Last update: March 1, 2024

Polyhedra (4)

Consider the following half-spaces in $\mathbb{R}^3$:

$$\begin{array}{rcrcrcl} x_1 & + & x_2 & + & x_3 & \leq & 4 \\ & & x_2 & & & \leq & 2 \\ & & & & x_3 & \leq & 3 \\ 3\,x_1 & & & + & x_3 & \leq & 6 \\ x_1 & & & & & \geq & 0 \\ & & x_2 & & & \geq & 0 \\ & & & & x_3 & \geq & 0 \end{array}$$

Questions

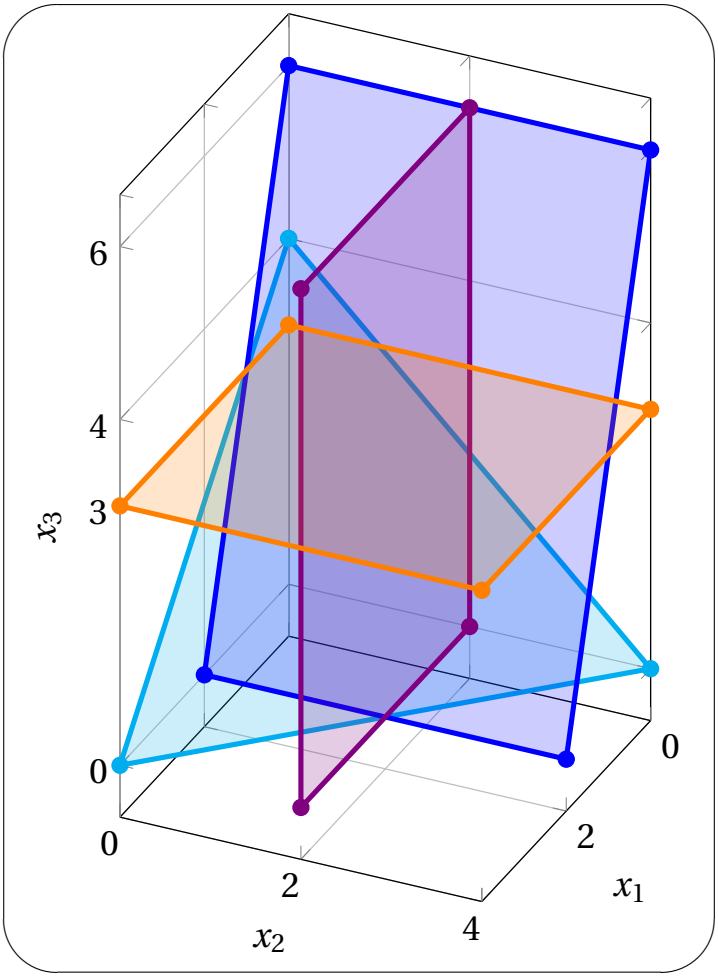
1. Draw a plot of their supporting hyper-planes.
2. Draw the polyhedron given by the intersection of the half-spaces. Is it a polytope?

Solution

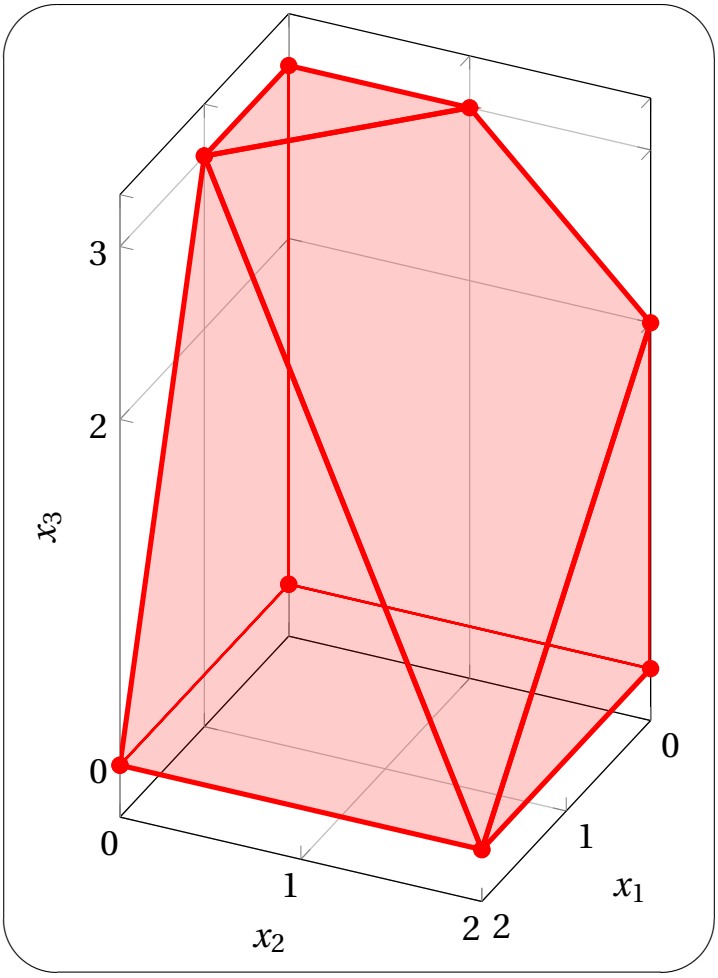
1. The supporting hyper-planes associated with the given half-spaces are:

$$
\begin{aligned}
x_1 + x_2 + x_3 &= 4 \\
x_2 &= 2 \\
x_3 &= 3 \\
3\,x_1 + x_3 &= 6 \\
x_1 &= 0 \\
x_2 &= 0 \\
x_3 &= 0
\end{aligned}
$$

Their plot in $\mathbb{R}^3$ is:



2. The polyhedron given by the intersection of the four half-spaces in $\mathbb{R}^3$ is:



Since the polyhedron is bounded, it is also a polytope.