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Question 1:- A computer vision model ....

Binomial distribution

(a) Ten images are evaluated. Write the probability mass function (pmf) of X.

$n = 10 \qquad P = 0.80$

Probability Mass function

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$= \binom{10}{k} (0.80)^k (\cancel{0}1-0.80)^{10-k}$$

for $k = 1, 2, \ldots, 10$

$$\binom{10}{1}(0.80)^1(1-0.80)^{10-1} + \binom{10}{2}(0.80)^2(1-0.80)^8 + \binom{10}{3}(0.80)^3(1-0.80)^7 + \binom{10}{4}(0.80)^4(1-0.80)^6 + \binom{10}{5}(0.80)^5(1-0.80)^5 + \binom{10}{6}(0.80)^6(1-0.80)^4 + \binom{10}{7}(0.80)^7(1-0.80)^3 + \binom{10}{8}(0.80)^8(1-0.80)^2 + \binom{10}{9}(0.80)^9(1-0.80)^1 + \binom{10}{10}(0.80)^{10}(1-0.80)^0.$$

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(b) Calculate $P(X \geq 8) = P(X=8) + P(X=9) + P(X=10)$

$\binom{10}{8}(0.80)^8(0.20)^2 + \binom{10}{9}(0.80)^9(0.20)^1 + \binom{10}{10}(0.80)^{10}$

$(0.20)^0$

$= 45 \times 0.1678 \times 0.04 + 10 \times 0.1342 \times 0.20 + 1 \times 0.1074 \times 1$

$= 0.3020 + 0.2684 + 0.1074 \quad = 0.6778$

(c) Find $E(X)$ and $Var(X)$

$E(X) = np = 10 \times 0.80 = 8$

$Var(X) = np(1-p) = 10 \times 0.80 \times 0.20 = 1.6$

(d) If at least 9 correct classification are required for a pass, what is probability the batch passes.

Calculate probability $P(X=10)$

$P(X=10) = \binom{10}{10}(0.80)^{10}(0.20)^0 = 1 \times (0.1074) \times 1 = 0.1074$

$P(X=9) = \binom{10}{9}(0.80)^9(0.20)^1 = 10 \times 0.1342 \times 0.20$

$= 0.2684.$

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Sum the probability.

$P(X \geq 9) = (0.2684) + (0.1074) = 0.3758$.

Question 2:- In a data science experiment ...
... distribution.

Given

$Y \sim$ Negative binomial $(r = 2, p = 0.30)$ where

$r = 2$ (no. of sucesses required)

$P = 0.30$ (probability of sucess per trial)

(a) Probability Mass function of Y.

The pmf of negative binomial function distribution

$$P(Y = k) = \binom{k-1}{r-1} p^r (1-p)^{k-r} \quad k = r, r+1, r+2 \ldots$$

$$P(Y = k) = \binom{k-1}{1} p^2 (1-p)^{k-2}$$

$$= \binom{k-1}{1} (0.30)^2 (0.70)^{k-2} \quad k = r, r+1, r+2 \ldots = 2, 3, 4 \ldots$$

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Simplified

$$P(Y=k) = (k-1) \times 0.09 \times [0.70]^{k-2}$$

where $k = 2, 3, 4 \dots$

(b) Compute $p(Y=5)$
using the pmf.

$$\binom{4}{1}(0.30)^2(0.70)^3 = 4 \times 0.09 \times 0.343 = 0.12348$$

(c) $E[Y] = \frac{r}{p} = \frac{2}{0.30} \approx 6.6667.$

Question 3:- A labblled dataset ---- hypergeometric distribution.

(a) Write the pmf of Z.

$Z \sim \text{Hypergeometric}(N=500, k=120, n=25)$

$N = 500$ (total items)

$k = 120$ (sucesses of population)

$n = 25$ (sample size)

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$$P(z=k) = \frac{\binom{K}{k}\binom{N-k}{n-k}}{\binom{N}{n}}$$ for $\max(0, n-(N-K)) \le k \le \min(n,k)$

for this problem

$$P(z=k) = \frac{\binom{120}{k}\binom{380}{25-k}}{\binom{500}{25}}$$ where $k = 0,1,2 \ldots 25$

Complete

$$P(z=6) = \frac{\binom{120}{6}\binom{380}{19}}{\binom{500}{25}}.$$

Calculating the binomial coefficient

$\binom{120}{6} \approx 2.361 \times 10^8$, $\binom{380}{19} \approx 1.263 \times 10^{36}$

$\binom{500}{25} \approx 1.353 \times 10^{42}$

$$\frac{(2.361 \times 10^8)(1.263 \times 10^{36})}{(1.353 \times 10^{42})} \approx 0.2204$$

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(c) Expected value $E(Z)$

$$E(Z) = n \cdot \frac{k}{N} = 25 \cdot \frac{120}{500} = 6.$$

Expectation with replacement (Binomial case)
If sampling were done with replacement, Z would follow a binomial distribution

$$Z \sim \text{Binomial}\left(n = 25,\ p = \frac{120}{500}\right) = 0.24$$

The expectation remains the same

$$E(Z) = n \cdot p = 25 \times 0.24 = 6.$$

Variance differs:-

Hypergeometric (without replacement)

$$Var(Z) = n \cdot \frac{k}{N} \cdot \frac{N-k}{N} = \frac{N-n}{N-1} = 25 \cdot 0.24 \cdot 0.76 \cdot \frac{475}{499}$$

$$\approx 4.34.$$

Binomial (with replacement)

$$Var(Z) = n \cdot p \cdot (1-p) = 25 \cdot 0.24 \cdot 0.76 = 4.56$$

$E(Z) = 6$ The expectation is same in both case, but variance decreases slightly without replacement.

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Question 4:- Incoming support ticket ... arriving.

(a) Probability of no ticket in 30 min

Given:

* Tickets arrives a a poisson process with rate
$\lambda = 4$ tickets/hour.

* N = no. of tickets in a given time interval.

Adjusted rate for 30 minutes ($t = 0.5$ hours)

$\lambda_{0.5} = \lambda \times t = 4 \times 0.5 = 2$ ticket per 30 min

Poisson PMF for $N=0$

$P(N=0) = e^{-\lambda_{0.5}} \cdot \lambda_{0.5}^{0} = e^{-2} \approx 0.1353$

(b) Probability of 6 tickets in one hour.

Rate for 1 hour ($t=1$)

$\lambda_1 = \lambda \times 1 = 4$ ticket per hour.

Calculate $P(N>6) = 1 - P(N \leq 6)$

$$P(N \leq 6) = \sum_{k=0}^{6} \frac{e^{-4} \cdot 4^4}{k!}$$

$$e^{-4}\left(1+4+\frac{16}{2}+\frac{64}{6}+\frac{256}{24}+\frac{1024}{120}+\frac{4096}{720}\right)$$

$$e^{-4}(1+4+8+10.667+10.667+8.533+5.6889)$$

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$\approx 0.0183 \times 49.556 \approx 0.8893$

$P(N>6) = 1 - 0.8893 = 0.1107$

(c) Mean and Variance for a two hour window.

Adjusted rate for 2 hour $(t=2)$

$\lambda_2 = \lambda \times 8 = 8$ tickets per 2 hour.

Mean $= \lambda_2 = 8$

Variance $= \lambda_2 = 8$.