Hometask 16

Solve **all** parts. Show every important step (formulas, substitution and reasoning) final numeric answers alone will not earn full credit

Q1. Suppose the processing time for a small machine-learning model is uniformly distributed between 5 and 15 seconds.

(a) Define the probability density function (pdf) for the processing time X.

(b) Find $P(7 \leq X \leq 12)$.

(c) Compute the expected value $E[X]$ and variance $\text{Var}(X)$.

(d) If the model is called twice independently, what is the probability that both processing times exceed 10 seconds?

Q2. The time (in minutes) between requests to an AI server follows an exponential distribution with a mean of 6 minutes.

(a) Write the pdf of the exponential random variable T.

(b) Find $P(T > 10)$.

(c) Calculate the median of T.

(d) Find $P(3 \leq T \leq 8)$.

Q3. Suppose the total waiting time to observe the third request at a server follows a Gamma distribution with shape parameter $k = 3$ and rate $\lambda = 0.5$ (requests per minute).

(a) Write the pdf of the Gamma distributed random variable W.

(b) Find the expected value $E[W]$ and variance $\text{Var}(W)$.

(c) Find $P(W \leq 5)$ using a Gamma cumulative distribution function (use a table or calculator).

(d) Comment briefly on the difference between the Gamma and Exponential distributions.