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Mathematics and Statistics for Data Science

Task 18.

Question 1:-

$\mu_{Math} = 70$ $\mu_{Physics} = 75$

$\sigma_{Math} = 10$ $\sigma_{Physics} = 12$.

$\rho = 0.7$ (correlation coefficient)

Let X = Math score

Y = Physics score.

(a)

The joint PDF of a ~~bivariable~~ bivariate normal distribution is

$$f(x,y) = \frac{1}{2\times\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left(\frac{-1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right).$$

Substitute the given values

$$f(x,y) = \frac{1}{2\times(10)(12)\sqrt{1-0.49}} \exp\left(\frac{-1}{2(1-0.49)}\left[\left(\frac{x-70}{10}\right)^2 - 2(0.7)\left(\frac{x-70}{10}\right)\left(\frac{y-75}{12}\right) + \left(\frac{y-75}{12}\right)^2\right]\right).$$

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(b) Standardize both variables

$Z_X = \frac{80-70}{10} = 1.0$ $\quad Z_Y = \frac{80-75}{12} = 0.4167$

$P(X>80, Y>80) = P(Z_X > 1.0, Z_Y > 0.4167)$

Since the variables are jointly normal with correlation 0.7, we need the bivariate normal integral.

using python, we get.

$P(X>80, Y>80) \approx 0.157$.

(c)

From properties of bivariate normal:

- $E[Y|X=x] = \mu_X + \rho \frac{\sigma_Y}{\sigma_X}(x-\mu_X)$
- $Var[Y|X=x] = \sigma_Y^2(1-\rho^2)$.

Substituting values

$E[Y|X=80] = 75 + 0.7 \cdot \frac{12}{10}(80-70)$

$= 75 + 8.4 = 83.4.$

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$$\text{Var}(Y|X=80) = 12^2 \cdot (1-0.49)$$
$$= 144(0.51)$$
$$= 73.44.$$

(d)

From part (c), conditional distribution

$$Y|X=80 \sim N(83.4, 73.44)$$

Standardize :

$$Z = \frac{85-83.4}{\sqrt{73.44}} \approx \frac{1.6}{8.57} \approx 0.1867$$

$$P(Y>85|X=80) \cdot P(Z>0.1867)$$
$$\approx 0.426$$

(e) Standardize : $Z = \frac{90-75}{12} = 1.25$

$$P(Y>90) \cdot P(Z>1.25) \approx 0.1056$$

Expected no. $= 200 \times 0.1056$
$= 21.12$

$\Rightarrow$ 21 Students.