# Visualizing Poynting vector energy flow in electric circuits

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## Abstract

According to standard Poynting vector arguments, energy in a circuit flows from the batteries to the resistors not through the connecting wires, but through empty space between the wires. A computer simulation helps to visualize this counterintuitive fact.

## I. THE PURPOSE

When a battery is connected to a resistor by wires to form a circuit, then current flows through the wires. In addition, electromagnetic energy flows from the battery to the resistor (the battery discharges, the resistor warms up). It is only natural to surmise that the energy, like the charge, flows through the wires.

According to Poynting vector theory[1] this surmise, although natural, is incorrect. Electromagnetic energy is located throughout space with energy density

$$u(\mathbf{r},t) = \frac{1}{2}\left(\epsilon_0 E^2(\mathbf{r},t) + \frac{1}{\mu_0}B^2(\mathbf{r},t)\right) \tag{1}$$

and flows with a power per cross-sectional area equal to the Poynting vector

$$\mathbf{S}(\mathbf{r},t) = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}. \tag{2}$$

Ideal wires have zero resistivity $\rho$ and hence, by Ohm's law $\mathbf{E} = \rho\mathbf{J}$, zero electric field as well. Instead of transporting power, the power flow within any ideal wire is exactly zero!

Many students find these results difficult to accept. If wires don't transmit power, then why do power companies go to enormous expense to put up transmission wires? Teaching strategies effective in making these points include:

- The drift velocity of charge carriers is constant throughout a single-loop circuit, so the energy can't be carried in the kinetic energy of charge carriers.

- The potential energy cannot be successfully assigned to individual charge carriers, but to either (1) pairs of charges or, better, to (2) fields[2].

- If energy is conserved in all reference frames, then it cannot just disappear at the battery and reappear simultaneously at the resistor, because (special relativity) the disappearance and reappearance would be simultaneous in one reference frame, but in other frames the disappearance would happen first and the reappearance happen after a time lag, and in still other frames the reappearance would happen before the disappearance had yet occurred!

- Energy can't be transported by the charge carriers because in alternating current circuits, there is no net charge transport but there certainly *is* energy transport.

- It's clear that electromagnetic energy can flow from the sun to the earth without passing through wires. Why not also from the battery to the resistor?
- The Feynman disk paradox[3] demonstrates that even static electromagnetic fields must (in some circumstances) possess momentum. If fields have momentum, it's a small jump to think that they have energy as well.

All of these strategies are good ones, but all of them appeal to the intellect and not to any intuitive "gut-feeling" for physics. The wires are clear and visual and appealing. Is there any way to make the energy flow through space visual and appealing?

The answer is "yes". Inspired by the work of Slavko Majcen, Ryan K. Haaland, and Scott C. Dudleye[4], we have produced a computer simulation to trace Poynting vector energy flows in circuits.

## II. THE MODEL

The program *PoyntPanel* takes a two-dimensional circuit as input, and then finds the associated electric, magnetic, and Poynting fields. While the primary role of *PoyntPanel* is to trace the Poynting energy flow using flow lines (similar to flow lines in moving water, or to electric field lines in electrostatics), it also finds the electric field. Hence it can be used to demonstrate field aspects of circuits, in contrast to the predominant emphasis on potentials.[5]

Talk about two dimensional character.

## III. THE SIMULATION

How to download and run the simulation. EPAPS (Electronic Physics Auxiliary Publication Service).

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[1] David J. Griffiths, *Introduction to Electrodynamics*, third edition (Prentice Hall, Upper Saddle River, New Jersey, 1999) section 8.1.2, pages 346–349.

[2] Griffiths, *ibid*, section 2.4.4, page 96.

[3] Richard P. Feynman, Robert B. Leighton, and Matthew Sands, *The Feynman Lectures on Physics*, volume II (Addison-Wesley, Reading, Massachusetts, 1964) sections 17–4 and 26–6, pages 17-6 and 27-11.

[4] Slavko Majcen, Ryan K. Haaland, and Scott C. Dudley, "The Poynting vector and power in a simple circuit," *Am. J. Phys.* **68**, 857–859 (2000).

[5] Mark A. Heald, "Electric fields and charges in elementary circuits," *Am. J. Phys.* **52**, 522–526 (1984). J. D. Jackson, "Surface charges on circuit wires and resistors play three roles," *Am. J. Phys.* **64**, 855–870 (1996). Igal Galilia and Elisabetta Goihbarg, "Energy transfer in electrical circuits: A qualitative account," *Am. J. Phys.* **73**, 141–144 (2005). Ruth Chabay and Bruce Sherwood, *Matter and Interactions*, volume II (John Wiley, New York, 2002) chapter 18.