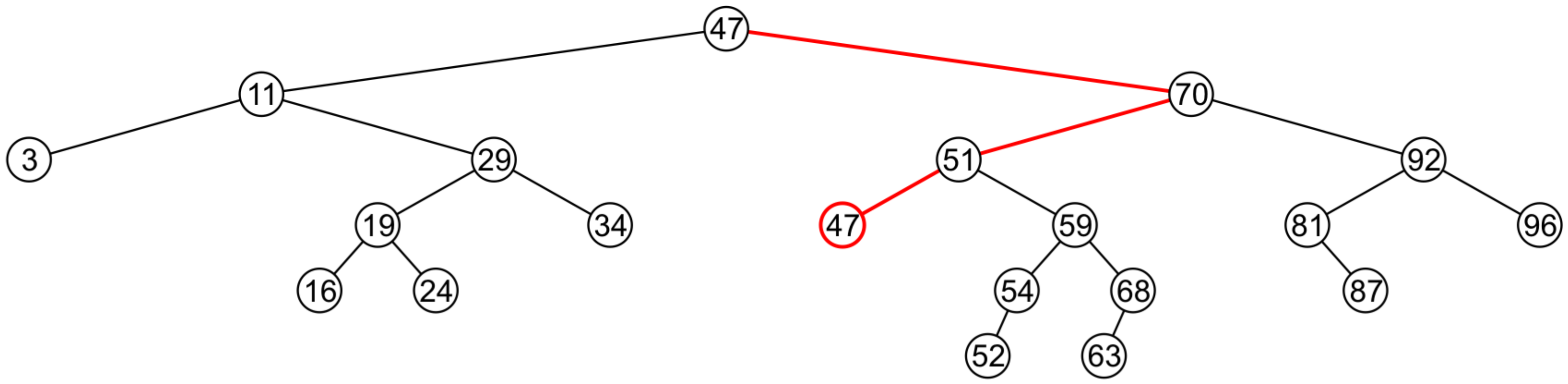
- In this case, we replace 42 with 47
  - We temporarily have two copies of 47 in the tree



Delete

We now recursively erase 47 from the right sub-tree

– We note that 47 is a leaf node in the right sub-tree

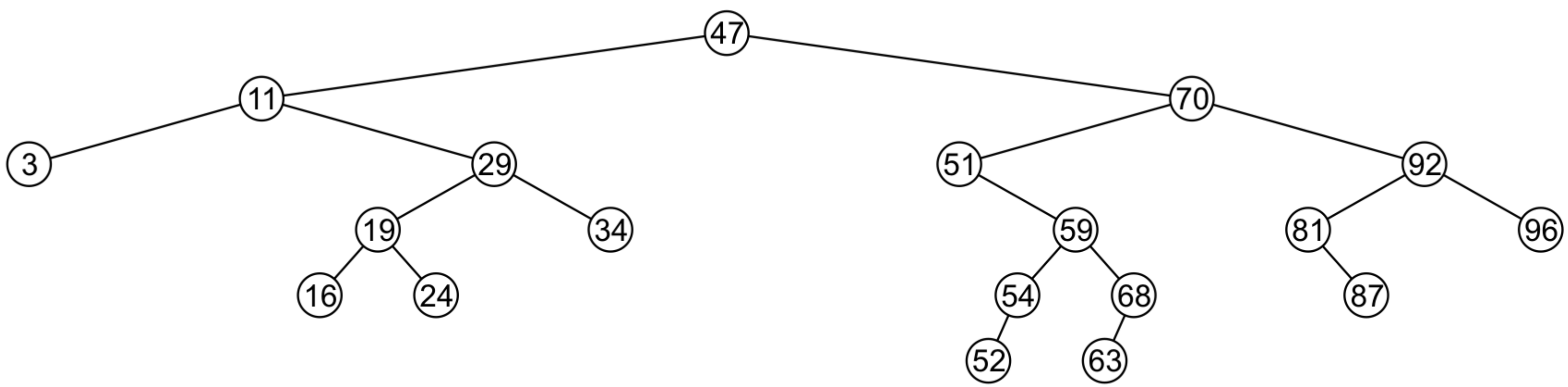


Delete

Leaf nodes are simply removed and `left_tree` of 51 is set to `nullptr`

- Notice that the tree is still sorted:

  47 was the least object in the right sub-tree

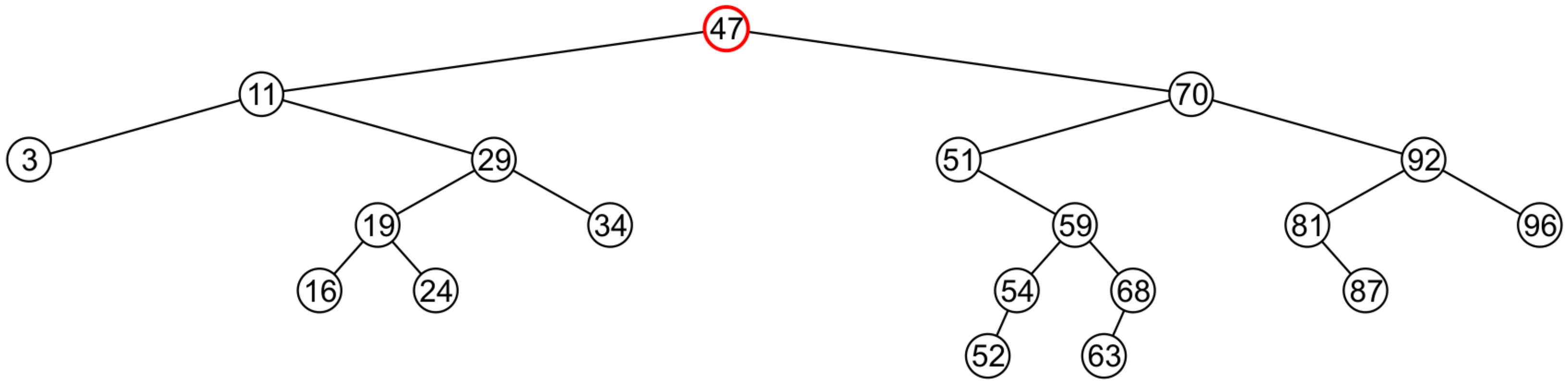


Delete

Suppose we want to erase the root 47 again:

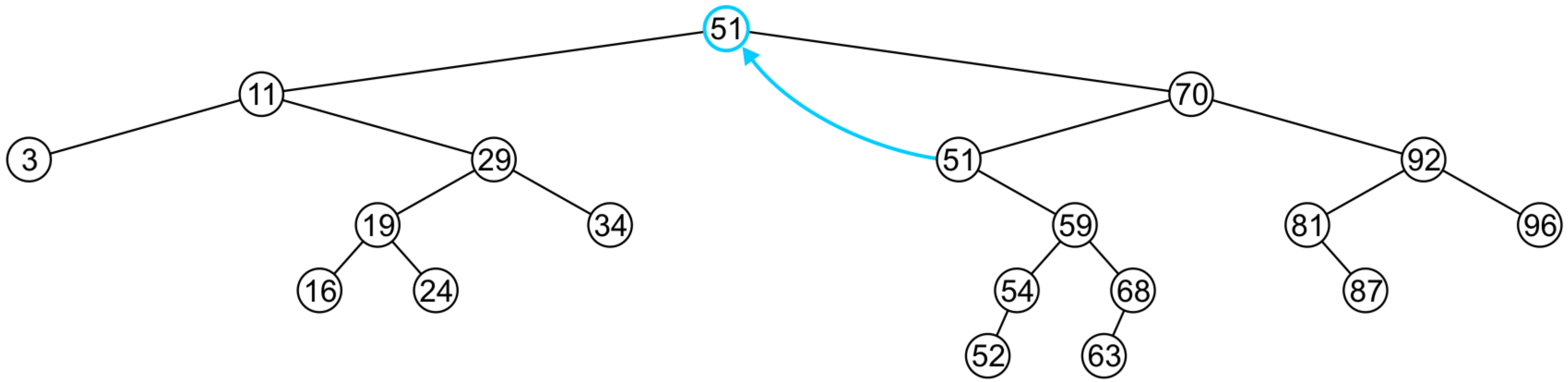
– We must copy the minimum of the right sub-tree

– We could promote the maximum object in the left sub-tree and achieve similar results



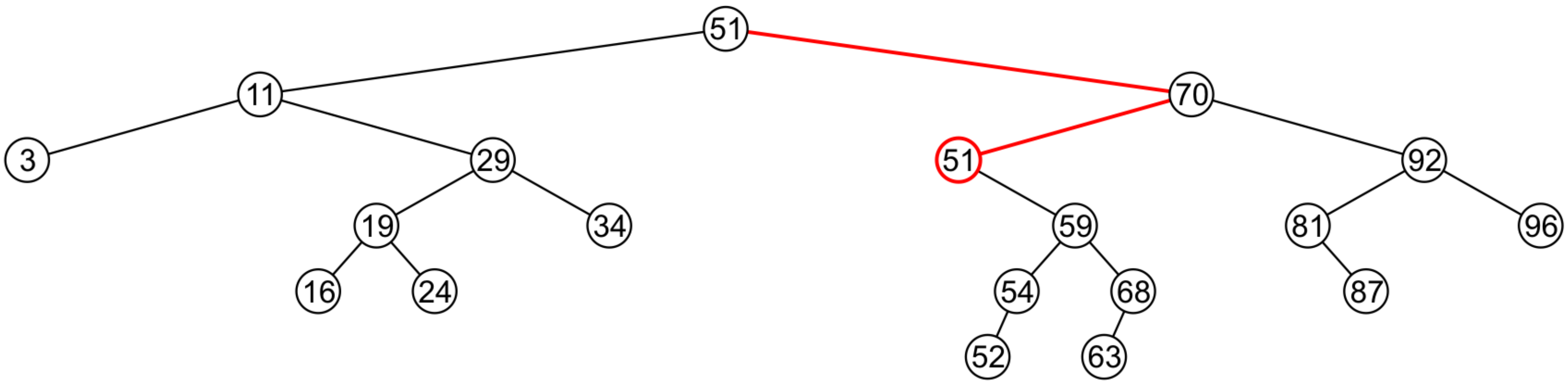
Delete

We copy 51 from the right sub-tree



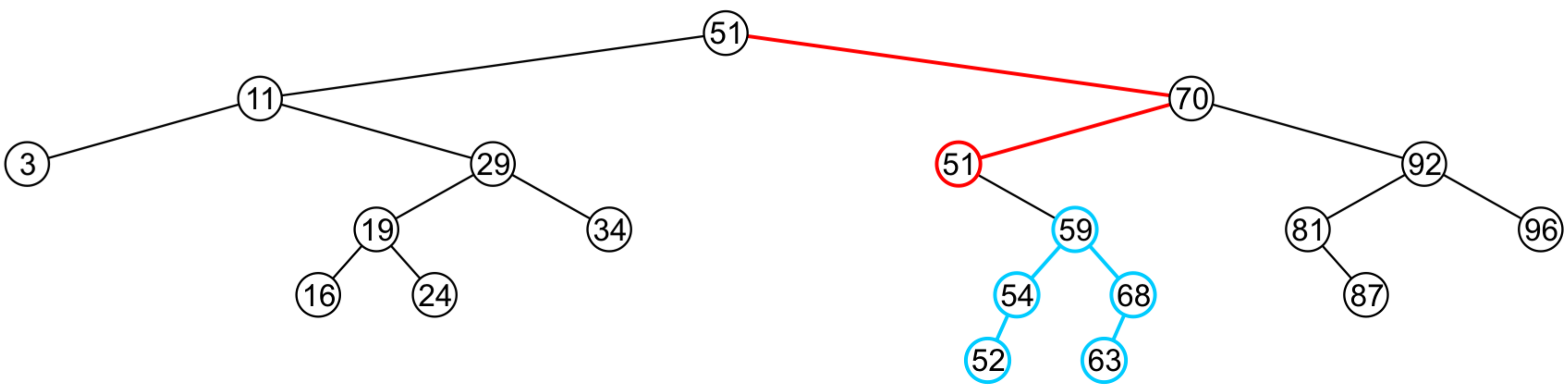
Delete

We must proceed by delete 51 from the right sub-tree



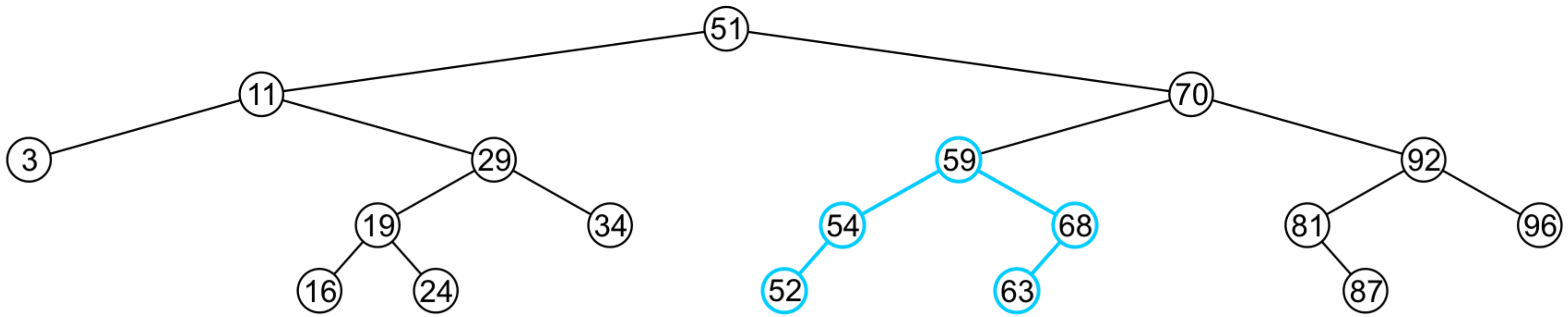
Delete

In this case, the node storing 51 has just a single child



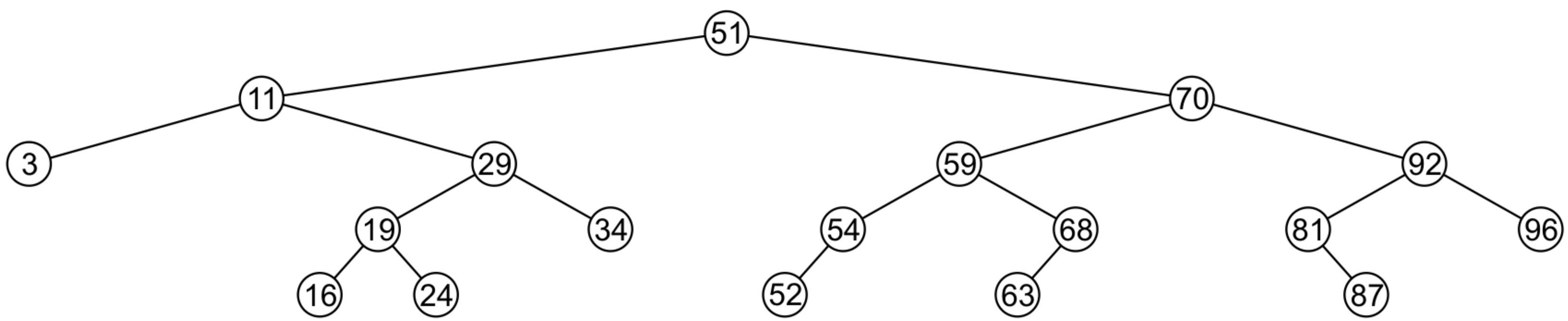
Delete

We delete the node containing 51 and assign the member variable `left_tree` of 70 to point to 59



Delete

Note that after seven removals, the remaining tree is still correctly sorted



Implementation (Delete)

```
Node * delete(int x, node* head)
{
        node *temp;
        if( head == NULL)
                return NULL;
        else if (data  <  head->data) //left leaf node
                head->left = delete (data, head->left);
        else if (data  >  head->data) //left right node
                head->right = delete (data, head->right);
        else if( head->left && head->right ) //full node
        {
                temp= min(head->right);
                head->data= temp->data;
                head->right= delete (head->data, head->right);
        }
```

Implementation (Delete) cont..

```
    else
    {
    temp=head;
    if(head->left == NULL) //neither node
            head=head->right;
    else if (head->right ==NULL) //neither node
            head=head->left;
    delete temp;
    }
    return head;
}
```

Traversal in a BST

- There are two types of traversals
  - Breadth First Search Traversal (BFS)
    - Make use of Queue ADT (Already covered)
  - Depth First Search Traversal (DFS)

    Make use of Stack ADT
    - Inorder
    - Preorder
    - Postorder

In-order Traversal

- It is a type of depth first search traversal.
- Following is the traversing order
  - Left node, root node, right node
- Example discussion

```
Void inorder (node *head)
{
        if (head == NULL)
                return;
        inorder (head->left);
        cout << head-data << " ";
        inorder (head->right);
}
```

Pre-order Traversal

- It is a type of depth first search traversal.
- Following is the traversing order
  - root node, left node, right node
- Example discussion

```
Void preorder (node *head)
{
        if (head == NULL)
                return;
        cout << head-data << " ";
        preorder (head->left);
        preorder(head->right);
}
```

Post-order Traversal

- It is a type of depth first search traversal.
- Following is the traversing order
  - Left node, right node, root node
- Example discussion

```
Void postorder (node *head)
{
        if (head == NULL)
                return;
        postorder (head->left);
        postorder(head->right);
        cout << head-data << " ";
}
```

Finding Height of a BST

```
Int height (node * head)
{
        if(head == NULL)
                return NULL;
        else
        {
                int h_left= height(head->left);
                int h_right = height (head ->right);
                if (h_left > h_right)
                        return (h_left + 1);
                else return ( h_right + 1);
        }
}
```

Summary

In this topic, we covered binary search trees

- Described Abstract Sorted Lists
- Problems using arrays and linked lists
- Definition of a binary search tree
- Looked at the implementation of:
  - Insert
  - Delete
  - Search
  - Traversals
  - Height