M. Atlas Malik HomeTask 11 24i-8020

Protein Intake	Exercise	Hydration	MM %age
80	5	3	45
100	7	3.5	50
90	6	3.2	48
110	8	4	55
95	6.5	3.8	50

→ learning rate $(\eta) = 0.0001$

→ Regularization parameter $(\alpha) = 0.05$

→ Initial weights : $w_1 = w_2 = w_3 = b = 0$

Iteration 1 : $\hat{y}_i = w_1 x_1 + w_2 x_2 + w_3 x_3 + b = 0$

$\Rightarrow \hat{y}_1 = 0 \; ; \; \hat{y}_2 = 0 \; ; \; \hat{y}_3 = 0 \; ; \; \hat{y}_4 = 0 \; ; \; \hat{y}_5 = 0$

$$X = \begin{bmatrix} 80 & 5 & 3 \\ 100 & 7 & 3.5 \\ 90 & 6 & 3.2 \\ 110 & 8 & 4 \\ 95 & 6.5 & 3.8 \end{bmatrix} \; ; \; y = \begin{bmatrix} 45 \\ 50 \\ 48 \\ 55 \\ 50 \end{bmatrix}$$

Error $= y - \hat{y} = [45, 50, 48, 55, 50]$

Update bias:

$$\frac{\partial L}{\partial b} = -\frac{2}{m} \sum_{i=1}^{n} (y_i - \hat{y}_i)$$

$$= -\frac{2}{5} \sum_{i=1}^{5} (y_i - \hat{y}_i)$$

$$= -\frac{2}{5} \left[(y_1 - \hat{y}_1) + (y_2 - \hat{y}_2) + (y_3 - \hat{y}_3) + (y_4 - \hat{y}_4) + (y_5 - \hat{y}_5) \right.$$

$$= -\frac{2}{5}\left[45+50+48+55+50\right]$$

$$\boxed{\frac{\partial L}{\partial b} = -99.2}$$

$$b_{new} = b_{old} - \eta \frac{\partial L}{\partial b}$$

$$= 0 - (0.0001)(-99.2)$$

$$\boxed{b_{new} = 0.0092}$$

Update weights:

$$\frac{\partial L}{\partial w_1} = \frac{-2}{m} \sum x_i (y_i - \hat{y}_i) + \alpha |w_i|$$

Update w_1 (protein)

$$\frac{\partial L}{\partial w_1} = -\frac{2}{5}\left[80(45)+100(50)+90(48)+110(55)+95(50)\right]$$

$$= -\frac{2}{5}(23720)$$

$$= -9488$$

$$w_{new} = w_{old} - \eta \frac{\partial L}{\partial w_1}$$

$$= 0 - (0.0001)(-9488)$$

$$\boxed{w_1 = 0.9488}$$

Update w_2 (exercise):

$$\frac{\partial L}{\partial w_2} = -\frac{2}{5}\left[5(45)+7(50)+6(48)+8(55)+6.5(50)\right]$$

$$= -\frac{2}{5}(628) = -651.2$$

$$w_{new} = w_{old} - \eta \frac{\partial L}{\partial w_2}$$

$$= 0 - (0.0001)(-651.2) = 0.06512$$

$$\boxed{w_2 = 0.06512}$$

Update w_3 (hydration):

$$\frac{\partial L}{\partial w_3} = -\frac{2}{5}(873.6) = -349.44$$

$$w_{new} = w_{old} - \eta \frac{\partial L}{\partial w_3}$$

$$= 0 - (0.0001)(-349.44)$$

$$= 0.034944$$

Compute MSE:

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$= \frac{1}{5}\left[(45-0)^2 + (50-0)^2 + (48-0)^2 + (55-0)^2 + (50-0)^2\right]$$

$$MSE = 2470.8$$

predicted y:

$\hat{y}_1 = x_{11} w_1 + x_{12} w_2 + x_{13} w_3 + b$

$= 80(0.9488) + 5(0.06512) + 3(0.034944) + 0.0092$

$\hat{y}_1 = 76.3443$

$\hat{y}_2 = x_{21} w_1 + x_{22} w_2 + x_{23} w_3 + b$

$= 100(0.9488) + 7(0.06512) + 3.5(0.034944) + 0.0092$

$= 95.468$

$\hat{y}_3 = x_{31} w_1 + x_{32} w_2 + x_{33} w_3 + b$

$= 90(0.9488) + 6(0.06512) + 3.2(0.034944) + 0.0092$

$= 85.9044$

$\hat{y}_4 = x_{41} w_1 + x_{42} w_2 + w_{43} w_3 + b$

$= 110(0.9488) + 8(0.06512) + 4(0.034944) + 0.0092$

$= 105.0386$

$\hat{y}_5 = x_{51} w_1 + x_{52} w_2 + x_{53} w_3 + b$

$= 95(0.9488) + 6.5(0.06512) + 3.8(0.034944) + 0.0092$

$= 90.7012.$

Iteration 2:

Error : $y_i - \hat{y}_i$

$= [45 - 76.3443, 50 - 95.468, 48 - 85.9044, 55 - 105.0386, 50 - 90.7012]$

$= [-31.3443, -45.468, -37.9044, -50.0386, -40.7019]$

Update bias: $\frac{\partial L}{\partial b} = \frac{-2}{m} \sum_{i=1}^{n} (y_i - \hat{y}_i)$

$$= -\frac{2}{5} \sum_{i=1}^{5} (y_i - \hat{y}_i)$$

$$= -\frac{2}{5} \left[-31.3443 - 45.468 - 37.9044 - 50.0386 - 40.7019 \right]$$

$$\frac{\partial L}{\partial b} = 82.18$$

$$b_{new} = b_{old} - \eta \frac{\partial L}{\partial b}$$

$$= 0.00992 - (0.0001)(82.18)$$

$$\boxed{b_{new} = 0.001702}$$

Update weights:

$$|w_i| = 1$$

$$\frac{\partial L}{\partial w_i} = \frac{-2}{m} \sum_{i=1}^{m} x_{ij} (y_i - \hat{y}_i) + \alpha |w_i|$$

$$\Rightarrow \frac{\partial L}{\partial w_1} = \frac{-2}{5} \Big[80(-31.3448) + 100(-45.468) + 90(-37.9044)$$
$$+ 110(-50.0386) + 95(-40.7019) \Big] + (0.5)(1)$$

$$\frac{\partial L}{\partial w_1} = 7934.7326$$

$$w_{new} = w_{old} - \eta \frac{\partial L}{\partial w_1}$$

$$= 0.09488 - (0.0001)(7934.7326)$$

$$\boxed{w_1 = 0.1553}$$

$$\Rightarrow \frac{\partial L}{\partial w_2} = -\frac{2}{5}\left[5(-31.3443)+7(-45.468)+6(-37.9044)+8(-50.0386)+6.5(-40.7019)\right]+0.05(1).$$

$$\frac{\partial L}{\partial w_2} = 546.96$$

$$w_{new} = w_{old} - \eta \frac{\partial L}{\partial w_2}$$

$$= 0.06512 - (0.0001)(546.96)$$

$$\boxed{w_2 = 0.10424}$$

$$\Rightarrow \frac{\partial L}{\partial w_3} = -\frac{2}{5}\left[3(-31.3443)+3.5(-45.468)+3.2(-37.9044)+4(-50.0386)+3.8(-40.7019)\right]+0.05$$

$$= 291.76$$

$$w_{new} = w_{old} - \eta \frac{\partial L}{\partial w_3}$$

$$= 0.034944 - (0.0001)(291.76)$$

$$\boxed{w_3 = 0.005768}$$

Compute MSE :

$$MSE = \frac{1}{m}\sum_{i=1}^{m}(y_i - \hat{y}_i)^2$$

$$= \frac{1}{5}\left[(-31.3443)^2+(-45.468)^2+(-37.9044)^2+(-50.0386)^2+(-40.7019)^2\right]$$

$$\boxed{MSE = 1729.41}$$

Predicted y:

$$\hat{y}_1 = x_{11}w_1 + x_{12}w_2 + x_{13}w_3 + b$$

$$= 80(0.1553) + 5(0.01042) + 3(0.00576) + 0.0017$$

$$\hat{y}_1 = 12.49$$

$$\hat{y}_2 = x_{21}w_1 + x_{22}w_2 + x_{23}w_3 + b$$

$$= 100(0.1553) + 7(0.01042) + 3.5(0.00576) + 0.0017$$

$$\hat{y}_2 = 15.62$$

$$\hat{y}_3 = x_{31}w_1 + x_{32}w_2 + x_{33}w_3 + b$$

$$= 90(0.1553) + 6(0.1042) + 3.2(0.00576) + 0.0017$$

$$\hat{y}_3 = 14.059$$

$$\hat{y}_4 = x_{41}w_1 + x_{42}w_2 + x_{43}w_3 + b$$

$$= 100(0.1553) + 8(0.01042) + 4(0.00576) + 0.0017$$

$$\hat{y}_4 = 17.1911$$

$$\hat{y}_5 = x_{51}w_1 + x_{52}w_2 + x_{53}w_3 + b$$

$$= 95(0.1553) + 6.5(0.01042) + 3.8(0.00576) + 0.0017$$

$$\hat{y}_5 = 14.844$$

Iteration 3: Error $= y_i - \hat{y}_i$

$$= [45-12.49, +50-15.62, 48-140.059, 55-17.191, 50-14.844]$$

$$= [32.50, 34.3752, 33.9404, 37.8089, 35.1552].$$

Update bias: $\frac{\partial L}{\partial b} = \frac{-2}{m}\sum_{i=1}^{m}(y_i - \hat{y}_i)$

$$= \frac{-2}{5}\left[32.5049 + 34.375 + 33.94 + 37.808 + 35.155\right]$$

$$\frac{\partial L}{\partial b} = 69.51$$

$$b_{new} = b_{old} - \eta\frac{\partial L}{\partial b}$$

$$= 0.001702 - (0.0001)(-69.51)$$

$$\boxed{b_{new} = 0.00865}$$

Update weights: $|w_i| = 1.$

$$\frac{\partial L}{\partial w_j} = \frac{-2}{m}\sum_{i=1}^{m} x_{ij}(y_i - \hat{y}_i) + \alpha|w_i|.$$

$$\frac{\partial L}{\partial w_1} = \frac{-2}{5}\Big[80(32.5049) + 100(34.375) + 90(33.94) +$$

$$110(37.8089) + 95(35.1552)\Big] + 0.05$$

$$= -6636.46$$

$$W_{new} = W_{old} - \eta\frac{\partial L}{\partial w_1}$$

$$= 0.1553 - (0.0001)(-6636.46)$$

$$\boxed{w_1 = 0.8189}$$

$$\frac{\partial L}{\partial w_2} = -\frac{2}{5}\Big[5(32.5049) + 7(34.375) + 6(33.9404)$$

$$+ 8(37.8089) + 6.5(35.1552)\Big] + 0.05$$

$= -455.06$

$$W_{new} = W_{old} - \eta \frac{\partial L}{\partial w_2}$$

$$= 0.01042 - (0.0001)(-455.06)$$

$$\boxed{w_2 = 0.0559}$$

$$\Rightarrow \frac{\partial L}{\partial w_3} = \frac{-2}{5}\Big[3(32.5049) + 3.5(34.37) + 3.2(33.94)$$
$$+ 4(37.8089) + 3.8(35.1552)\Big] + 0.05$$

$$= -244.45$$

$$W_{new} = W_{old} - \eta \frac{\partial L}{\partial w_3}$$

$$= 0.00575 - (0.0001)(-244.45)$$

$$\boxed{w_3 = 0.0302}$$

Compute MSE :

$$MSE = \frac{1}{m} \sum_{i=1}^{m} (y_i - \hat{y}_i)^2$$

$$= \frac{1}{5}\Big[(32.5049)^2 + (34.3752)^2 + (33.94)^2 +$$
$$(37.808)^2 + (35.155)^2\Big]$$

$$\boxed{MSE = 1211.09}$$

Predicted y:

$\hat{y}_1 = 65.8908$

$\hat{y}_2 = 82.3957$

$\hat{y}_3 = 74.1417$

$\hat{y}_4 = 90.6557$

$\hat{y}_5 = 78.2823.$

$x_1 = 105 \quad x_2 = 7.5 \quad x_3 = 3.6$

$$\hat{y} = x_1 w_1 + x_2 w_2 + x_3 w_3 + b$$

$$= 105(0.8189) + 7.5(0.0559) + 3.6(0.0302) + 0.00865$$

$$\hat{y} = 86.5231$$

So, the pridicted muscle mass percentage for an individual consuming 105 g of protein, exercising 7.5 hours per week, and hydrating with 3.6L per day is 86.52%.

Regularization:

Regularization is a technique used to prevent overfitting by adding a penalty to the loss function in Lass regression, the penalty is based on the absolute values of the weights (L1 Norm)