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Question 1: Suppose the processing time ..... 15 seconds

(a) Define the probability density function (pdf) for the processing is uniformly distribution b/w 5 and 15 seconds.

The processing time X is uniformly distributed b/w 5 and 15 seconds. For a continuous uniform distribution over time interval [a, b], the probability density function is given by.

$$f_X(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \le x \le b \\ & \text{otherwise} \end{cases}$$

for $X \sim$ uniform $(5, 15)$

$a = 5 \quad b = 15$

$$f_X(x) = \begin{cases} \frac{1}{15-5} = \frac{1}{10} & \text{if } 5 \le x \le 15 \\ & \text{otherwise.} \end{cases}$$

(b) Find $P(7 \le x \le 12)$

For a uniform distribution, the probability $P(c \le x \le d)$ is the area under the pdf from c to d, which is

$$P(c \le x \le d) = (d-c) \cdot \frac{1}{b-a}$$

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$= P(7 \le x \le 12) = (12-7) \cdot \frac{1}{10}$

$= 5 \cdot \frac{1}{10} = \frac{1}{2}.$

(c) Compute the expected value $E(X)$ and variance $Var(x)$.

For a uniform distribution Uniform (a, b)

The expected value is

$E(x) = \frac{a+b}{2}$ Here $\frac{5+15}{2} = \frac{20}{2} = 10$

$Var(x) = \frac{(b-a)^2}{12} = \frac{(15-5)^2}{2} = \frac{100}{12} = \frac{25}{3} \approx 8.333$

(d) If the model is called twice independently, what is the probability that both processing time exceed 10 seconds?

First find $P(X > 10)$ for one call. for a uniform distribution : $\frac{b-c}{b-a} = P(x > c)$

$P(x > 10) = \frac{15-10}{15-5} = \frac{5}{10} = \frac{1}{2}$

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Since the two calls are independent, the probabitity that both exceed 10 seconds is

$$P(X_1>10) \cdot P(X_2>10) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

Question 2: The time (in minuits) b/w requests to an AI server follows an expontial.

Given:- The time T b/w requests follow exponential distribution with a mean $H = 6$ minuits.

For an exponential distribution, the rate parameter λ is the reciprocal of the mean

$$\lambda = \frac{1}{\mu} = \frac{1}{6}.$$

(a) Probability density function of T.

The pdf of an exponential random variable T with rate λ is

$$f_T(t) = \begin{cases} \lambda e^{-\lambda t} & \text{if } t \geq 0 \\ 0 & \text{if } t < 0. \end{cases}$$

Subsituting $\lambda = \frac{1}{6}$

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$$f_T(t) = \begin{cases} \frac{1}{6}e^{-t/6} & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$$

(b) Find $P(T > 10)$

The comulative distribution function of an exponential random variable is

$$P(T \leq t) = 1 - e^{-\lambda t}$$

Thus.

$$P(T > 10) = 1 - P(T \leq 10) = e^{-\lambda \cdot 10} = e^{-10/6} = e^{-5/3}$$

$$e^{-5/3} \approx 0.1889.$$

(c) Calculate the median of T.

The median m is the value such the $P(T \leq m) = 0.5$ using the CDF

$$1 - e^{-\lambda m} = 0.5 \Rightarrow e^{-\lambda m} = 0.5 \Rightarrow -\lambda m = \ln(0.5)$$

Solving for m.

$$m = -\frac{\ln(0.5)}{\lambda} = \mu \ln(2) \approx 6 \times 0.6931 \approx 4.1587 \text{min.}$$

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(d) Find $P(3 \leq T \leq 8)$

using the CDF

$P(3 \leq T \leq 8) \approx 0.6665 - 0.2636 = 0.3429.$

Question 3:- Suppose the total waiting time ---

---- (requests per min).

(a) Write the pdf of Gamma Distribution random variable W.

The pdf of a Gamma-distribution random variable W with shape k and rate λ is:

$$f_W(w) = \begin{cases} \dfrac{\lambda^a x^{a-1} e^{-\lambda x}}{\Gamma(\alpha)} & \text{if } w \geq 0, \\ 0 & \text{if } w < 0. \end{cases}$$

when $\Gamma(k)$ is the Gamma function, and for integer k, $\Gamma(k) = (k-1)!$

Subsituting $k = 3$ and $\lambda = 0.5$

$$f_W(w) = \begin{cases} (0.5)^3 w^2 e^{-0.5w} & \text{if } w \geq 0 \\ 0 & \text{if } w < 0. \end{cases}$$

$$f_W(w) = \begin{cases} 0.125 w^2 e^{-0.5w} = \dfrac{w^2 e^{-0.5w}}{16} & \text{if } w \geq 0 \\ 0 & \text{if } w < 0. \end{cases}$$

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(b) Expected value E[W] and variance Var(W)

For a Gamma distribution

$$E[W] = \frac{k}{\lambda}, \quad Var(W) = \frac{k}{\lambda^2}$$

$$E[W] = \frac{3}{0.5} = 6 \text{ minutes}$$

$$Var(W) = \frac{3}{(0.5)^2} = \frac{3}{0.25} = 12 \text{ min}^2.$$

(c) Find $P(W \leq 5)$ using Gamma CDF

The commulative distribution function (CDF) for a Gamma distribution

$$P(W \leq w) = \frac{\gamma(k, \lambda w)}{\Gamma(k)}$$

where $\gamma(k, \lambda w)$ is the lower incomplete Gamma function.

For $k = 3$, $\lambda = 0.5$ and $w = 5$

$$P(W \leq 5) = \frac{\gamma(3, 0.5 \times 5)}{2} = \frac{\gamma(3, 2.5)}{2}.$$

(d) Comment briefly on the difference b/w the Gamma and Exponential distribution

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Gamma Distribution

* Generalizes the exponential distribution
* Models the time until the k-th event in a poisson process.
* Has two parameters: shape k and rate λ.
* For $k=1$ Gamma reduces to exponential.
* Not memoryless for $k>1$.

Exponential Distribution

* Models the time b/w consecutive events in a poission process
* Memory less property: $P(T>s+t \mid T>s) = P(T>t)$

Key Difference.

Exponential is a special case of Gamma with $k=1$. Gamma can model waiting times for multiple events, while exponential model waiting times for a single event.